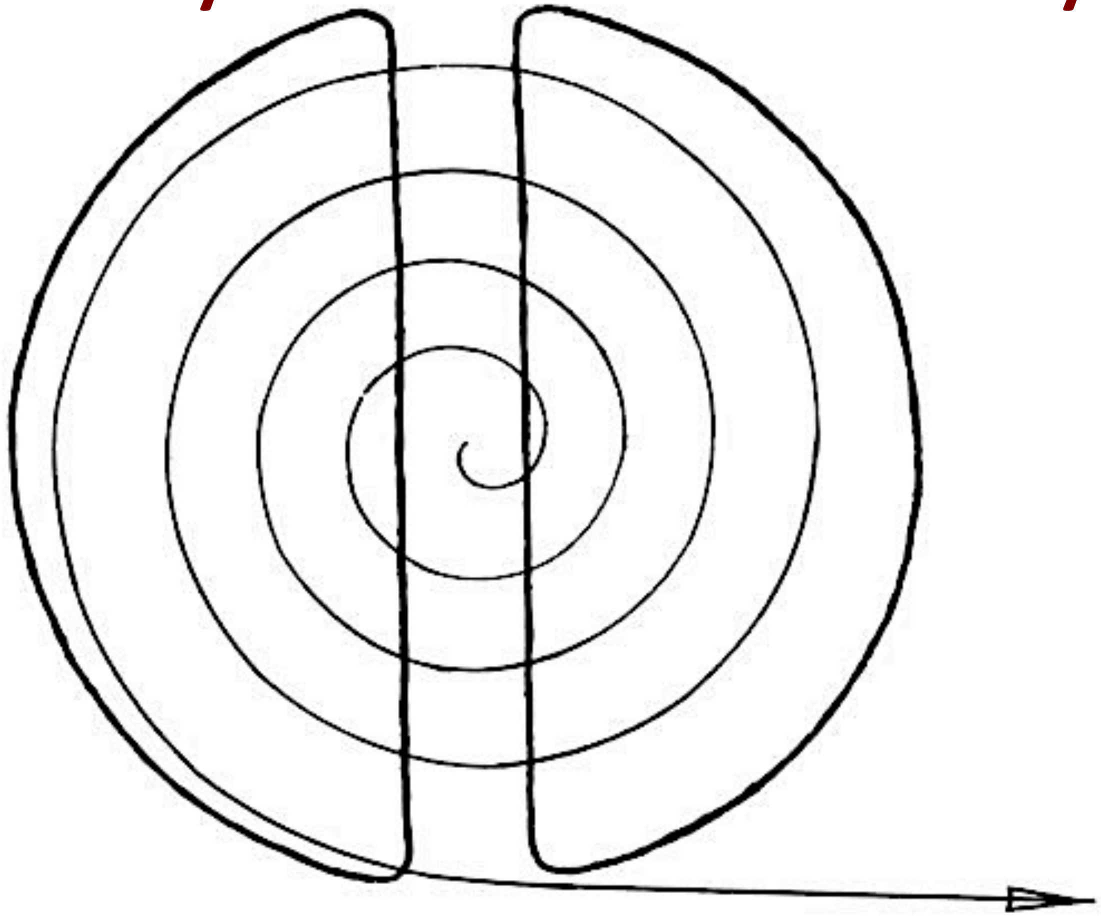
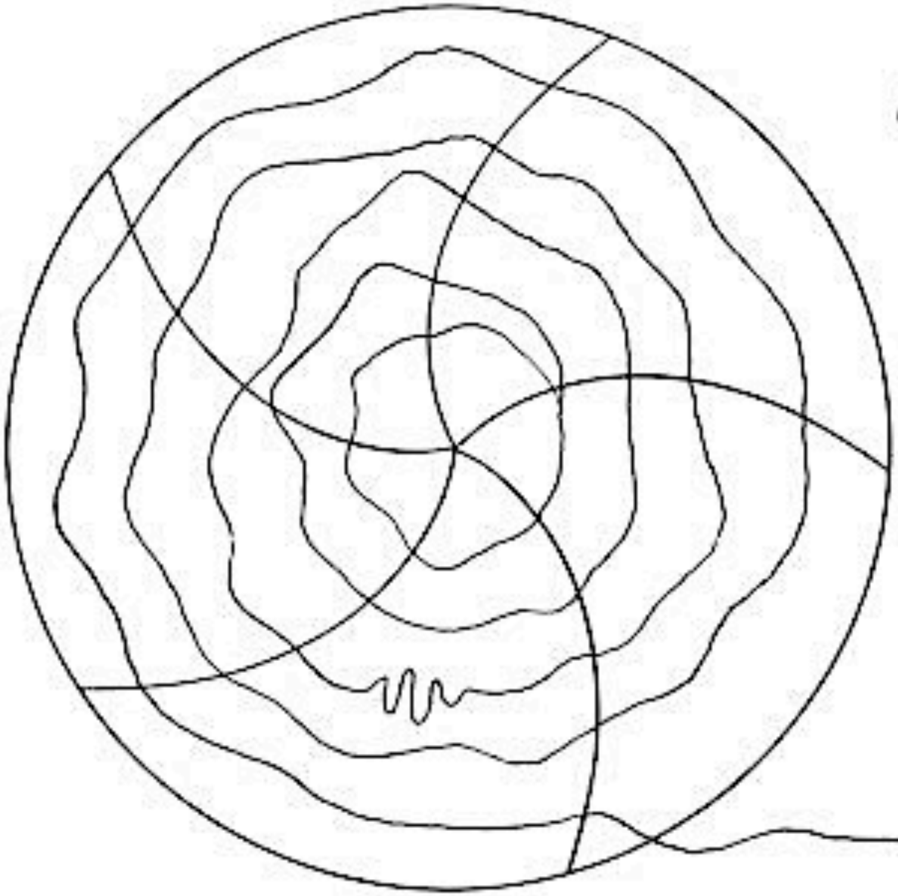
The cyclotron as seen by…



. . . the inventor

Dave Judd and Ron MacKenzie

XBD9705-02291.TIF



$$r = r_0\left[1+\left(\frac{frw}{c}\right)\cos(3\theta+\delta_0+\delta_1 r)+\right.$$

$$\left(\frac{frw}{c}\right)^2\cos(5\theta+\delta_3-\delta_5 r^3)+$$

$$\left(\frac{frw}{c}\right)^2\cos(7\theta+\delta_7-\delta_9 r^3)+$$

$$\left.- - - - - - -\right]\times\left\{\frac{e^{7/5\, r^2 \ln Z}}{1+\left(\frac{a}{b}\right)^{7/4}}\right\}$$

$$\frac{d\phi}{dt} = \left[\sin(\omega t - k\phi) - \sin k\phi_0 - \tfrac{3}{5} f f_1 f_2 f_3'\right]\frac{eV_0}{2\pi\omega}$$

. . . the theoretical physicist

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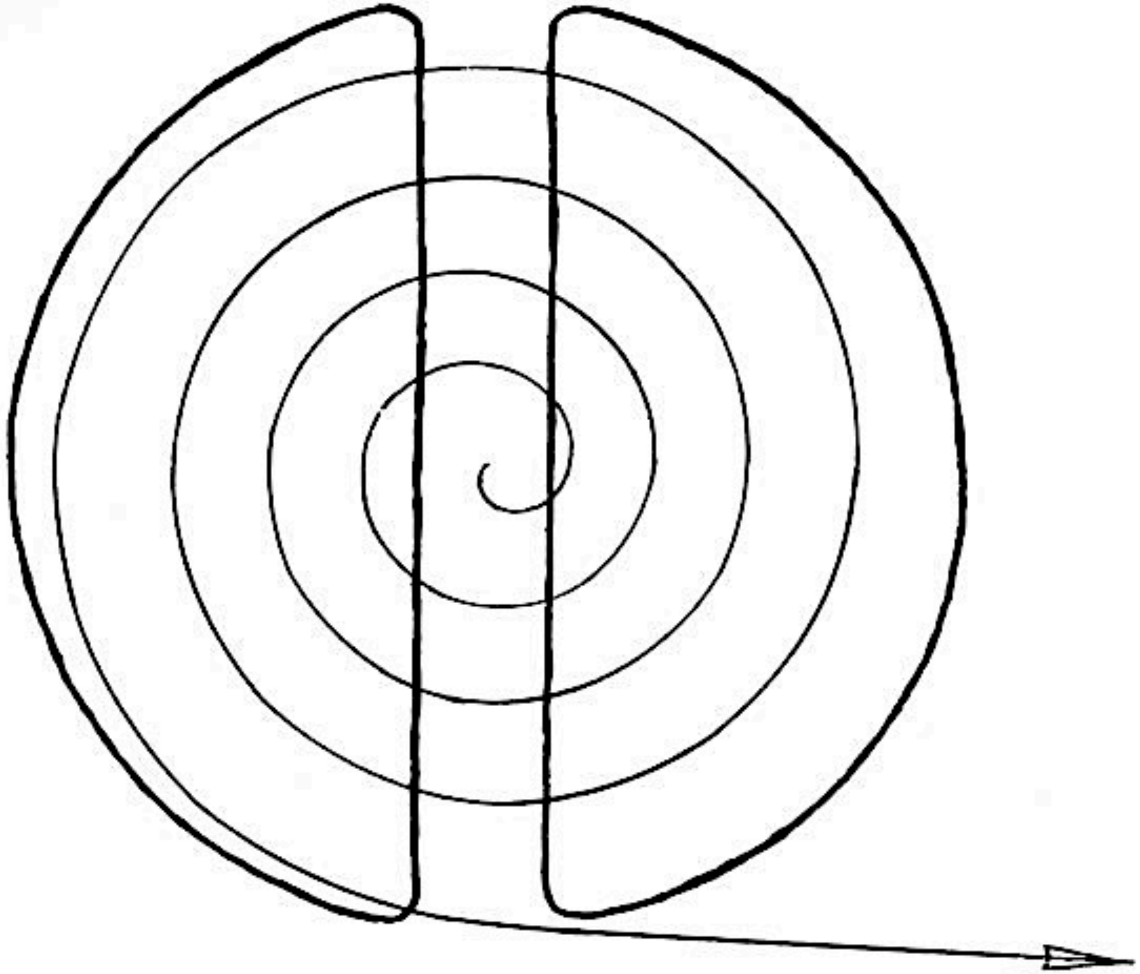

. . . the student

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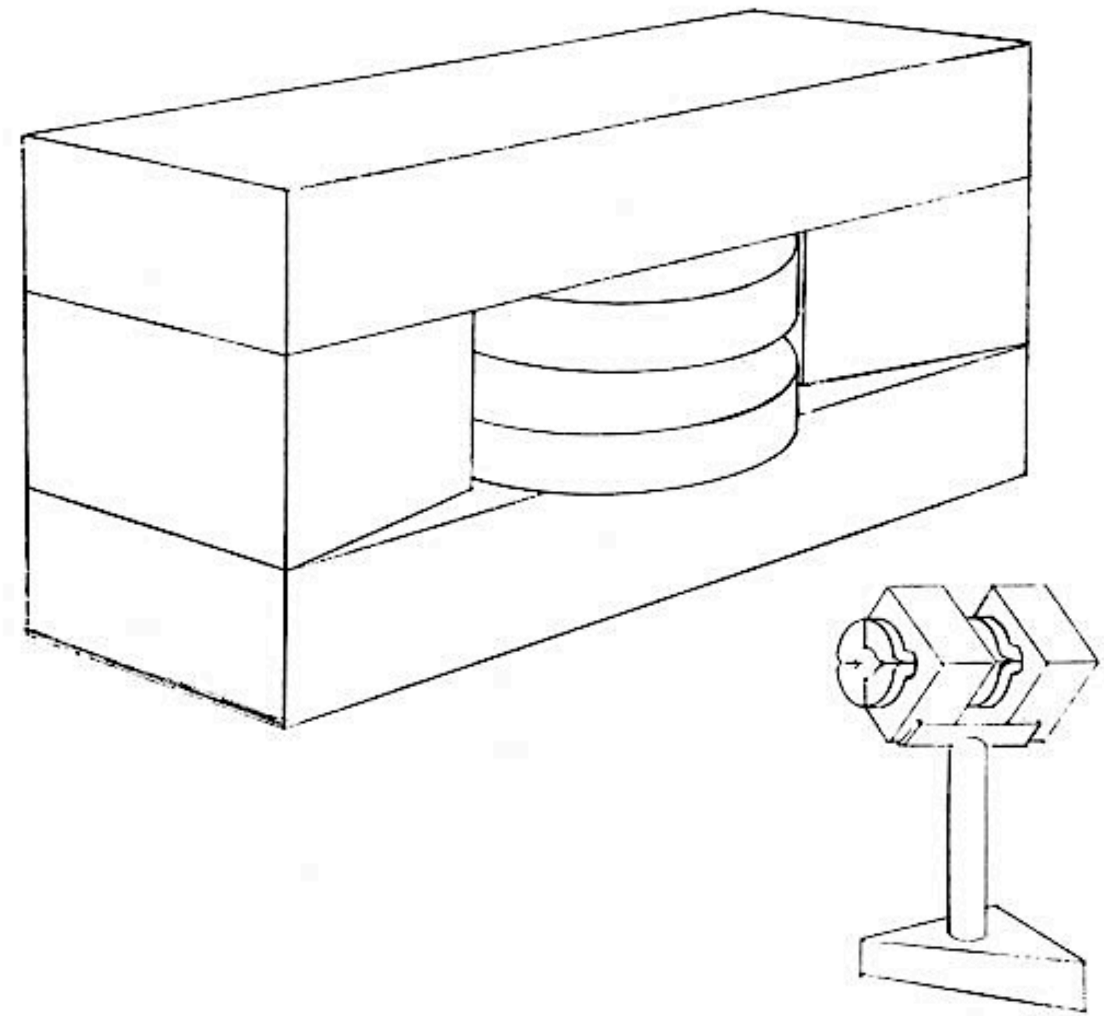

. . . the mechanical engineer

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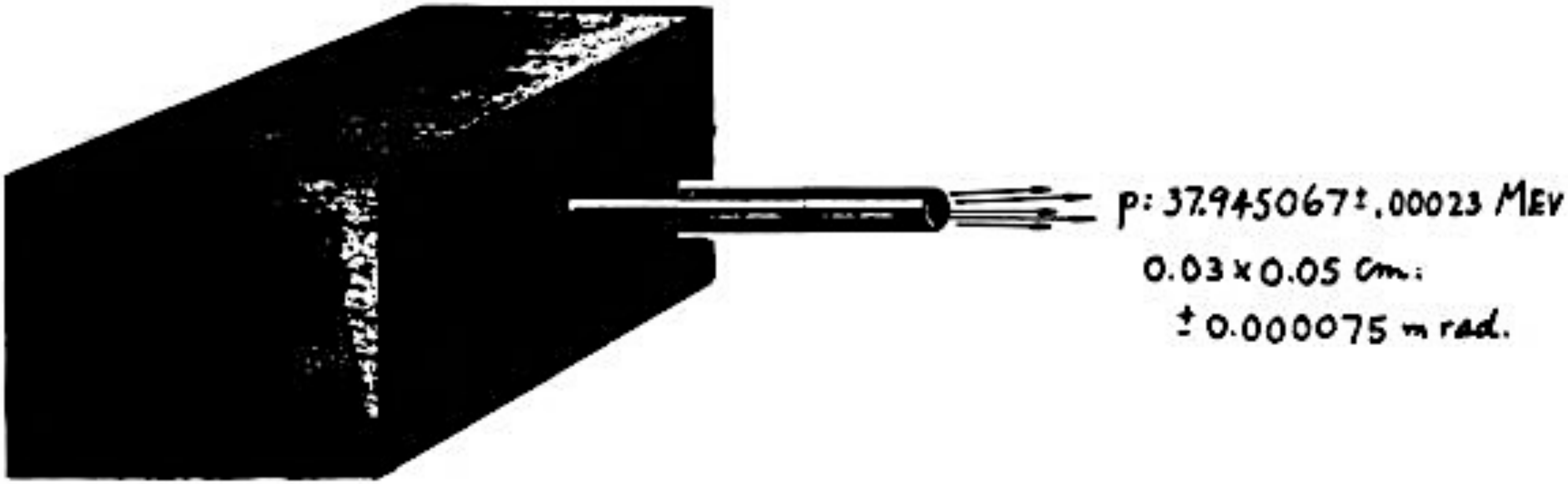

. . . the experimental physicist

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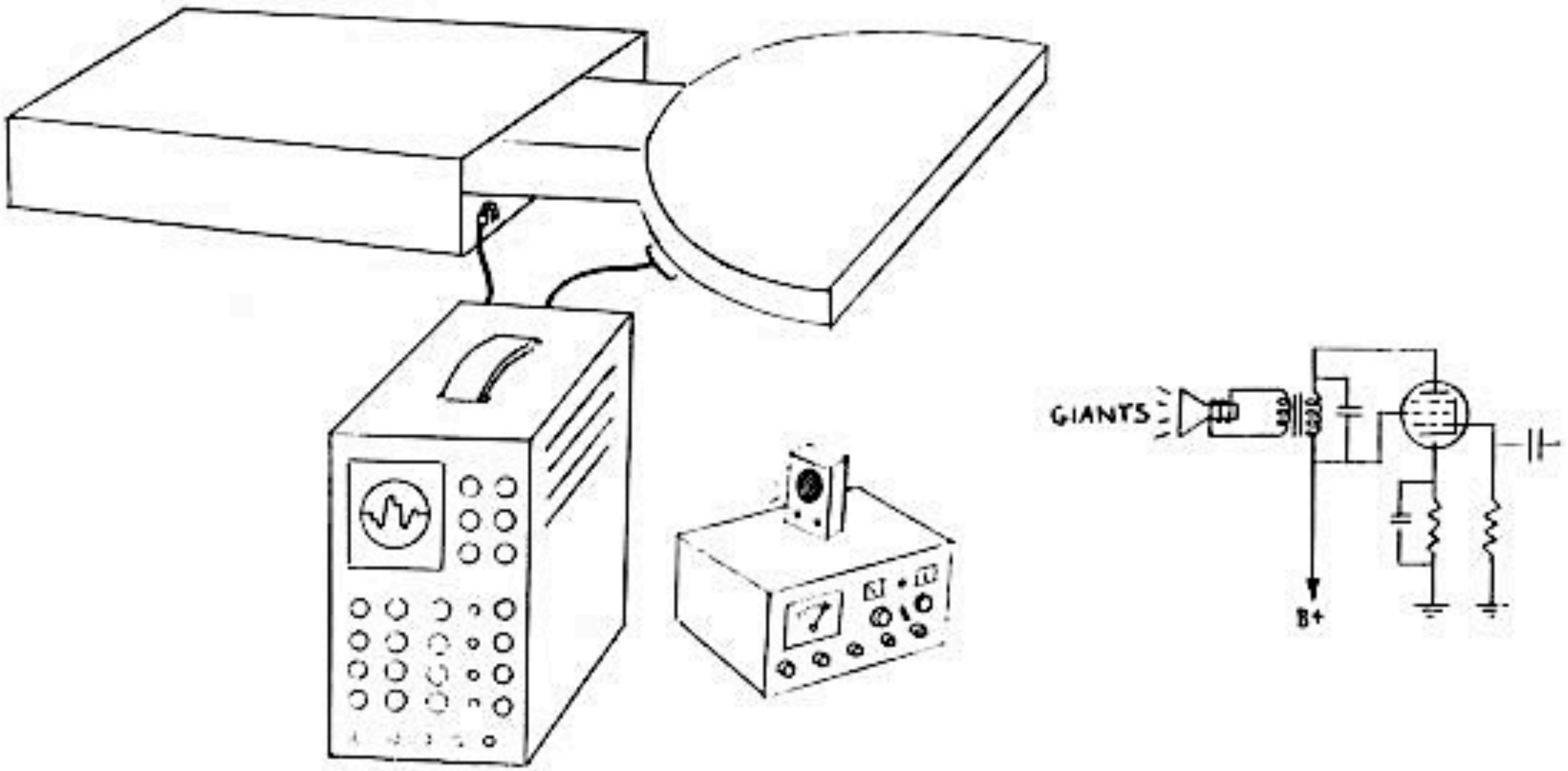

. . . the electrical engineer

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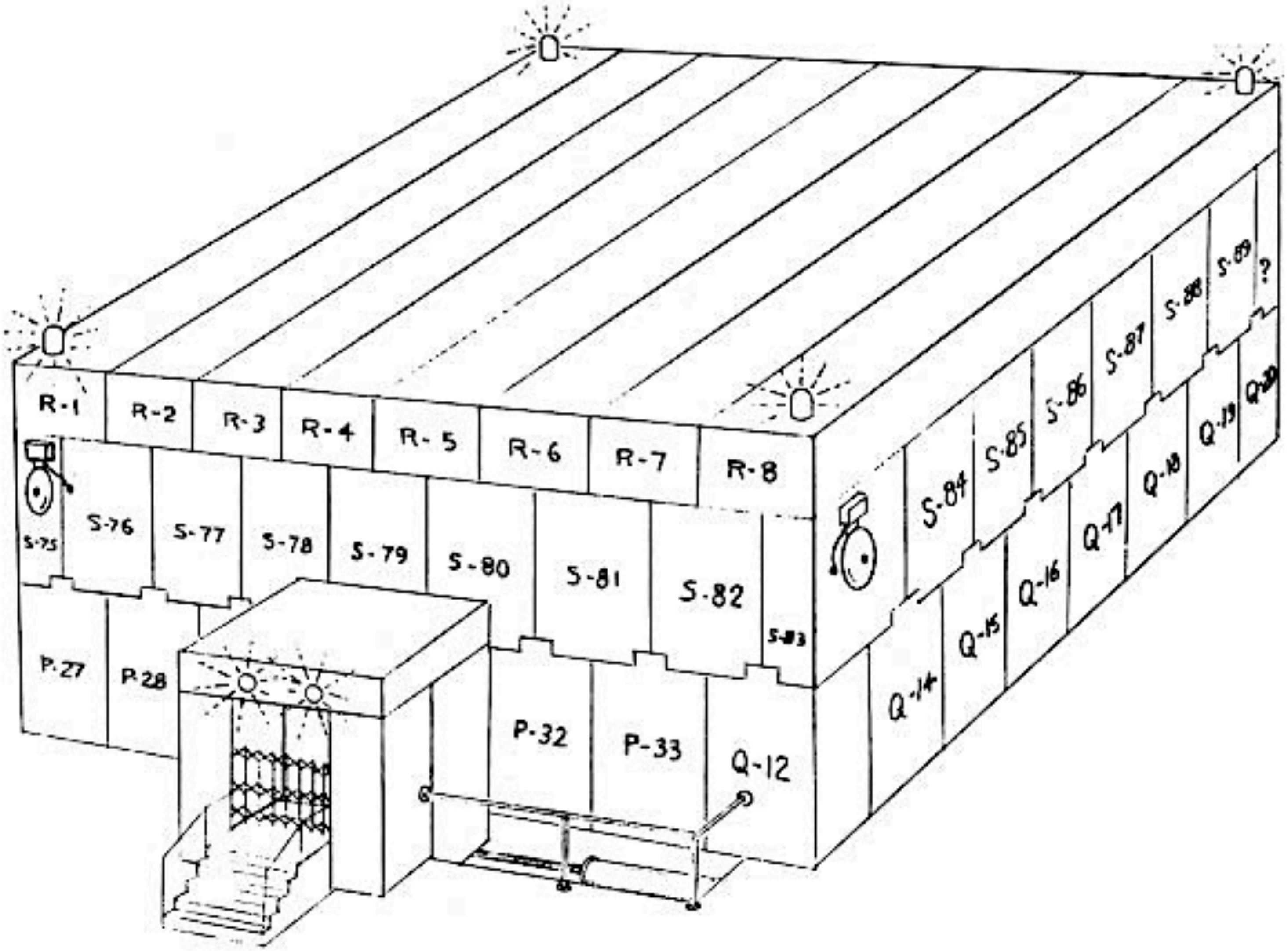

. . . the health physicist

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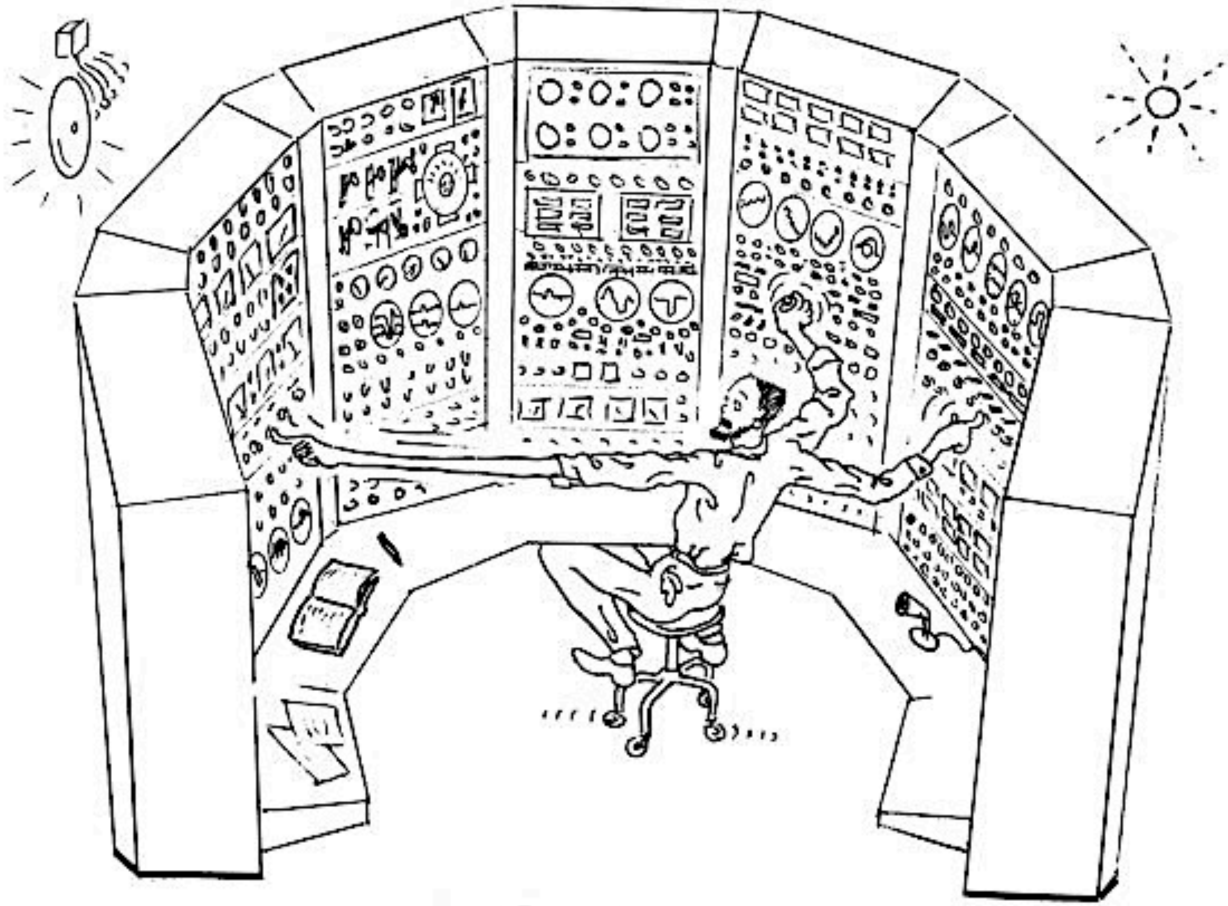

. . . the operator

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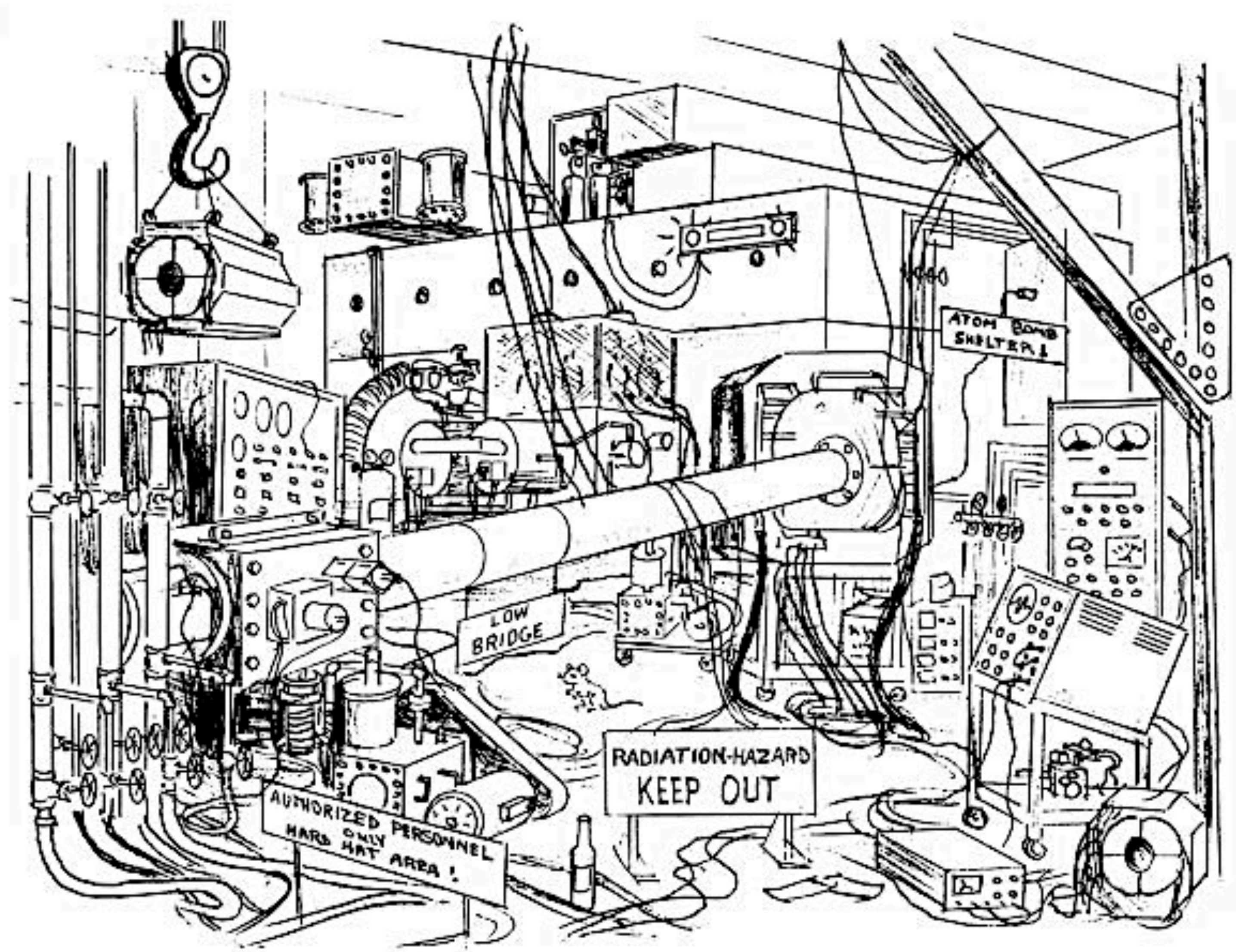

. . . the visitor

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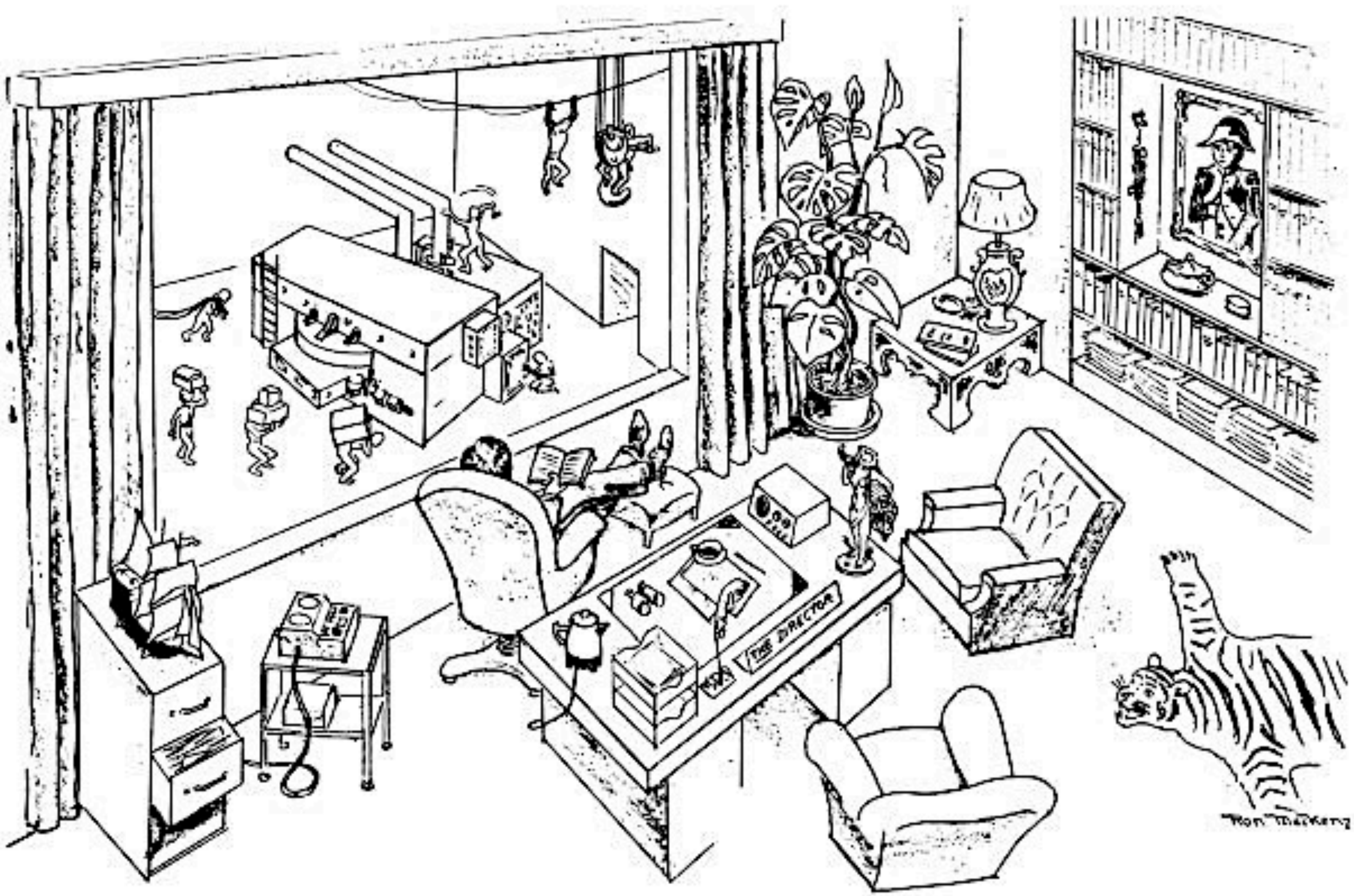
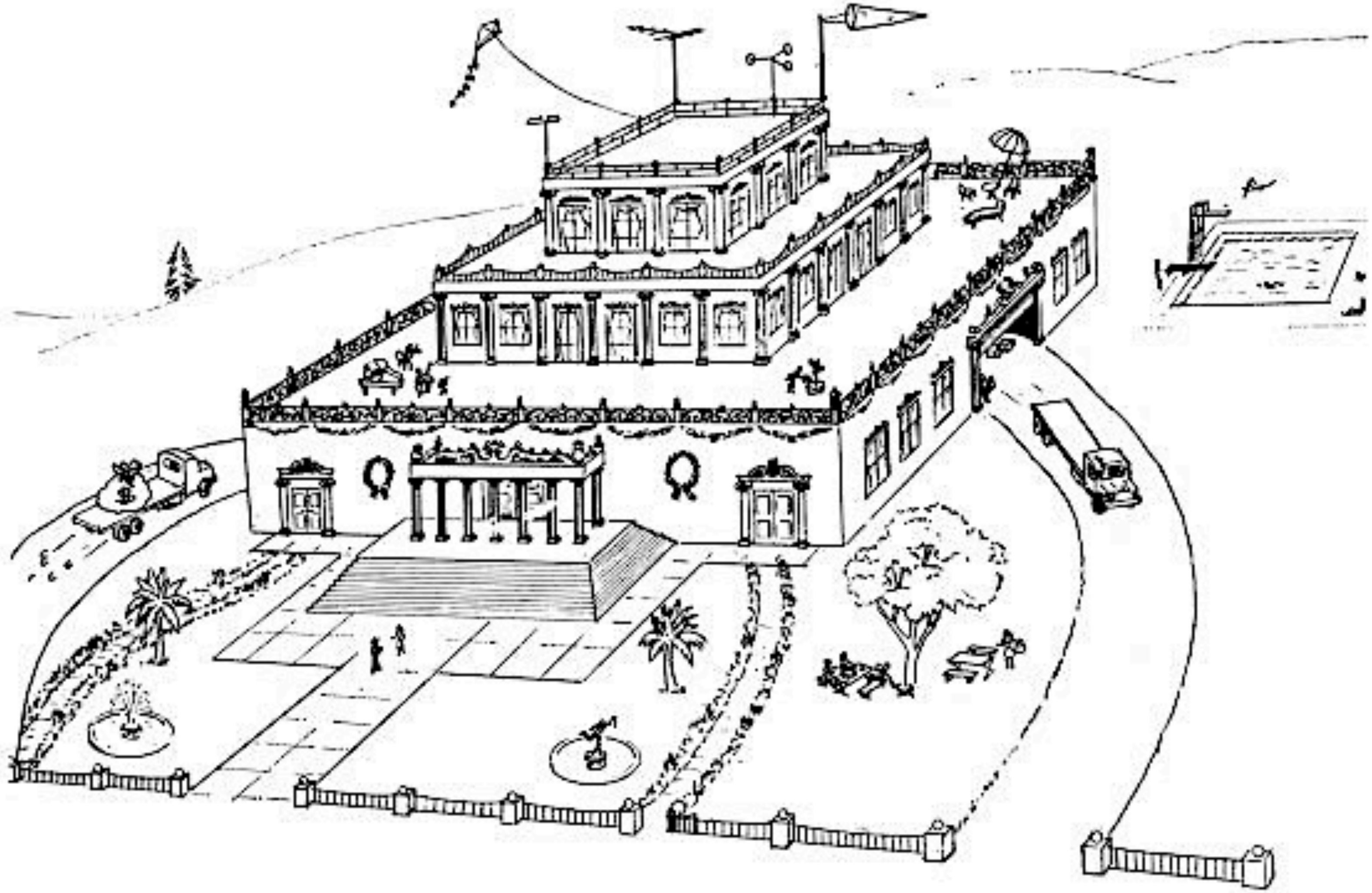


. . . the laboratory director

XBD9705-02303.TIF



. . . the governmental funding agency

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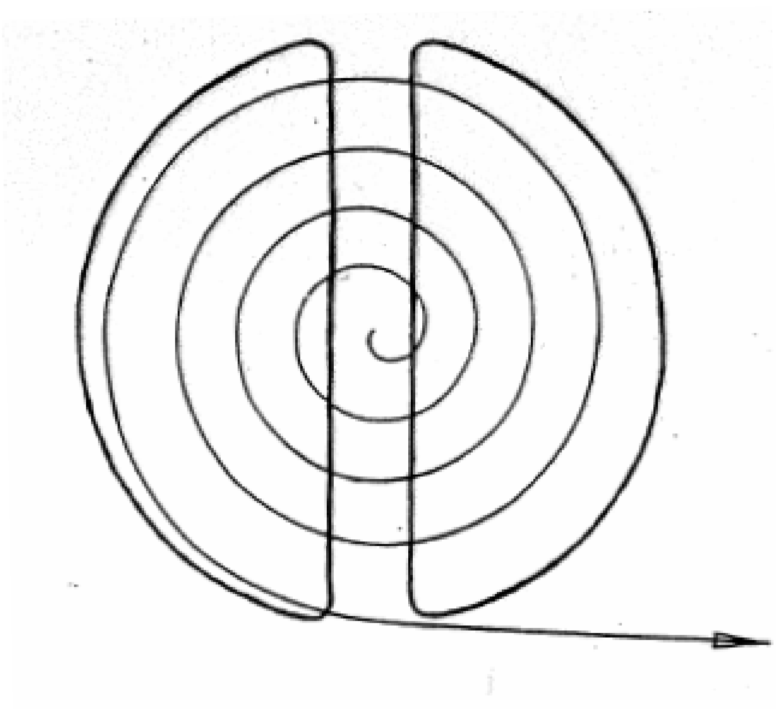
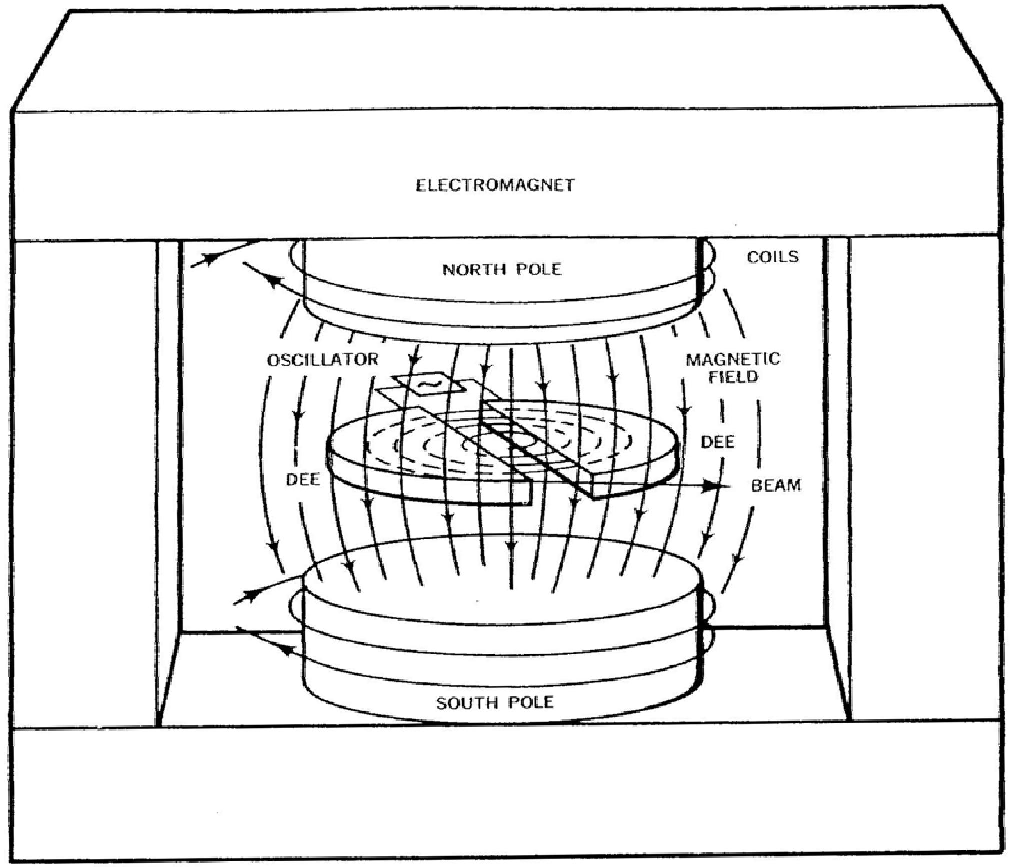
original Cyclotrons

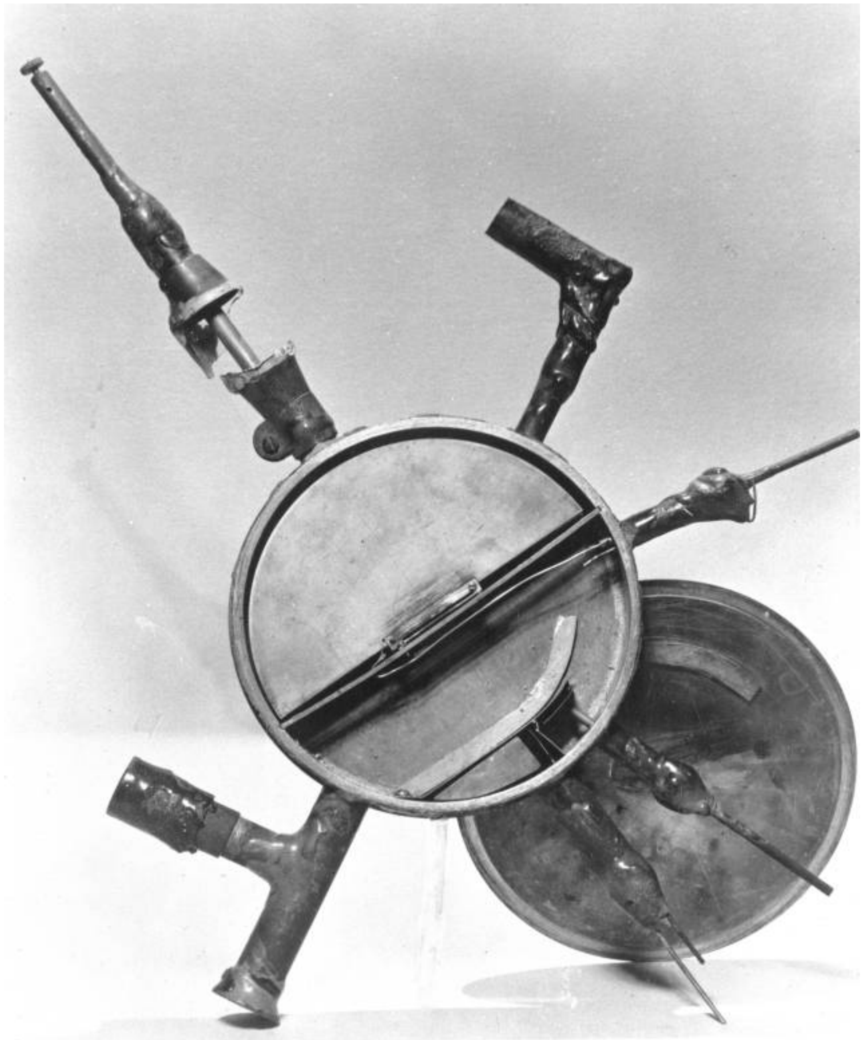


the cyclotron as seen by the inventor



the first classical cyclotrons

The first Cyclotron 1931



E.O.Lawrence,

M.S.Livingston

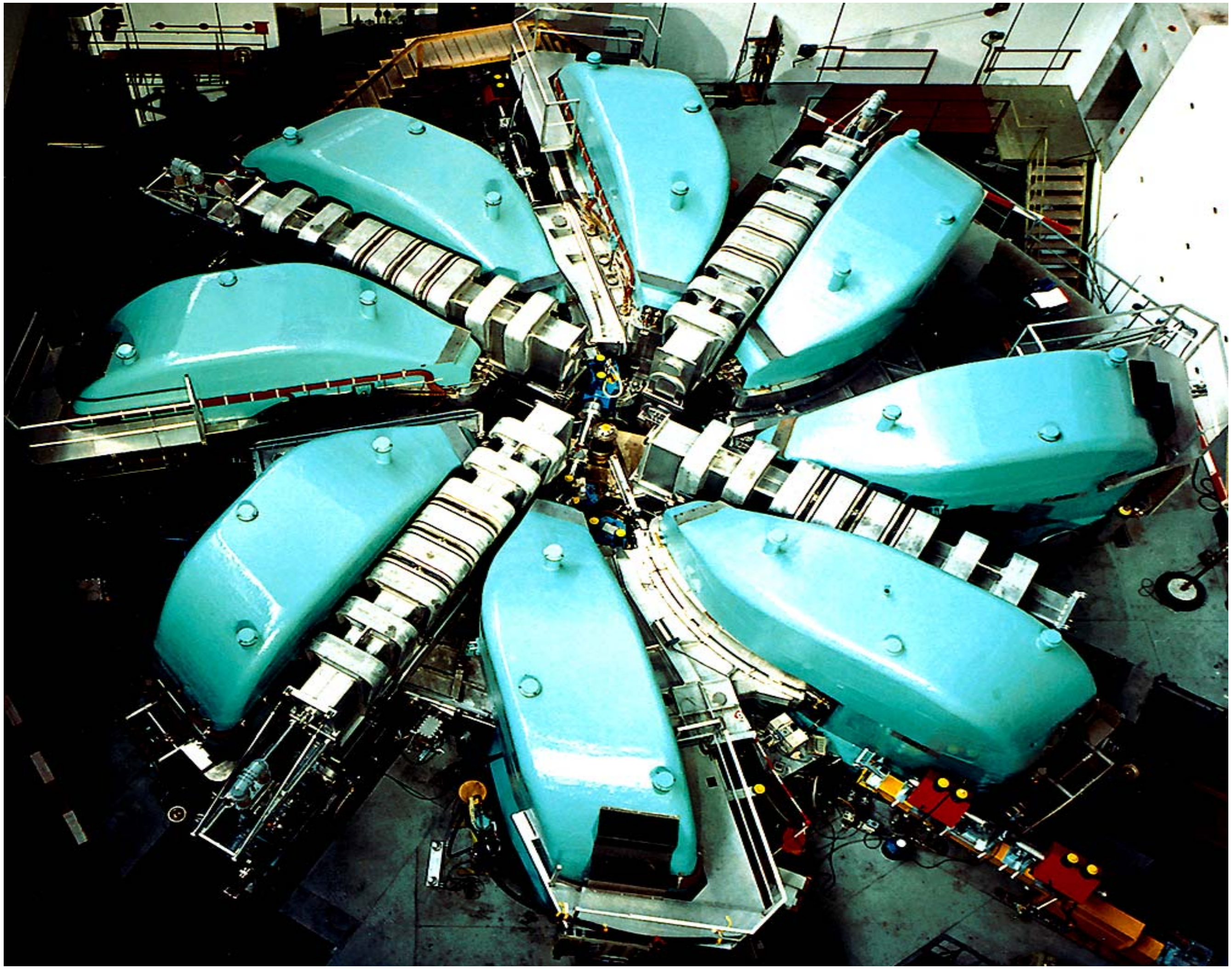
Berkeley, California

4 inch diameter

1 kV on the Dee

80 keV Protons

43 years later (1974)



Ring Cyclotron

590 MeV Protons

15 m Diameter

Hans Willax,

Jean Paul Blaser,

Villigen, Switzerland

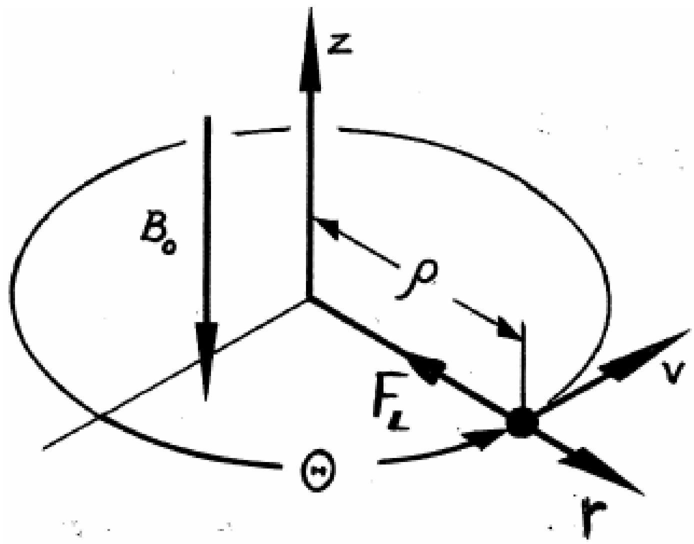
History of the Cyclotron

1929	Idea by E.O.Lawrence in Berkeley (inspired by R.Wideroe!)		
1931	4 inch cyclotron	80 keV	p
1932	10 inch cyclotron	1.2 MeV	p
1934	26 inch cyclotron	7 MeV	p
1939	60 inch cyclotron	16 MeV	d
1946	184 inch synchrocyclotron	200 MeV	d
		400 MeV	α
1938	Idea for sectored cyclotron (AVF) by Thomas		
1962	88 inch sector cyclotron	K=160 MeV	ion
1974	SIN/PSI Ringcyclotron	590 MeV	p
1982	supercond.cyclotron MSU	K=500 MeV	ion

2008: ca. 90 indiv. cyclotrons, ca. 200(?) commercial cyclotrons

circular orbit

In a homogeneous magnetic field B the particle has a circular orbit with radius ρ



Balance between Lorentz-force F_L and centrifugal force F_Z:

$$F_L = q\,v\,B\,, \qquad F_r = \frac{m\,v^2}{\rho} \qquad \text{(non relativ.)}$$

with $p = m\,v$:

$$p = q\,B\rho$$

valid relativistically!
(Bρ) = „magnetic rigidity"

Basis of all circular accelerators (Cyclotron, Synchrotron, Storage Ring, Spectrometer etc.)

for electrons with E ≥ 10 MeV:

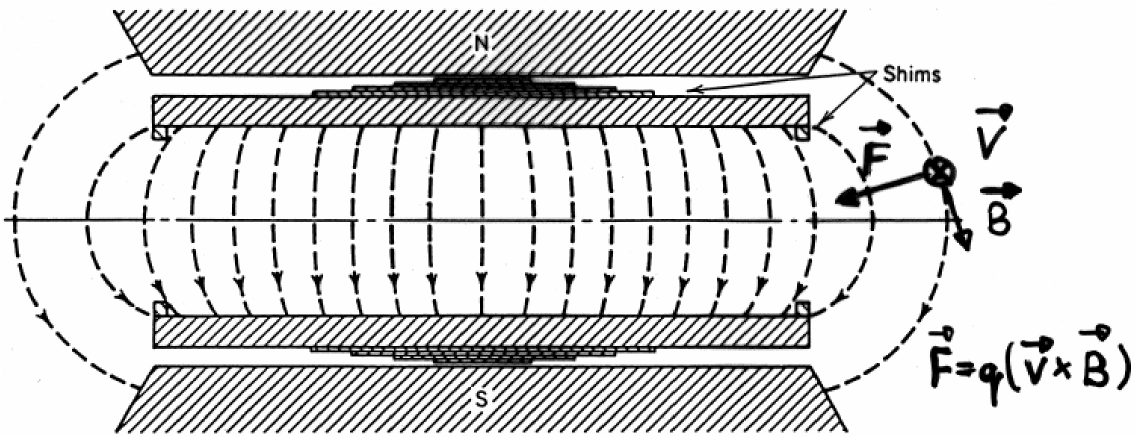
E[GeV] = pc= 0.3 Bρ [Tm]

classical Cyclotron

In homogeneous magnetic field the circular orbits are vertically unstable

vertical stability with

radially decreasing field B(r)



Definition of field index n with „logarithmic derivative"

$$\left(\frac{dB_0}{B_0}\right) \equiv -n\left(\frac{dr}{r}\right)$$

Focusing frequencies :

stable for $0 < n < 1$

$Q_r = \sqrt{1-n}\ , \quad Q_y = \sqrt{n}\ ,$

$Q_r^2 + Q_y^2 = 1$

=> weak focusing,
horizontally and vertically

Larmor Frequency

Revolution frequency ω_0 in homogeneous magnetic field:

ω_0 = v/R , p = mv = q B R (non rel.) :

$$\omega_0 = \frac{q}{m} B \qquad (= Larmor\ frequency)$$

ω_0 is independent of radius R and energy E !

⇒ Basis for classical Cyclotron (non rel.)

relativistic formula for all energies, with $E_{tot} = \gamma\, mc^2$ and $\omega_0 \equiv 2\pi\nu_0$

$$\nu_0 = \left(\frac{q}{2\pi\, m}\right)\frac{B}{\gamma}$$

$$\frac{q}{2\pi\, m} = 15.25 \text{ MHz/T} \qquad \text{for protons}$$

$$\frac{q}{2\pi\, m} = 28 \text{ GHz/T} \qquad \text{for electrons}$$

Isochronism

Acceleration of a particle with RF frequency ν_{RF} on harmonic h:

$$\nu_{RF} = h\,\nu_0$$

If this RF frequency stays constant during acceleration, we talk about an **isochronous cyclotron**. The condition for this is an average field which increases proportional to γ:

$$\Rightarrow B_0(R) \sim \gamma(R)$$

For an azimuthally symmetric field this leads to vertical instability. The way out is:

1) **magnetic sectors** give vertical focusing => $B(r,\vartheta)$, Thomas 1938

   => $B_0(R)$ = field averaged over the whole orbit

2) **synchro-cyclotron** with $\nu_{RF}(t)$ => pulsed beam, reduced intensity

Cyclotrons

classical Cyclotron — † — CW-beam — non relativistic energy limit

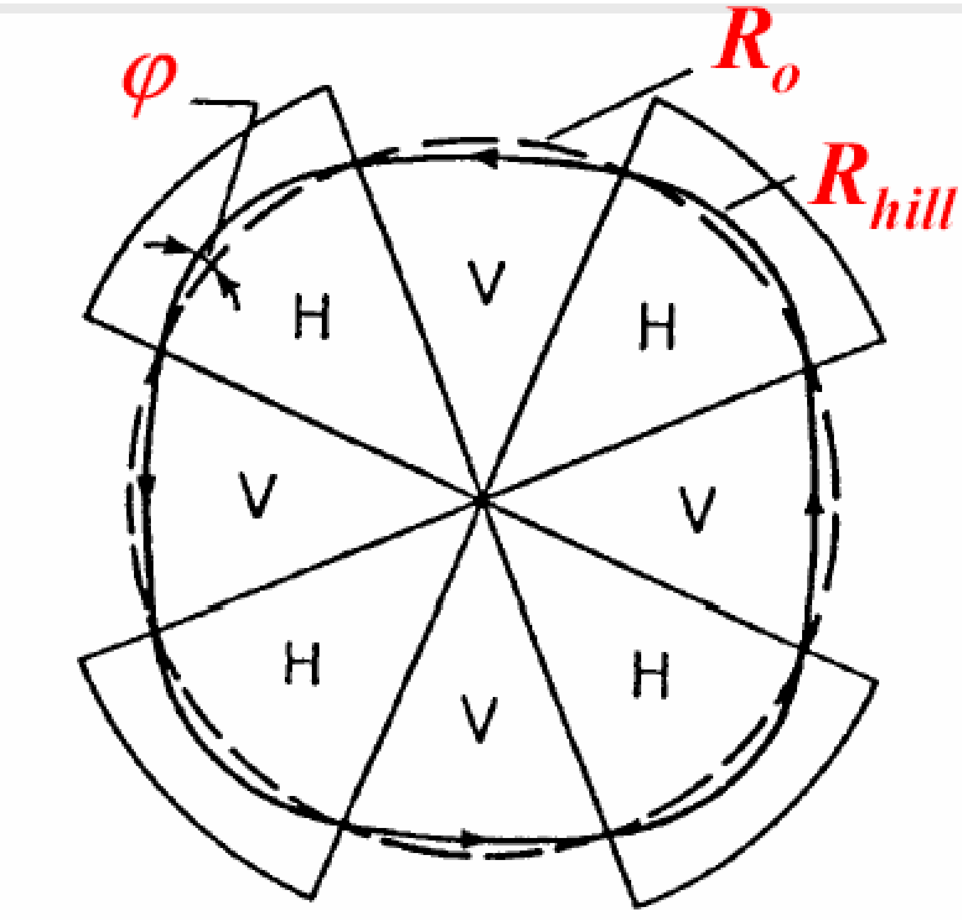
Synchro-cyclotron
- single pole (dying out!)
- FFAG with sectors

pulsed beam

Isochronous Cyclotron
- single pole with sectors
- Ring Cyclotron

CW-beam

Thomas Cyclotron (1938)



Sectors on the pole plates of an H-magnet

=> vertical edge focusing between

Hill (H) and Valley (V)

focal length f_y through edge angle Ψ :

$$\frac{1}{f_y} = \frac{[B(H) - B(V)]}{B\rho} \tan\Psi, \quad (f_x = -f_y)$$

Edge Focusing

horizontally:

the deflection of a particle with parallel displacement x is delayed by the path length

$ds = x \tan \Psi \quad \Rightarrow x' = ds/R$

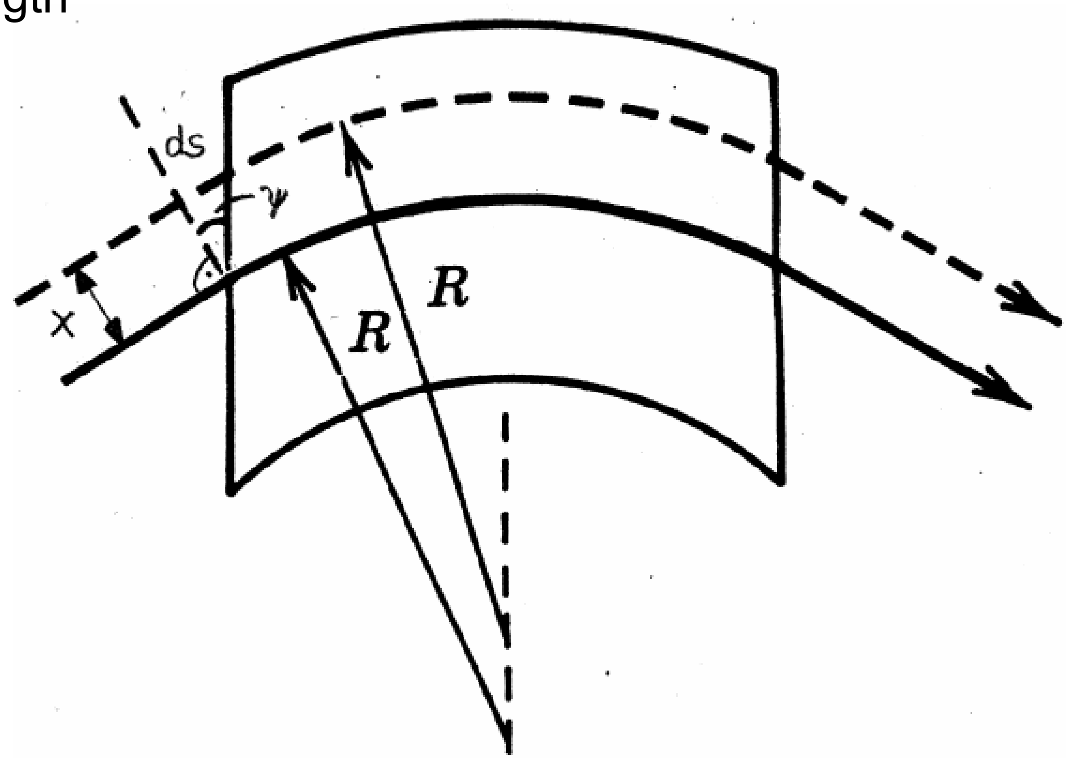
the effect is the same as a defocusing quadrupole

with strength: $1/f_x = -(1/R) \tan \Psi$

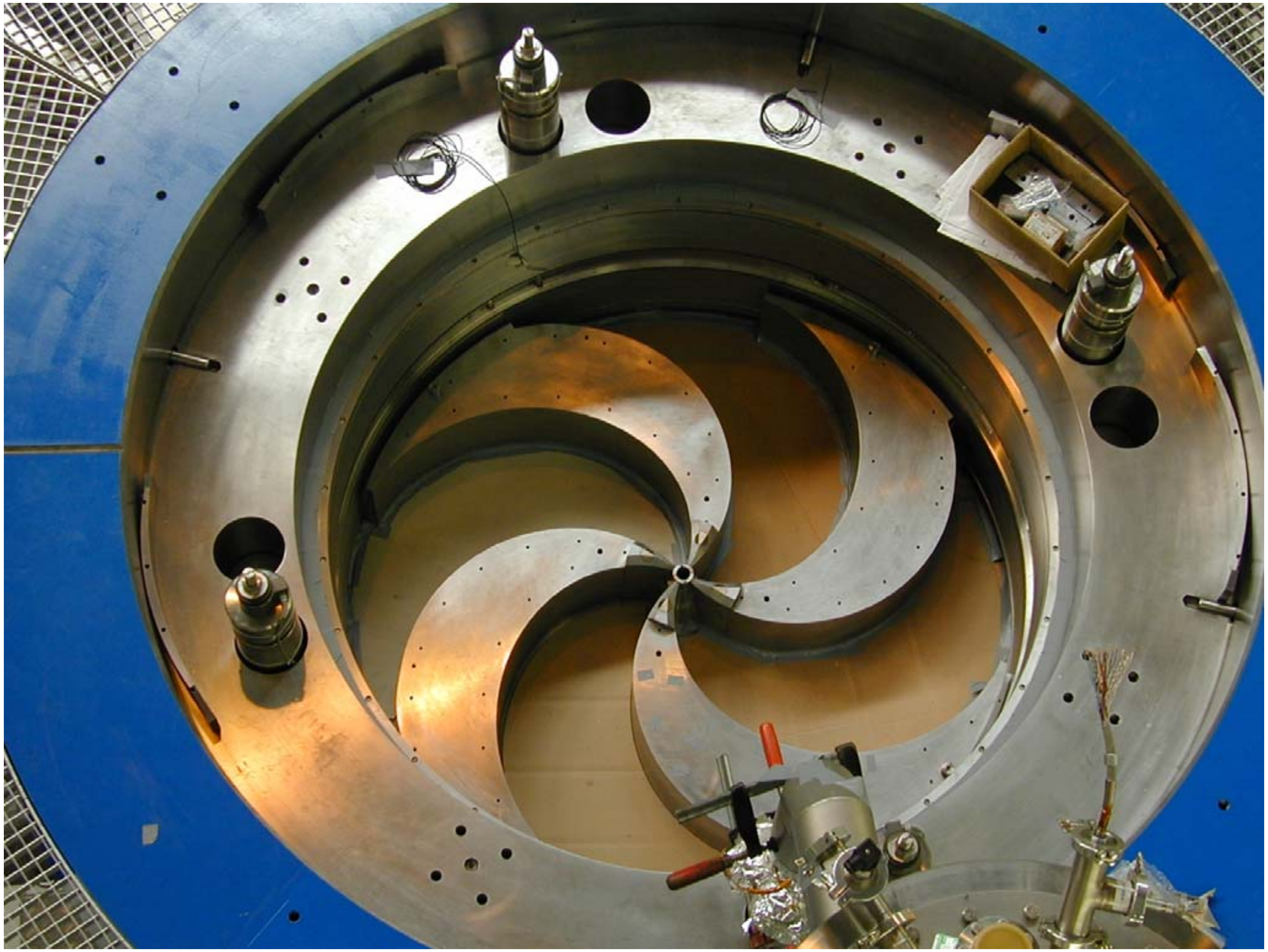
vertically:

focusing with $f_y = - f_x$

Sectormagnet with edge angle Ψ



Comet Cyclotron, Spiral Sectors



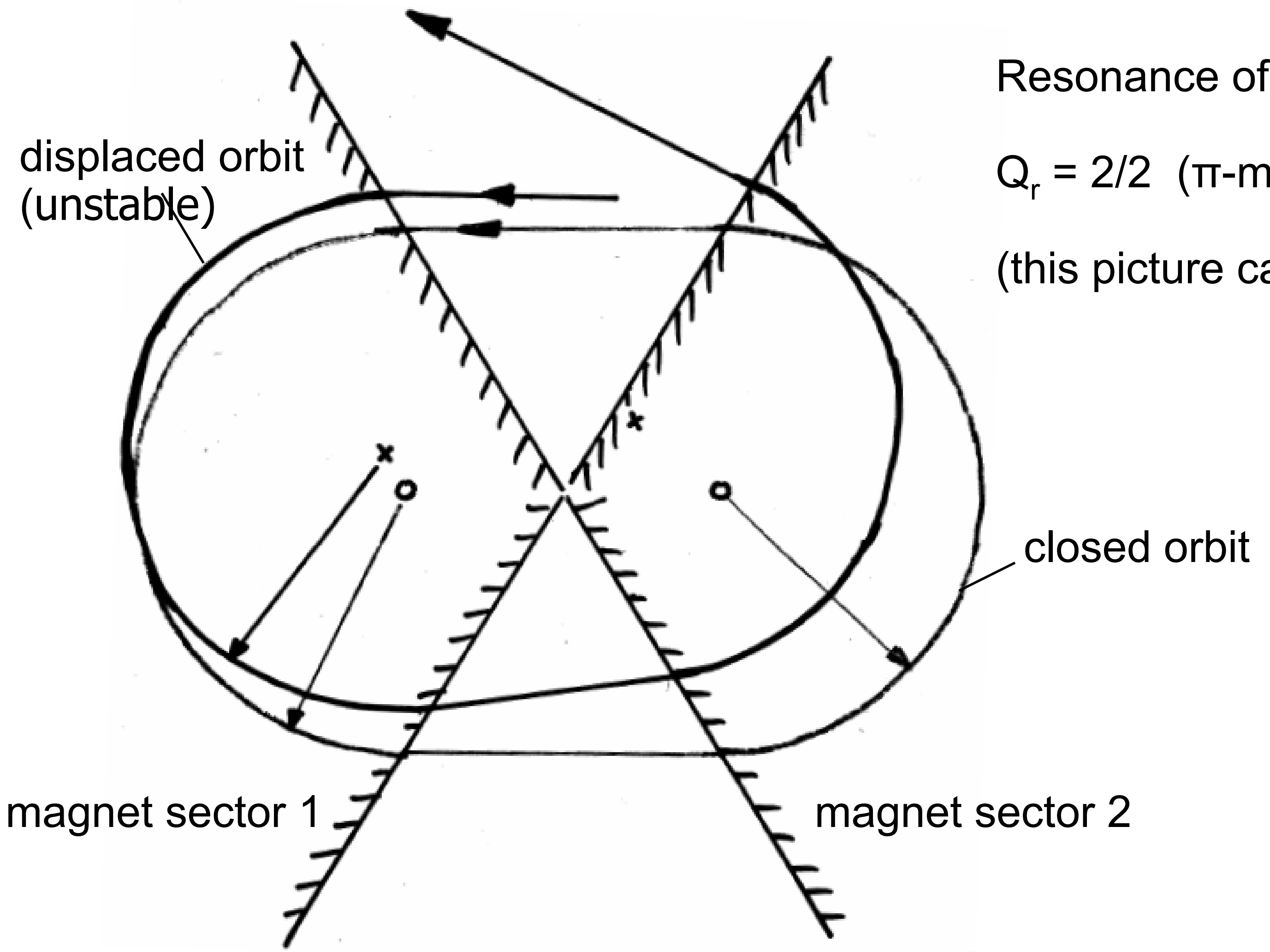
250 MeV Protons for Therapy (ACCEL/ PSI)

superconducting Magnet with 4 Sectors

The spiral structure is responsible for the vertical beam focusing

0.5 m

unstable 2-sector Cyclotron



Resonance of horiz. focusing frequency:

Q_r = 2/2 (π-mode stopband)

(this picture can be constructed by hand!)

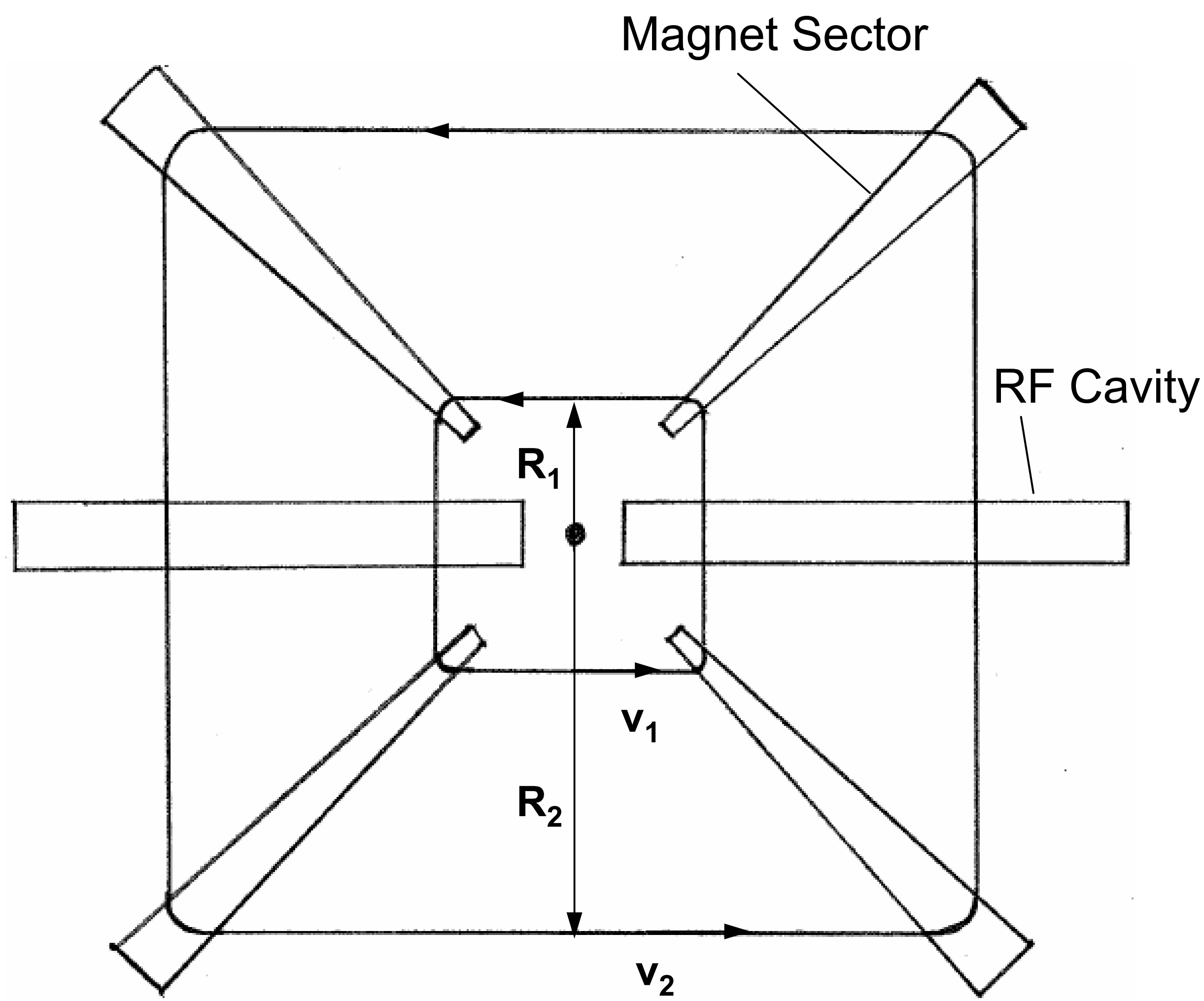
Advantages of Ring Cyclotron
(Hans Willax 1963)

magnetic field and RF system are decoupled

- **many cavities (incl. flattop!) with high voltage**
  - fast crossing of resonances
  - good turn separation, low extraction losses
  - **high intensity**
- **strong vertical focusing**
  - small magnet gap
  - low power consumption
  - total magnet weight as low as for a compact warm magnet
- **straight sections**
  - easy construction of injection and extraction elements (no kickers !)
  - lots of space for diagnostic and correction elements

but: requires injector!

„Square Cyclotron"



isochronous condition :

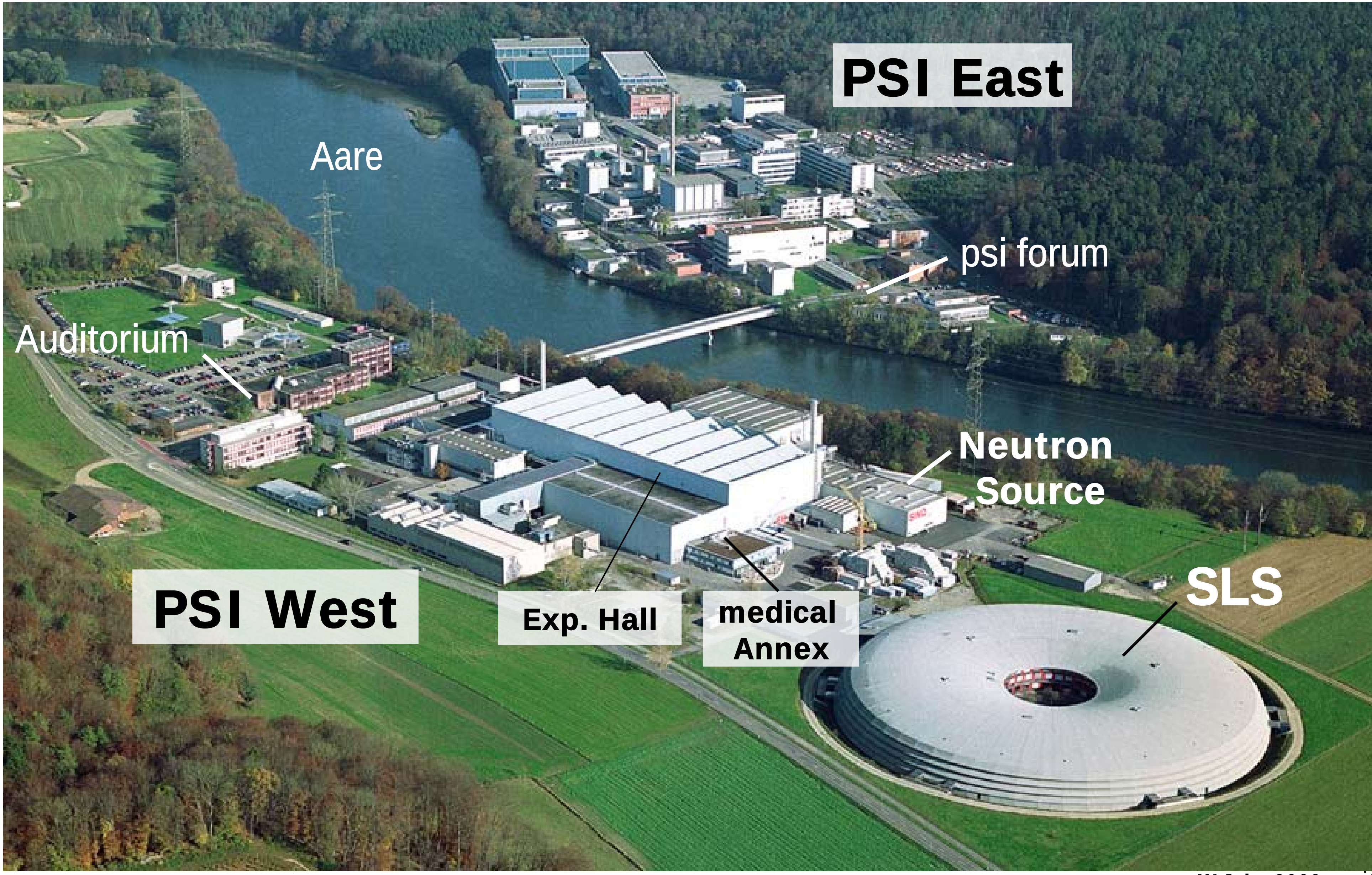
constant revolution time,

independent of energy

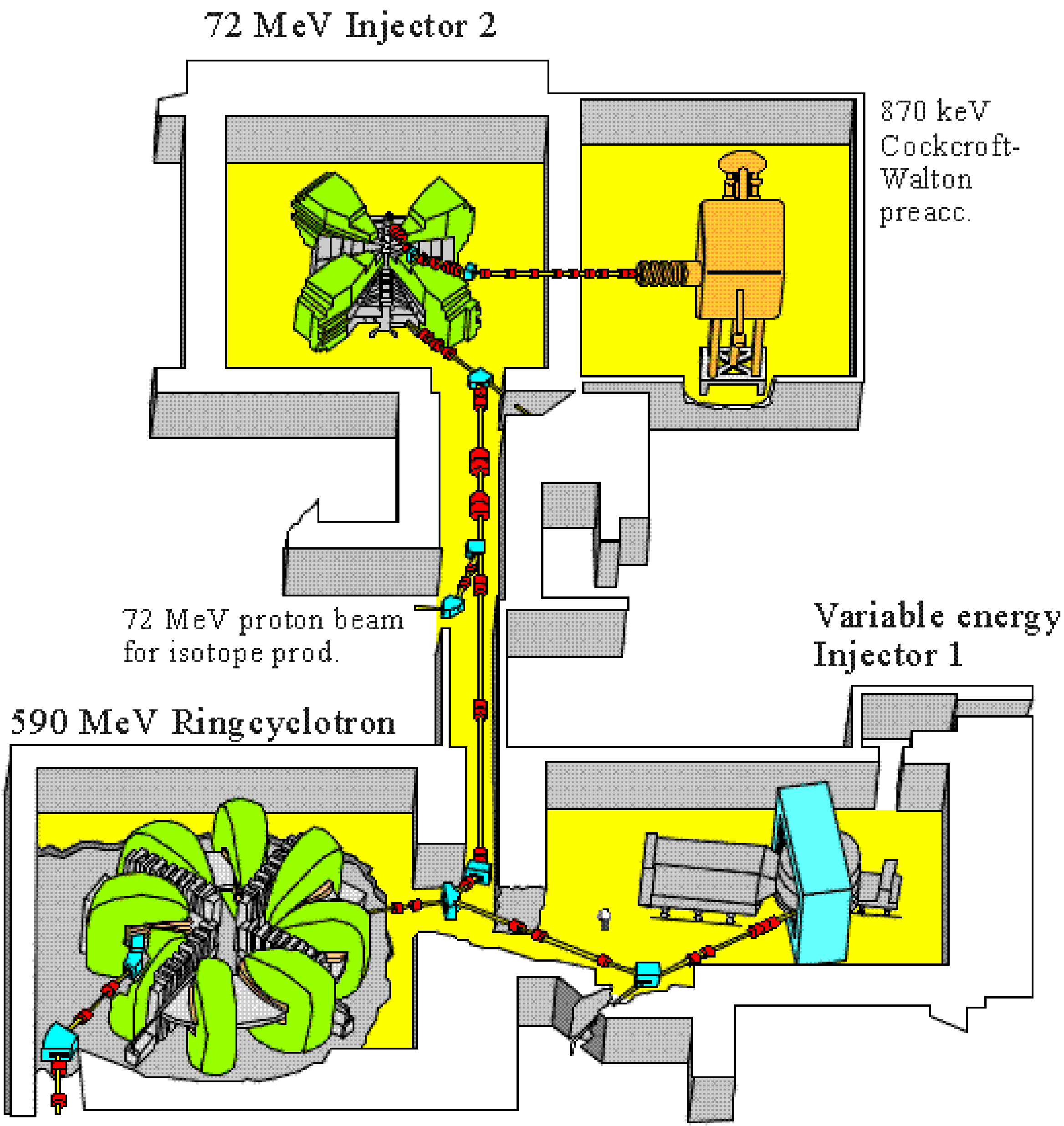
$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{R_1}{R_2}$$

the magnet sectors in this hypothetical example are too narrow and would lead to vertical overfocusing !

Aerial View


PSI East
Aare
psi forum
Auditorium
Neutron Source
SLS
PSI West
Exp. Hall
medical Annex

Accelerator Facilities with 4 Cyclotrons



Injector 1:

Nuclear Physics + Eye Tumours

Injector 2:

Injection + Isotopes for Hospitals

Ring Cyclotron:

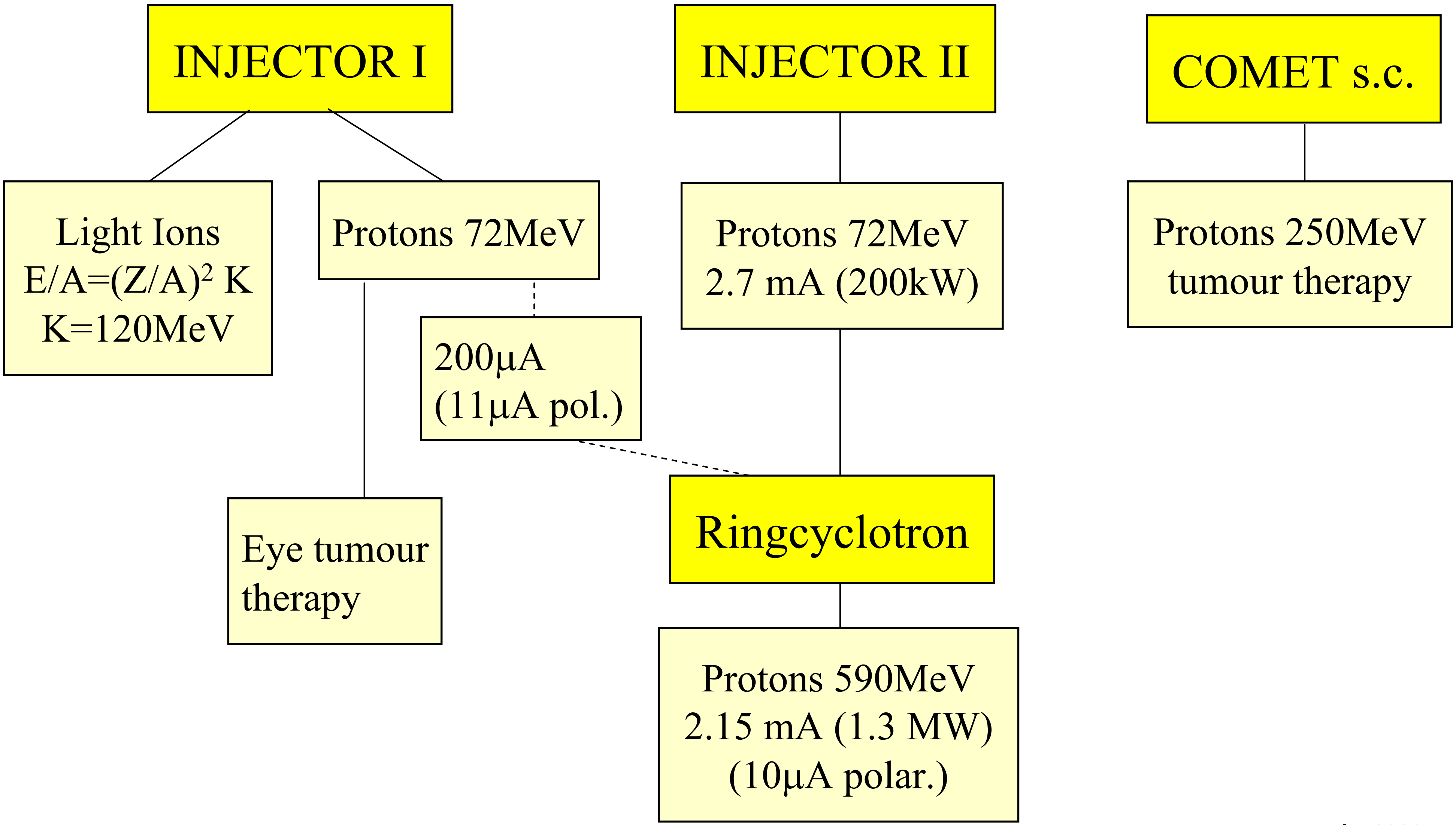
Muons, Pions, Neutrons,

Proton Therapy

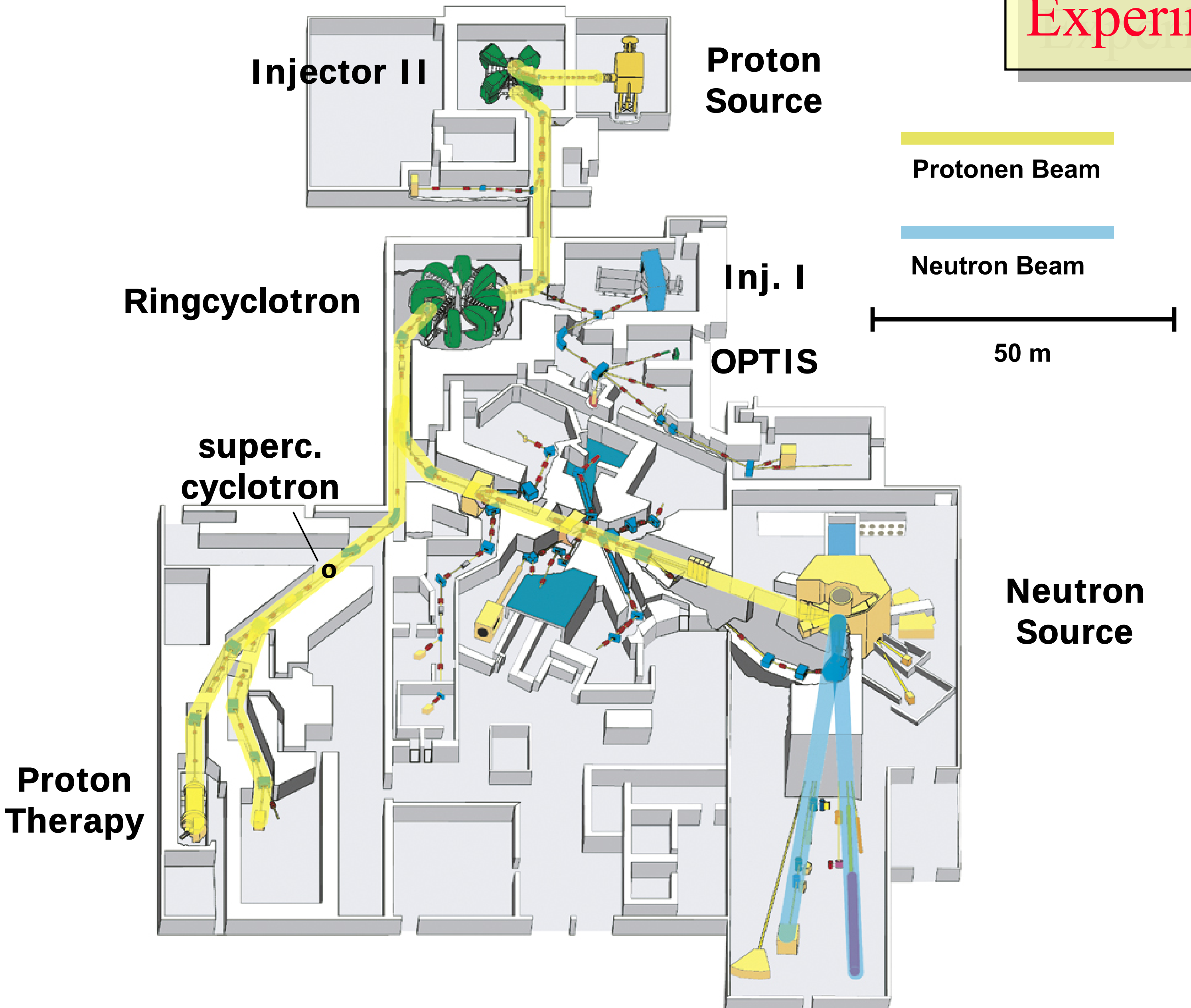
superconducting Cyclotron:

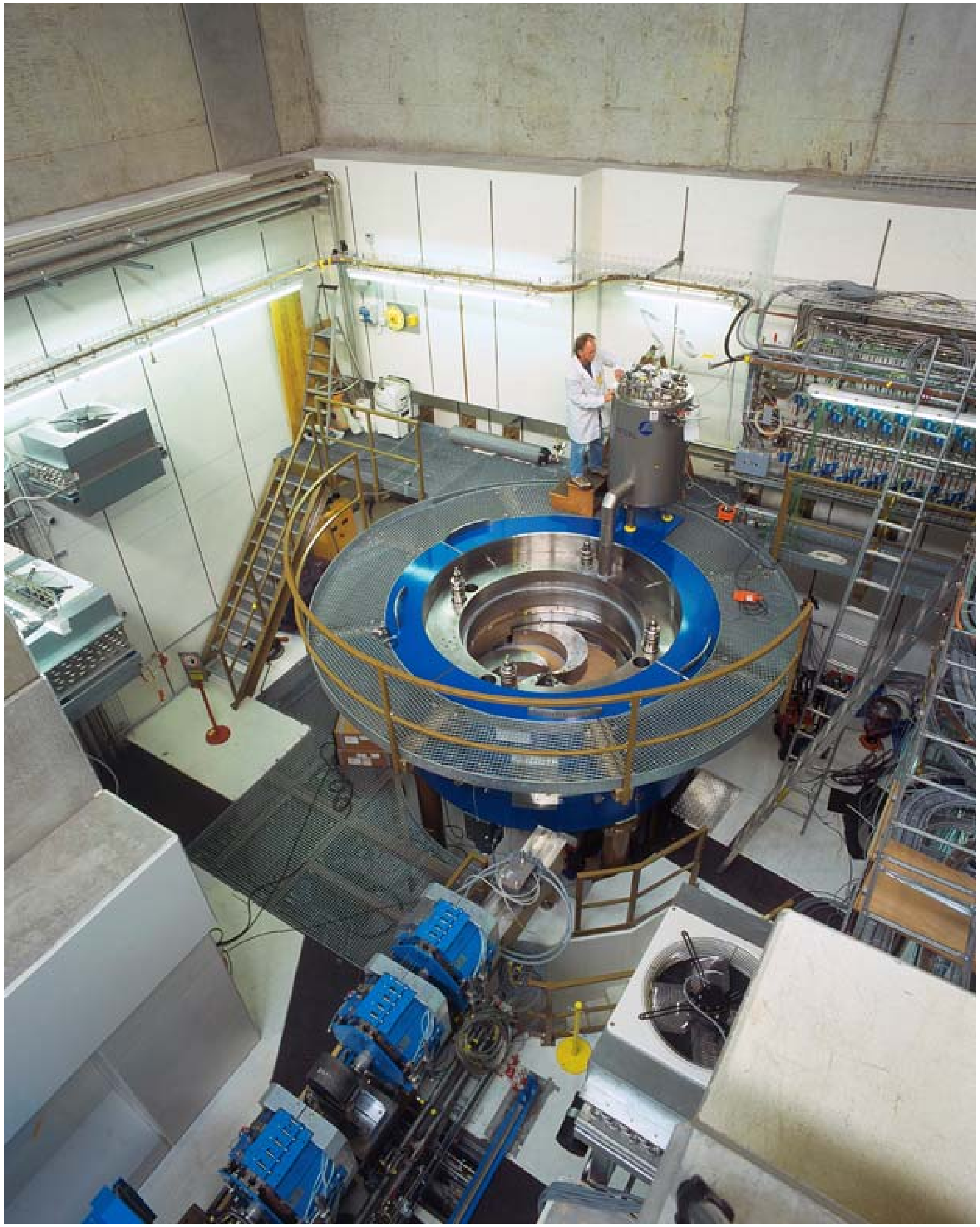
Proton Therapy

The PSI Cyclotrons



Experimental Hall





Comet Cyclotron

Radiation Therapy with

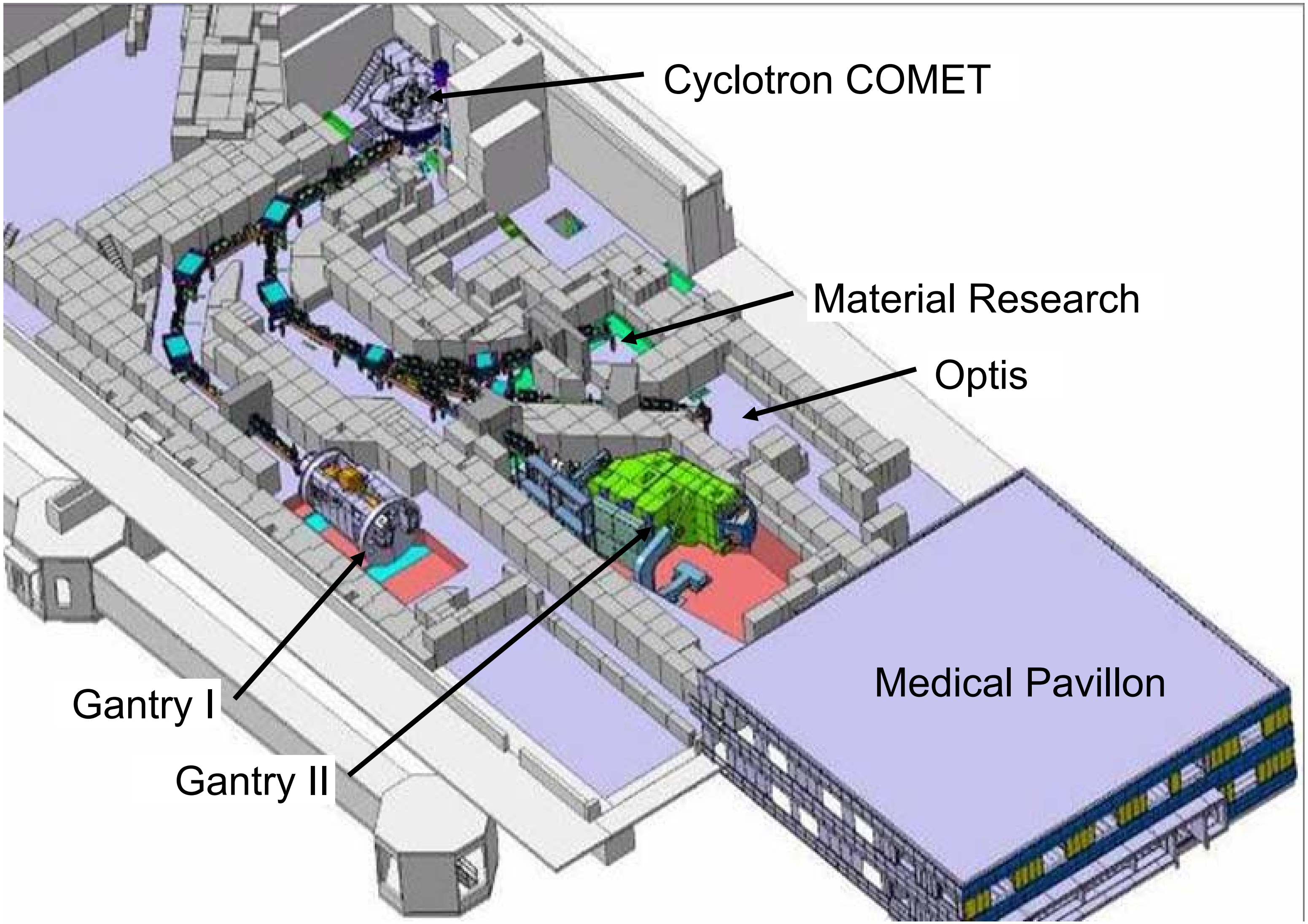
250 MeV Protons

superconducting Cyclotron:

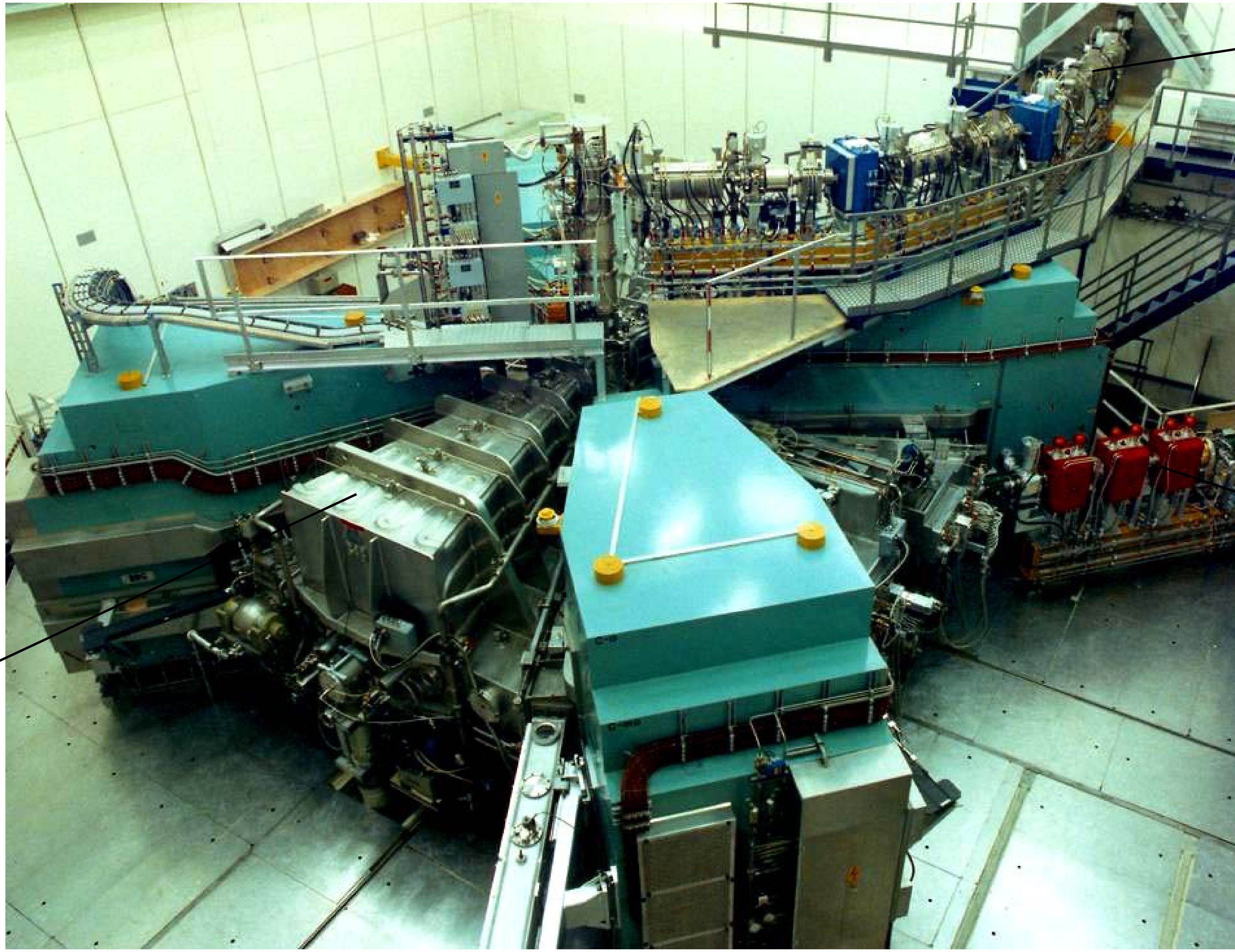
Magnet, 3m Ø

Collaboration: ACCEL & PSI

The PROSCAN Facility



Injector II



Injection Line

870 keV

Extraction Line

72 MeV Protons

(after 100 turns)

Resonator

50 MHz

Recipe for high Intensity

- continuos beam (cw)

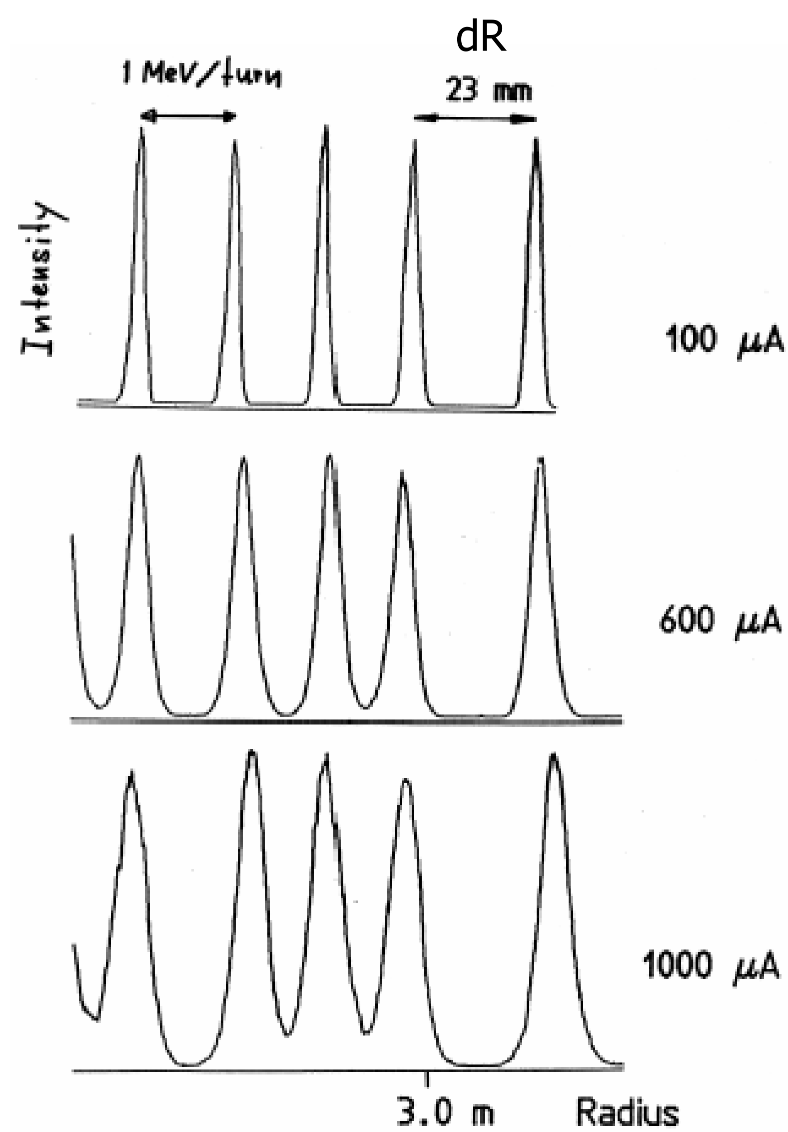
- very low extraction losses

  => separated turns with
  large turn separation dR
  at extraction

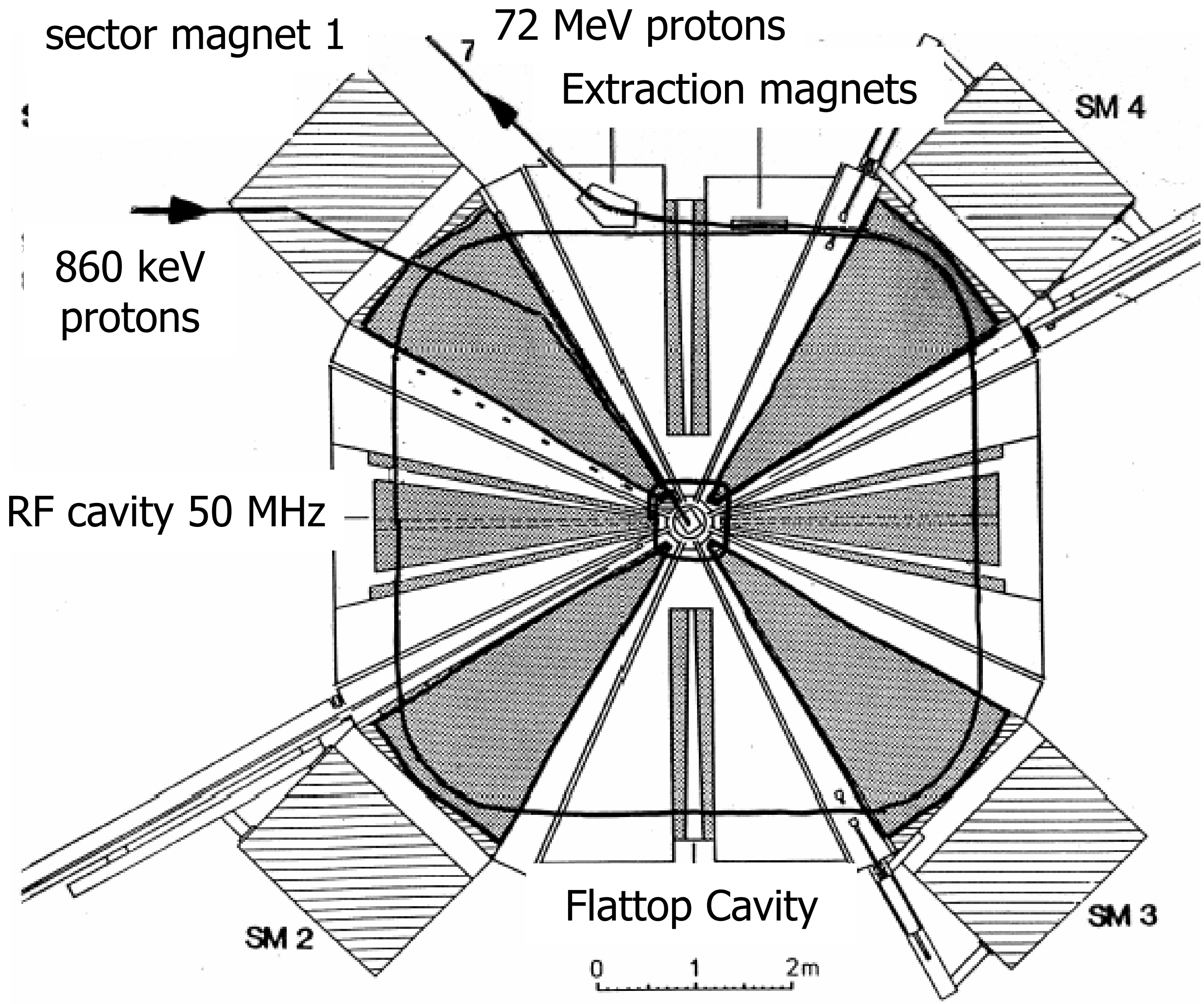
  => high energy gain per turn,
  powerful RF-system with
  high voltage cavities

  dR ~ Radius R
  => **large machine radius** !!

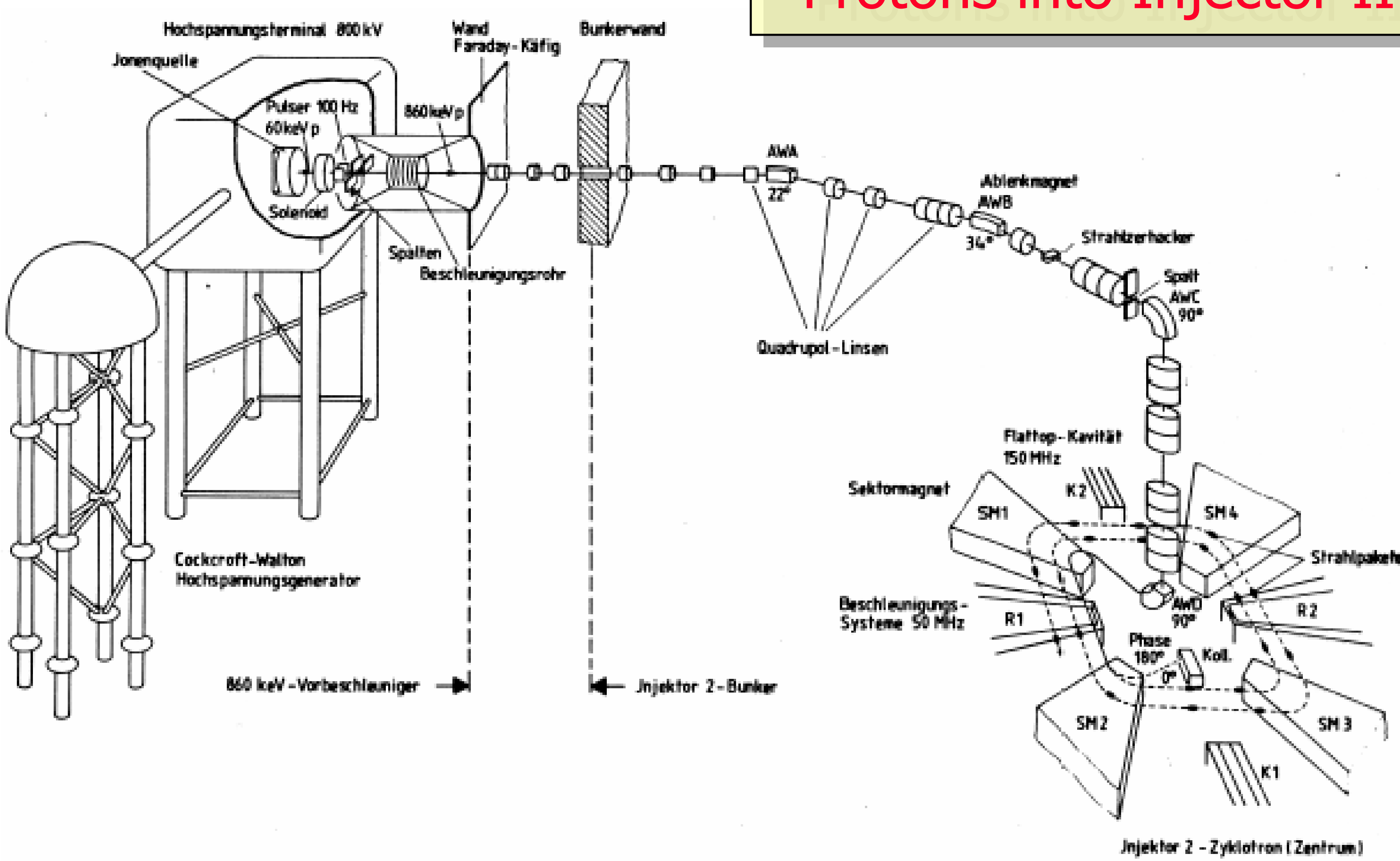
the last 5 turns in the Injector II



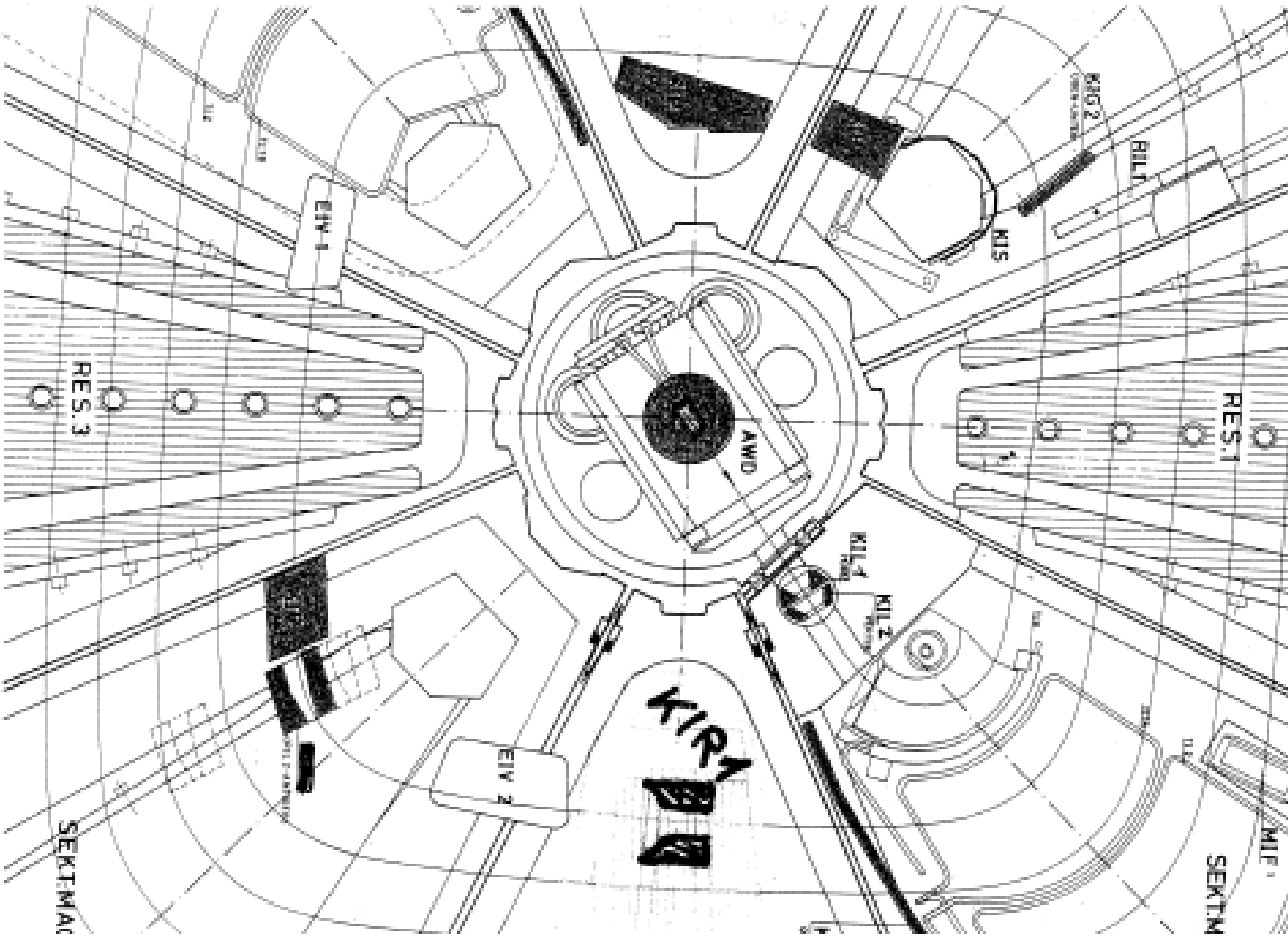
Layout Injector II



Injection of 870 keV Protons into Injector II



Injector II, Central Region

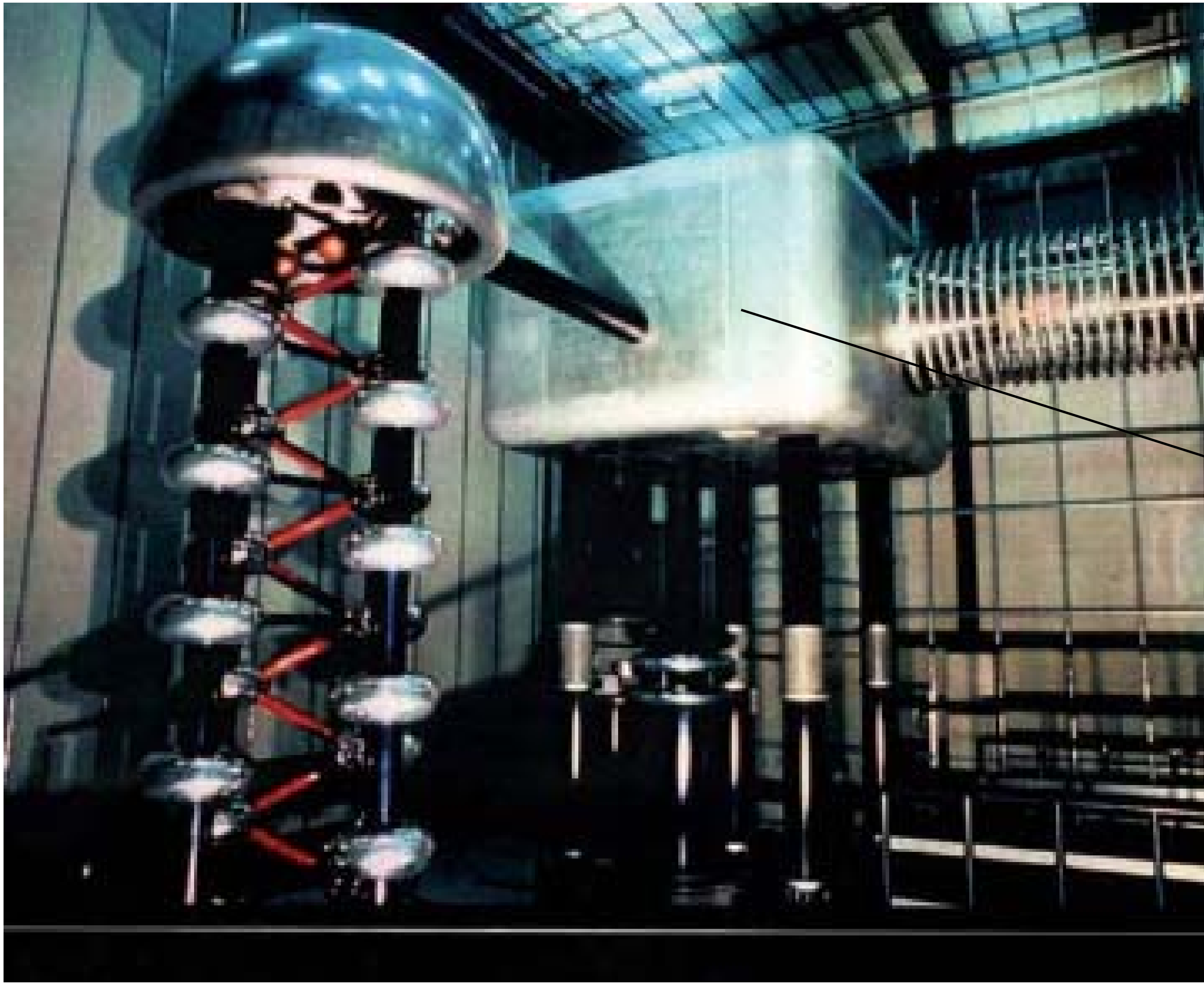


on axis injection of 870 keV p

acceleration gaps

200 mm

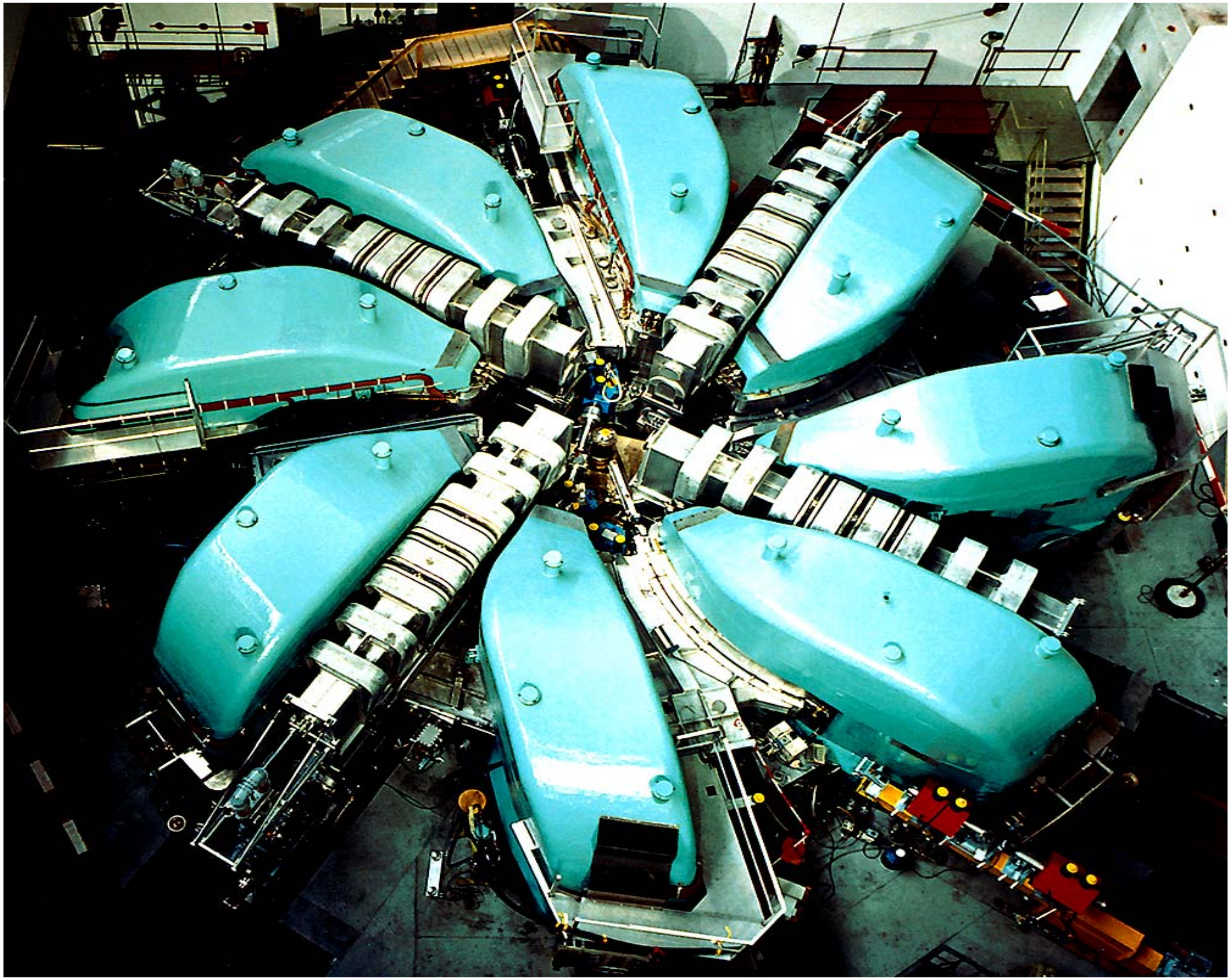
Cockkroft-Walton Pre-accelerator



Voltage: 810 kV

Acceleration Tube

Proton Source
inside Faraday
Cage on 60 kV



Ringcyclotron

590 MeV Protons

1.3 MW Beam Power

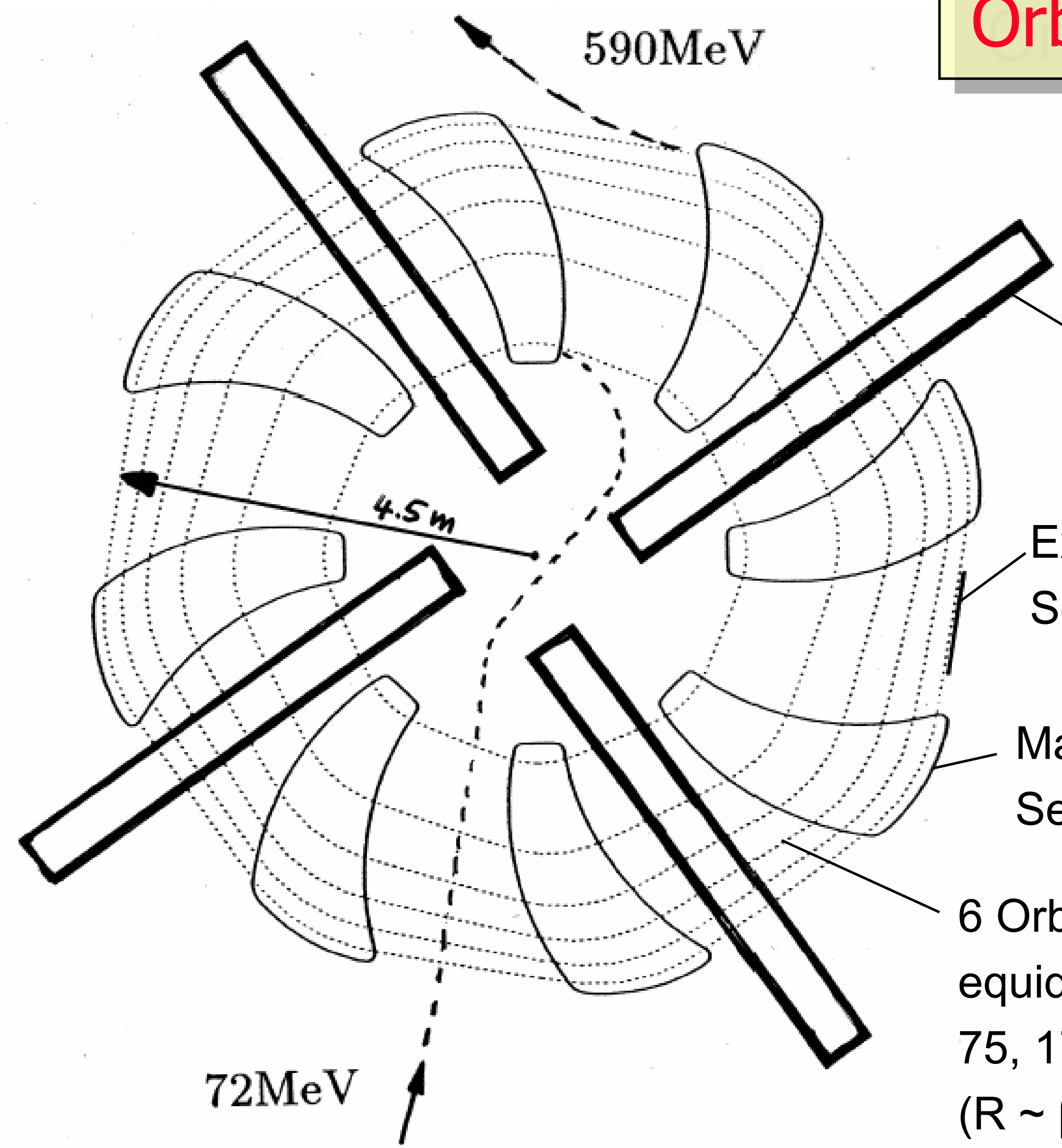
(world record!)

8 Magnet à 250 Tons

4 Cavities à 700 kV
(upgraded to 1MV
in 2008)

Extraction ≈ 99.97 %

Orbits in Ring Cyclotron



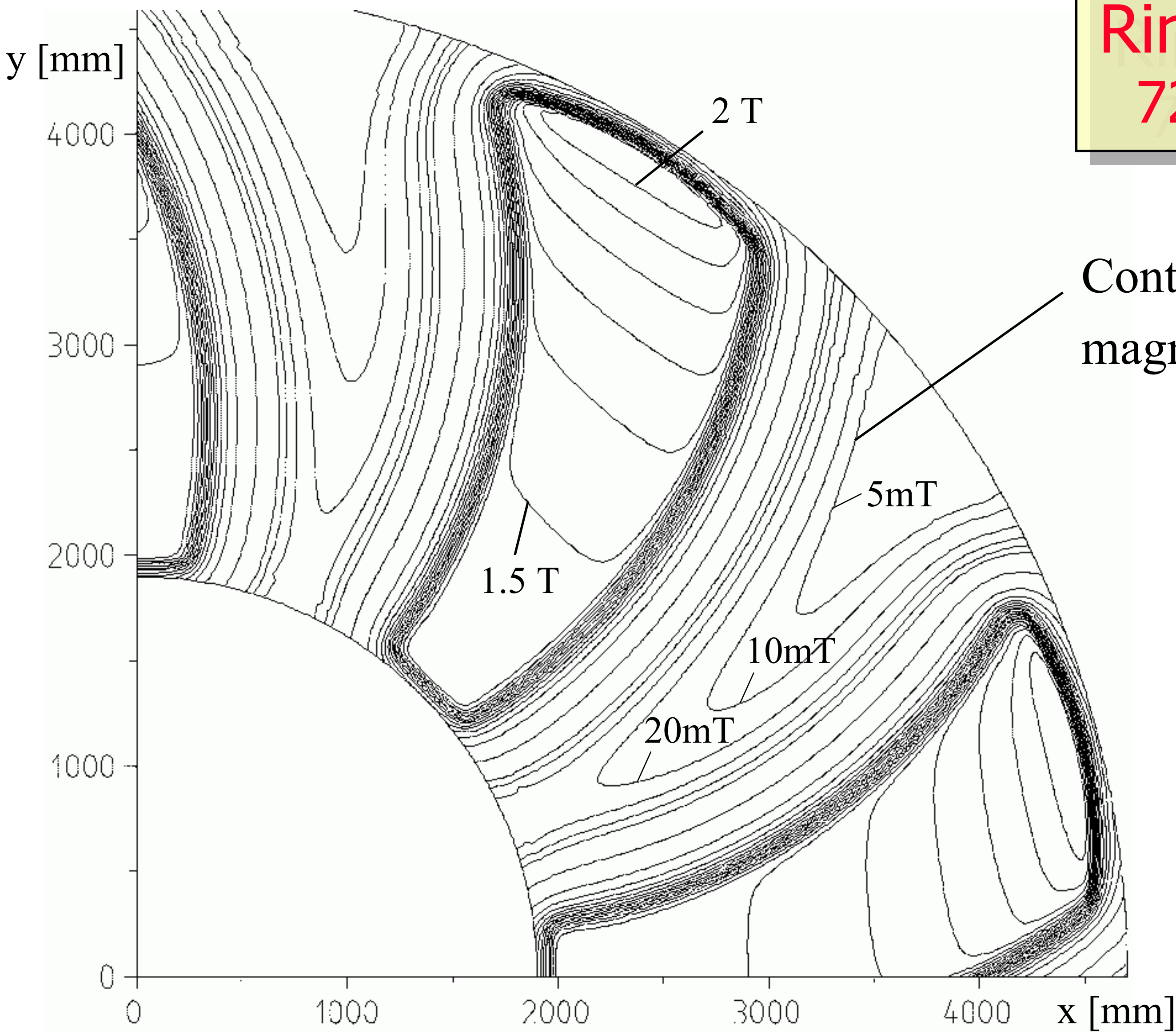
RF Cavity

Extraction Septum

Magnet Sector

6 Orbits plotted at equidistant energies:
75, 177, 279, 381, 483, 585 MeV
(R ~ β)

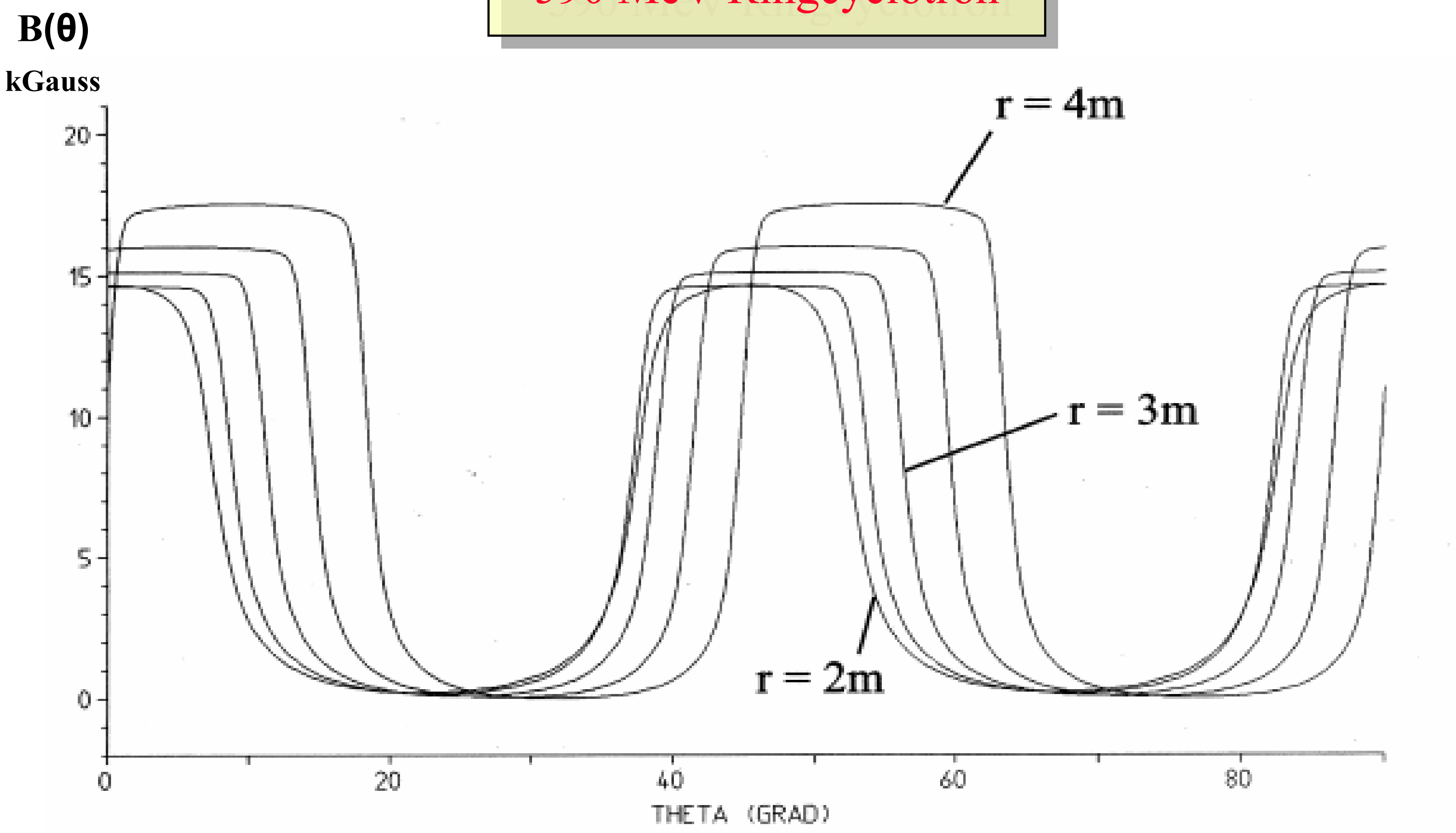
Ring Cyclotron
72-590 MeV p



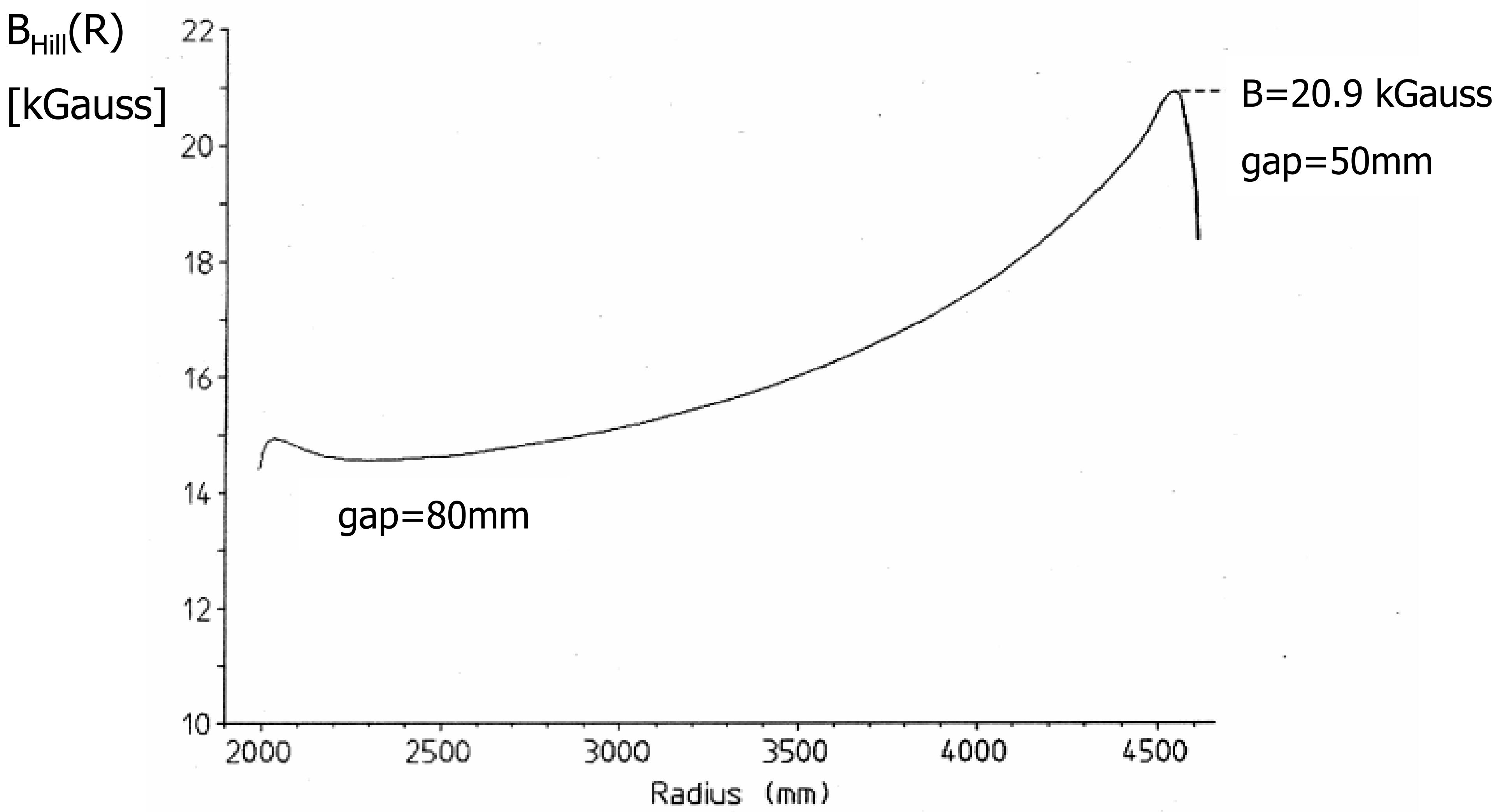
Contour lines of the magnetic field

scaling of average field:
$B_0(R) \sim \gamma$
increases 55%
from 72-590 MeV

Profile magnetic field
590 MeV Ringcyclotron



Hillfield 590 MeV Ringzyklotron



$$Q_x^2 = \quad \nu_r^2 = \underline{1 + K} + \tfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)\Big\{\Big[1 + \frac{\lambda^2}{(Nn)^2}\frac{(Nn)^2 - 2\sigma^2}{(Nn)^2 - \sigma^2}\Big]^2 \frac{(Nn)^2}{[(Nn)^2 - 4\sigma^2]^2} +$$

$$+ \frac{2(Nn)^2}{[(Nn)^2 - \sigma^2]\,[(Nn)^2 - 4\sigma^2]}\Big[1 + \frac{\lambda^2}{(Nn)^2}\frac{(Nn)^2 - 2\sigma^2}{(Nn)^2 - \sigma^2}\Big] - \frac{1 + 3K + K'}{2\sigma^2}\Big[\frac{3(Nn)^2 - 2 + K'}{[(Nn)^2 - \sigma^2]^2}\Big] +$$

$$+ \frac{3K' + K''}{2\,[(Nn)^2 - \sigma^2]^2} - 4\sigma^2\frac{[(Nn)^2 - 1 + (K'/2)]}{[(Nn)^2 - \sigma^2]^2\,[(Nn)^2 - 4\sigma^2]} - \sigma^2\Big[\frac{3(Nn)^2 + 2\lambda^2}{Nn[(Nn)^2 - 4\sigma^2]\,[(Nn)^2 - \sigma^2]}\Big]^2 +$$

$$+ \frac{3\sigma^2}{2}\Big[\frac{Nn}{(Nn)^2 - \sigma^2}\Big]^2\Big\} + \sum_{n=1}^{\infty}(a_n a'_n + b_n b'_n)\Big\{\frac{(Nn)^2 - 2\sigma^2}{(Nn)^2 - 4\sigma^2}\Big[1 + \frac{\lambda^2}{(Nn)^2}\frac{(Nn)^2 - 2\sigma^2}{(Nn)^2 - \sigma^2}\Big] +$$

$$+ \frac{(Nn)^2 - 2\sigma^2}{[(Nn)^2 - \sigma^2]\,[(Nn)^2 - 4\sigma^2]} - \frac{1 + 3K + K'}{2\sigma^2\,[(Nn)^2 - \sigma^2]} + \frac{1}{(Nn)^2 - \sigma^2}\Big[1 - \frac{\lambda^2}{(Nn)^2}\frac{(Nn)^2 - 2\sigma^2}{(Nn)^2 - \sigma^2}\Big] -$$

$$- \frac{2\sigma^2\,[3(Nn)^2 + 2\lambda^2]}{(Nn)^2\,[(Nn)^2 - \sigma^2]\,[(Nn)^2 - 4\sigma^2]^2}\Big\} + \tfrac{1}{2}\sum_{n=1}^{\infty}\frac{a'^2_n + b'^2_n}{(Nn)^2 - 4\sigma^2} + \tfrac{1}{2}\sum_{n=1}^{\infty}\frac{a_n a''_n + b_n b''_n}{(Nn)^2 - \sigma^2}. \qquad (3)$$

Fig. 3. General formula for radial frequency, ν_r. → useless!

$$Q_z^2 = \quad \nu_z^2 = -K + \tfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)\Big\{\frac{(Nn)^2}{(Nn)^2 - (1 + K)} + \tfrac{1}{2}\Big(K + \frac{3K'}{1 + K}\Big)\Big[\frac{Nn}{(Nn)^2 - (1 + K)}\Big]^2 -$$

$$-\Big[K'\frac{2 + K}{1 + K} + \frac{K''}{2} - \frac{(K')^2}{2(1 + K)}\Big]\Big[\frac{1}{(Nn)^2 - (1 + K)}\Big]^2 + \frac{K'^2}{(Nn)^2[(Nn)^2 - (1 + K)]^2} - \frac{4K(K')^2}{(Nn)^4[(Nn)^2 - (1 + K)]^2}\Big\} +$$

$$+ \tfrac{1}{2}\sum_{n=1}^{\infty}(a_n a'_n + b_n b'_n)\Big\{\frac{2K'}{(Nn)^2[(Nn)^2 - (1 + K)]} - \frac{1 + K - K'}{(1 + K)[(Nn)^2 - (1 + K)]} - \frac{8KK'}{(Nn)^4[(Nn)^2 - (1 + K)]}\Big\} -$$

$$- \tfrac{1}{2}\sum_{n=1}^{\infty}\frac{a_n a''_n + b_n b''_n}{(Nn)^2 - (1 + K)} + \tfrac{1}{2}\sum_{n=1}^{\infty}(a'^2_n + b'^2_n)\Big\{\frac{1}{(Nn)^2} - \frac{4K}{(Nn)^4}\Big\}. \qquad (4)$$

Fig. 4. General formula for axial frequency, ν_z. → useless!

how to scare young students!!

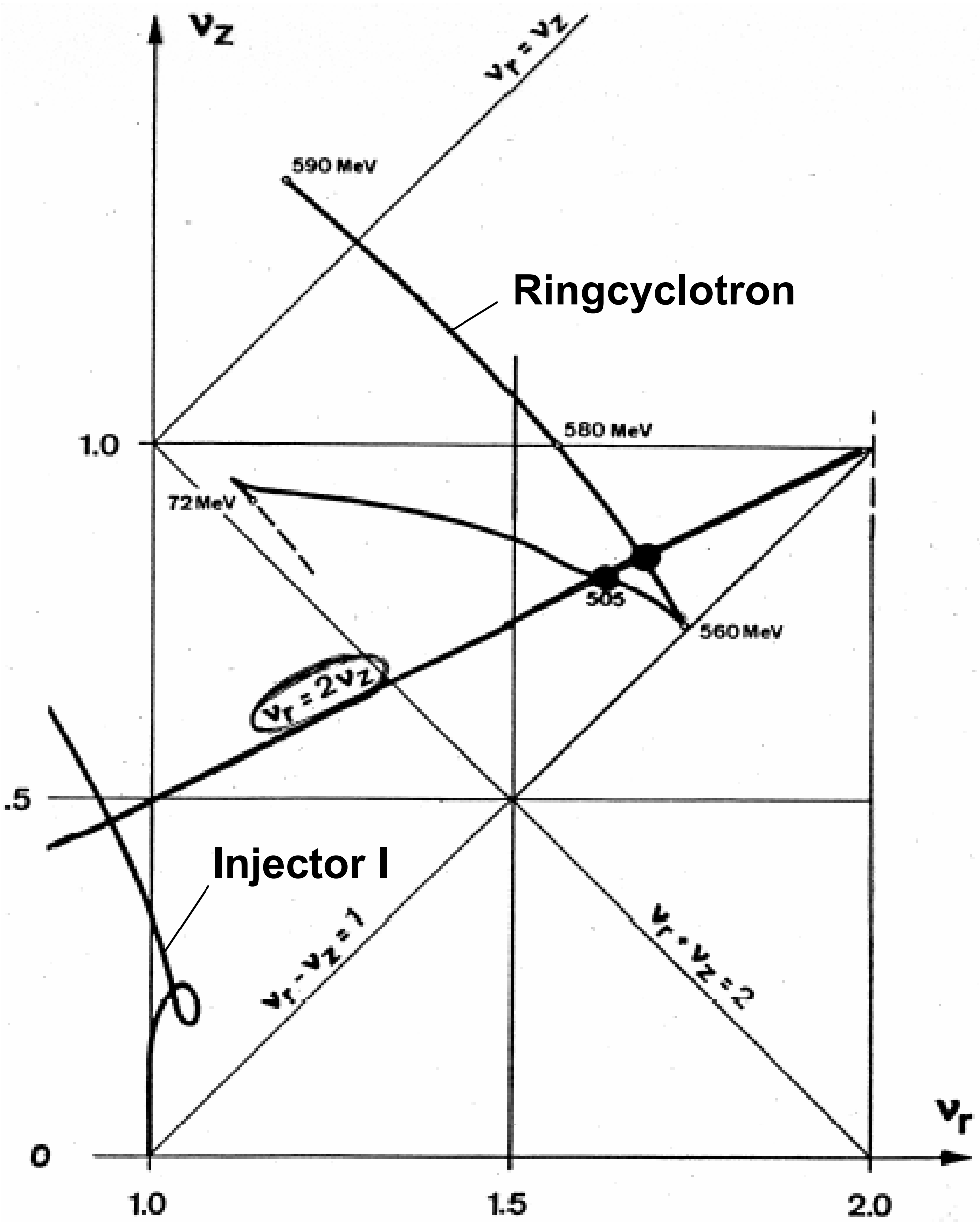
(Al Garren 1962)

better approach to get focusing frequencies:

1. simple approximations
2. numerical calculations

$Q_x^2 \approx 1 + k$

$Q_z^2 \approx -k + F\,(1 + 2\tan^2\delta)$

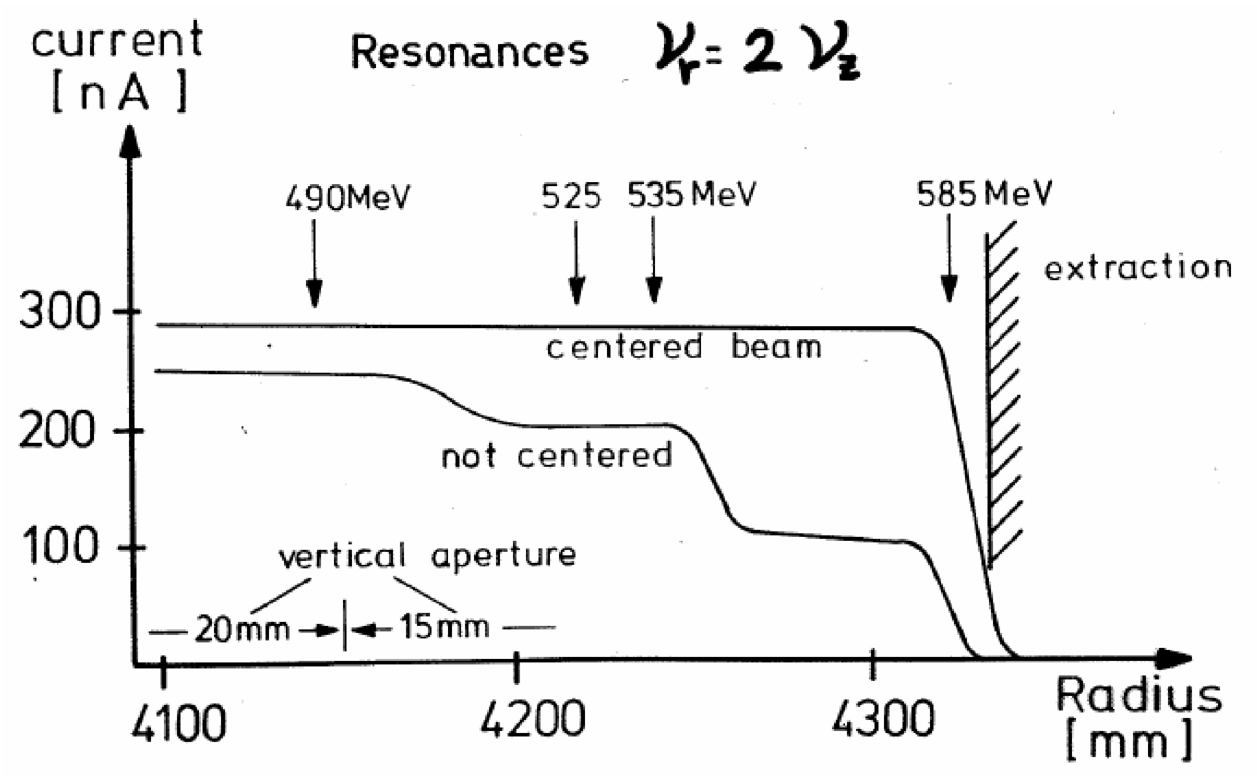


Resonance Diagram of focusing frequencies

In the Ring Cyclotron the coupling resonance $\nu_r=2\nu_z$ is crossed twice before extraction

In the Injector I the resonance $\nu_r=1$ is used to enhance the extraction efficiency

coupling resonance



Ring cyclotron 590 MeV p

a large horizontal oscillation is transformed into a large vertical one at the coupling resonance $\nu_r=2\nu_z$

This can lead to beam losses

radial scaling

In an isochronous cyclotron: ω_0 = const.

$R = v/\omega_0 \sim \beta$, ($\beta = v/c$)

absolute radius limit at v=c: $R_\infty = c/\omega_0$

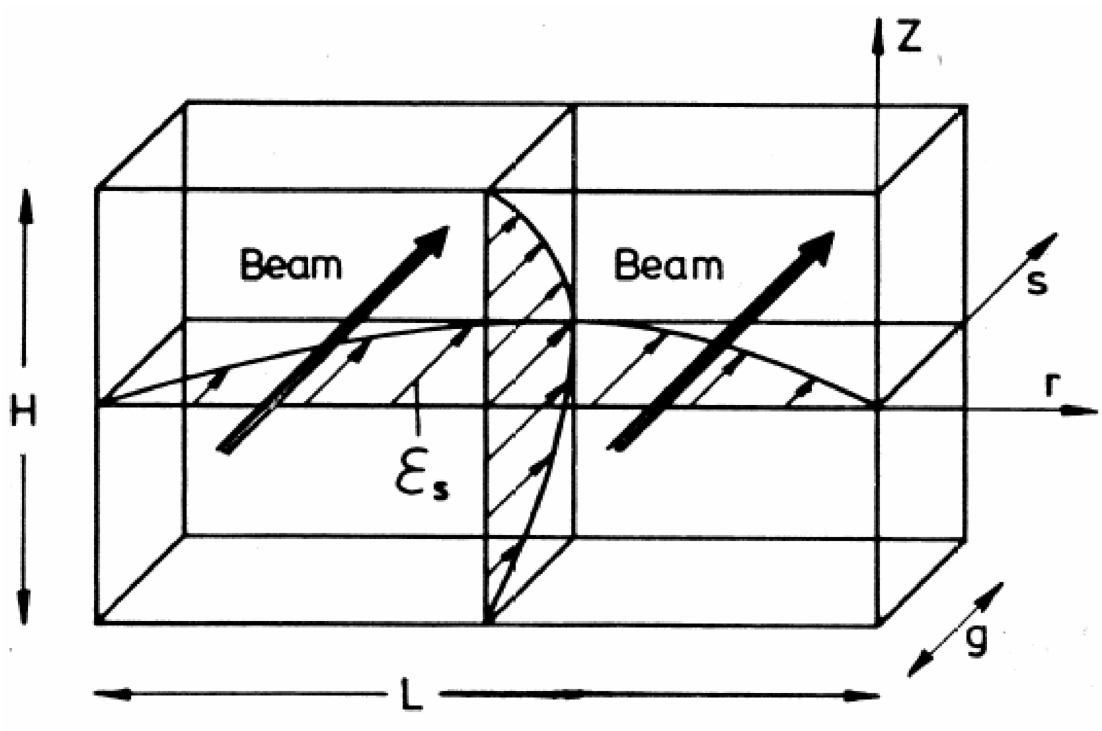
$$R = \beta\, R_\infty$$

$$R_\infty[m] = h\,\left(\frac{47.7\,MHz)}{\nu_{RF}}\right)$$

Example: protons at PSI: ν_{RF} = 50.7 MHz , R_∞ = h· 0.94m

	E[MeV]	β_{max}	h	R_∞[m]	R_{max}[m]	B_0[T] (center)
Injektor I	72	0.37	3	2.83	1.05	1.1
Injektor II	72	0.37	10	9.40	3.5	0.33
Ring	590	0.79	6	5.65	4.5	0.55

RF Cavity



Ring Cyclotron 590 MeV , 50.7 MHz

original version:

aluminum , V=720 kV

300 kW power loss

216 turns

at 2 mA: 300 kW power/cavity is

delivered to the beam

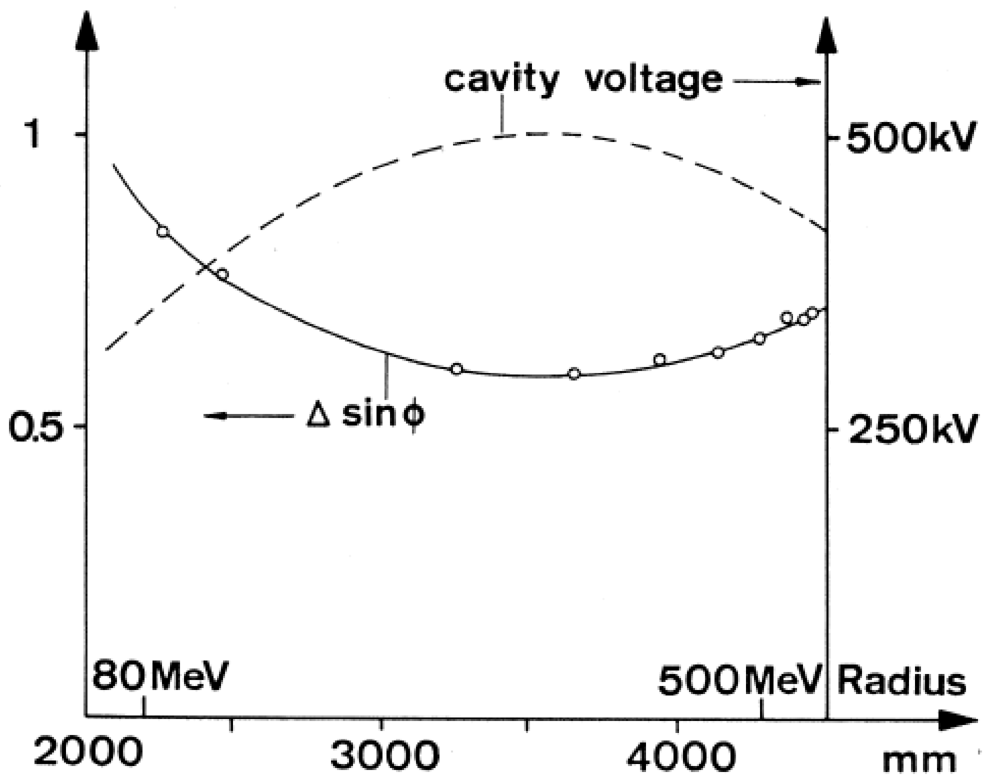
new cavity:

copper , V = 1 MV

400 kW power loss

160 turns , current limit > 3 mA ?

Phase Compression / Phase Expansion due to Variation in Cavity Voltage



The radial variation of the cavity voltage produces a phase dependent magnetic field. This effects the revolution time and thus the phase of a particle.

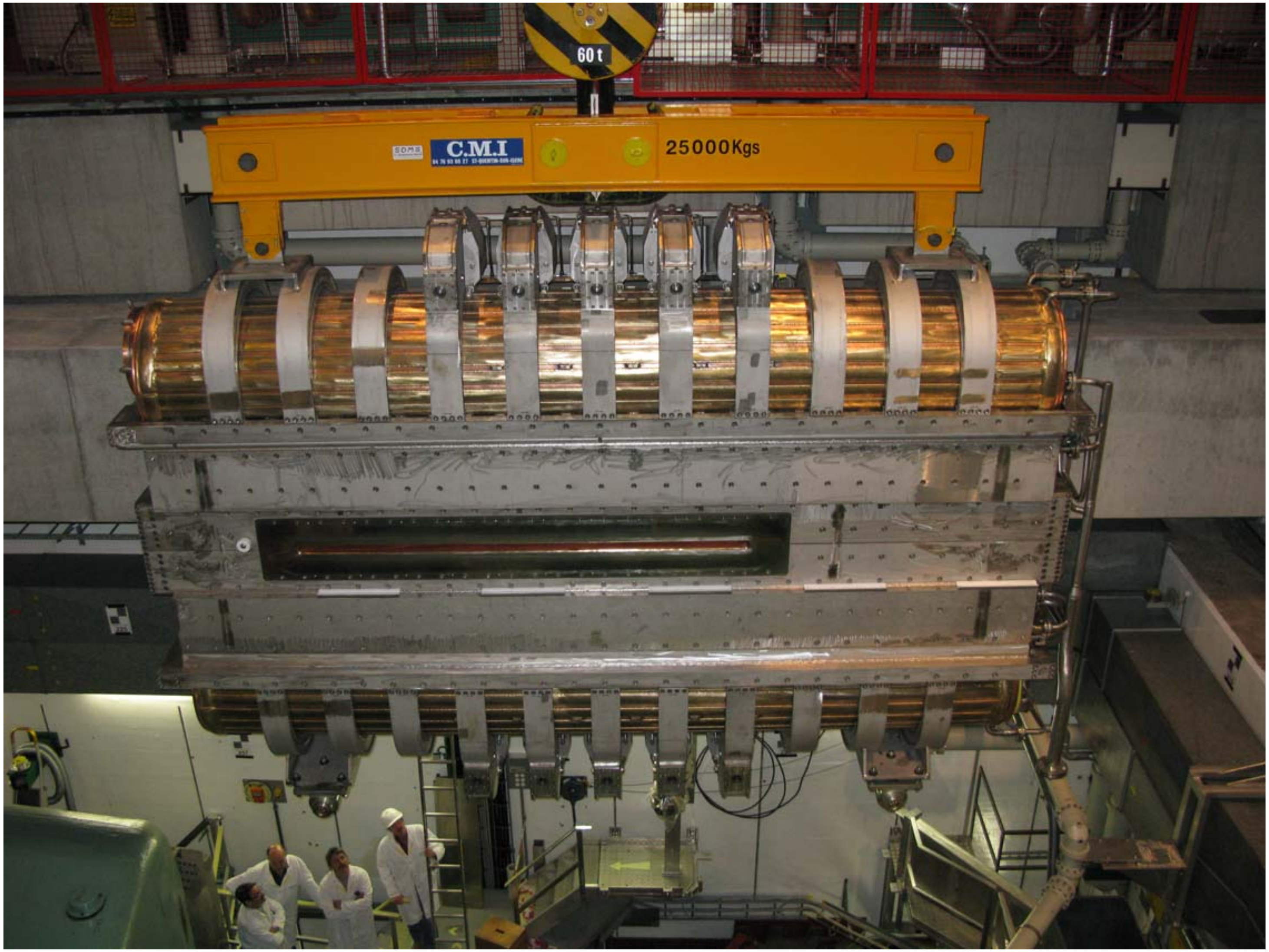
$$E_G(R)\,\Delta\sin\Phi(R) = const.$$

$\mathrm{E_G}$ = peak energy gain/turn

Φ = phase of particle

W.Joho, Particle Accelerators 1974, Vol.6, pp. 41-52

New Copper Cavity

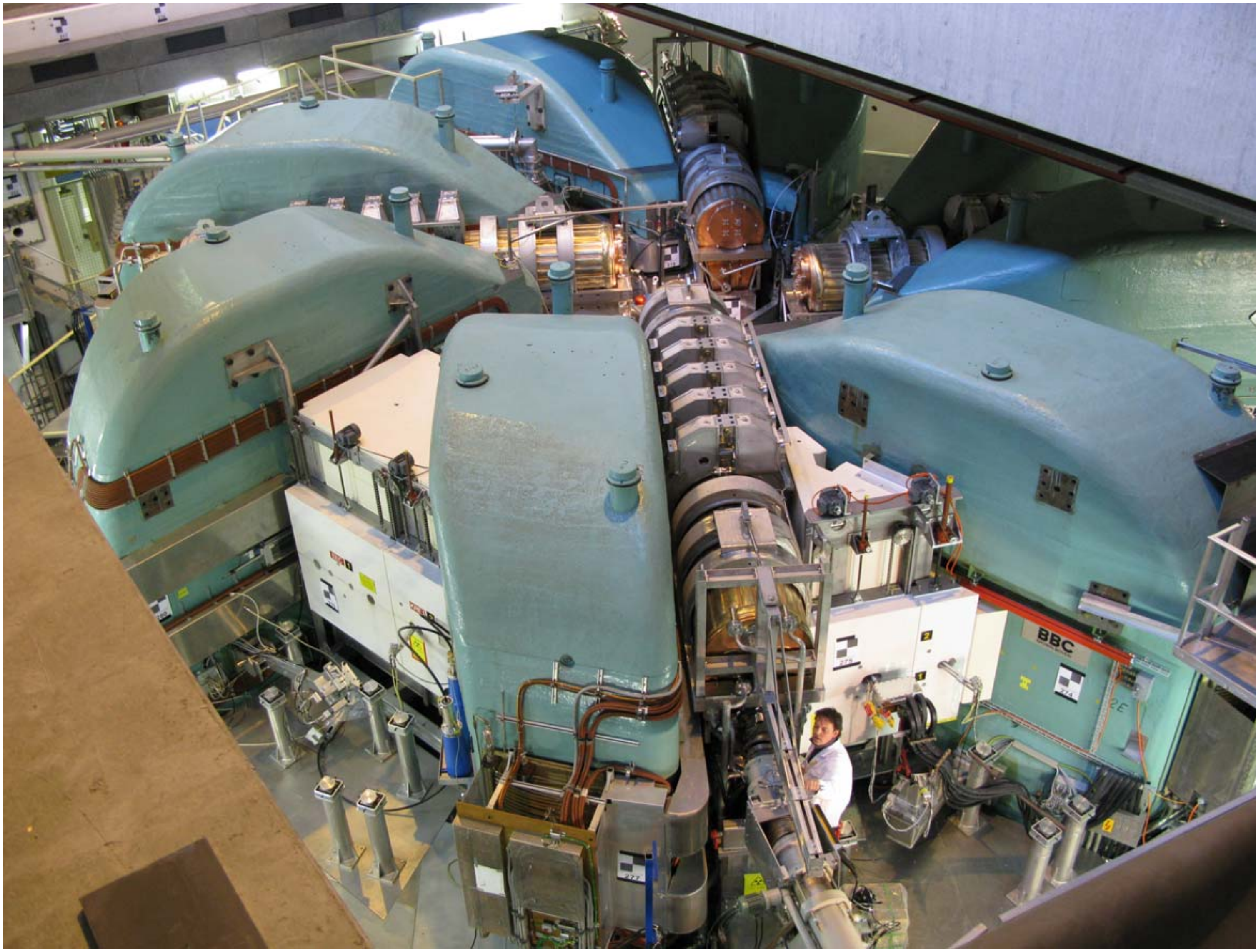


50 MHz, CW

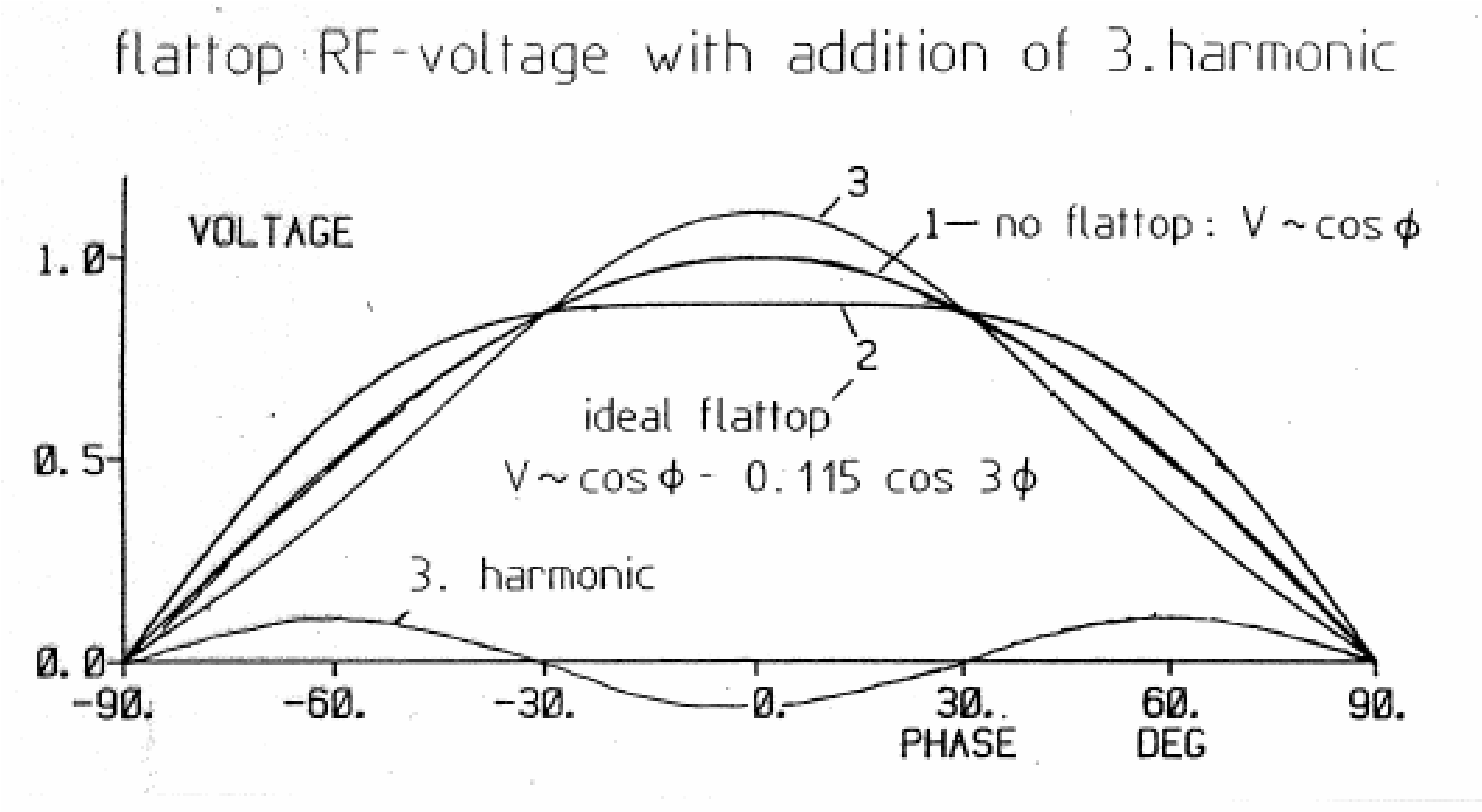
Voltage: 1 MV

(old cavity 0.72 MV)

4 new Cu Cavities in Ringcyclotron (2008)



Flattop Voltage gives minimum energy spread



Extraction from a Cyclotron

The intensity limit of a Cyclotron is given by the beam losses.

Important is the radial distance dR/dn between the last two turns before extraction

=> large turn separation with:

- high RF voltage (intensity limit ~ V^3 !!)
- large machine radius R !

=> compact cyclotrons (supercond. !) have limited intensity

$$(1)\quad E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 R^2 \sim R^2$$

(non relativistic)

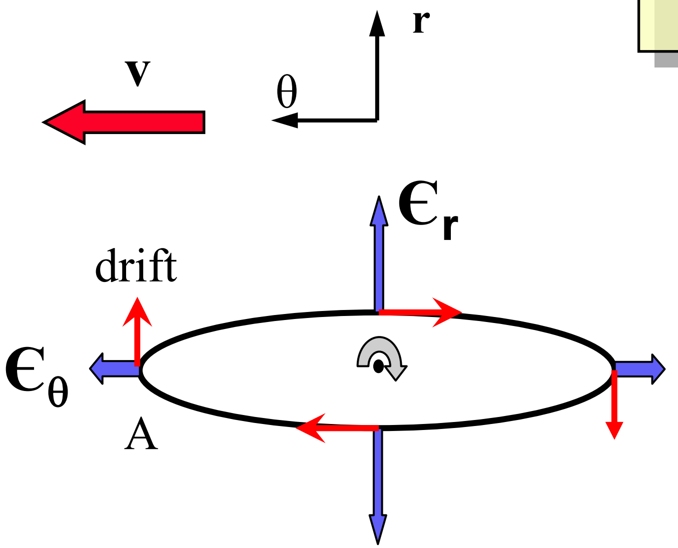
$$(2)\quad E \approx n q \bar{V} \quad \sim n \quad \text{(turn number)},$$

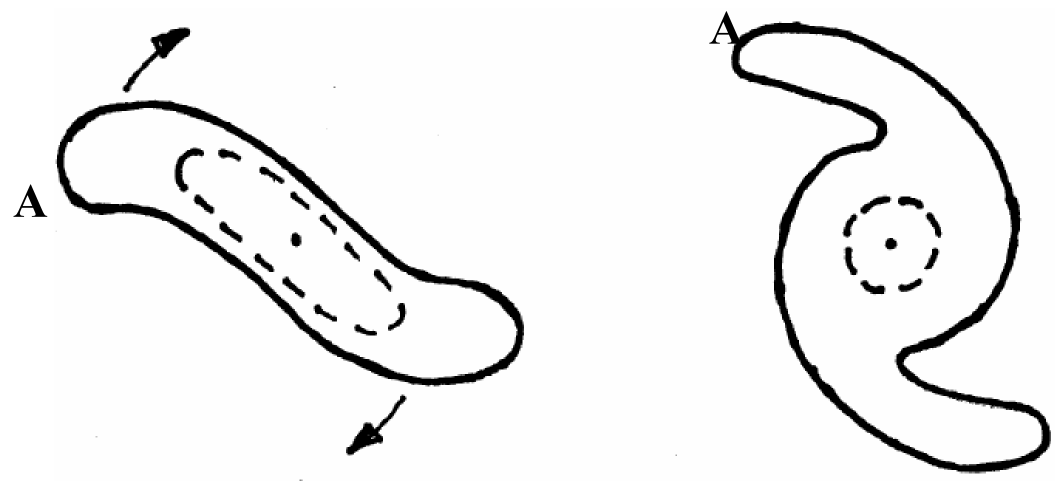
$$\bar{V} = \text{average RF - voltage per turn}$$

$$\Rightarrow \quad R \sim \sqrt{n}, \qquad \frac{dR}{dn} = \frac{R}{2n}$$

$$\frac{dR}{dn} = \frac{\gamma}{\gamma+1} R \frac{\bar{V}}{(E/e)} \frac{1}{Q_r^2} \quad \text{(exact)}$$

Longitudinal Space Charge in a Cyclotron Beam





Particle at position A:

=> gains additional energy from space charge forces

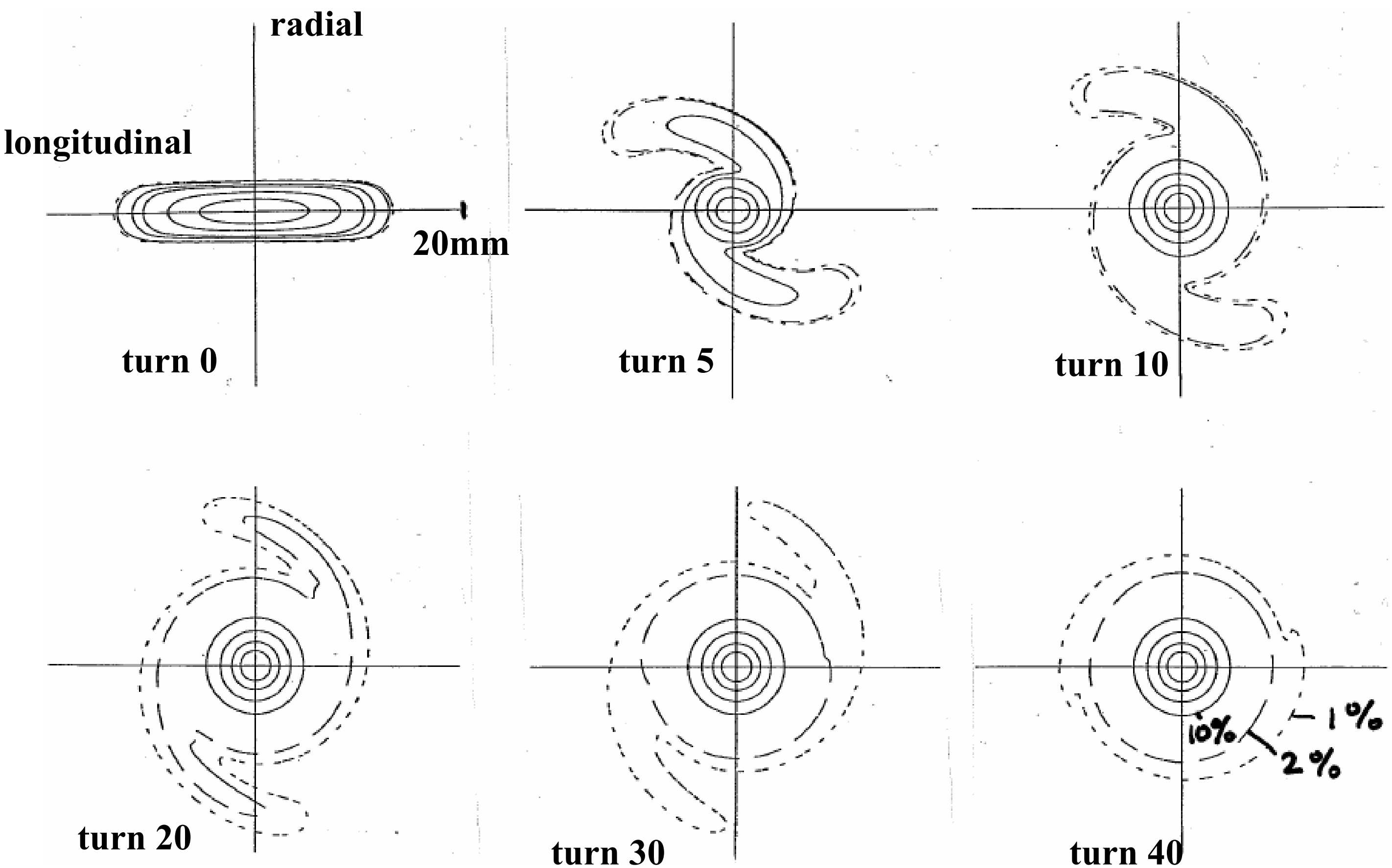
=> moves to higher radius due to isochronous condition

=> rotation of the bunch

=> nonlinearities produce spiral shaped halos

=> production of a rotating sphere

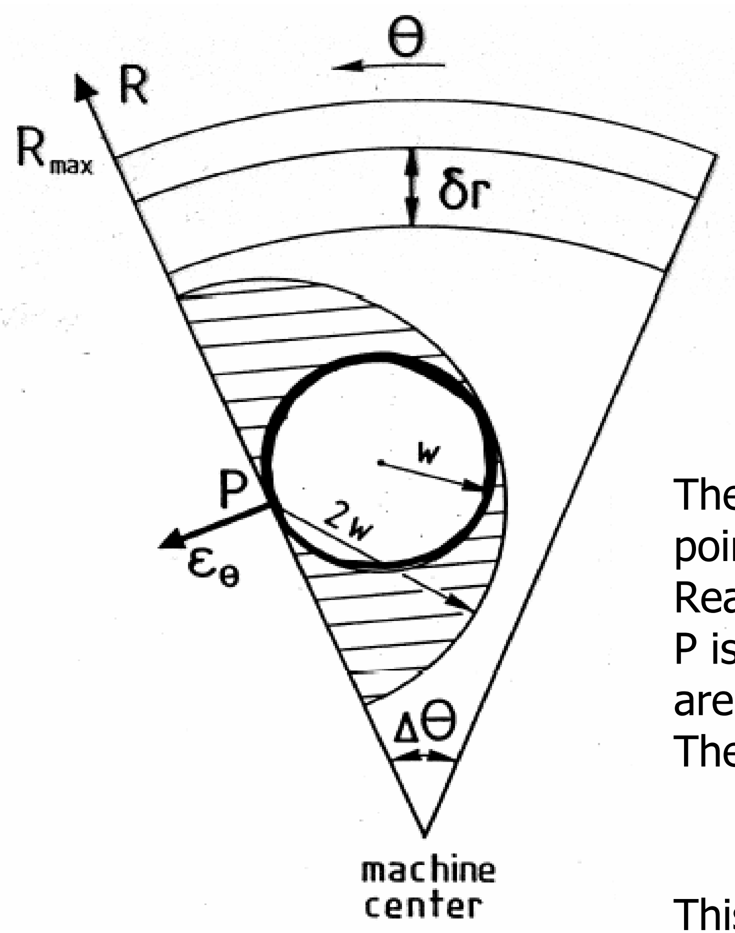
Longitudinal Space Charge in Cyclotron



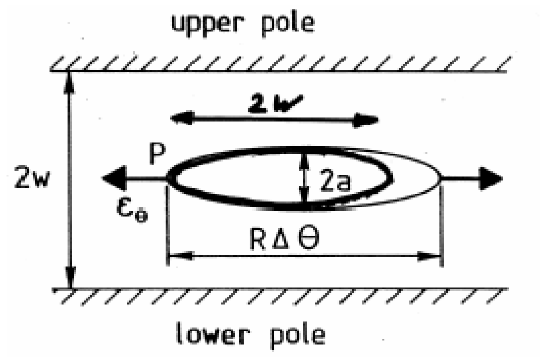
Simulation of a 1mA beam, circulating in Injector II at 3 MeV for 40 turns without acceleration.

The core stabilizes faster than the halos (calculations by Stefan Adam)

Space Charge Fields in a Cyclotron



top view



side view

Disc-Model

(W.Joho, Int. Cyclotron Conf. Caen 1981)

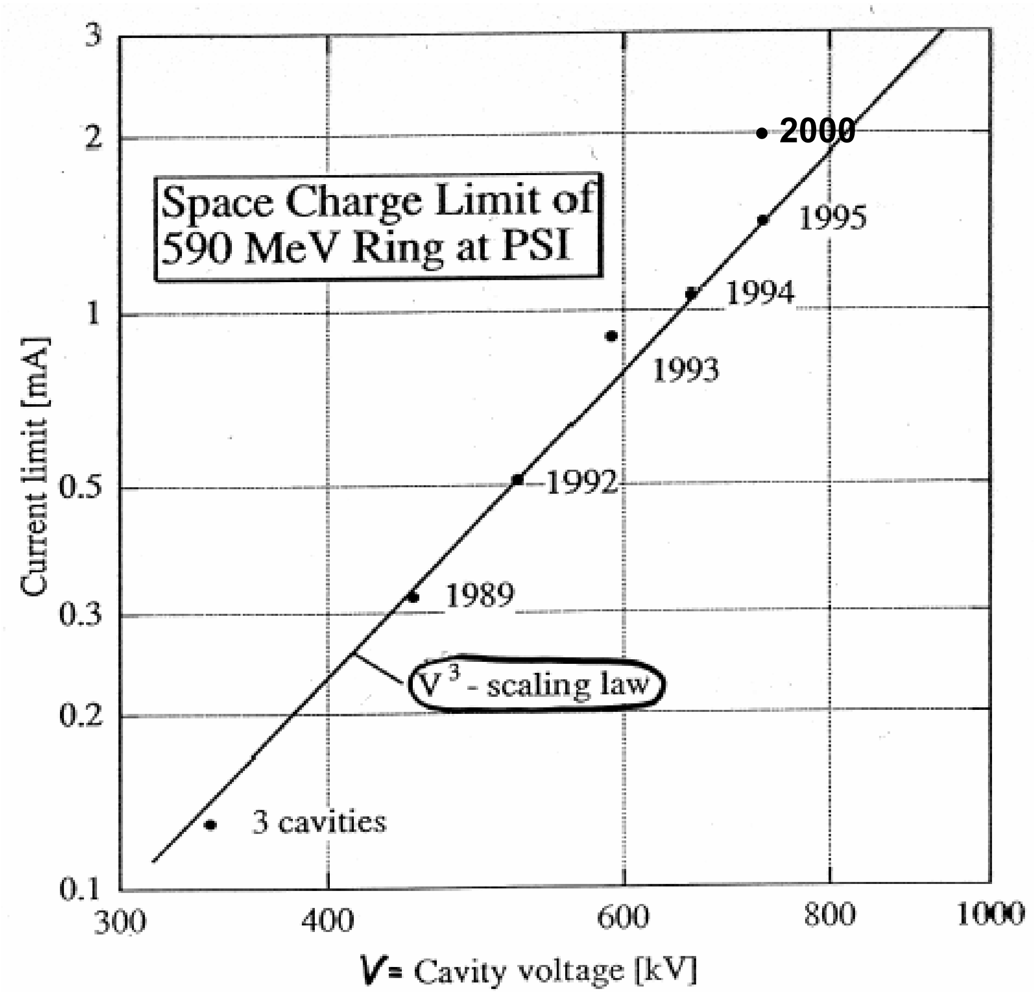
circulating protons fill a cake-like piece with azimuthal extension Δθ. Neighbouring orbits are assumed to overlap radially.

The azimuthal electric field at the edge of the „piece of cake" at point P is approximated by the calculable field of a disc with radius w. Reasoning: the charge of the protons outside of the half circle around P is screened by the upper and lower poles and protons in the hashed area give only a small contribution to the azimuthal field ϵ_θ.
The proton at P gains through ϵ_θ an additional energy/turn:

$$dE/dn = 2\pi R\, \epsilon_\theta$$

This simple model predicts, that the intensity limit from longitudinal space charge forces increases with $\mathbf{V^3}$!!
(V=cavity voltage/turn)

Current Limit in Ring Cyclotron



Longitudinal space charge forces

increase the energy spread

=> higher extraction losses

=> limit on beam current

Remedy:

higher voltage V on the RF cavities

=> lower turn number n (V·n = const.)

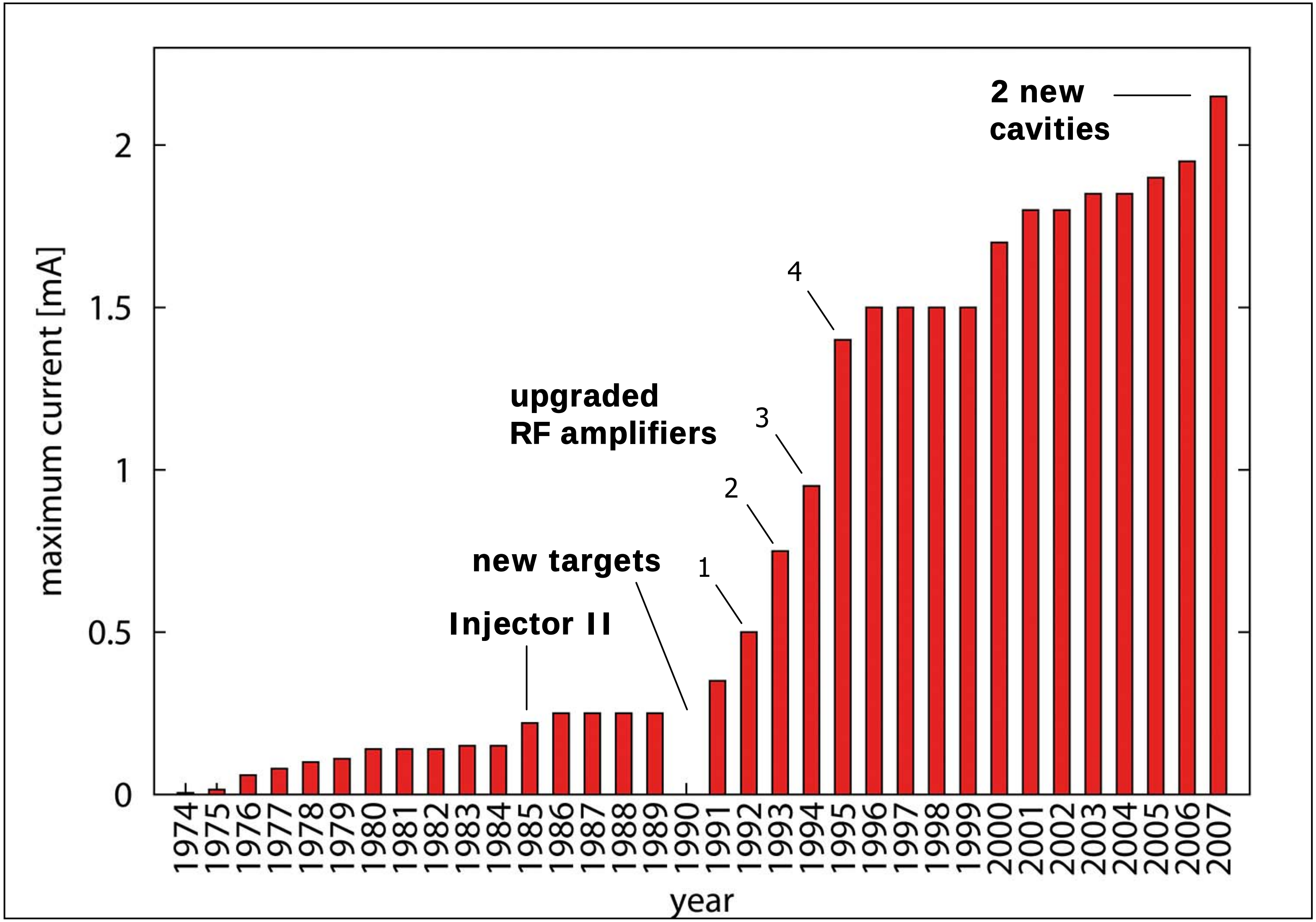
current limit ~ V^3 !

There are 3 effects, each giving a factor V(~1/n):

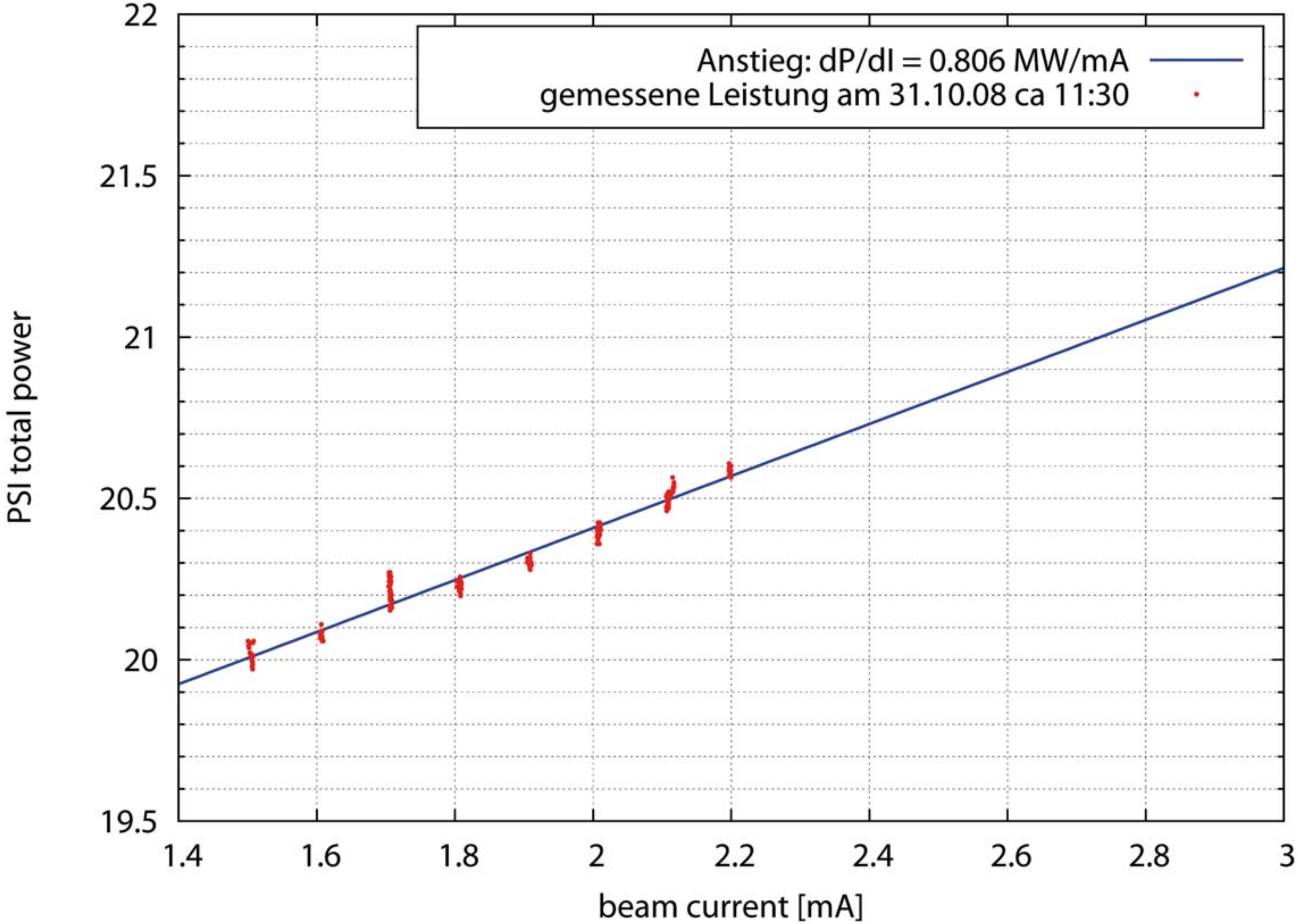
1) beam charge density ~ n
2) total path length in the cyclotron ~ n
3) turn separation ~ V

W.Joho, 9th Int. Cyclotron conference CAEN (1981)

maximum current in ring cyclotron



Power for Current in Ring Cyclotron



beam current needs
0.806 MW/mA

beam power
=0.59 MW/mA

=> RF efficiency=73%

Graphite Target Wheel

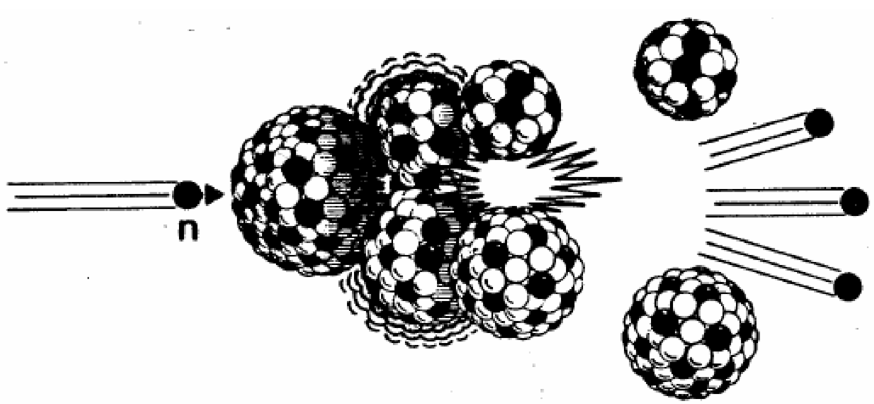
1.3 MW
Proton Beam

creates Pions
and Muons

„slow“ Neutrons for Material Research

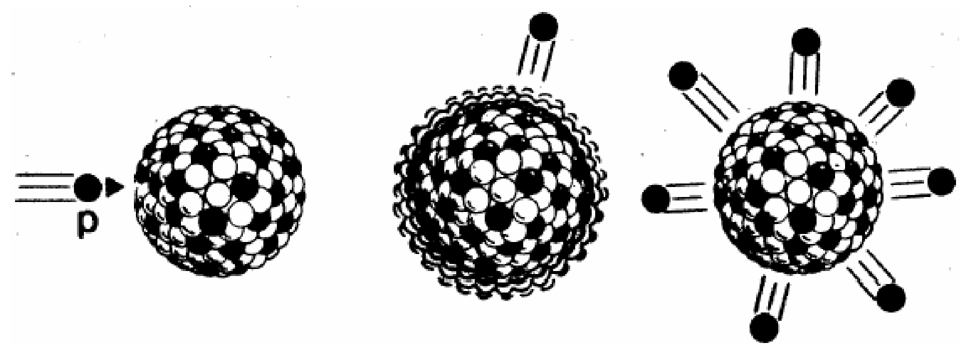
- Production of fast Neutrons
- slowing down in Moderator

1. Fission of Uranium (U^{235}) in a Reactor



2. Spallation of heavy Nuclei (e.g. lead) by Bombardment with Protons from an Accelerator

=> safe and fast turning off !



Spallation Neutrons

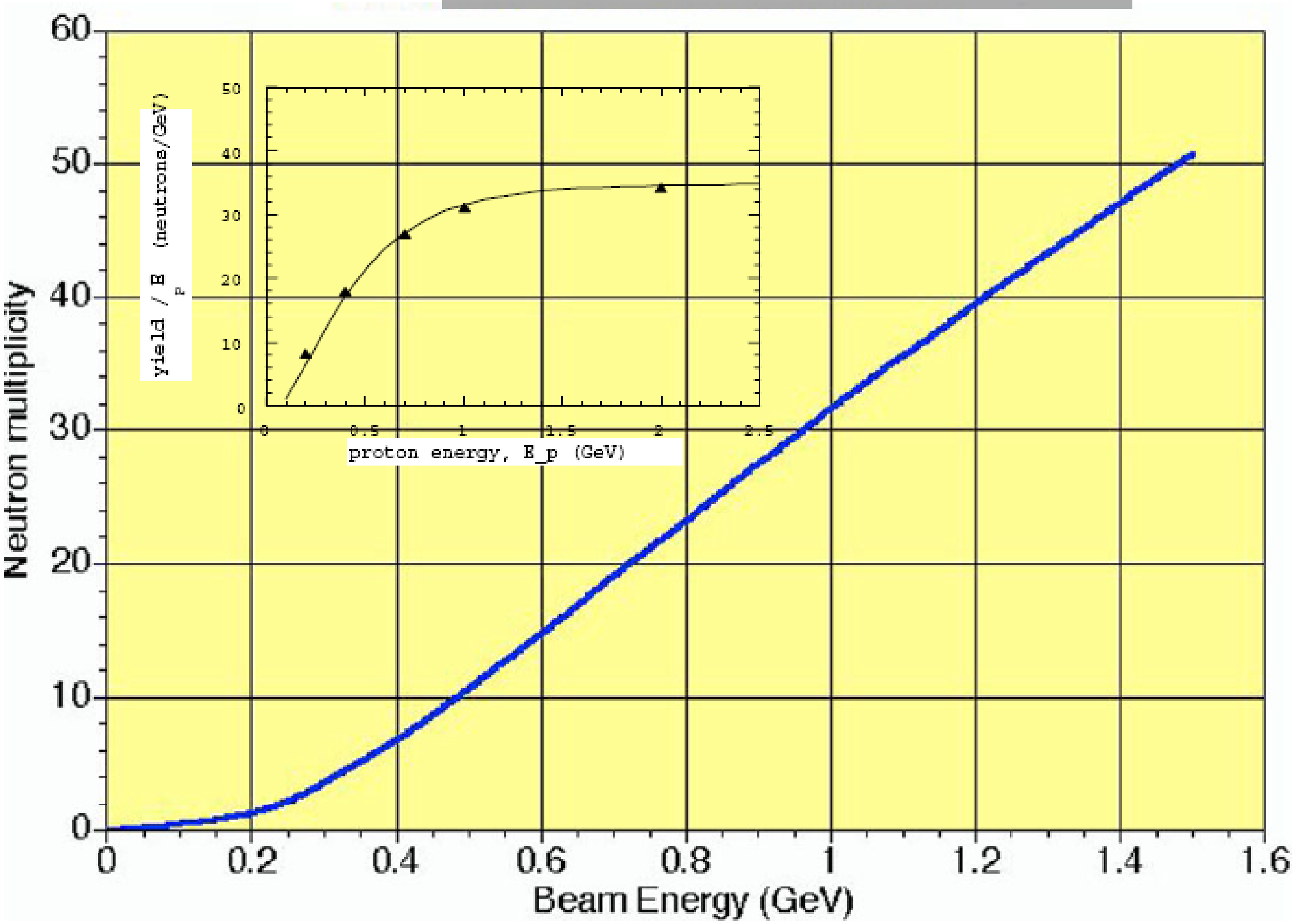
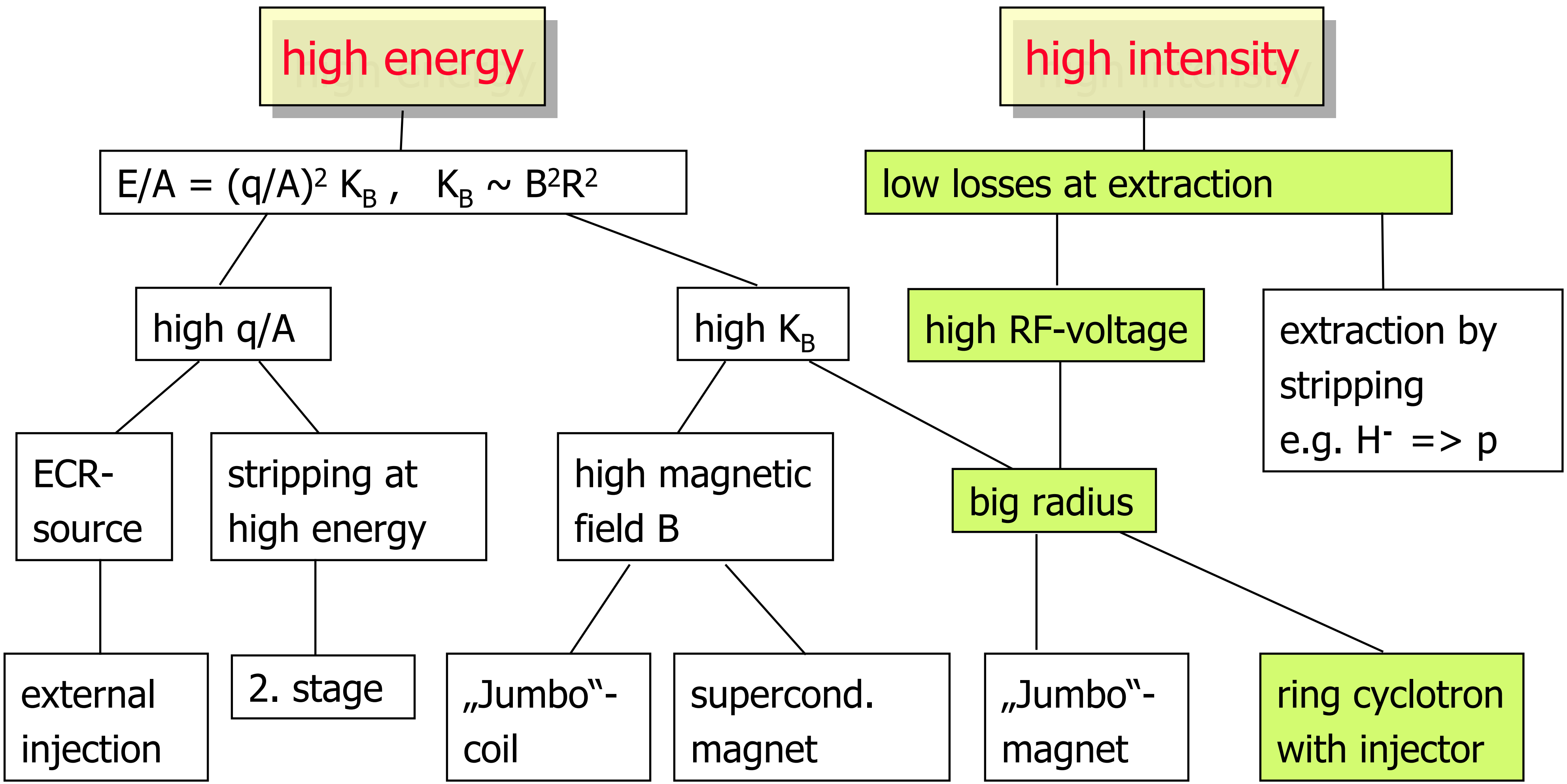


Figure 1. Calculated neutron multiplicity on lead as a function of proton energy[5]; the insert shows a calculated energy-normalised yield.

Strategy for Cyclotrons



why is the PSI Ring Cyclotron such an efficient accelerator ?