Why a Muon Collider?

- First – why a lepton collider?
  - In proton (or proton-antiproton) collisions, composite particles (hadrons), made up of quarks and gluons, collide
    - The fundamental interactions that take place are between individual components in the hadrons
    - These components carry only a fraction of the total energy of the particles
    - For p-p collisions, the effective interaction energies are O(10%) of the total center-of-mass (CoM) energy of the colliding protons
    - Thus a 14 TeV CoM energy at the LHC probes an energy scale E < 2 TeV
  - Electrons (and positrons) as well as muons are fundamental particles (leptons)
    - Leptons are point-like particles
    - Their energy and quantum state are well understood during the collision
    - When the leptons and anti-leptons collide, the reaction products probe the full CoM energy
    - Thus a few TeV lepton collider can provide a precision probe of the full energy range of fundamental processes that are discovered at the LHC

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (I)

- Now – why a muon collider?
- s-Channel Production
  - When 2 particles annihilate with the correct quantum numbers to produce a single final state. Examples:

    $e^+e^- \rightarrow Higgs$ **OR** $\mu^+\mu^- \rightarrow Higgs$

  - The cross section for this process scales as m^2 of the colliding particles, so:

$$\sigma\left(\mu^+\mu^- \rightarrow H\right) = \left(\frac{m_\mu}{m_e}\right)^2 \times \sigma\left(e^+e^- \rightarrow H\right) = \left(\frac{105.7\,MeV}{0.511\,MeV}\right)^2 \times \sigma\left(e^+e^- \rightarrow H\right)$$

$$\sigma\left(\mu^+\mu^- \rightarrow H\right) = 4.28 \times 10^4 \sigma\left(e^+e^- \rightarrow H\right)$$

  - Thus a muon collider offers the potential to probe the Higgs resonance directly
    - The luminosity required is not so large
    - A precision scan capability is particularly interesting in the case of a richer Higgs structure (eg, a Higgs doublet)

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (II)

- Synchrotron Radiation
  - In a circular machine, the energy loss per turn due to synchrotron radiation can be written as:

$$\Delta E_{turn} = \left(\frac{4\pi mc^2}{3}\right)\left(\frac{r_0}{\rho}\right)\beta^3\gamma^4$$

where ρ is the bending radius

$$\rho \propto \frac{\beta\gamma}{B} \Rightarrow \boxed{\Delta E_{turn} \propto B\gamma^3}$$

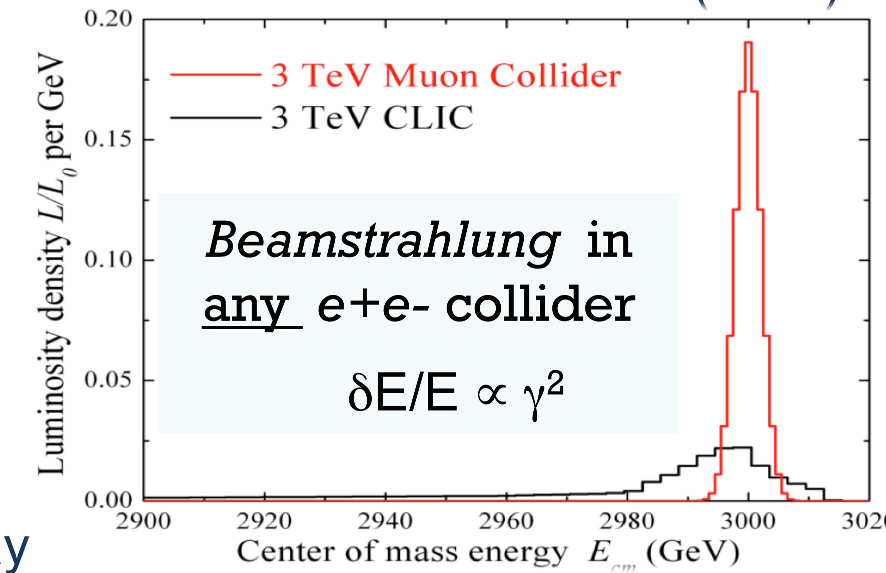
  - If we are interested in reaching the TeV scale, an e^+e^- circular machine is not feasible due to the large energy losses

    Solution 1: e^+e^- linear collider

    Solution 2: Use a heavier lepton – eg, the muon

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (III)

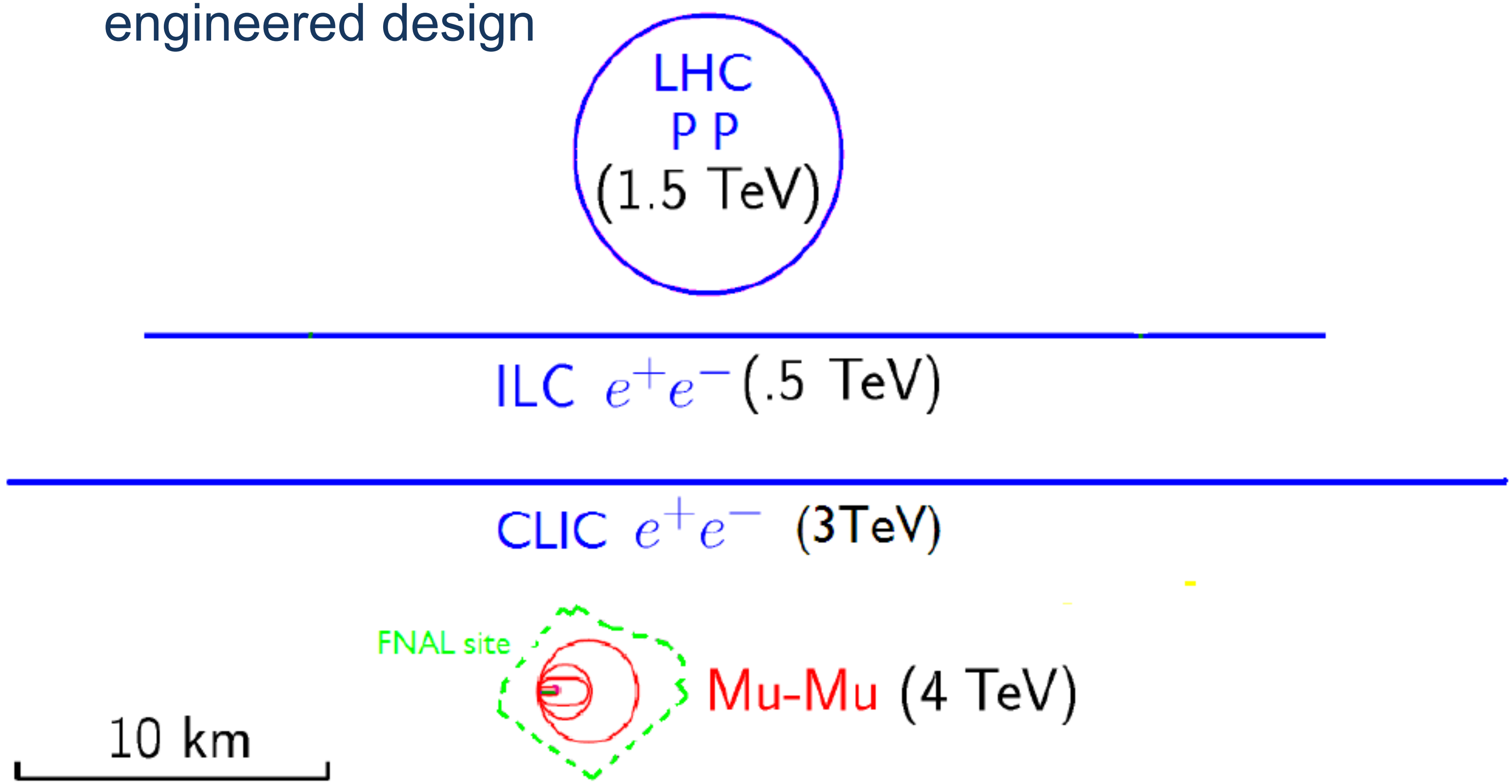
- Beamstrahlung
  - When electrons and positrons collide, the interaction of the particles in one beam with the electromagnetic fields of the other beam results in the radiation of photons (synchrotron radiation) ⇨ *beamstrahlung*
  - This broadens the energy distribution of colliding particles and lowers the fraction of collisions that are near the nominal center-of-mass (CoM) energy
  - The beamstrahlung effect is negligible for a muon collider ⇨ most luminosity is produced near the nominal CoM energy



- Implications for a Higgs Factory
  - With negligible beamstrahlung, it may be possible to directly probe the width of the Higgs
  - Expected width of a standard Higgs is ~4.5 MeV
  - 125 GeV muon collider lattices with $\Delta E/E \sim 3\times10^{-5}$ (3.8 MeV) have been designed

- Circular machines offer a number of advantages
  - Many crossings at an interaction point
    - Luminosity multiplier
    - For a TeV-scale muon collider, expect to have O(1000) crossings for each bunch
  - Multiple detectors can be used
    - Luminosity multiplier
    - Improved systematics understanding of the detectors
  - The additional integrated luminosity from multiple crossings allows larger transverse emittances than are needed for a linear collider. Machine tolerances become much easier
  - Acceleration can utilize multiple passes through the RF system
  - Overall, the beam and wall power for a circular machine can be significantly less than that for a linear collider

- The footprint of a muon collider can be much smaller than other facilities
  - Provides for a more flexible sight choice
  - Has the potential to provide cost savings in a fully engineered design



Muon Collider Luminosity

- For a muon collider, we can write the luminosity as:

$$\boxed{\mathcal{L} = \frac{N^2 f_{coll}}{4\pi\sigma_x\sigma_y} = \frac{N^2 n_{turns} f_{bunch}}{4\pi\sigma_\perp^2}}$$

- For the 1.5 TeV muon collider design, we hav
  - N = 2×10^{12} particles/bunch
  - $\sigma_{x,y}$ ~ 4.18 μm ⇔ β^* = 10 mm, $\varepsilon_{x,y}(norm)$ = 25 μm-rad
  - n_{turns}~1000
  - f_{bunch}=15 Hz

$$\boxed{\mathcal{L} \approx \frac{N^2 n_{turns} f_{bunch}}{4\pi\sigma_\perp^2} \approx 2.7\times10^{34}\,cm^{-2}s^{-1}}$$

- But this is optimistic since we've assumed N is constant for ~1000 turns when it's actually decreasing. The anticipated luminosity for this case is $\sim 1\times10^{34}\ cm^{-2}s^{-1}$.

Challenges for a $\mu^+\mu^-$ Collider

- Make muons from the decay of pions
- With pions made from protons on a target
- To avoid excessive proton power, we must capture a large fraction of pions made

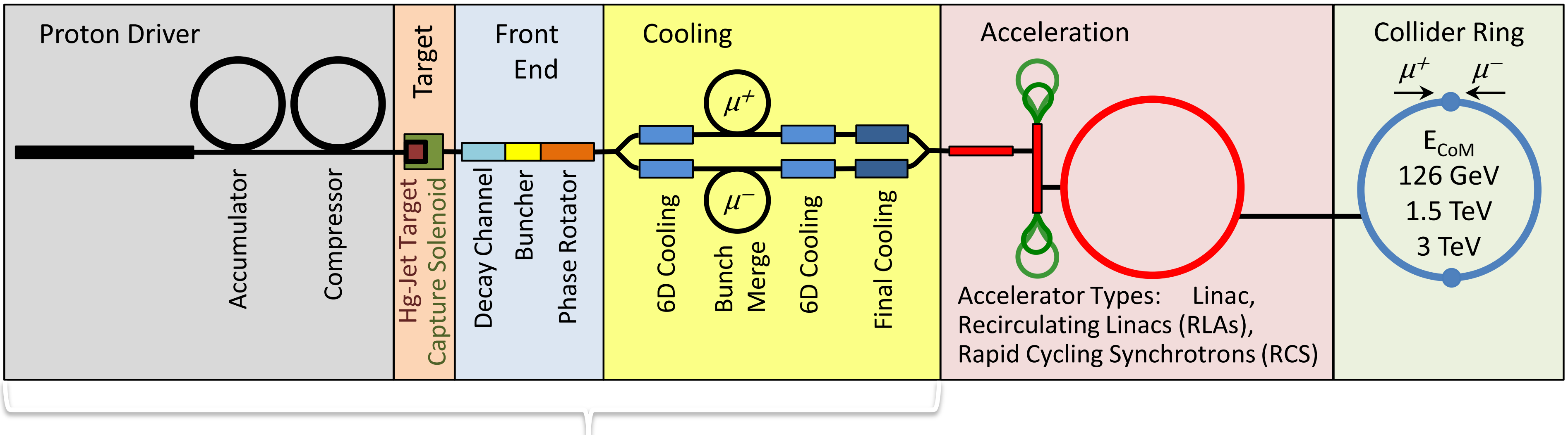
- Capture both forward and backward decays and loses polarization
- The phase space of the pions is now very large:
  - a transverse emittance of 20 pi mm and
  - a longitudinal emittance of 2 pi m
- Emittances must be somehow be cooled by a factor $\approx\ 10^7$!
  - $\approx$ 1000 in each transverse direction and
  - $\approx$ 40 in longitudinal direction

Cooling Options

- Electrons are typically cooled (damped) by synchrotron radiation but muons radiate too little ($\Delta E \propto 1/m^3$)
- Protons are typically cooled by:
  - a co-moving cold electron beam too slow
  - Or by stochastic methods too slow

- Ionization cooling is probably the only hope
- Although optical stochastic cooling has been studied does not look good

Muon Collider Block Diagram



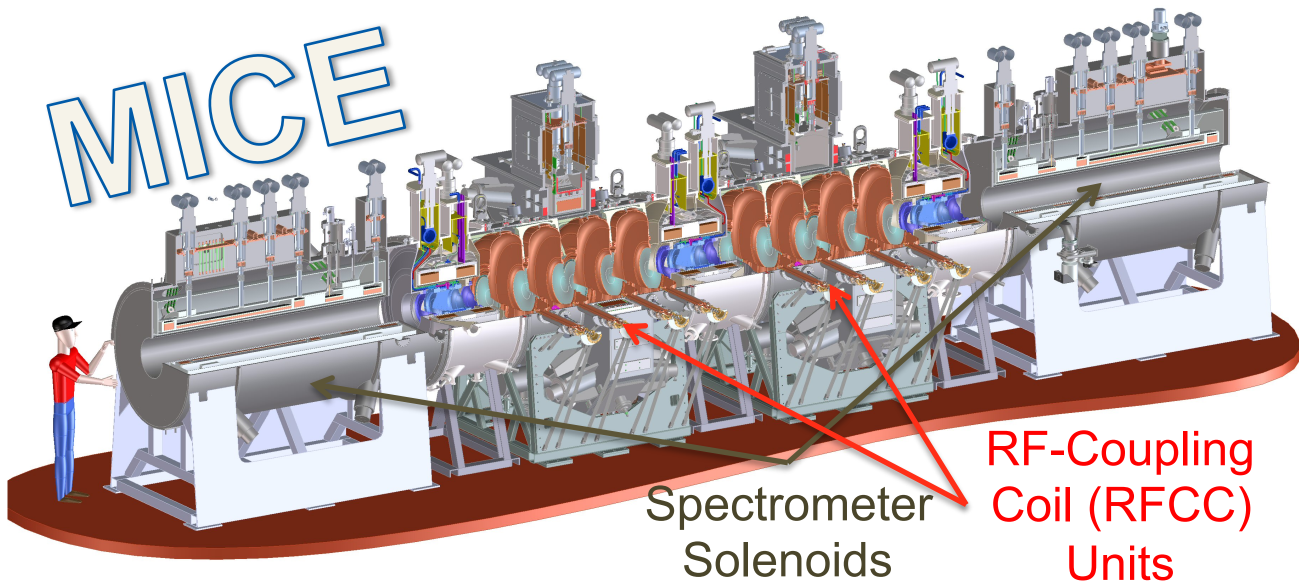
Proton source:
For example Fermilab's PROJECT X at 4 MW, with 2±1 ns long bunches

Goal:
O(10^{21}) muons/year within the acceptance of an accelerator

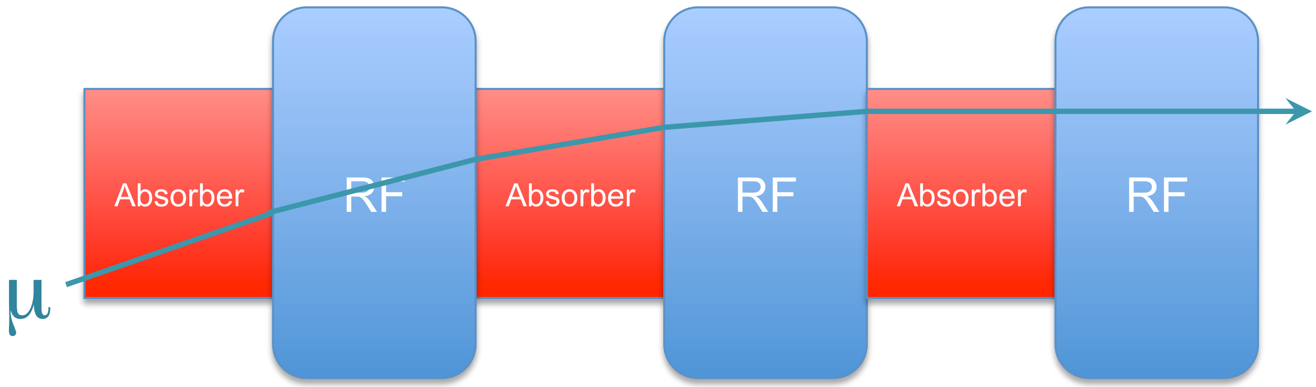
Collider: $\sqrt{s}$ = 3 TeV
Circumference = 4.5km
$L = 3\times10^{34}$ cm^{-2}s^{-1}
μ/bunch = 2×10^{12}
$\sigma(p)/p$ = 0.1%
$\varepsilon_{\perp N}$ = 25 μm, $\varepsilon_{//N}$=72 mm
β^* = 5mm
Rep. Rate = 12 Hz

- Tertiary production of muon beams ➪
  - Initial beam emittance intrinsically large
  - Cooling mechanism required, but no radiation damping
- Muon Cooling ➪ Ionization Cooling
  - dE/dx energy loss in materials
  - RF to replace p_{long}

The Muon Ionization Cooling Experiment: Demonstrate the method and validate our simulations



Technical Challenges - Cooling

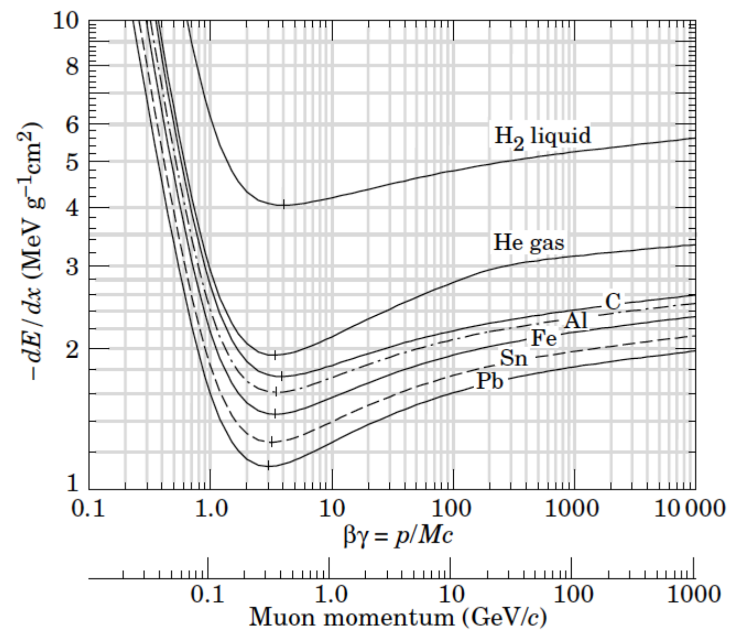


Ionization Energy Loss: $$\Delta E = \left\langle \frac{dE}{dx} \right\rangle \Delta s$$

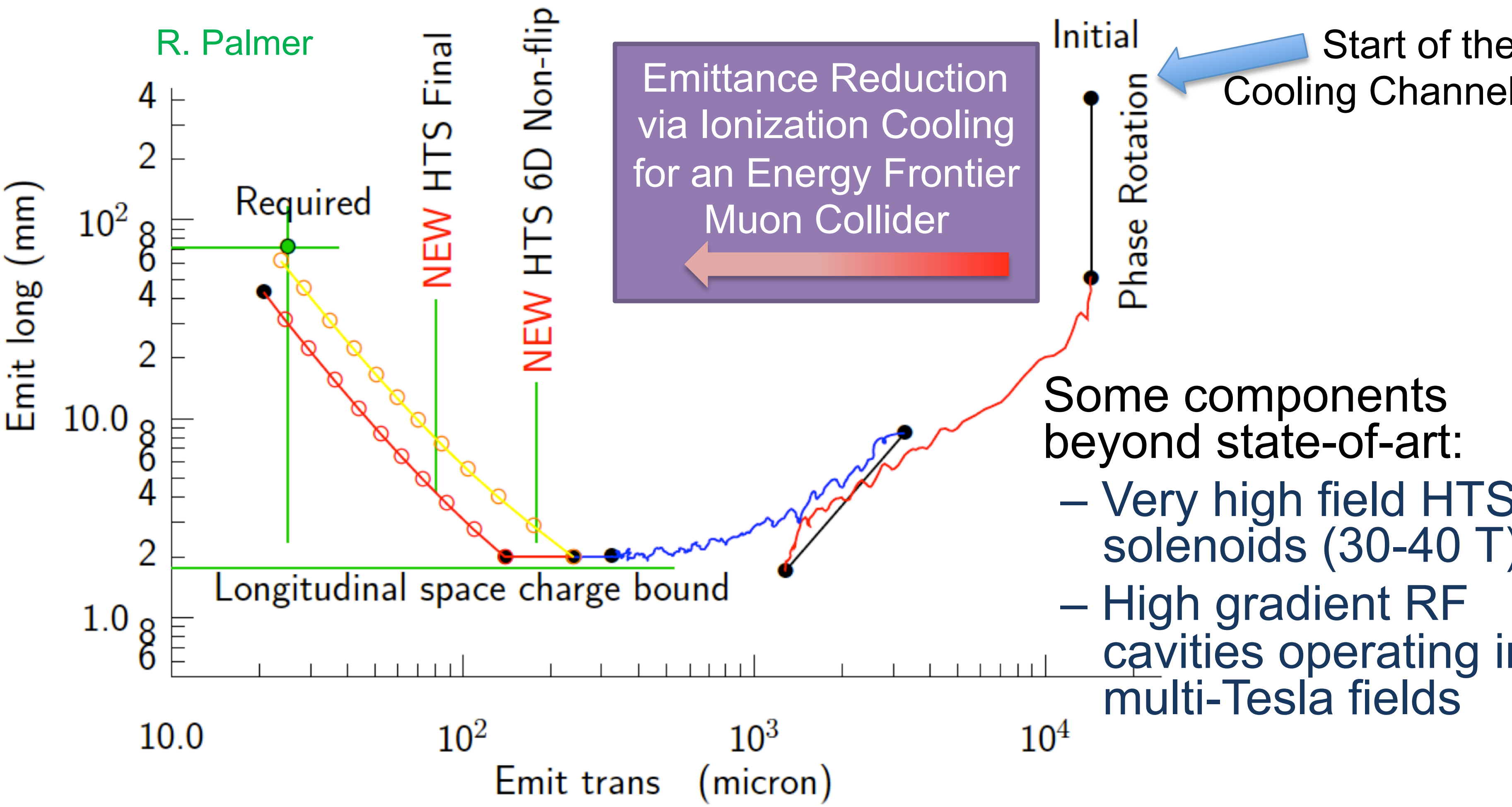
Competes with multiple scattering.

Equilibrium emittance:

$$\frac{d\varepsilon_N}{ds} \approx -\frac{1}{\beta^2}\left\langle \frac{dE_\mu}{ds} \right\rangle \frac{\varepsilon_N}{E_\mu} + \frac{\beta_\perp (0.014GeV)^2}{2\beta^3 E_\mu m_\mu X_0}$$



- Development of a cooling channel design to reduce the 6D phase space by a factor of $O(10^6)$ $\longrightarrow$ luminosity of $O(10^{34})$ cm^{-2} s^{-1}

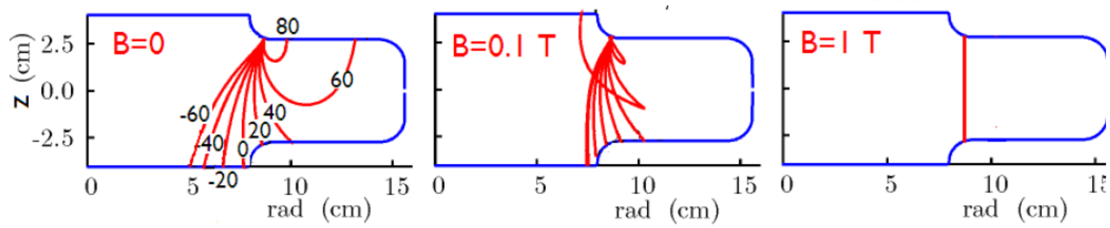


Some components beyond state-of-art:

- Very high field HTS solenoids (30-40 T)
- High gradient RF cavities operating in multi-Tesla fields

Technical Challenges – RF

- A Viable Cooling Channel requires
  - Strong focusing and a large accelerating gradient to compensate for the energy loss in absorbers
    - ⇨ Large B- and E-fields superimposed



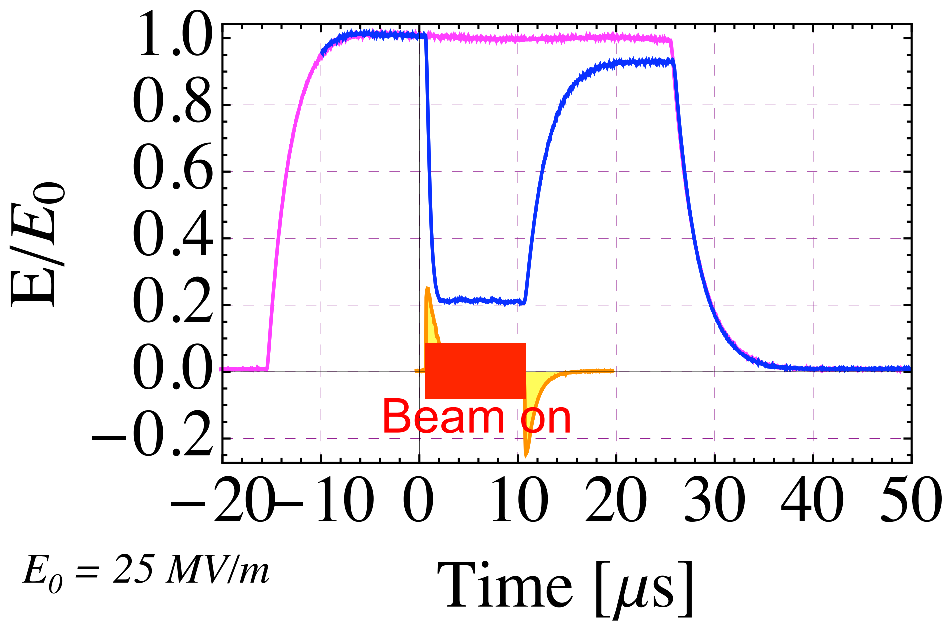
1. "Dark Current" electrons accelerated and focused by magnetic field
2. Damage spots by thermal fatigue causing breakdown

- Operation of RF cavities in high magnetic fields is a necessary element for muon cooling

- Control RF breakdown in the presence of high magnetic fields
- The MuCool Test Area (MTA) at Fermilab is actively investigating operation of RF cavities in the relevant regimes
- Development of concepts to mitigate this problem are being actively pursued

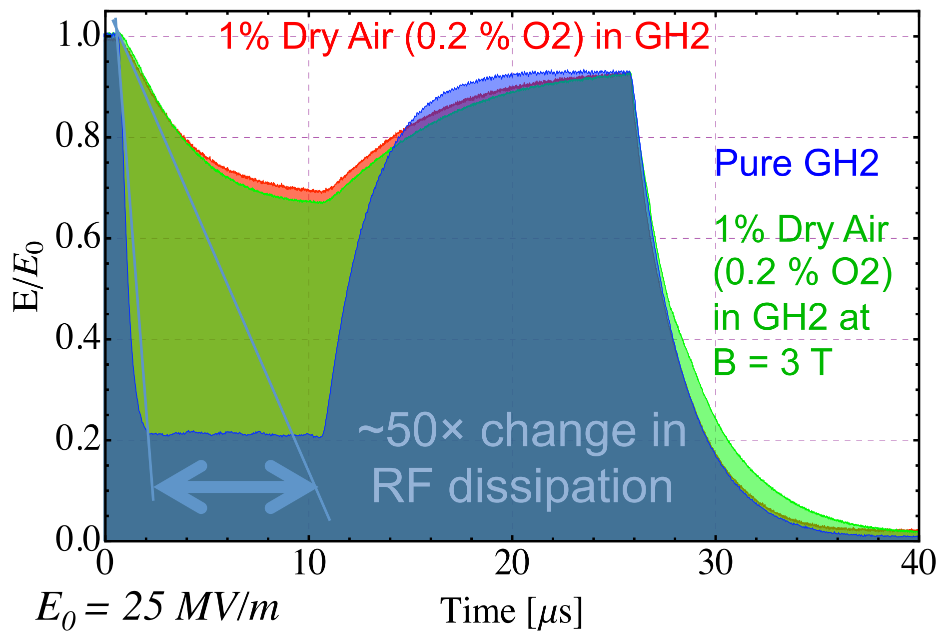
Recent Progress IV: High Pressure RF

- Gas-filled cavity
  - Can moderate dark current and breakdown currents in magnetic fields
  - Can contribute to cooling
  - Is loaded, however, by beam-induced plasma



$E_0 = 25\ MV/m$

- Electronegative Species
  - Dope primary gas
  - Can moderate the loading effects of beam-induced plasma by scavenging the relatively mobile electrons

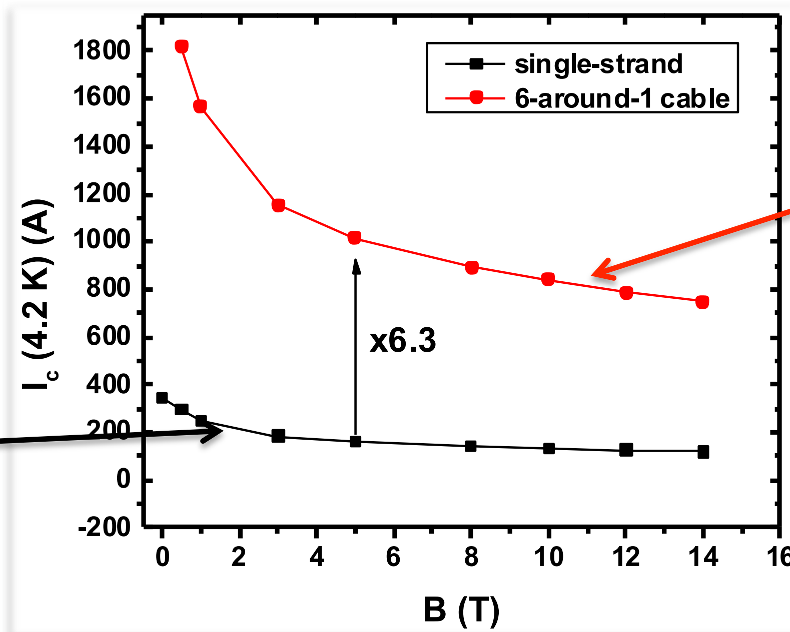


$E_0 = 25\ MV/m$

Recent Progress V: High Field Magnets

Progress towards a demonstration of a final stage cooling solenoid:

- Demonstrated 15+ T (16+ T on coil)
  - ~25 mm insert HTS solenoid
  - BNL/PBL YBCO Design
  - Highest field ever in HTS-only solenoid (by a factor of ~1.5)
- Will soon begin preparations for a test with HTS insert + mid-sert in NC solenoid at NHFML ⇨ >30 T



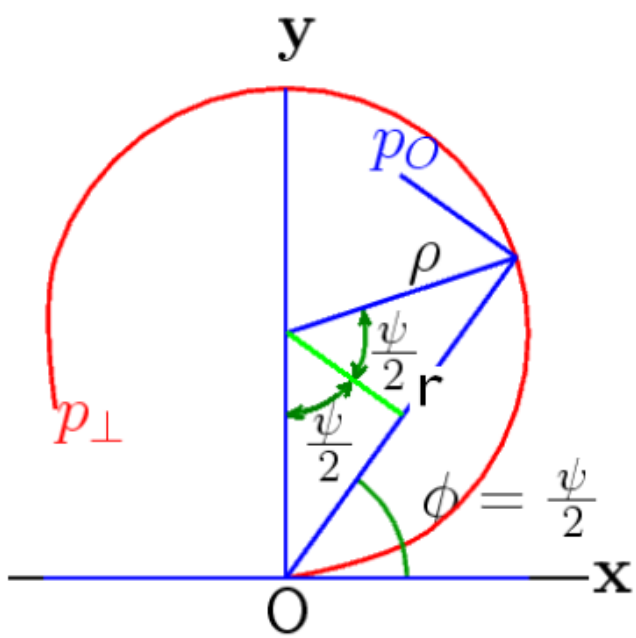
PART II

A CLOSER LOOK AT MUON COOLING

with acknowledgments to Bob Palmer

SOLENOID FOCUSING

- Field: Assume $B_z = constant$
- Consider a particle starting on at the origin O, with no longitudinal momentum, but finite transverse momentum. Since the particle starts at the origin, it has no initial angular momentum in this frame. The particle's motion can be described as:



$$\rho = \frac{p_\perp c/e}{cB_z}$$

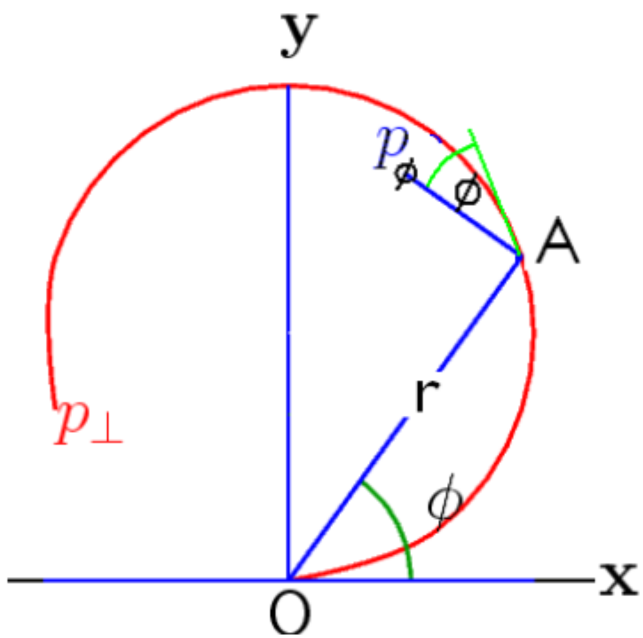
$$x = \rho \sin(\psi)$$

$$y = \rho\left[1 - \cos(\psi)\right]$$

$$r = 2\rho \sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi)$$

- Note that r is sinuosoidal, as is x, but oscillates with half the frequency of x.

- If we now solve for the angular momentum of the particle, we obtain:



$$p_\phi = -p_\perp \sin(\phi)$$

$$r = 2\rho \sin(\phi)$$

$$\Rightarrow p_\phi = -p_\perp \frac{r}{2\rho}$$

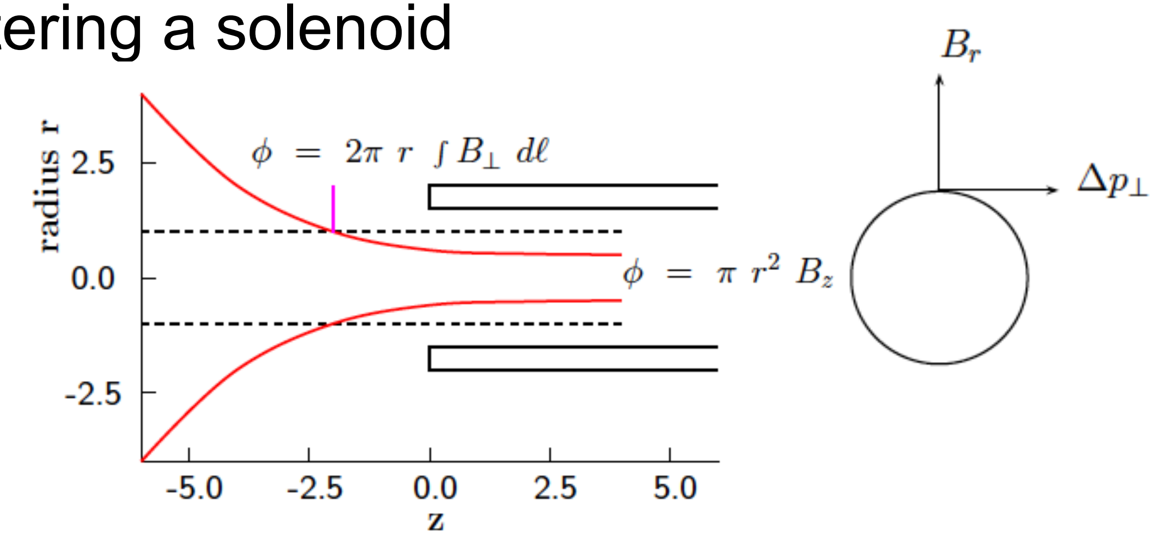
- Using the expression for ρ from the previous slide, we then obtain:

$$p_\varphi \, c/e = -\frac{rc}{2} B_z$$

The Larmor Frame

- Consider a particle entering a solenoid

$$\Delta p_\phi \, c/e = \int B_r dz$$
$$= -\frac{rc}{2}\Delta B_z$$



- If the particle has no initial angular momentum, this implies:

$$p_\phi \, c/e = -\frac{rc}{2}\Delta B_z$$

which is exactly the condition required for a helix that passes through the axis of the solenoid.

- Define a coordinate system u, v which rotates about the axis by the angle ϕ. For a particle in that frame which initially has no angular momentum, it will remain in the u=0 plane – this is the Larmor Frame

- For a particle moving in an axially symmetric solenoidal field, $B_z(z)$
  - Define a transverse frame with axes u and v rotating rotating about the axis which moves longitudinally with the particle
$$\frac{d\phi}{dz} = -\frac{cB_z(z)}{2\left(\frac{p_z c}{e}\right)}$$
  - In this frame the focusing force is given by:
$$\frac{1}{\eta} = \frac{d^2 r}{dz^2} = -\left[\frac{cB_z(z)}{2(p_z c/e)}\right]^2 r \qquad \Rightarrow \text{Solenoid Focusing} \propto \frac{B^2}{p^2}$$
where r is the distance from the axis
  - The equivalent expression for a quadrupole is
$$\frac{1}{\eta} = \frac{d^2 r}{dz^2} = -\left[\frac{Gc}{(p_z c/e)}\right] r$$

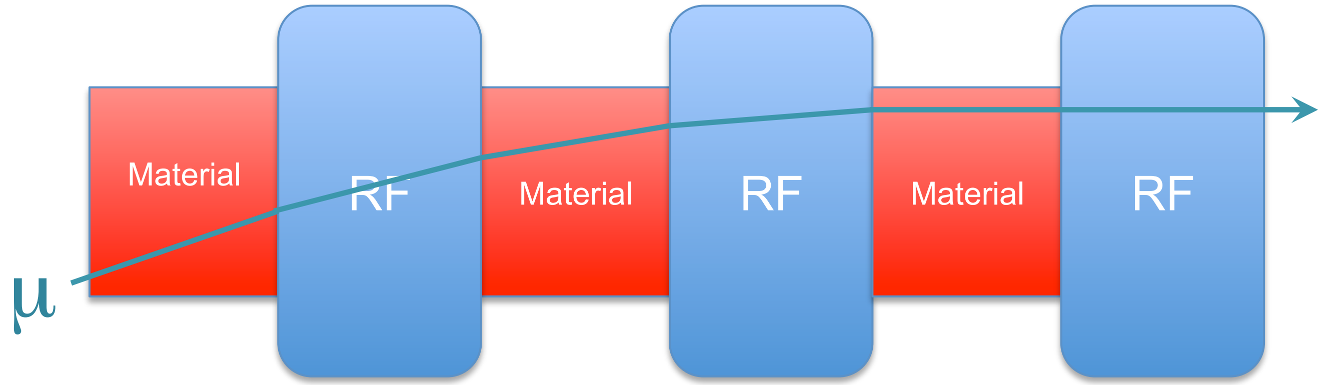
⇨ NOTE: solenoid focusing (unlike a quad) is independent of sign and is stronger for lower momenta

Solenoid Focusing Summary

- In a long solenoid: a particle moves along a helix of wavelength λ_{helix}
- In the Larmor Plane: a particle oscillates with wavelength $\lambda_{Larmor} = 2\lambda_{helix}$
- For motion in the Larmor plane
  - It focuses towards the axis
  - It has a focusing force proportional to B^2/ρ^2
  - A particle that starts in the Larmor plane stays in the Larmor plane
- At sufficiently low momentum, solenoid focusing is always stronger than quadrupole focusing
- Solenoids focus in *both* planes, unlike quadrupoles which focus in one plane and defocus in the other
- A solenoid can focus very large transverse emittances, with large angles (a radian or more), and thus are very well-suited for focusing in an ionization cooling channel

IONIZATION COOLING

Transverse Ionization Cooling



$$p_{\perp,f}(material) < p_{\perp,i} \qquad p_{\perp,f}(RF) \approx p_{\perp,i}$$

$$p_{\parallel,f}(material) < p_{\parallel,i} \qquad p_{\parallel,f}(RF) > p_{\parallel,i}$$

- Emittance Cooling $\quad \varepsilon_{x,y} = \beta_v \gamma \sigma_{\theta_x,\theta_y} \sigma_{x,y}$
  - In the absence of Coulomb scattering (and any other emittance growth mechanisms), σ_θ and $\sigma_{x,y}$ are not affected by energy loss. However, p and $\beta\sigma$ are reduced. Thus we have:

$$\frac{d\varepsilon}{\varepsilon} = \frac{dp}{p} = \frac{dE}{\beta_v^2 E}$$

Thus, the cooling rate improves at lower energies

Equilibrium Emittance

- Equating the expressions for the emittance growth rate due to scattering and the emittance damping rate due to cooling gives:

$$\frac{dE}{\beta_v^2 E} = C(mat, E)\frac{\beta_\perp dE}{\varepsilon \gamma \beta_v^3}$$

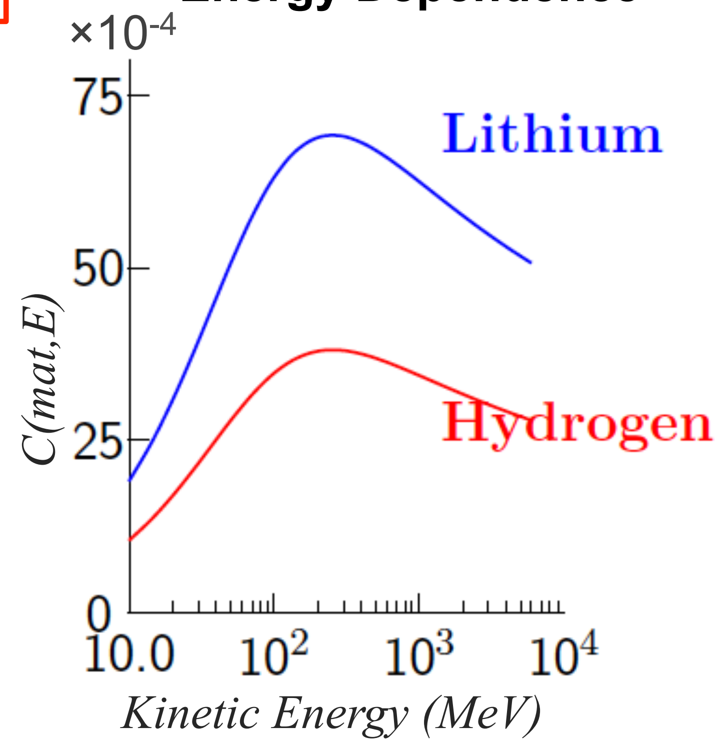
$$\Rightarrow \boxed{\varepsilon_0 = C(mat, E)\frac{\beta_\perp}{\beta_v}}$$

where ε_0 is the *equilibrium emittance*

Values at Ionization Minimum

Material	T (°K)	ρ (kg/m^3)	dE/dx (MeV/m)	L_R (m)	C_0 ($\times10^{-4}$)
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

Energy Dependence



Cooling Channel Optimization (I)

- Material:
  - Liquid H_2 has, by far, the best performance, but comes with challenges
    - Cryogenic liquid
    - Safety issues
    - Requires windows
  - LiH is the second best material
    - Doesn't need windows nor cryogenics
- Cooling Channel Energy:
  - At lower energies, C is smaller, but longitudinal heating occurs
  - The initial cooling for a collider or neutrino factory uses an energy near the minimum ionizing point
  - In the final cooling section, an energy of ~10 MeV is employed (see next section)

Cooling Channel Optimization: Rate of Cooling

$$\frac{d\varepsilon}{\varepsilon} = \left(1 - \frac{\varepsilon_{\min}}{\varepsilon}\right)\frac{dp}{p}$$

- Choice of β:
  - Naively, cooling rate appears best with $\varepsilon_{\min} \ll \varepsilon$, but this can cause problems due to non-linearities when large values of σ_θ result
- Beam Divergence Angles and Required Aperture
  - Recall

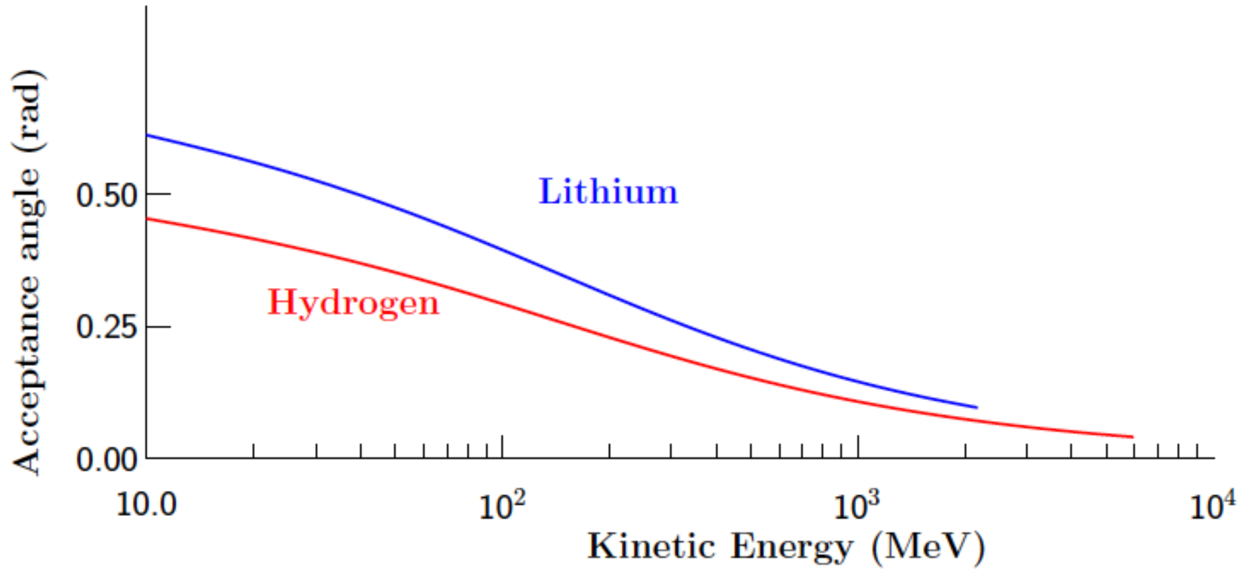
$$\sigma_\theta = \sqrt{\frac{\varepsilon_\perp}{\beta_\perp}\left(\frac{1}{\beta_v \gamma}\right)} = \sqrt{\frac{C(mat,E)}{\beta_v^2 \gamma}}$$

  - If we assume that we obtain only half of the maximum cooling rate and also require that the angular aperture provide at least a 3σ spread, we obtain:

$$A_\theta = 3\sqrt{\frac{2C(mat,E)}{\beta_v^2 \gamma}}$$

Cooling Channel Optimization: Aperture

- Plot of A_θ for Li and H_2 as a function of energy



- NOTE: For low energies, the required angular acceptance is VERY large!
- In realistic lattice configurations it is doubtful whether $A_\theta > 0.3$ is feasible

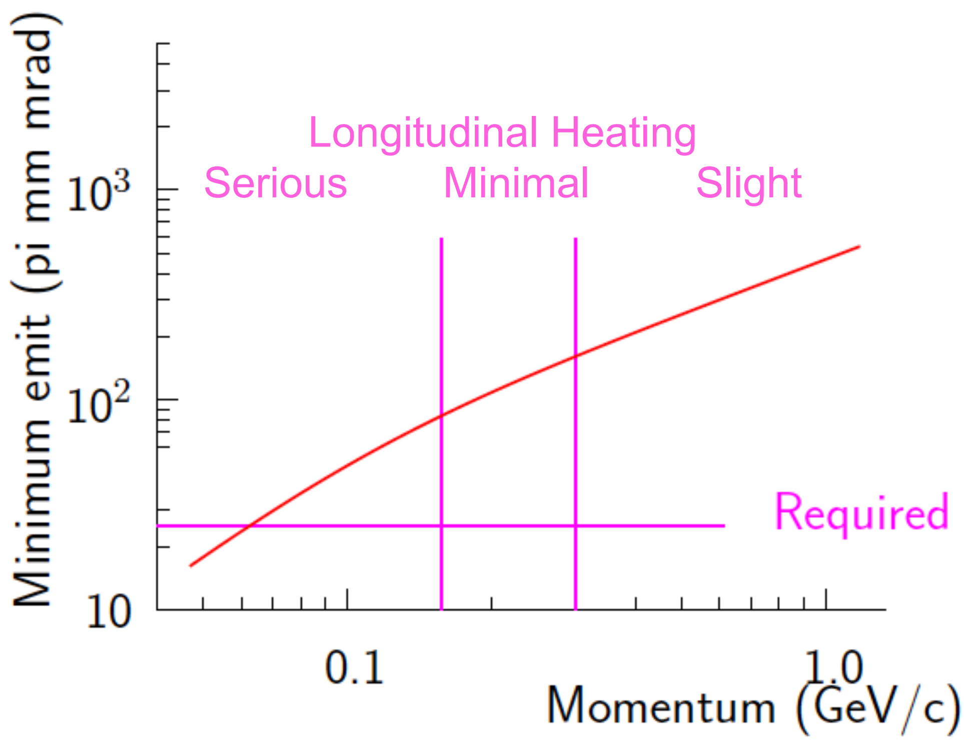
Cooling Channel Optimization: Focusing and Final Cooling

- Recall $$\beta_\perp = \frac{2 p_z c / e}{c B_z}$$

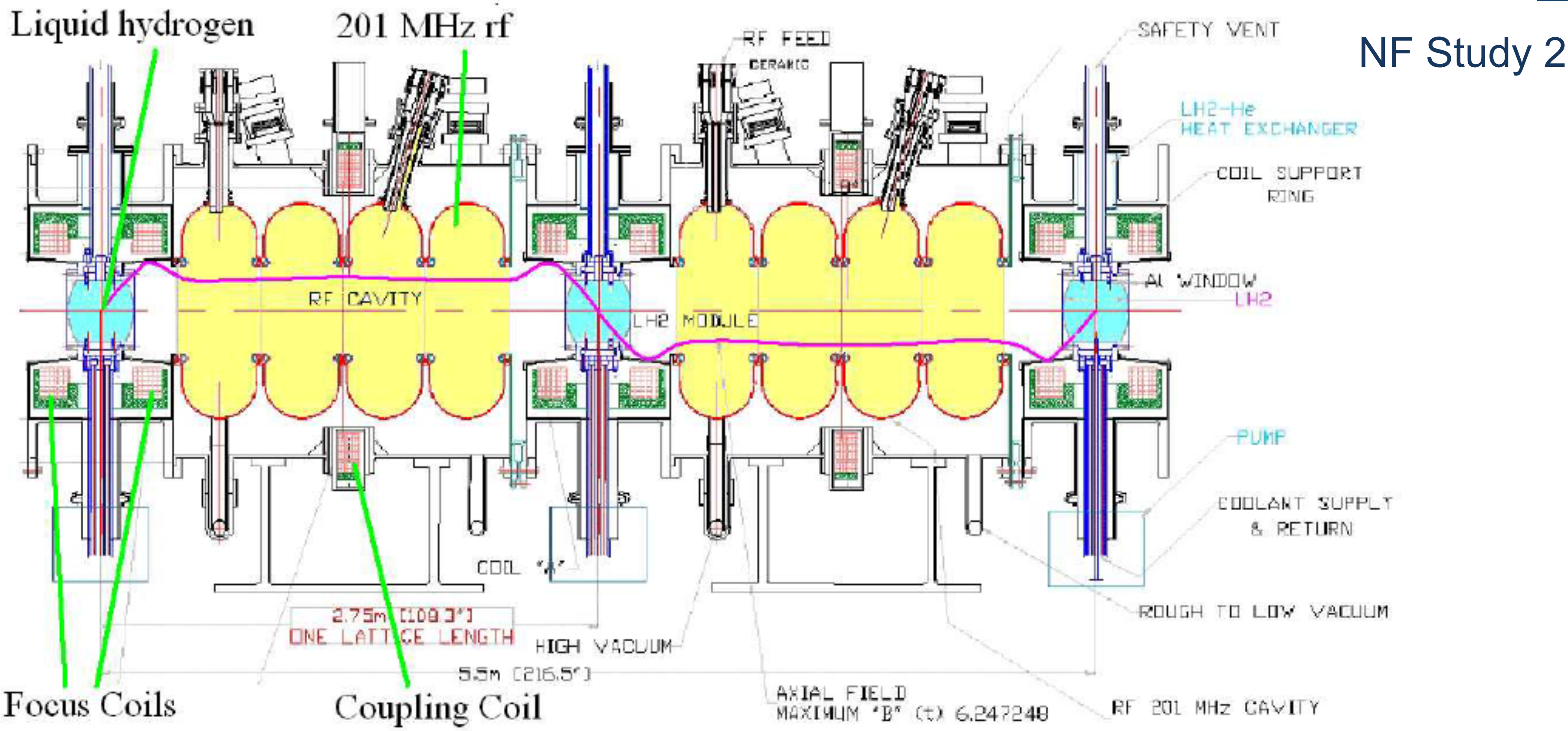
- Thus we can write $$\varepsilon_{x,y}(\min) = \frac{2C(mat, E)\gamma m_\mu c^2 / e}{c B_z}$$

- Note: Transverse emittance target for collider is ~25μm

- The plot at the right shows that the target transverse emittance cannot be obtained without going to low energies and allowing some heating of the longitudinal emittance
  ⇨ a careful balance for the final cooling stage. Higher B-fields (~30-40 T) help here



SFOFO for Neutrino Factory and MICE Demo



- This is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- Study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls, keeping σ_θ and ϵ/ϵ_o more constant, thus maintains cooling rate

Summary of Transverse Cooling

- The optimum “absorber” material for a cooling channel is hydrogen (gas or liquid)
  - This offers operational and safety challenges
- Cooling requires very large angular acceptances
  - Solenoid focusing is well-suited for this requirement
  - Betas can be lowered by adding periodicity [but at the expense of reduced momentum acceptance]
- Final cooling with a target transverse emittance of 25μm is possible if have high magnetic fields and operate at low energies ⇨ must accept an increase in longitudinal emittance in ths situation

LONGITUDINAL COOLING

Partition Functions

- When dealing with synchrotron radiation, typically work with the “radiation integrals” and “partition functions”. You will learn about this in detail during the DR lectures.
- For now, we would like to introduce the partition functions to look at the features of 6D cooling

$$J_{x,y,z} = \frac{\left(\Delta\varepsilon_{x,y,z} / \varepsilon_{x,y,z}\right)}{\left(\Delta p / p\right)}$$

$$J_6 = J_x + J_y + J_z$$

where $\Delta\varepsilon$ and Δp are changes due to the energy loss mechanism

- For discrete function electron synchrotrons, you will learn in the DR lectures that $J_x \approx J_y = 1$ *and* $J_z = 2$
- For muon ioniztion cooling, $J_x = J_y = 1$ but J_z is small or negative

Generalized Expression for the Transverse Emittance

- We saw previously that

$$\frac{\Delta\varepsilon_{x,y}}{\varepsilon_{x,y}} = \frac{\Delta p}{p}$$

  which corresponds to $J_x = J_y = 1$

- More generally, with $J_{x,y} \neq 1$ we can write

$$\frac{\Delta\varepsilon_{x,y}}{\varepsilon_{x,y}} = \frac{1}{J_{x,y}} \frac{\Delta p}{p}$$

- In this case, the expression for the minimum emittance then becomes:

$$\varepsilon_{x,y}(\min) = \frac{\beta_\perp}{J_{x,y}\beta_v} C\left(mat, E\right)$$

Longitudinal Cooling/Heating from the shape of the dE/dx Curve

- The longitudinal emittance can be written as:

$$\varepsilon_z = \gamma\beta_v \frac{\sigma_p}{p}\sigma_z = \frac{1}{m}\sigma_p\sigma_z = \frac{1}{m}\sigma_E\sigma_t = c\sigma_\gamma\sigma_t$$

where σ_t is the rms bunch length in time.

- Note that a particle interaction with matter will not change σ_t, so that the change in emittance will only come from the energy change in the interaction:

$$\frac{\Delta\varepsilon_z}{\varepsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \Delta s \frac{d\left(d\gamma/ds\right)}{d\gamma}}{\sigma_\gamma} = \Delta s \frac{d\left(d\gamma/ds\right)}{d\gamma}$$

- We also can write:

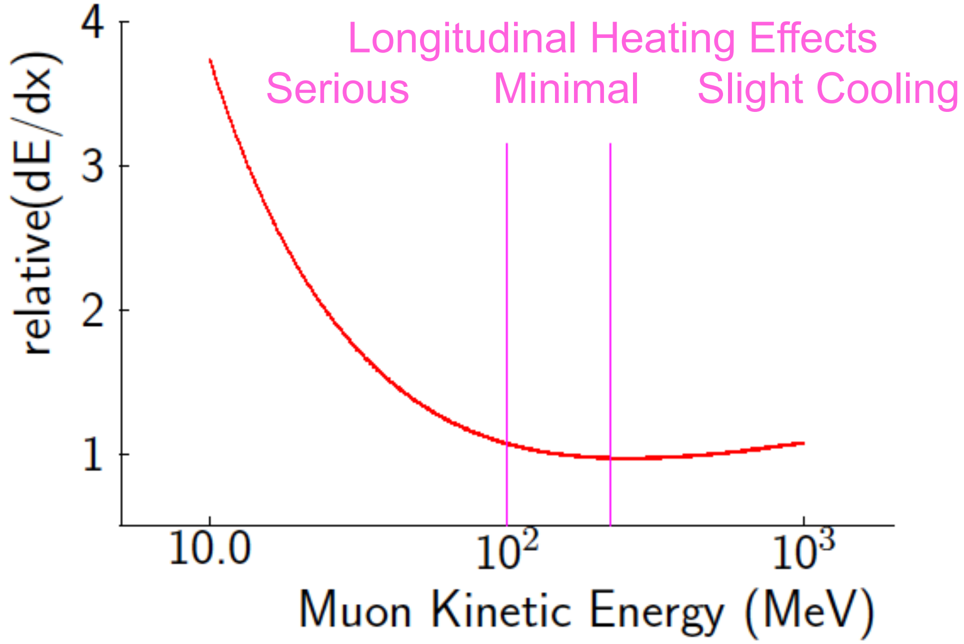
$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

Longitudinal Heating and Cooling

- The partition function, J_z, can then be written as:

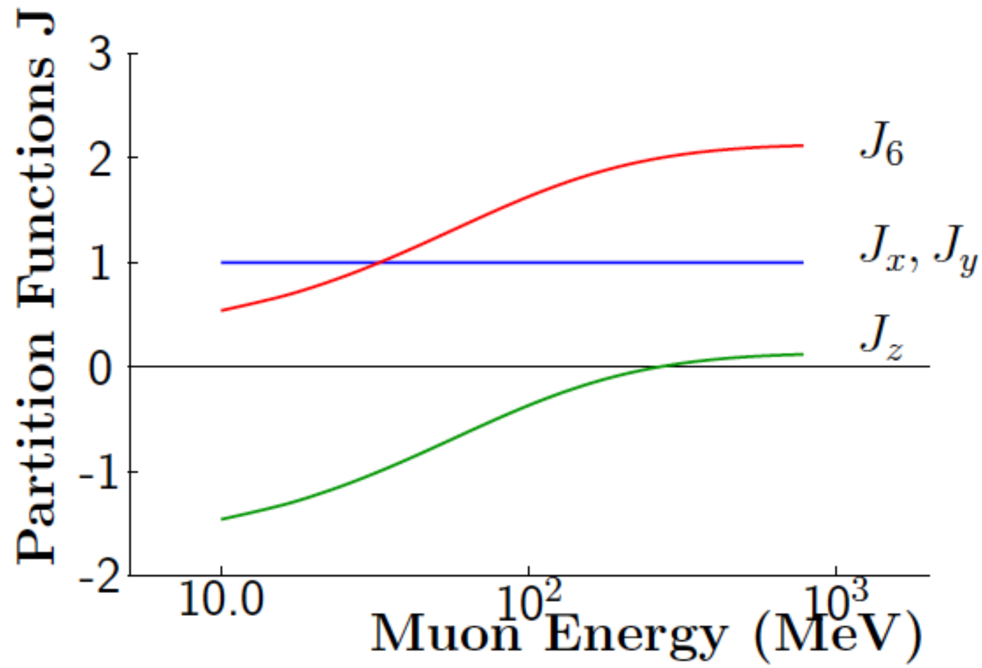
$$J_z = \frac{\Delta\varepsilon_z/\varepsilon_z}{\Delta p/p} = \frac{\left(\Delta s \dfrac{d(d\gamma/ds)}{d\gamma}\right)}{\dfrac{\Delta s}{\beta_v^2\gamma}\left(\dfrac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2\gamma \dfrac{d(d\gamma/ds)}{d\gamma}\right)}{\left(\dfrac{d\gamma}{ds}\right)}$$

- The relative energy loss as a function of energy is shown for the example of Li:



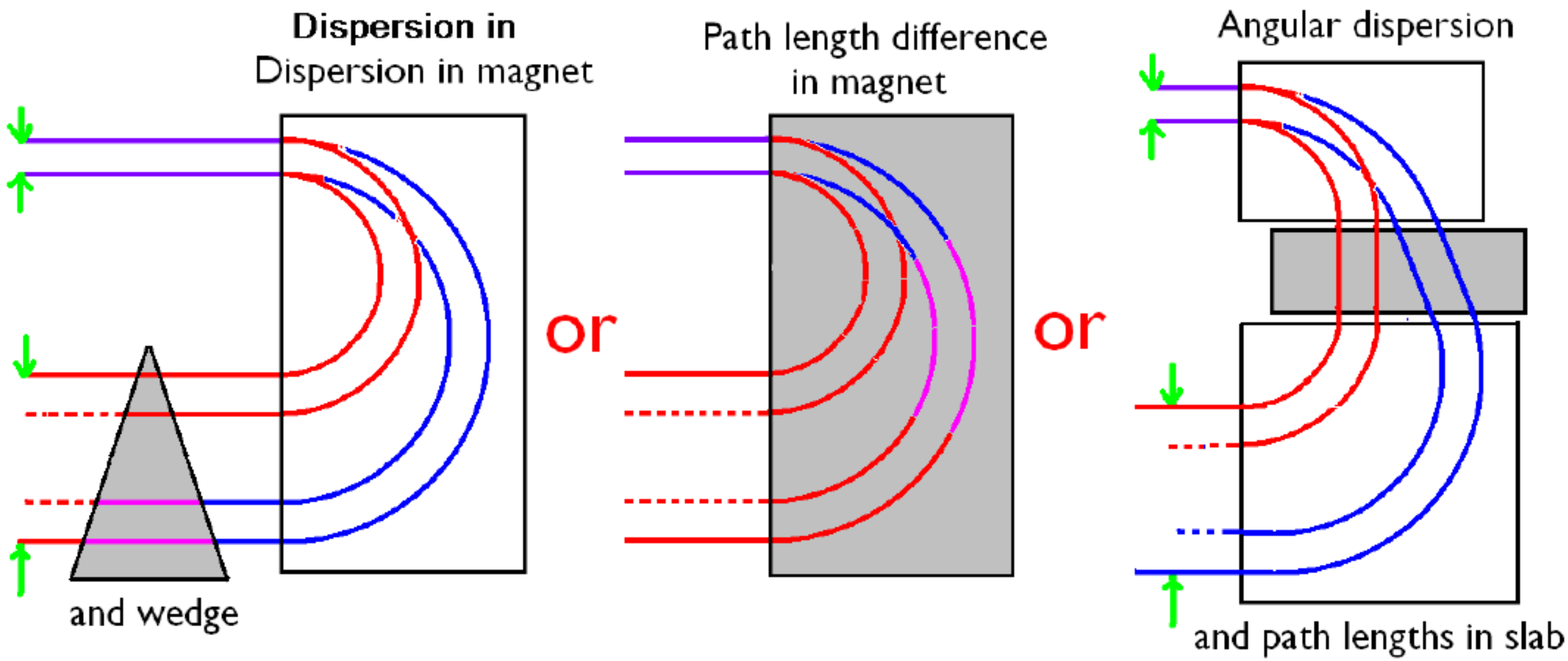
Energy Dependence of the Ionization Cooling Partition Functions

- J_z is seen to be strongly negative a low energies (longitudinal heating) and becomes slightly positive above 300 MeV/c



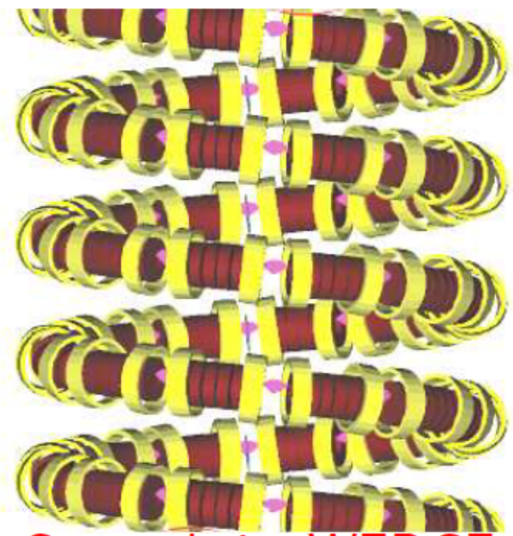
- More acceleration per cooling decrement at higher energies makes cooling at energies of ~200 MeV preferable

- Final cooling still requires moving to very low beam energies. However, the overall 6D cooling remains significant at these low energies.

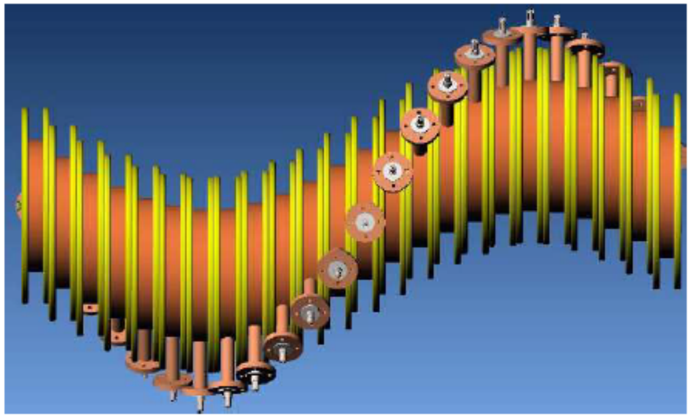


- In these examples, $\Delta p/p$ is reduced but σ_y is increased
  ⇨ the longitudinal emittance is reduced but the transverse emittance increases

6D Cooling Candidate Designs



Guggenheim WEDGE

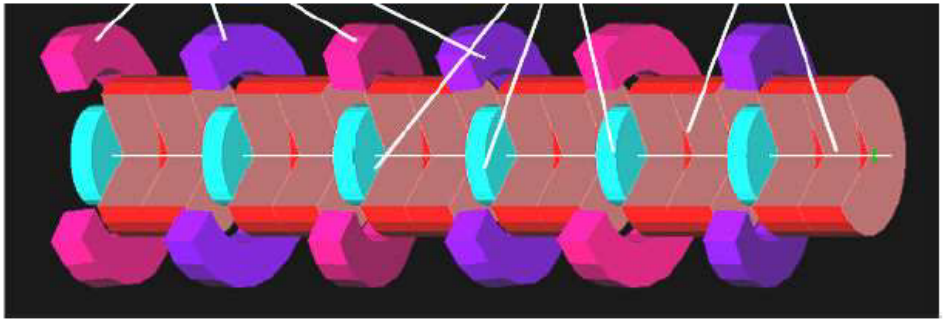


Helical Cooling Channel PATH

Alternating tilted solenoids

Hydrogen aborbers

rf



Snake SLAB

- Each of these examples has been simulated
- No design has had all of its outstanding issues resolved at the level that one can be selected as an official baseline design
- MAP is targeting an initial baseline selection within the next 18 months

Last year P5 recommended ending the MAP project.

The succession of cooling systems to reach the 6D phase emittance for a TeV collider are 6 impossible things…

For each problem, there just might be a clever solution

*Devise a way to go straight to **GO** without going to hell and back, and perhaps you could win a Nobel!*