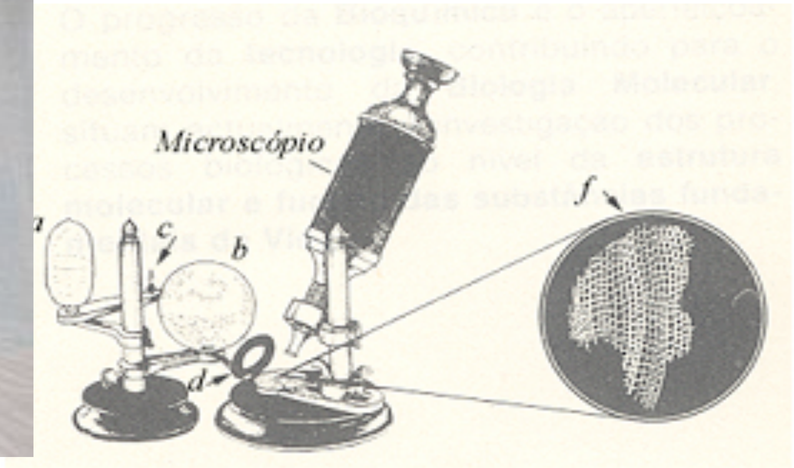
Synchrotron Radiation

Liu Lin

LNLS – Brazilian Synchrotron Light Laboratory

Observation tools – origins of modern science



Robert Hooke

1638-1703

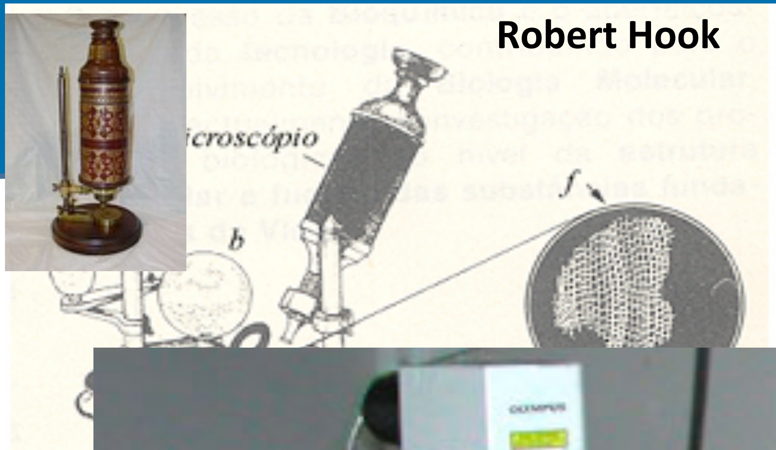
Antonie van Leeuwenhoek

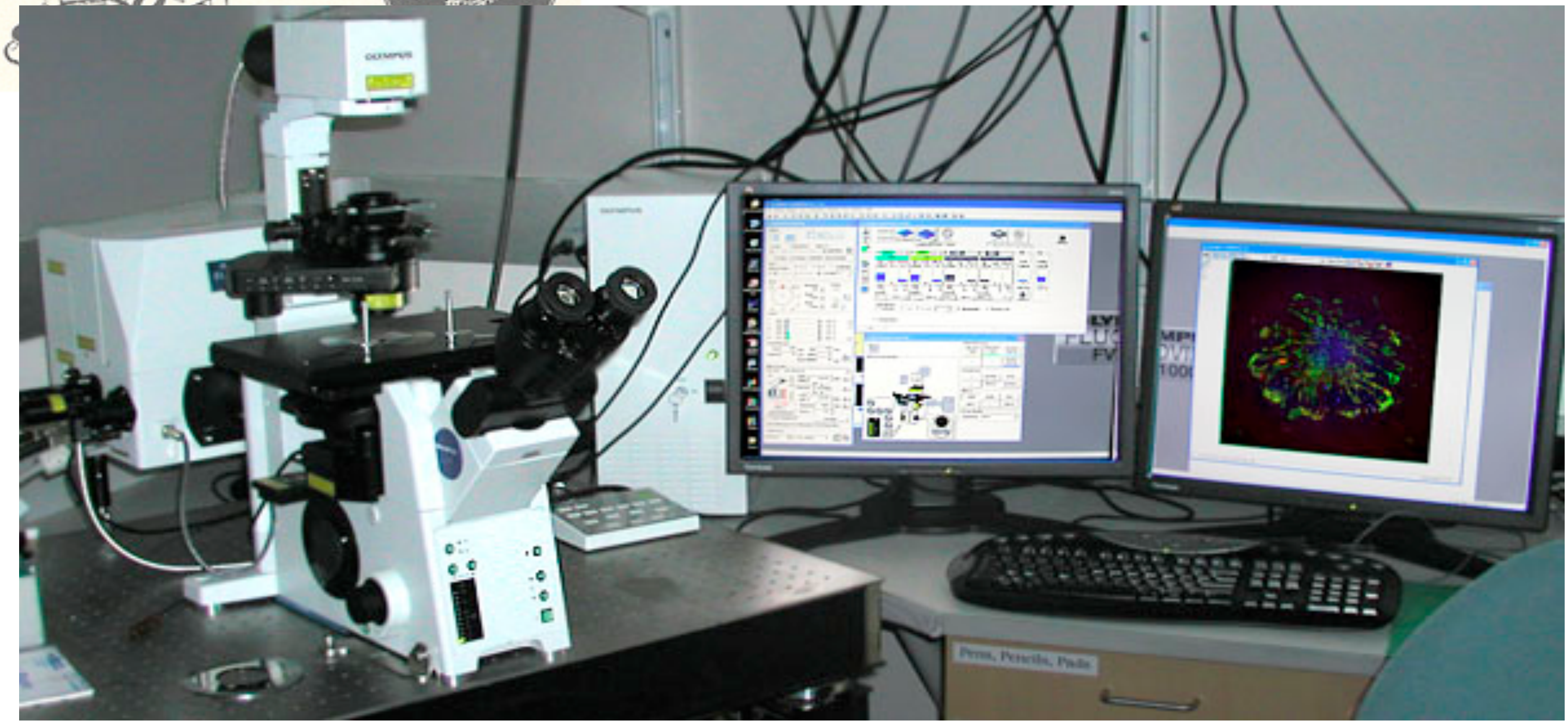
1632-1723

Galileo Galilei

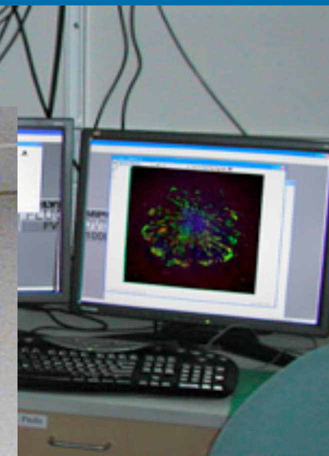
1564-1642

Robert Hook

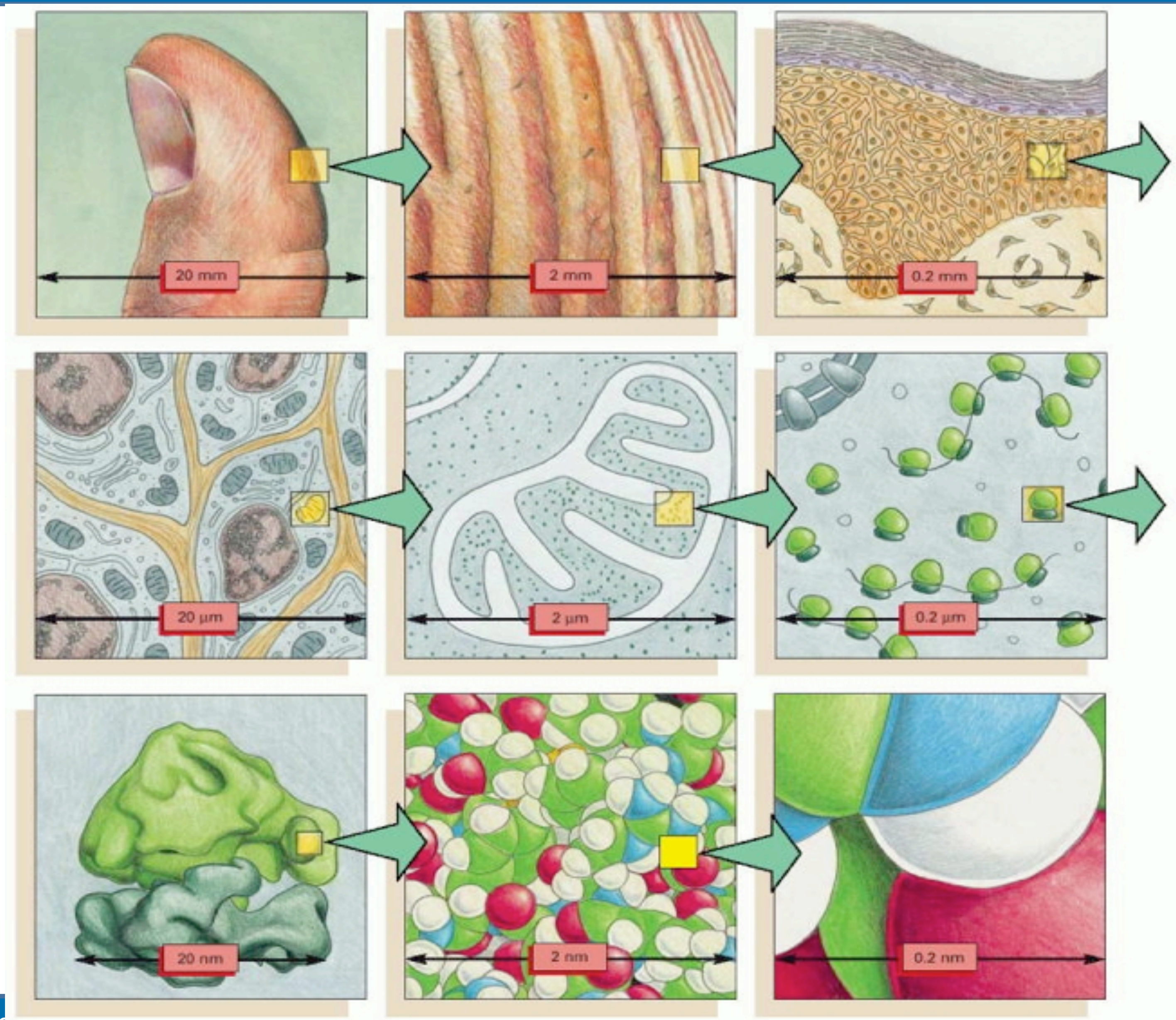




Robert Hook



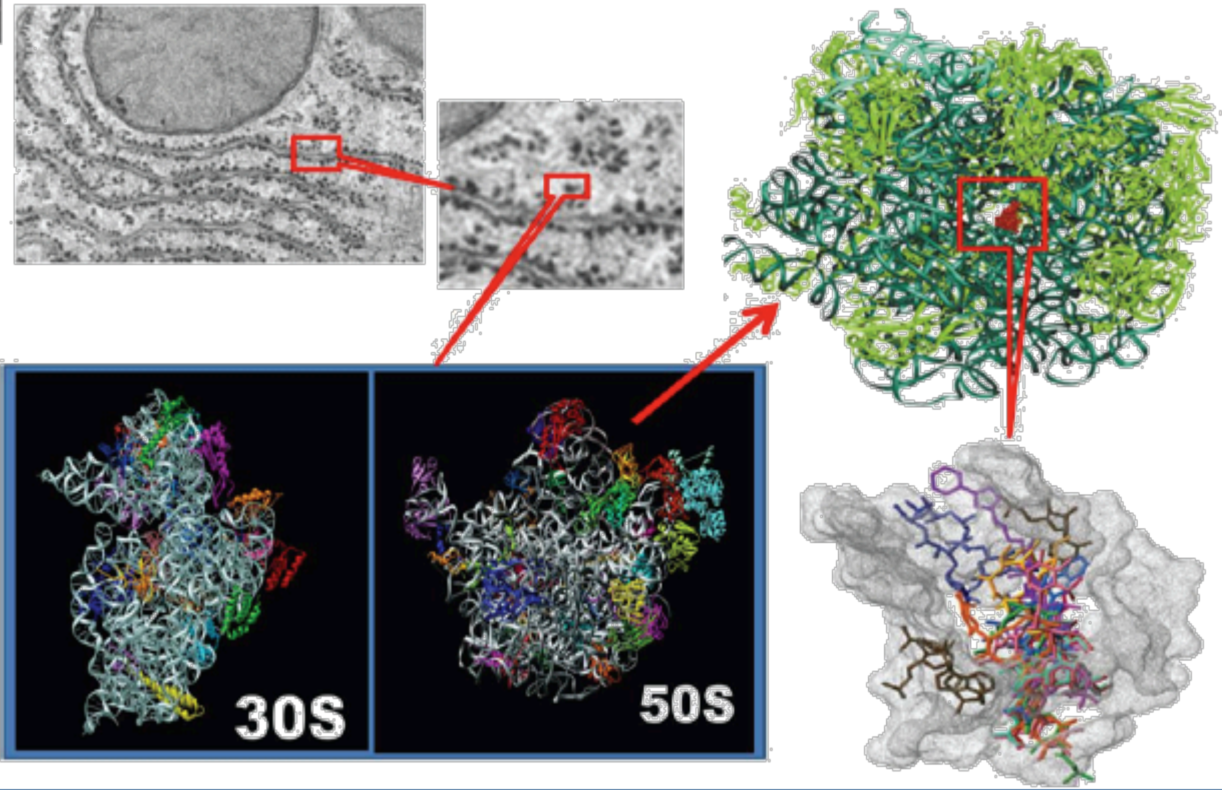
And if we want to know the atomic structure?



V. Ramakrishnan; T. A. Steitz; Ada E. Yonath
Prêmio Nobel de Química de 2009

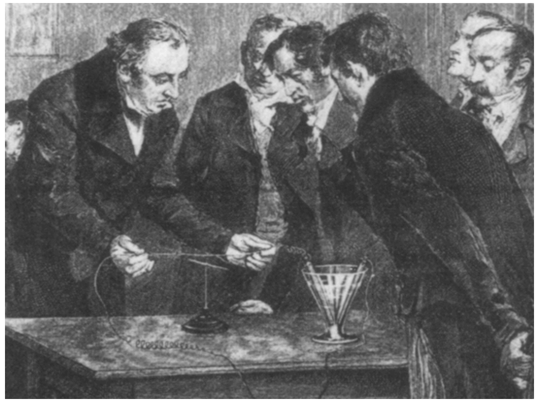
THE USE OF SYNCHROTRON RADIATION WAS FUNDAMENTAL FOR THIS RESEARCH!

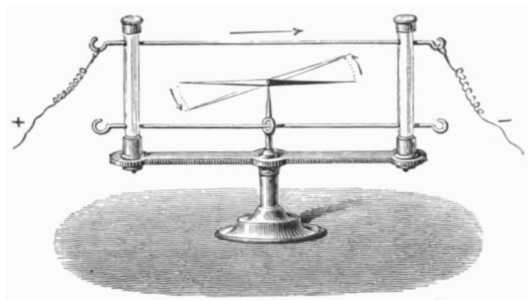
- Ribossom atomic structure.
- Besides the importance as fundamental knowledge, important for the development of antibiotics.



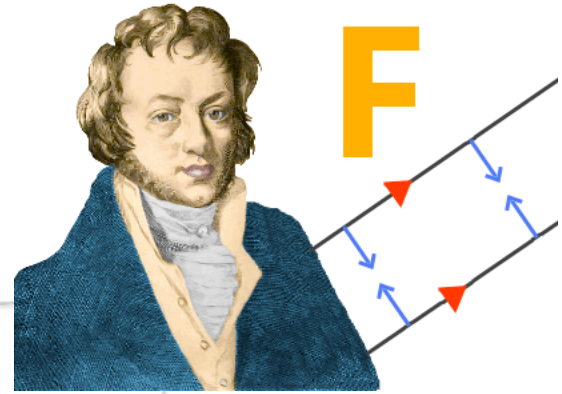
Electrodynamics: Historical understanding

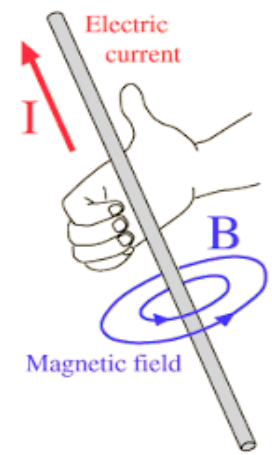
Hans Christian Ørsted (1777-1851)





André-Marie Ampère (1775 – 1836)

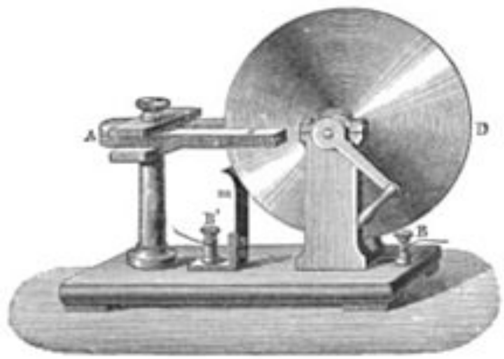


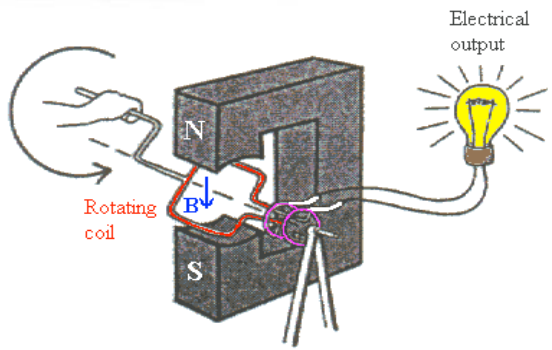


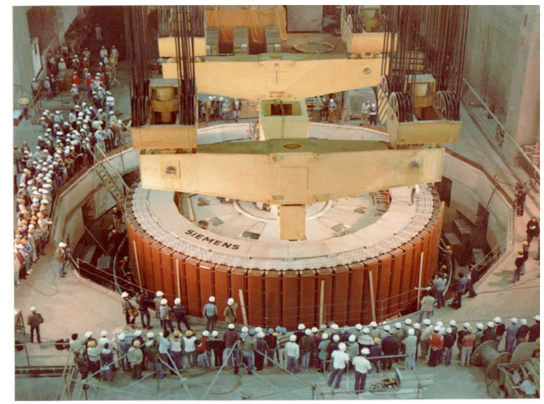
- In 1820 Øersted discovered that **electric currents create magnetic fields**. He noticed that a compass needle was deflected from magnetic north when an electric current from a battery was switched on and off. The electric battery was invented in 1800 by Alessandro Volta.
- Ampère was fascinated by Oersted's discovery and decided to understand why electric current produced a magnetic field. He discovered that magnetic attraction and repulsion is produced between two parallel wires carrying electric currents. **All magnetism was generated electrically**.

Electrodynamics: Historical understanding

Michael Faraday (1791 – 1867)



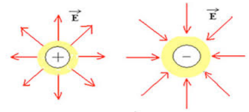


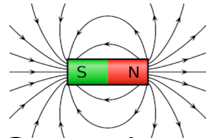


- In 1831 Faraday discovered that a varying magnetic field causes electricity to flow in an electric circuit. Previously, people had only been able to produce electric current with a battery. Now Faraday had shown that movement could be turned into electricity.
- Most of the power in our homes today is produced using this principle.

Electrodynamics: Historical understanding

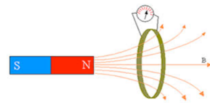
James Clerk Maxwell (1831–1879)





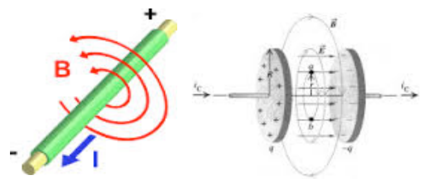
Gauss Law

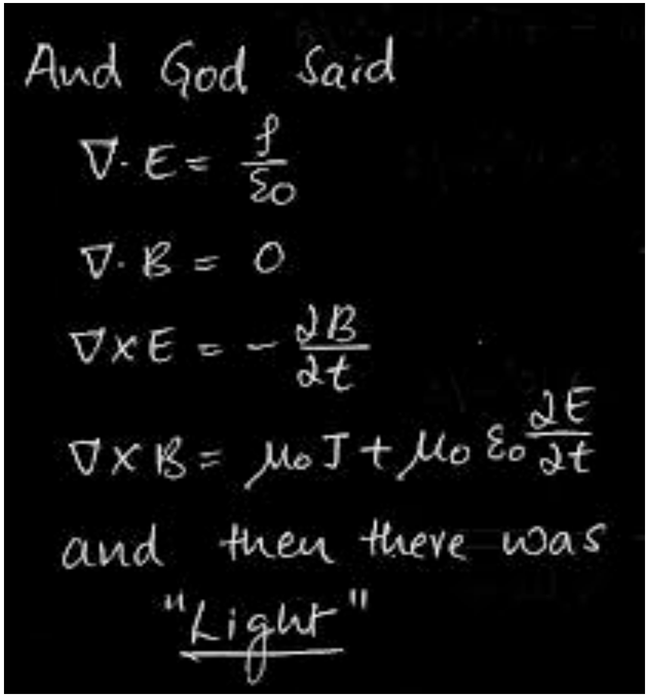
Gauss Law for Magnetism



Faraday Law

Ampere-Maxwell Law

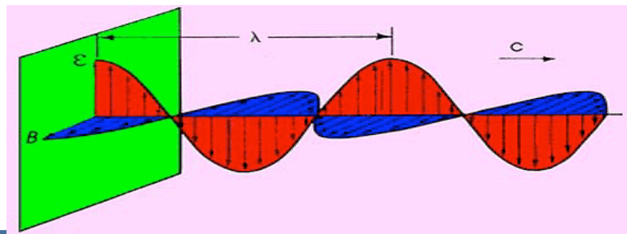




$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

for the electric Field — for the magnetic Field

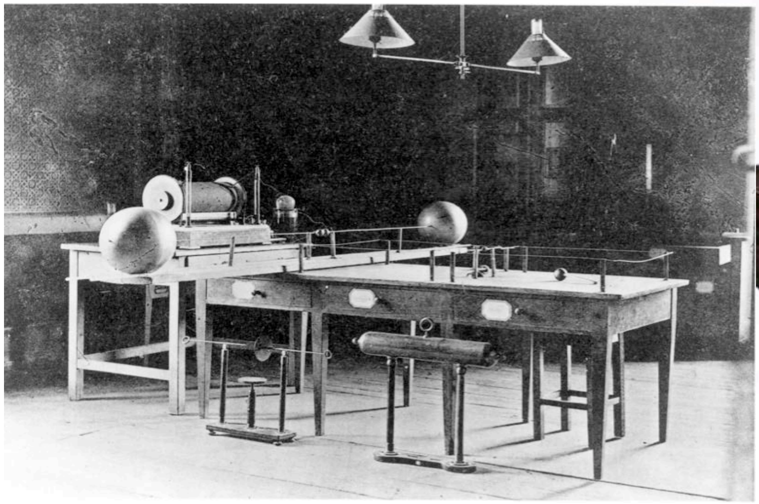
$$\text{where } \frac{1}{c^2} = \varepsilon_0 \mu_0$$

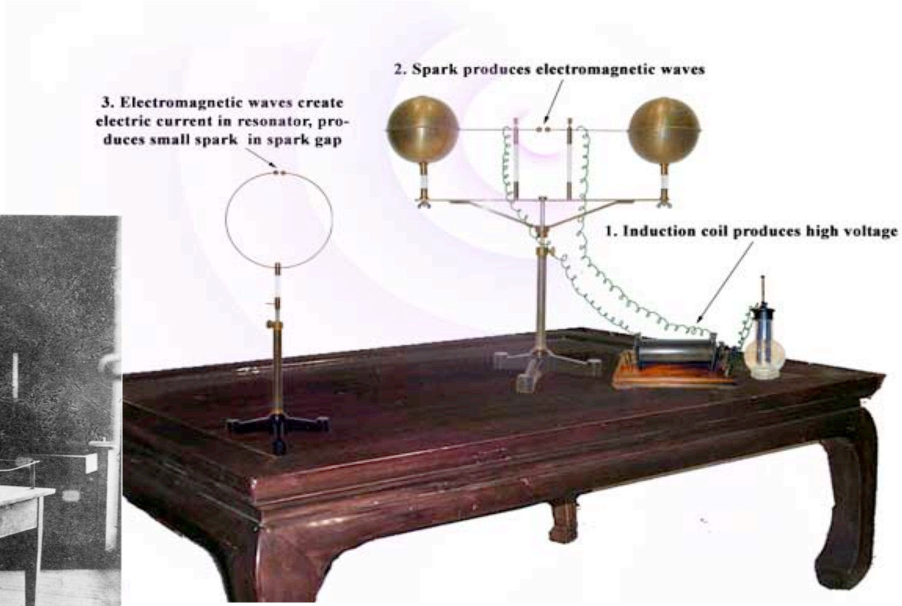


- Maxwell bla blabla bla
- Maxwell's equations made evident that changing charge densities would result in electromagnetic waves that would radiate outward with the speed identical to the speed of light.

Electrodynamics: Historical understanding

Heinrich Hertz (1857 – 1894)

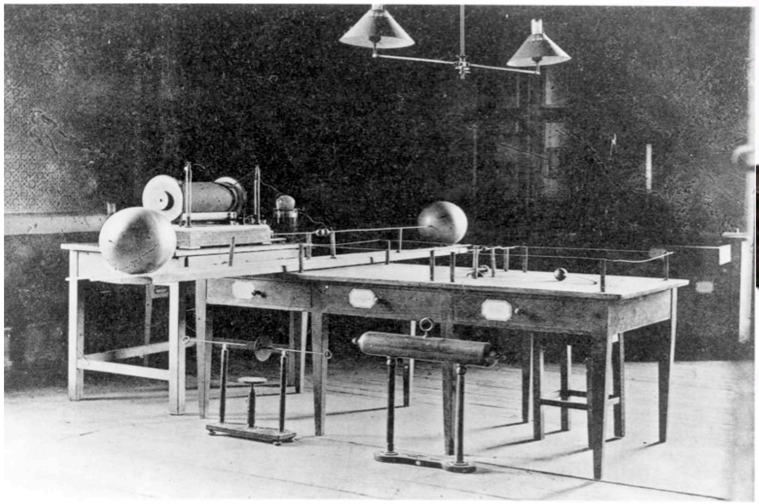


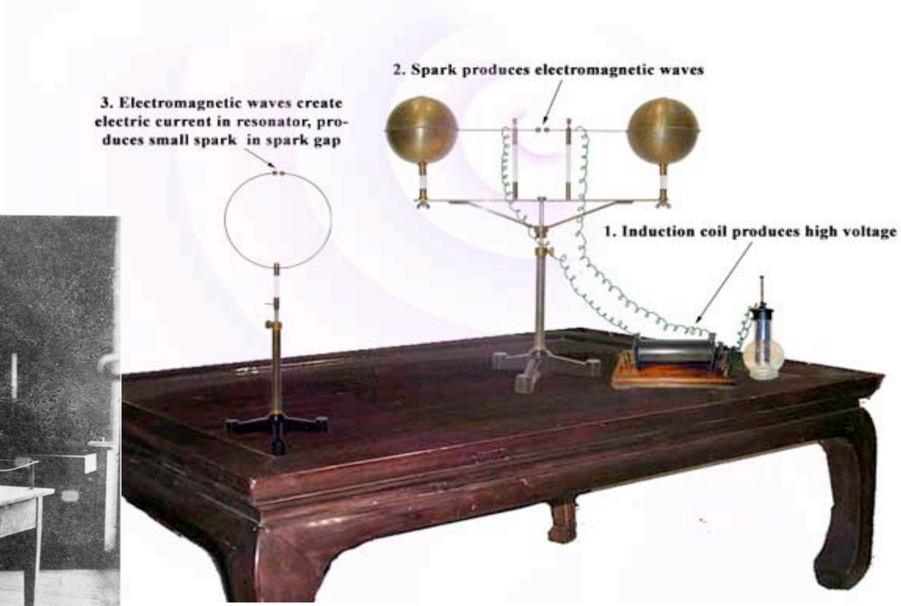


- In 1887 Heinrich Hertz demonstrated the existence of such waves by inducing extremely small sparks in a resonant antenna.

Electrodynamics: Historical understanding

Heinrich Hertz (1857 – 1894)

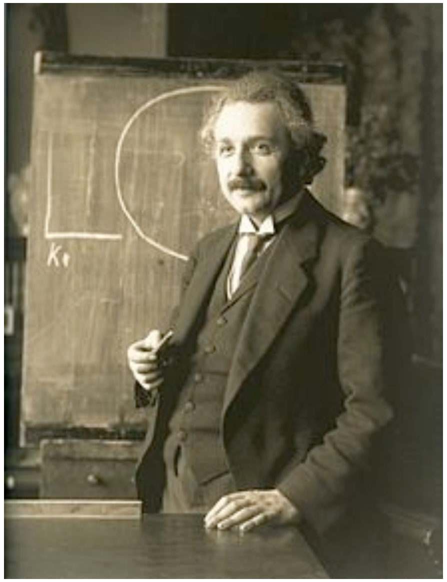


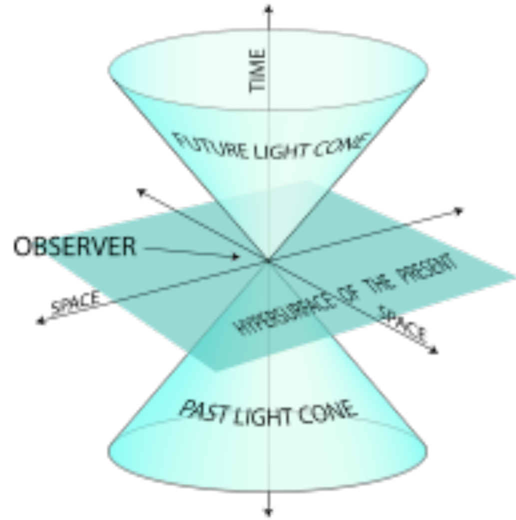


- In 1887 Heinrich Hertz demonstrated the existence of such waves by inducing extremely small sparks in a resonant antenna.
- When asked about the practical importance of his experiments he replied: “It's of no use whatsoever, this is just an experiment that proves Maestro Maxwell was right.”

Electrodynamics: Historical understanding

Albert Einstein (1879–1955)

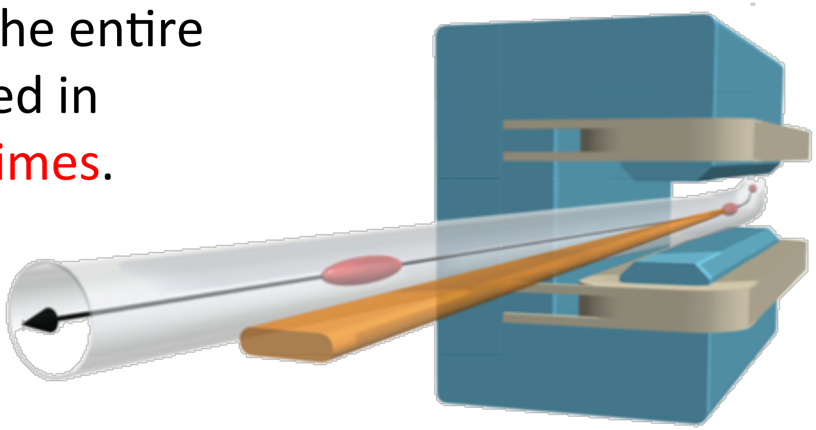




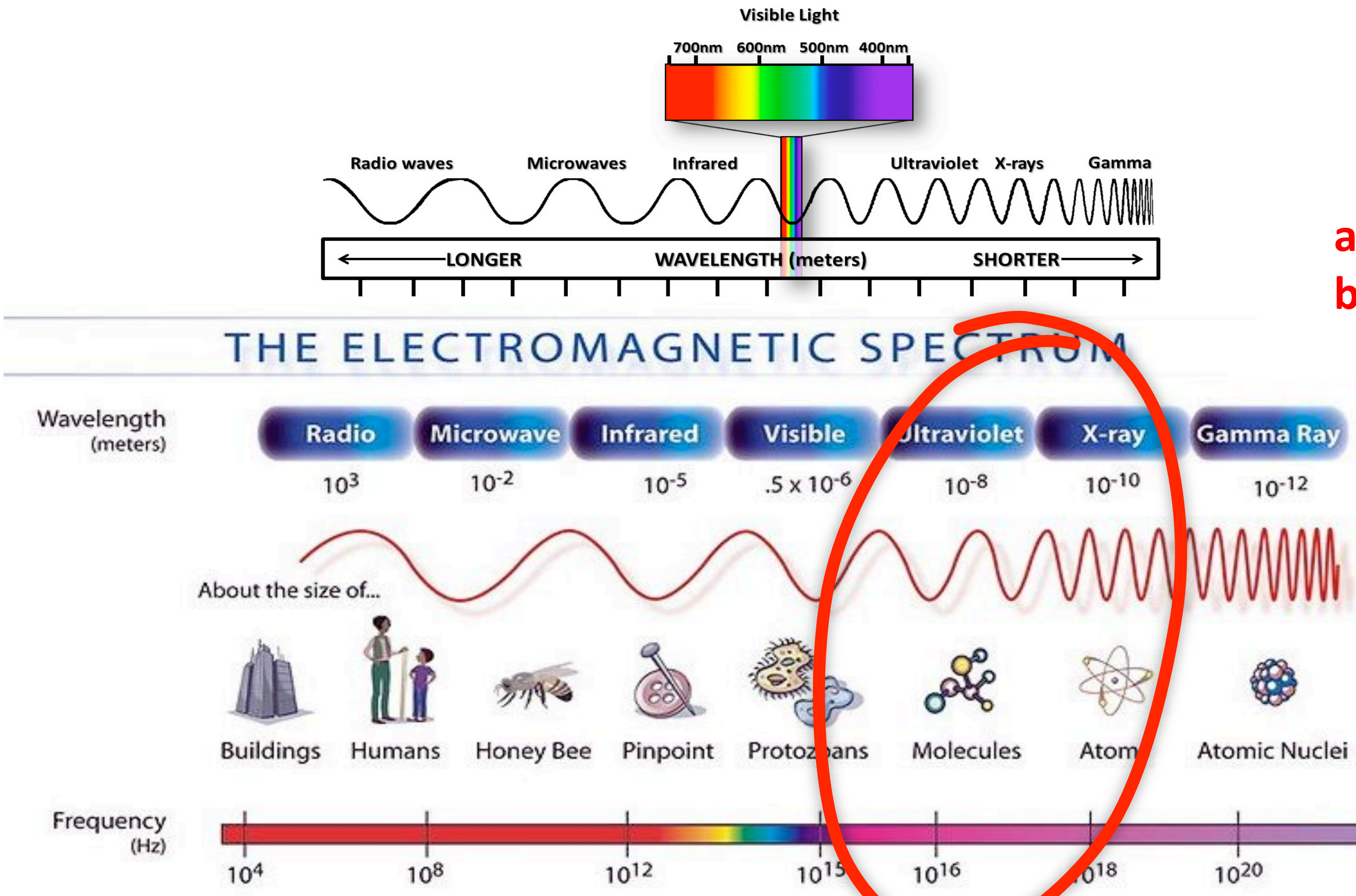
- In 1905 Albert Einstein published the paper founding Special Theory of Relativity, in which he proposed that light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body. It then follows that time, length and mass actually depend on the speed we are moving at. For example, as we move faster, time passes more slowly and lengths become contracted.
- Transformations between reference frames follow Lorentz Transformation, that preserve the symmetries of the laws of electromagnetism.

Synchrotron Radiation

- Accelerator-based synchrotron radiation was first observed at GE in 1947 in a type of accelerator known as synchrotron. Since then, the electromagnetic radiation generated in these machines is called synchrotron radiation.
- We now use the name synchrotron radiation (light) to describe radiation that is emitted from charged particles traveling at relativistic speeds ($v \approx c$) when they change direction, regardless of the accelerating mechanism and shape of the trajectory.
- Although synchrotron radiation can cover the entire electromagnetic spectrum, we are interested in radiation in the UV, soft and hard-X ray regimes.



The electromagnetic spectrum



accelerator based sources

The atomic scale: X-Rays

X-rays are used since more than 100 years for basic and applied research on atomic level

Wilhelm Roentgen discovered X-rays in November, 1895

Hand of Roentgen's Wife, Anna Berthe

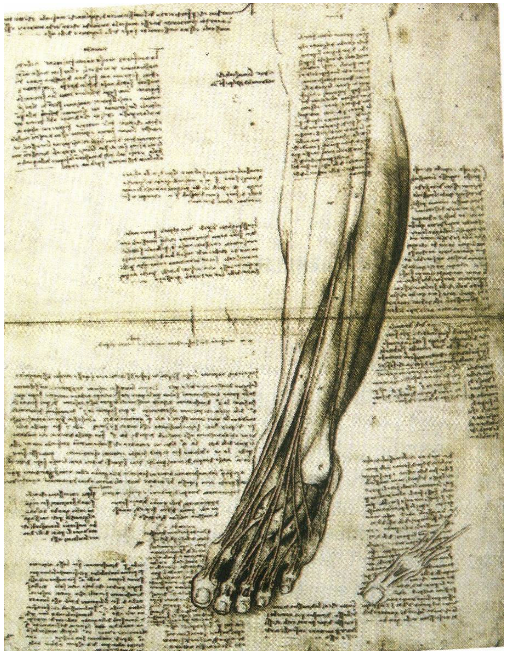
Hand of Roentgen's Assistant

H. Wiedemann

Example: understanding of human body

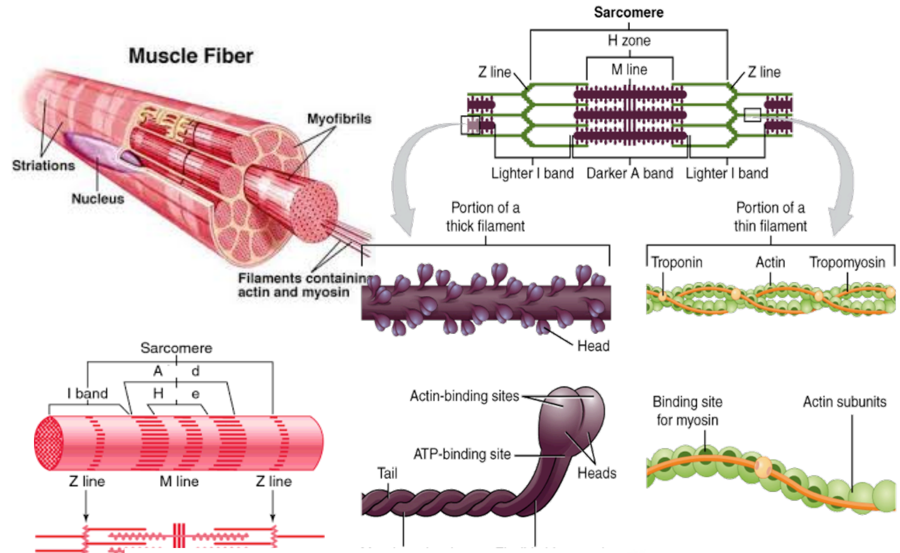
visible
naked eye

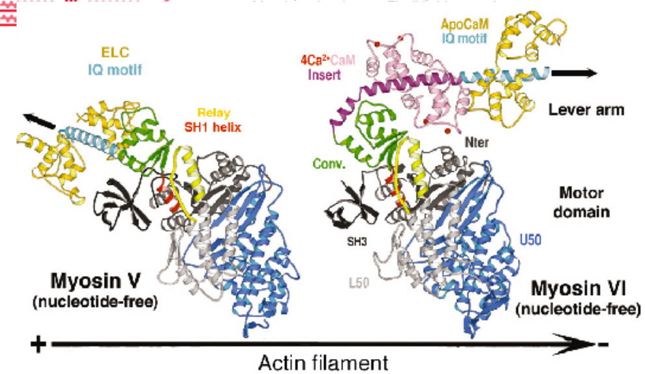
x-rays
with instruments



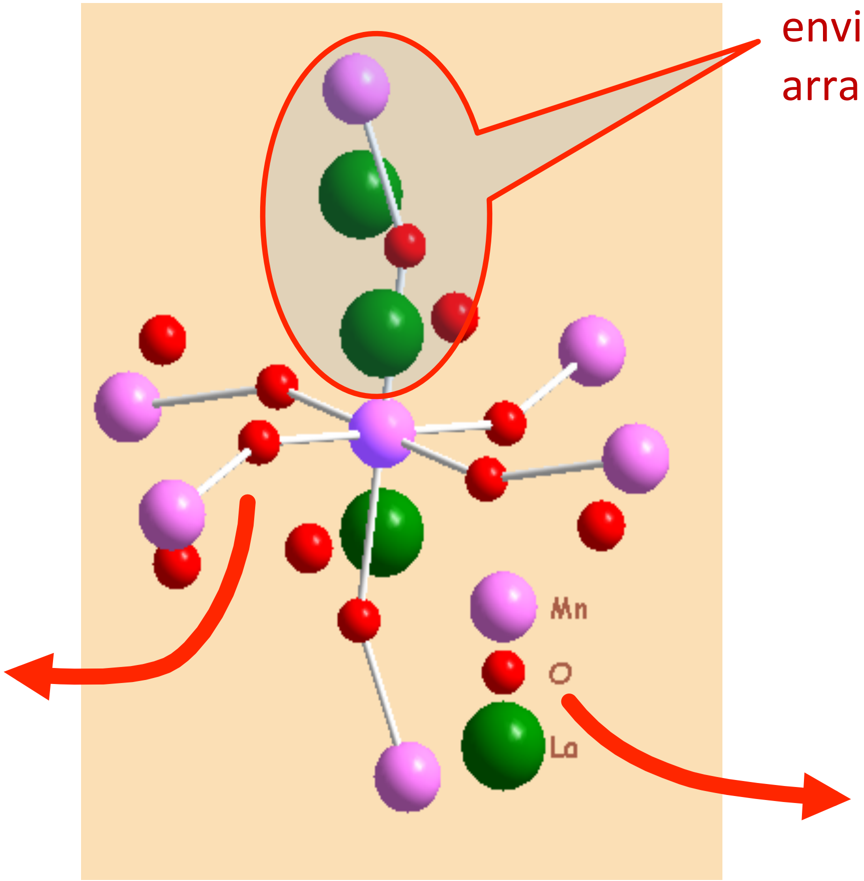
Leonardo da Vinci, 1507/1508

Andreas Vealius, 1543





The atomic scale: why UV and X-Rays ?



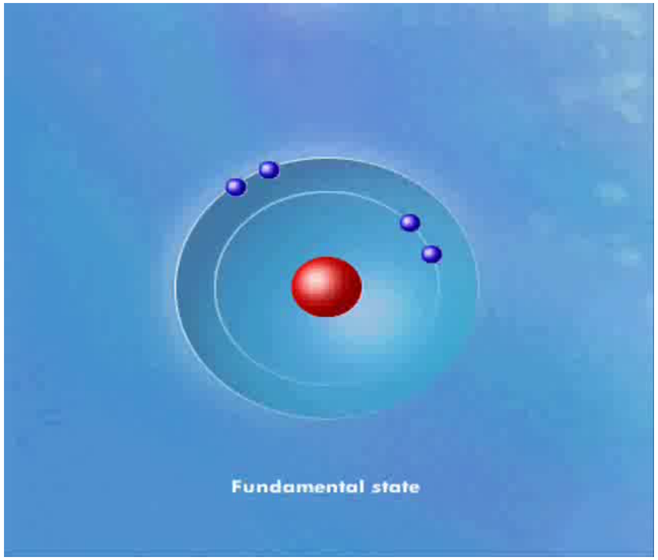
Type of atoms, chemical environment, structural arrangement: **X-rays**

Distance between atoms: **nanometer**

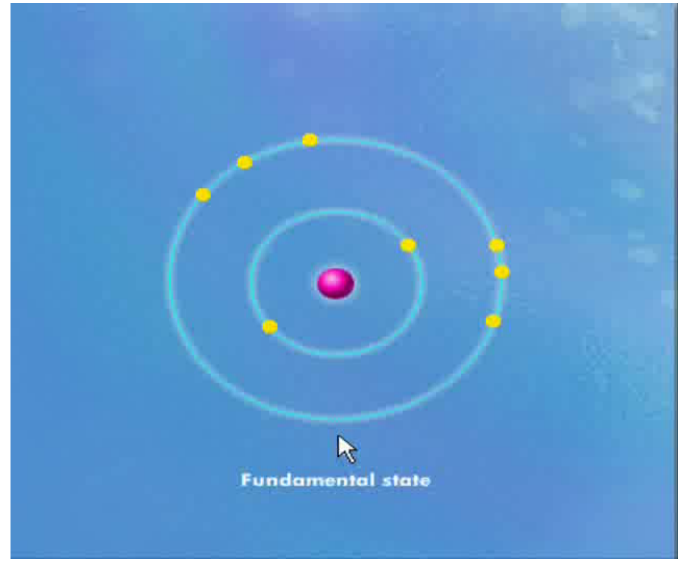
Chemical bonds: **UV**

Interaction of X-Rays and matter

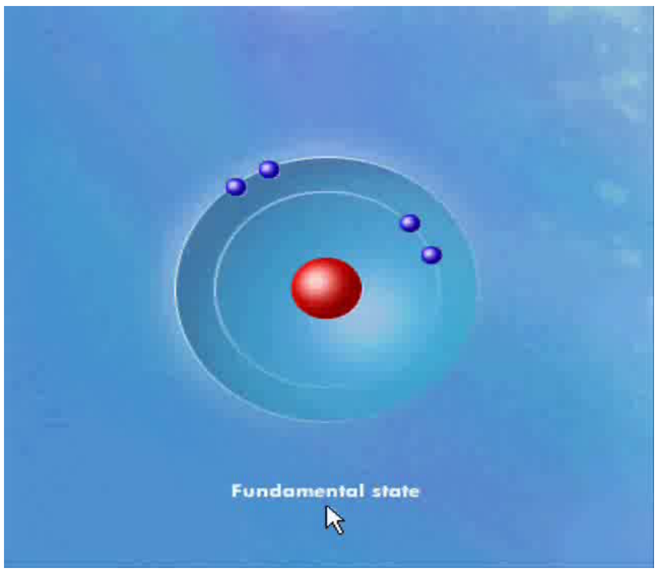
excitation



fluorescence



ionization



scattering

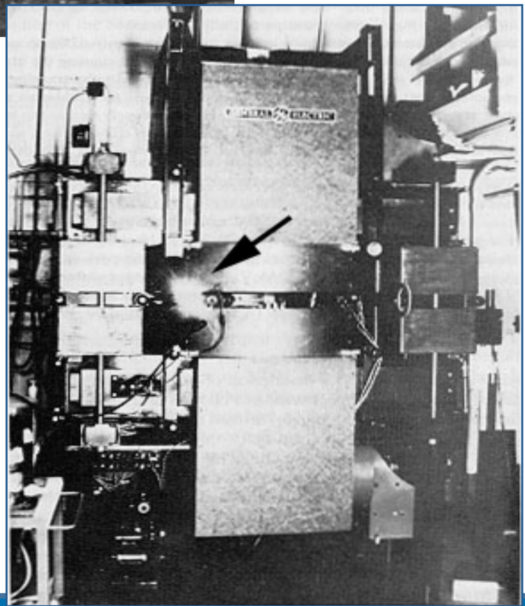
How to obtain X-rays?

Crab Nebula, 6000 light years away

First light observed 1054 AD

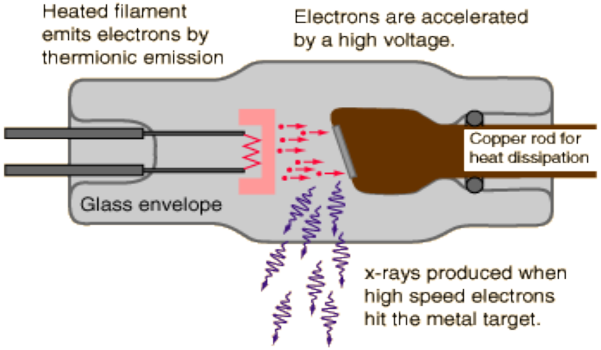
Synchrotrons

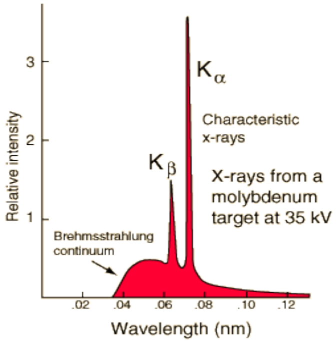
70 MeV synchrotron from GE New York State



First light observed 1947

X-ray tubes



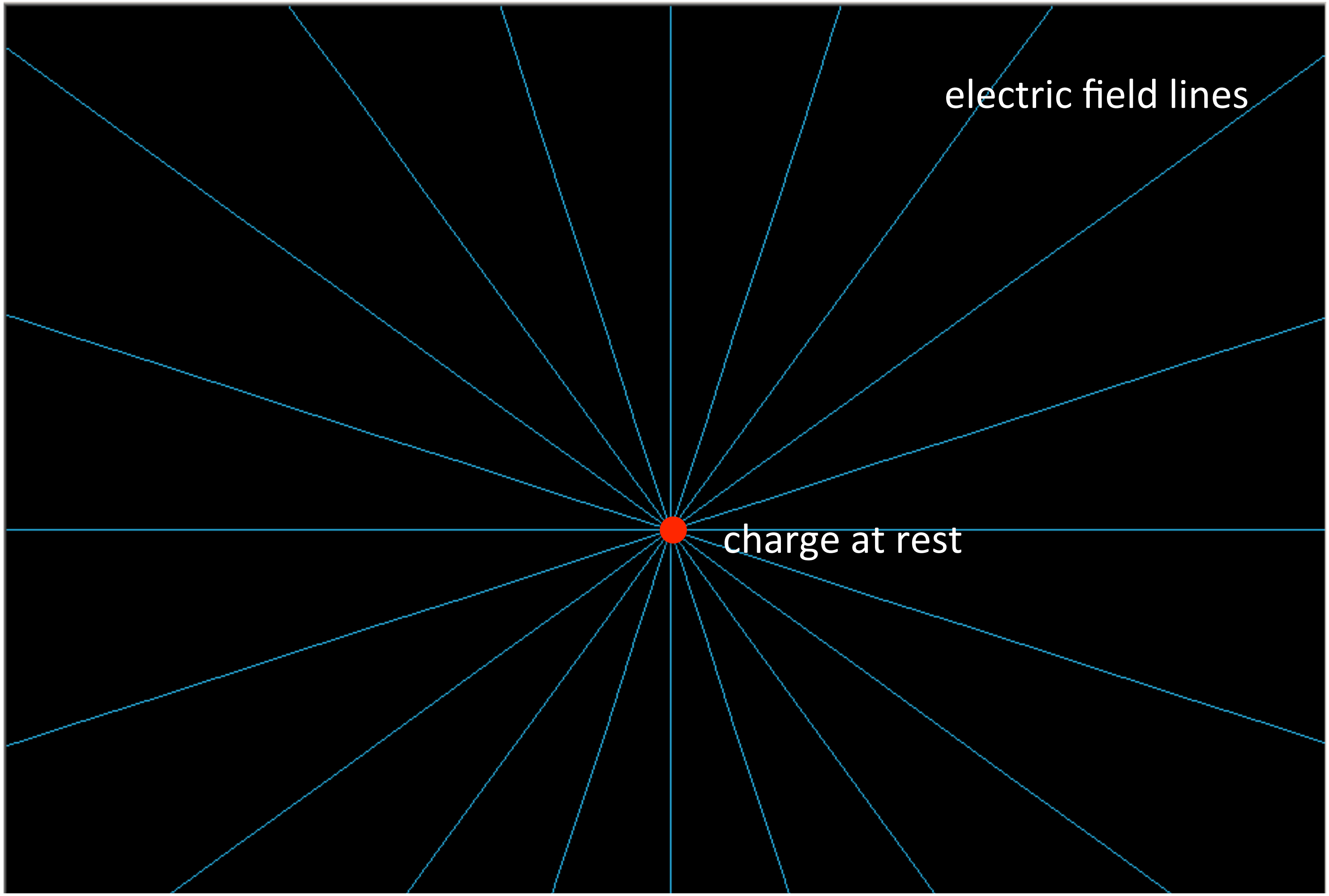


Radiation from moving charges

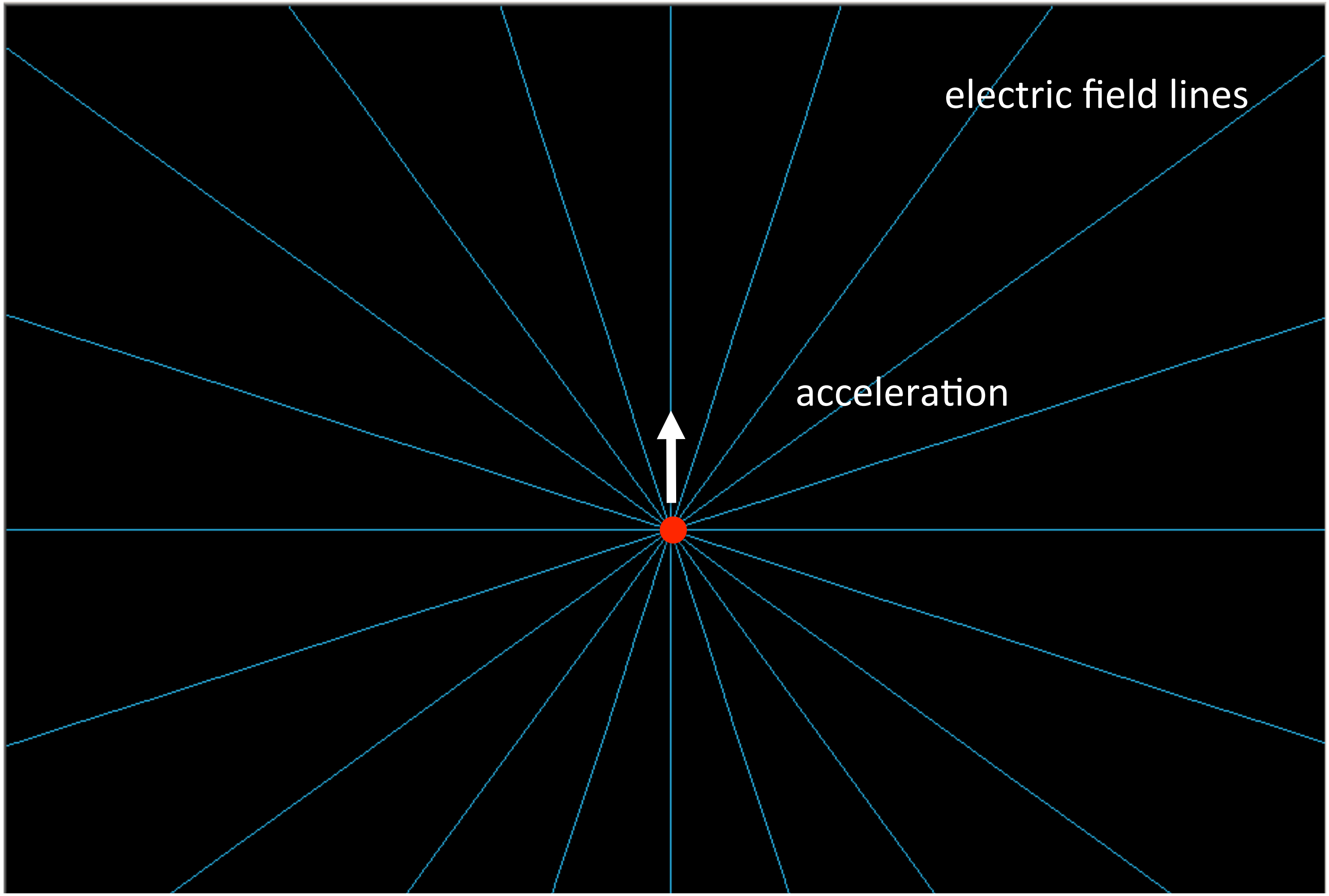
When an electrically charged body is suddenly moved, the electromagnetic field is at first changed only near that body, and then the changes in this field propagate at the speed of light. In fact, in this case the changes in the electromagnetic field ***are*** what we know as light.

Following "movies" were generated by the free software: Radiation2D by T. Shintake

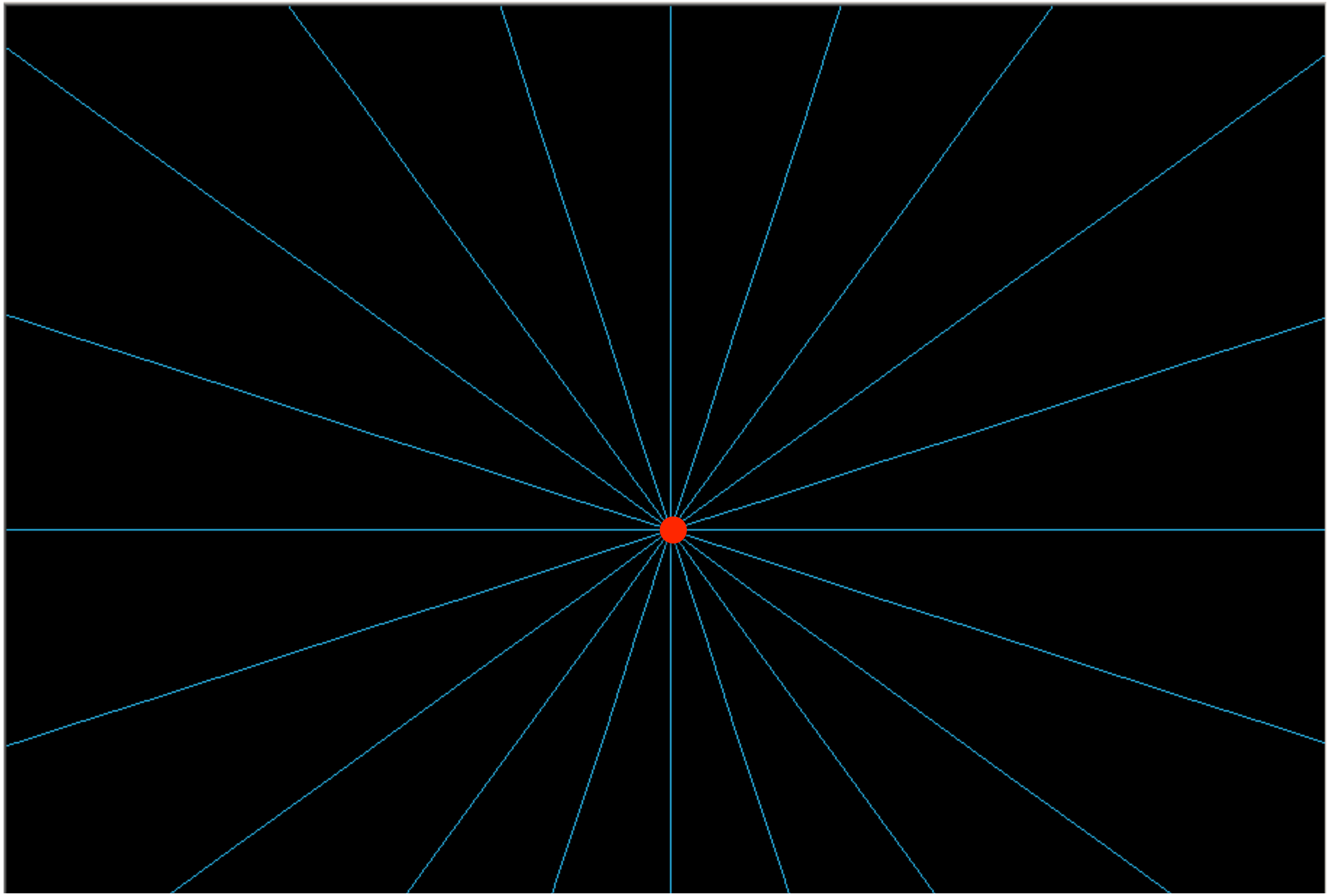
Radiation from moving charges



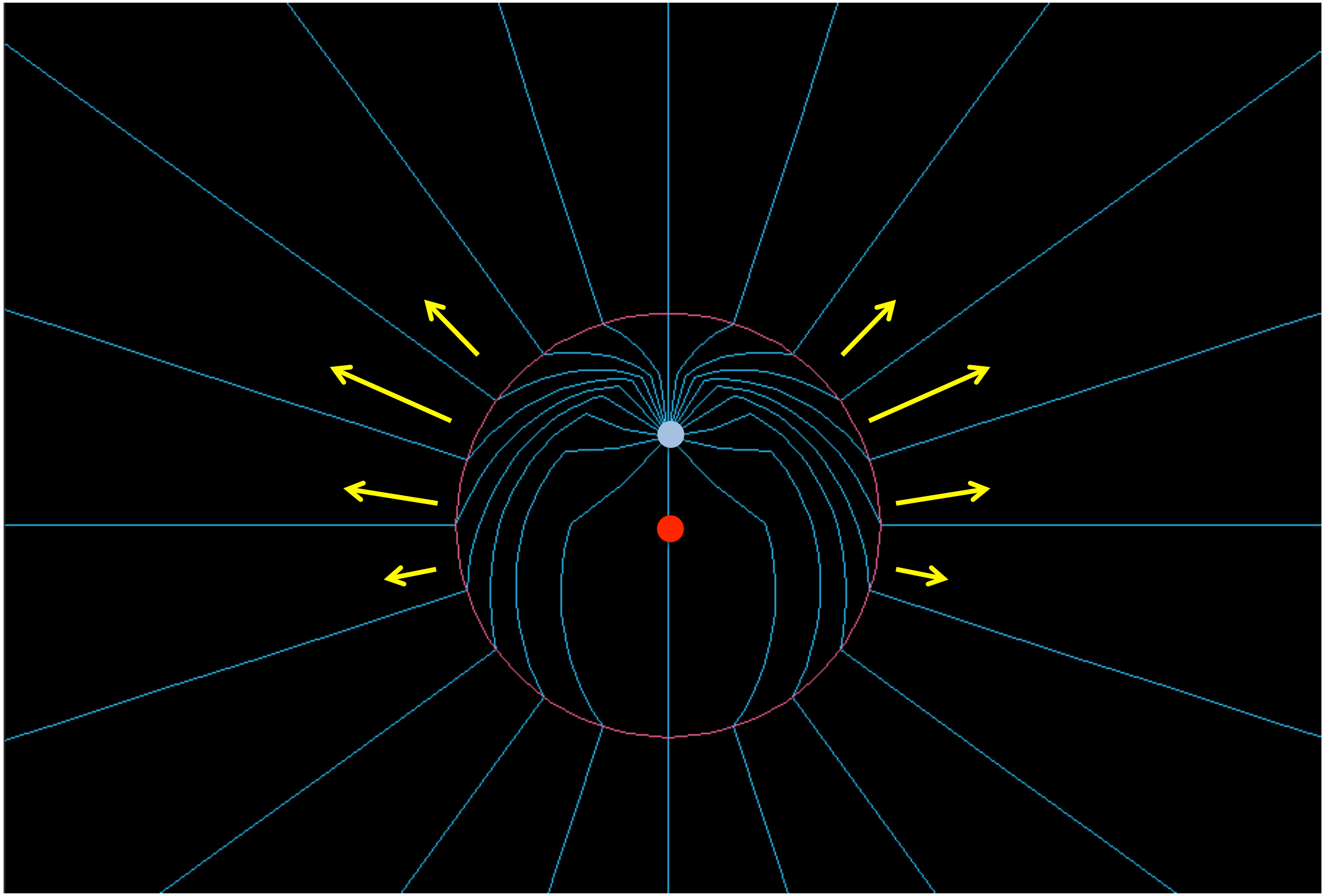
Radiation from moving charges



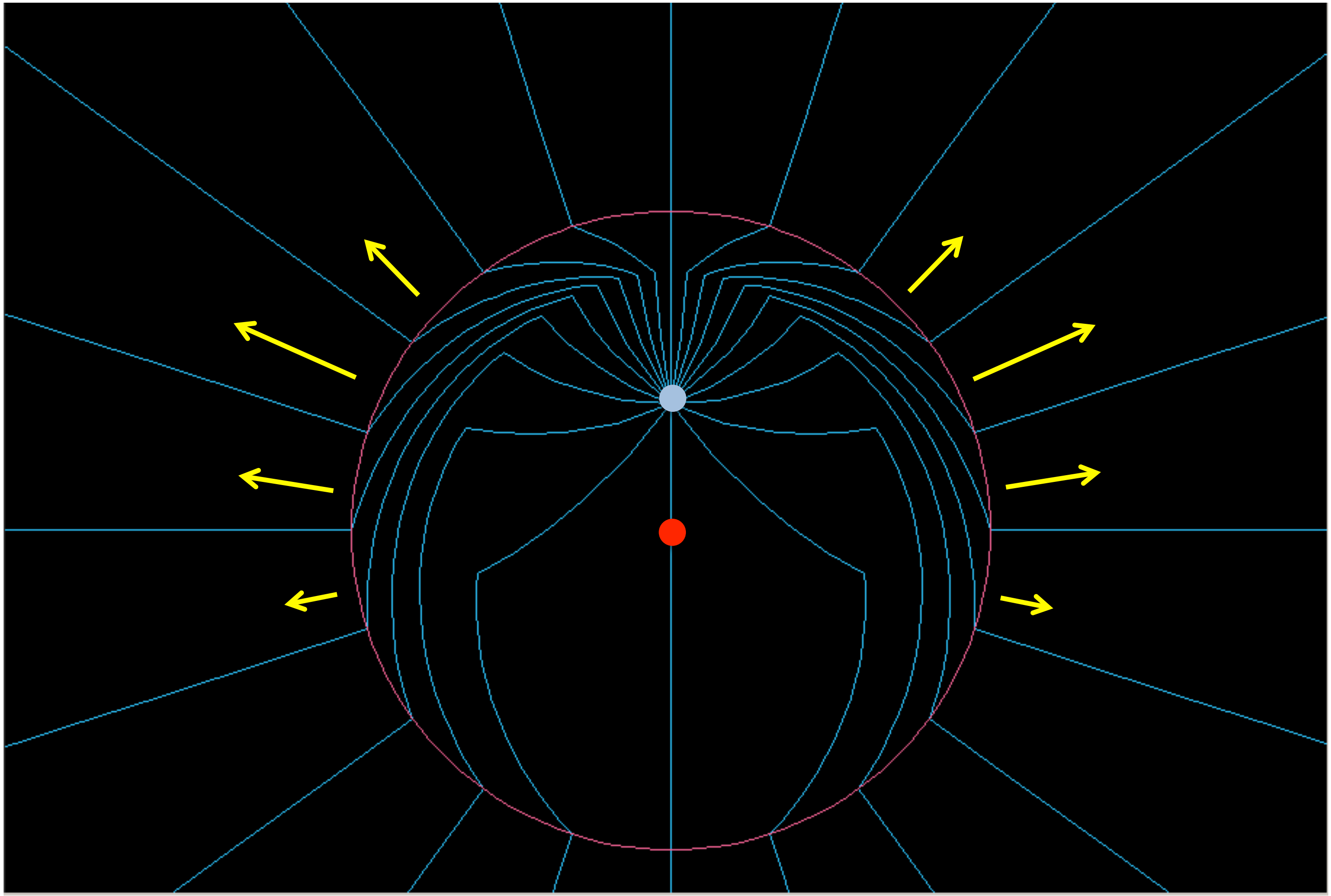
Radiation from moving charges



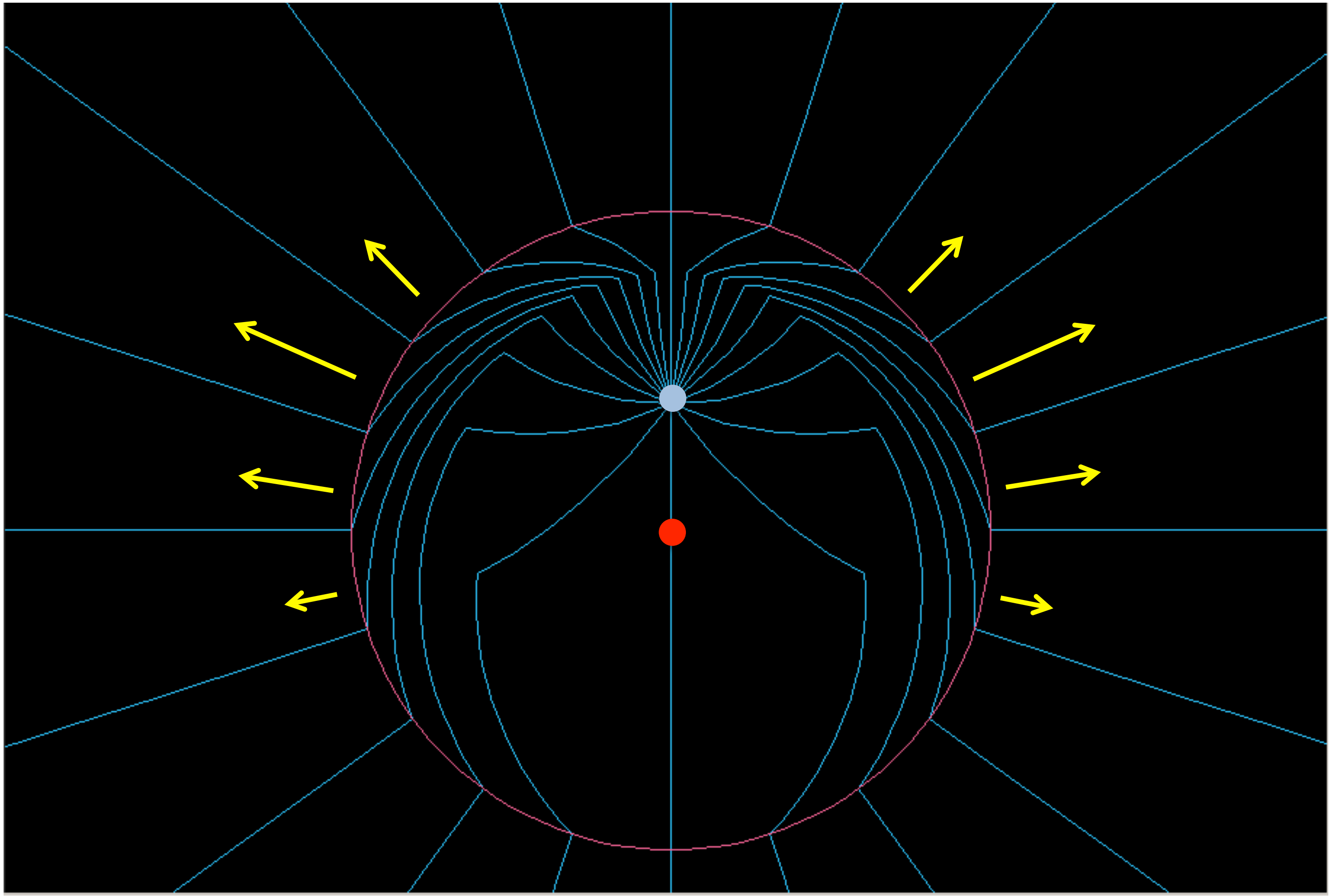
Radiation from moving charges



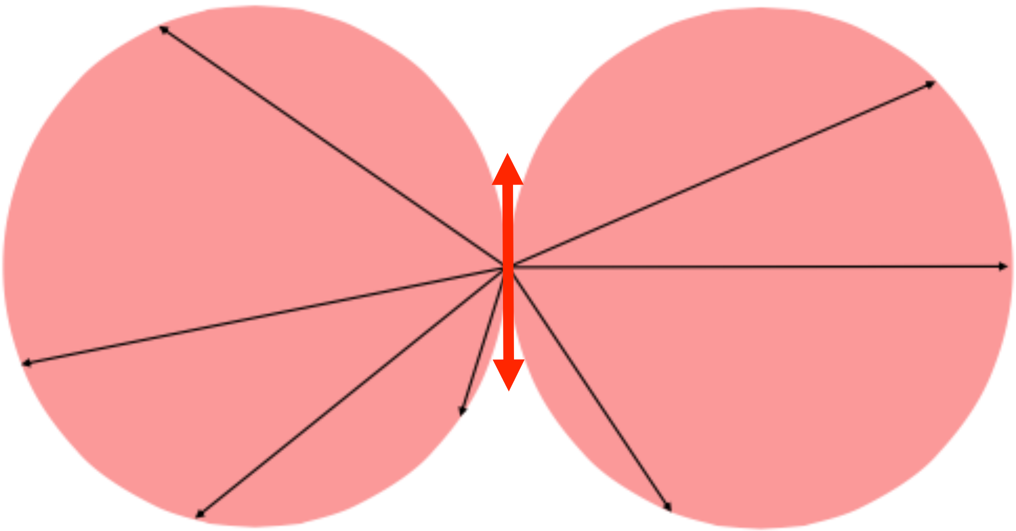
Radiation from moving charges



Radiation from moving charges



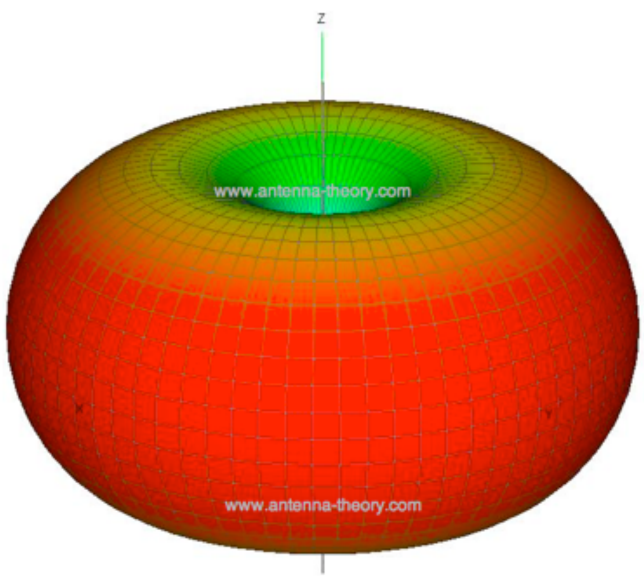
Radiation from moving charges

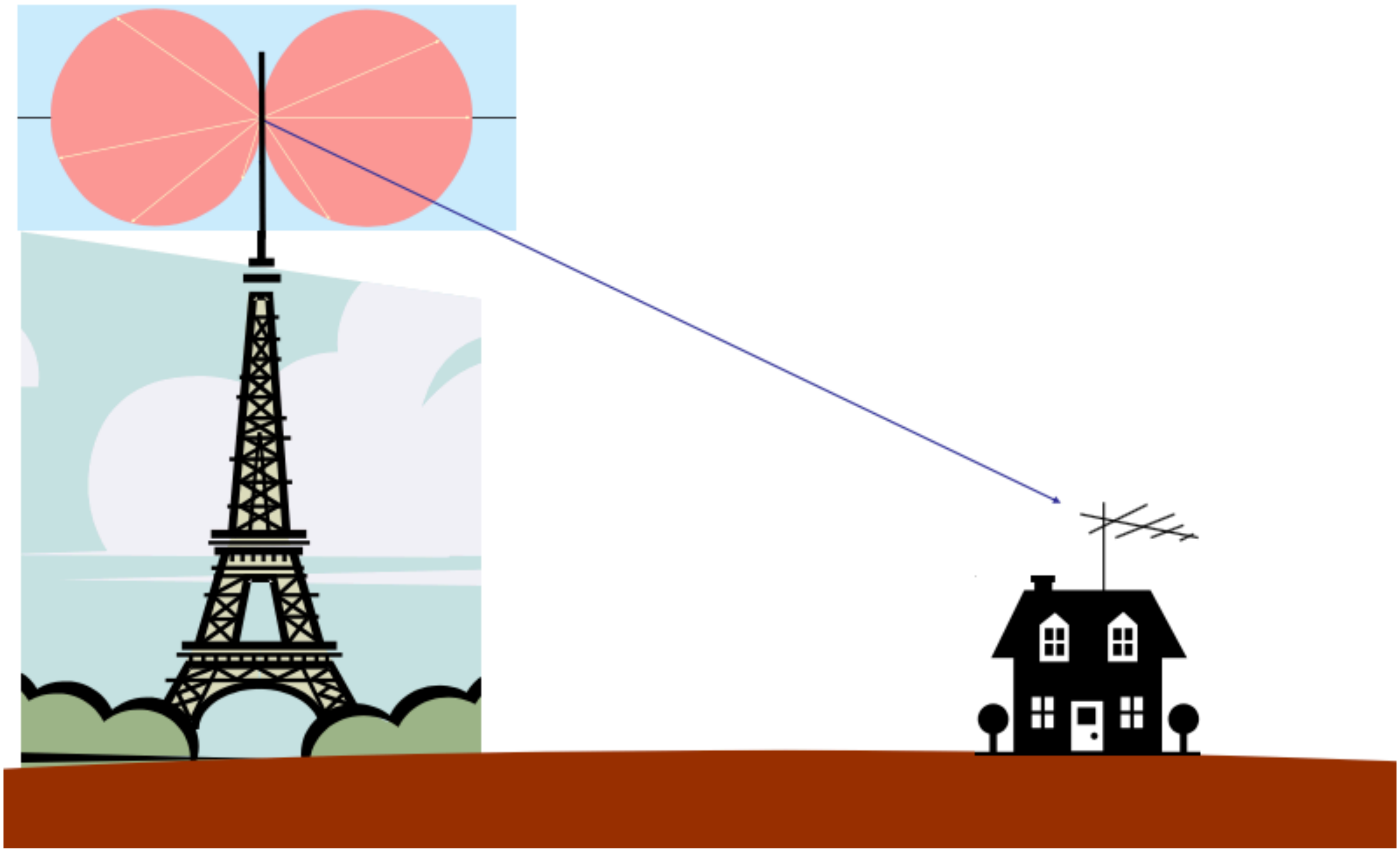


acceleration

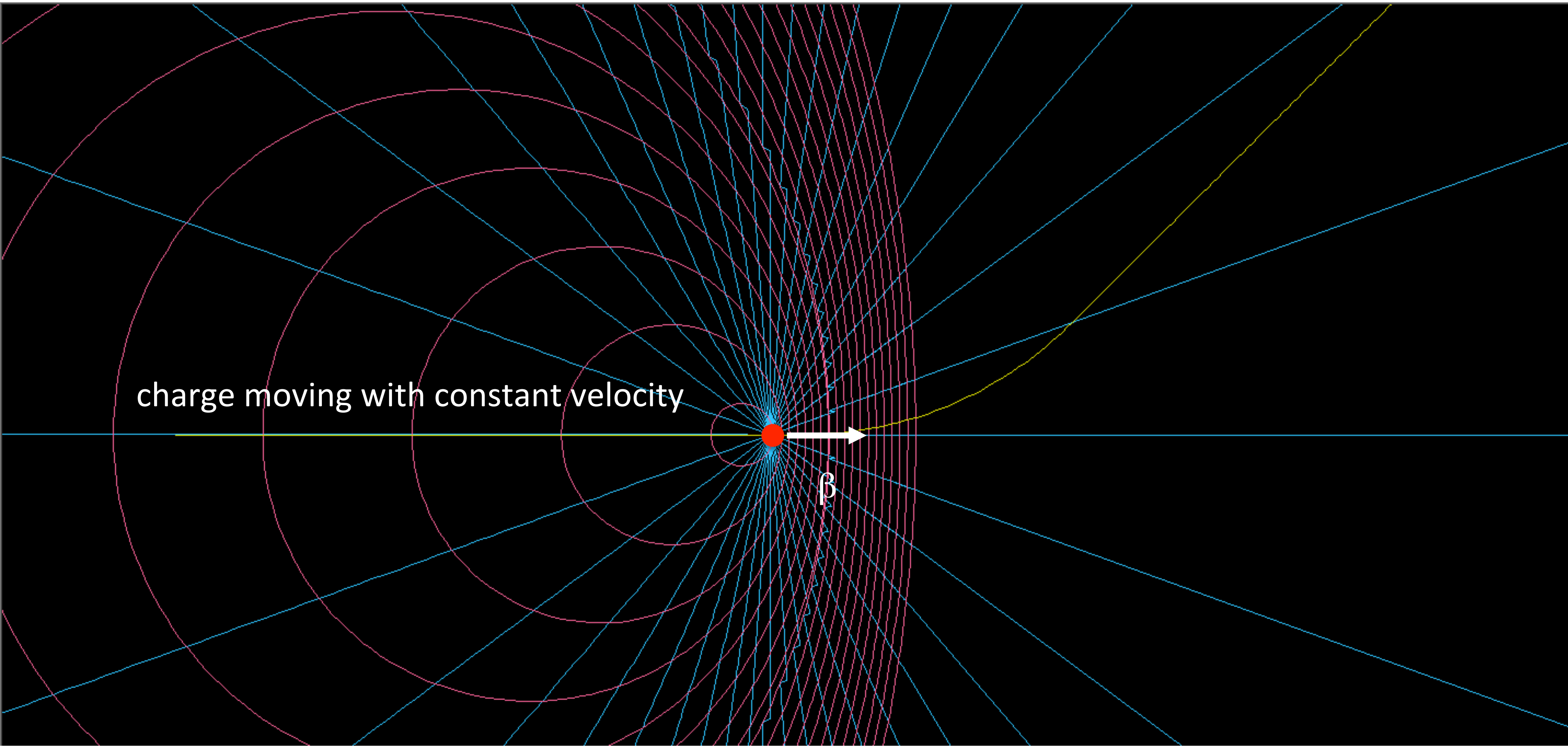
radiation lobe

actually: radiation doughnut

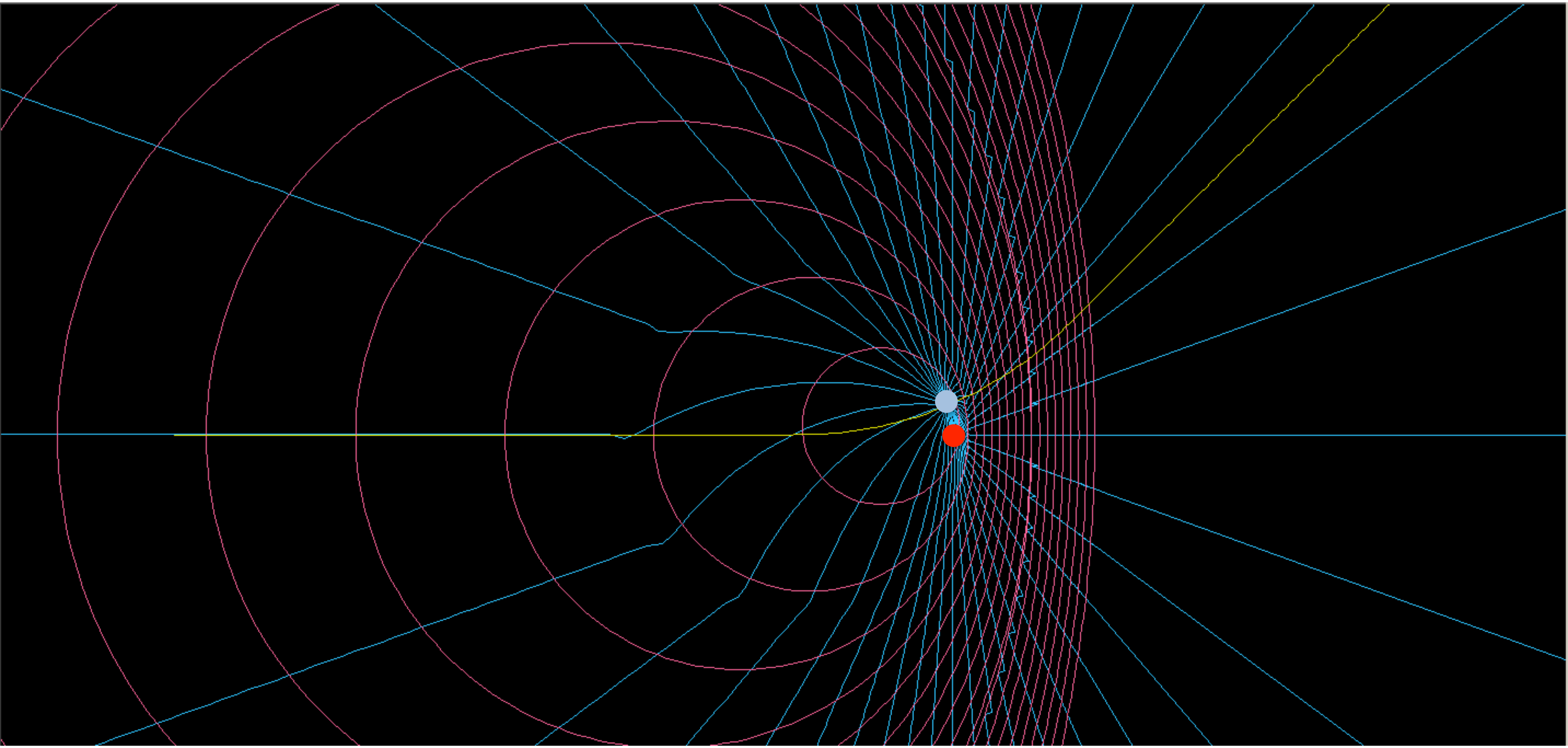




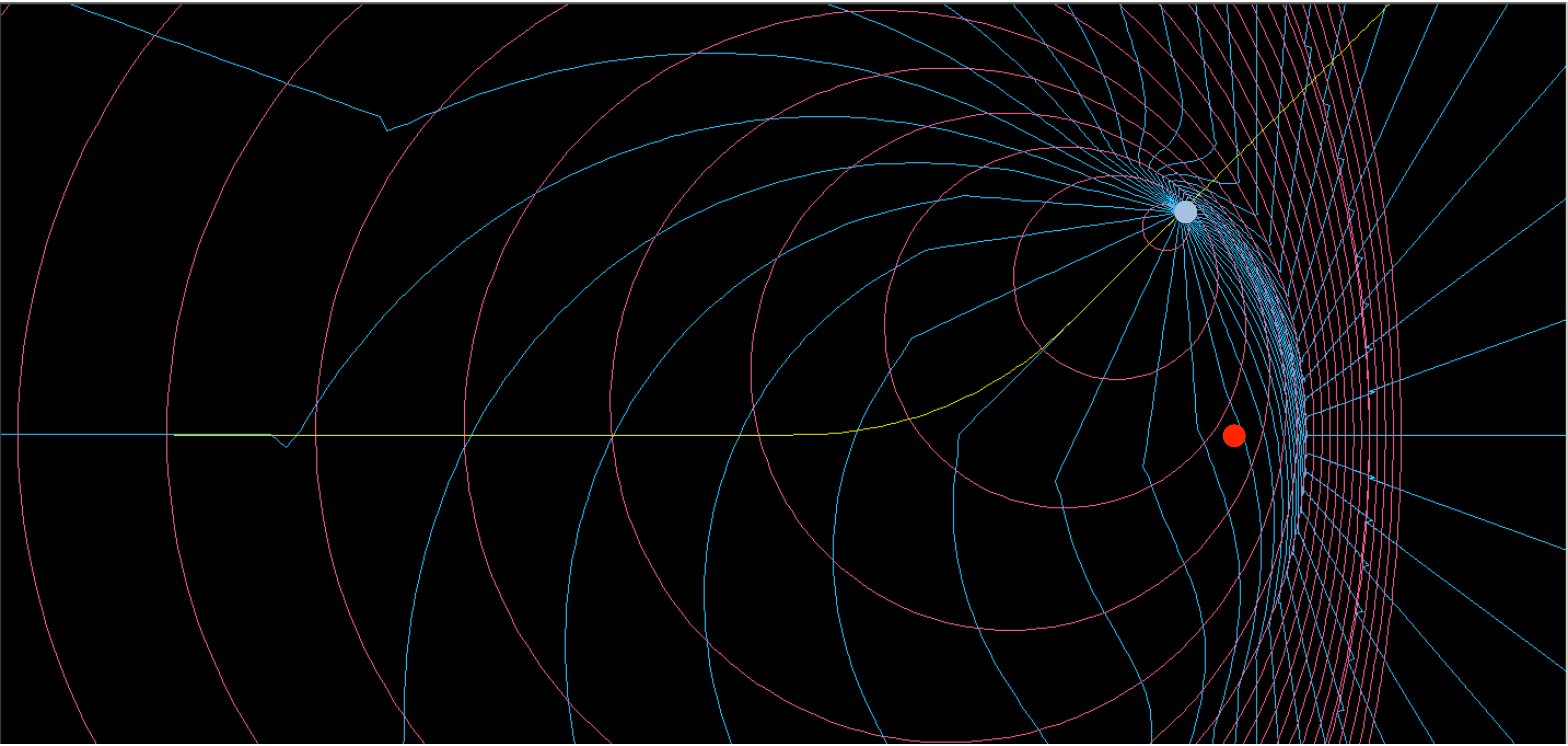
Radiation from moving charge: Curved trajectory



Radiation from moving charge: Curved trajectory

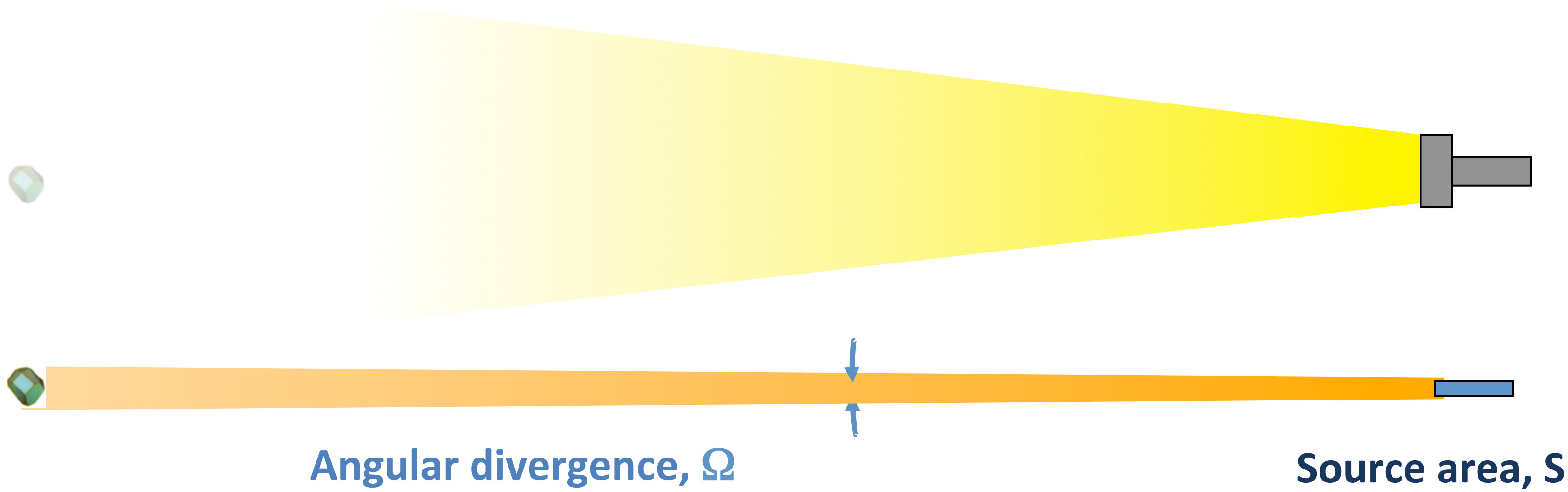


Radiation from moving charge: Curved trajectory



A "good" source of light – high brightness

- Intense, high flux (photons/s).
- Small and collimated.



$$Brightness = \frac{Flux}{S \times \Omega}$$

Why relativistic electrons ?

Relativistic electrons: $v \approx c,\ \beta = \frac{v}{c} \approx 1$ $\gamma = \frac{E}{E_0} \gg 1$ $\gamma^2 = \frac{1}{1-\beta^2}$

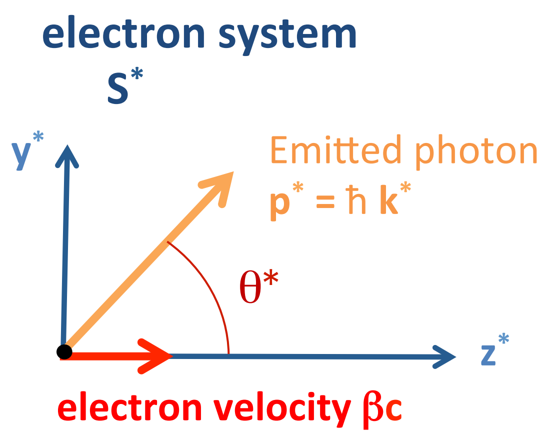
- A relativistic electron beam is easier to focus to a small transverse size, the electrostatic repulsion is cancelled by attractive magnetic forces.

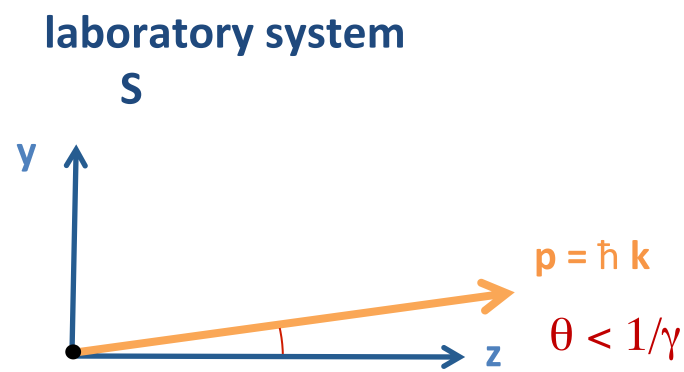
  $F_{res} \propto 1/\gamma^2 \Rightarrow$ **small source size**.

- Electrons emit more light than heavier particles of the same energy.

  $Power \propto \gamma^4 \Rightarrow$ **high flux**.

- **Collimation** of the emitted radiation is proportional to $1/\gamma$





Emission of radiation: spectral and spatial distribution

Consider EM wave in particle system: $E^* = E_0^* e^{i\Phi^*}$

Phase of the wave is: $\Phi^* = \omega^*[t^* - \frac{1}{c}(n_x^* x^* + n_y^* y^* + n_z^* z^*)]$

Phase is product of two 4-vectors: $(i\frac{1}{c}E, \mathbf{p})(ict, \mathbf{r}) = -tE + \mathbf{pr}$

with $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k} = \hbar k\hat{\mathbf{n}}$

$$(i\frac{1}{c}E, \mathbf{p})(ict, \mathbf{r}) = -tE + \mathbf{pr} = -t\hbar\omega + \hbar k\hat{\mathbf{n}}\mathbf{r} = -t\hbar\omega + \hbar\frac{\omega}{c}\hat{\mathbf{n}}\mathbf{r}$$

Product of two 4-vectors is Lorentz invariant, radiation phase is Lorentz invariant:

$$\omega^*[ct^* - n_x^* x^* - n_y^* y^* - n_z^* z^*] = \omega[ct - n_x x - n_y - n_z z]$$

Now apply Lorentz transformation:

$$\omega^*[(-\beta\gamma z + \gamma ct) - n_x^* x - n_y^* y - n_z^*(\gamma z - \beta\gamma ct)] = \omega[ct - n_x x - n_y - n_z z]$$

Relativistic Doppler effect

$$\omega^*[(-\beta\gamma z + \underline{\gamma ct}) - n_x^* x - n_y^* y - n_z^*(\gamma z - \underline{\beta\gamma ct})] = \omega[\underline{ct} - n_x x - n_y - n_z z]$$

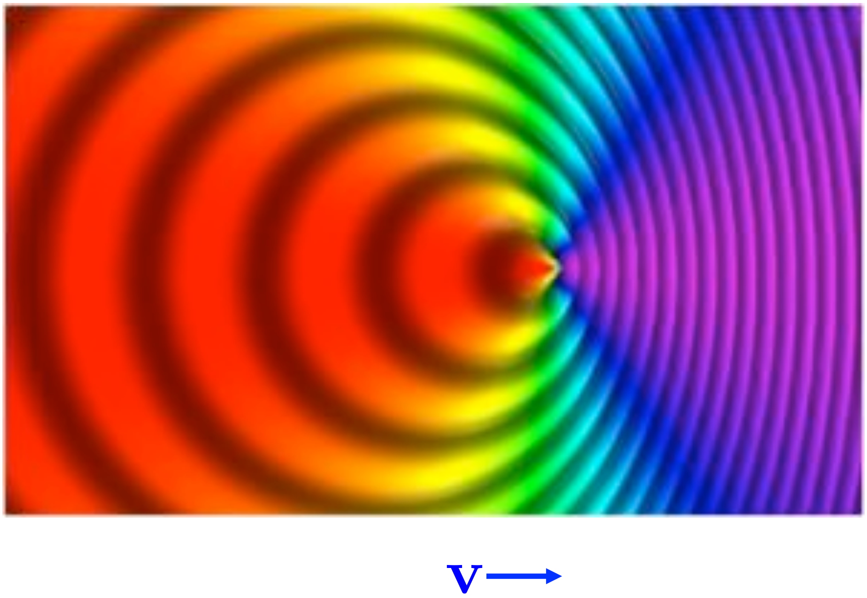
Collecting *ct*-term: $\omega^*\gamma(1 + n_z^*\beta) = \omega$

Relativistic Doppler effect

$$\omega^*[(-\beta\gamma z + \underline{\gamma ct}) - n_x^* x - n_y^* y - n_z^*(\gamma z - \underline{\beta\gamma ct})] = \omega[\underline{ct} - n_x x - n_y - n_z z]$$

Collecting *ct*-term: $\omega^* \gamma(1 + n_z^* \beta) = \omega$

$$\boxed{\omega = \gamma(1 + \beta\ cos\theta^*)\omega^*}$$



In the direction of movement: $\theta^* = 0$

$$\omega = \gamma(1 + \beta)\omega^*$$

For highly relativistic particles: $\beta \approx 1$

$$\boxed{\omega \approx 2\gamma\ \omega^*}$$

The frequency shift can be very large!

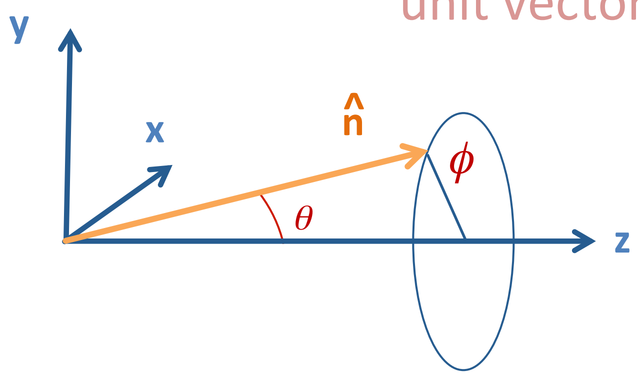
$$\gamma \gg 1 \qquad \gamma \approx 1957 \frac{E}{[GeV]}$$ for electrons

Collimation

$$\omega^*[(-\beta\gamma z + \gamma ct) - n_x^* x^* - n_y^* y^* - n_z^*(\gamma z - \beta\gamma ct] = \omega[ct - n_x x - n_y - n_z z]$$

Collecting x, y, z -terms: $$n_{x,y} = \frac{n^*_{x,y}}{\gamma(1+n_z^*\beta)}$$ and $$n_z = \frac{(\beta + n_z^*)}{(1+n_z^*\beta)}$$

unit vector $\hat{n}$



$$n_x = sin\theta\ sin\phi$$
$$n_y = sin\theta\ cos\phi$$
$$n_z = cos\theta$$

$$n_x^{*2} + n_y^{*2} = sin^2\theta^*$$
$$n_x^2 + n_y^2 = sin^2\theta$$

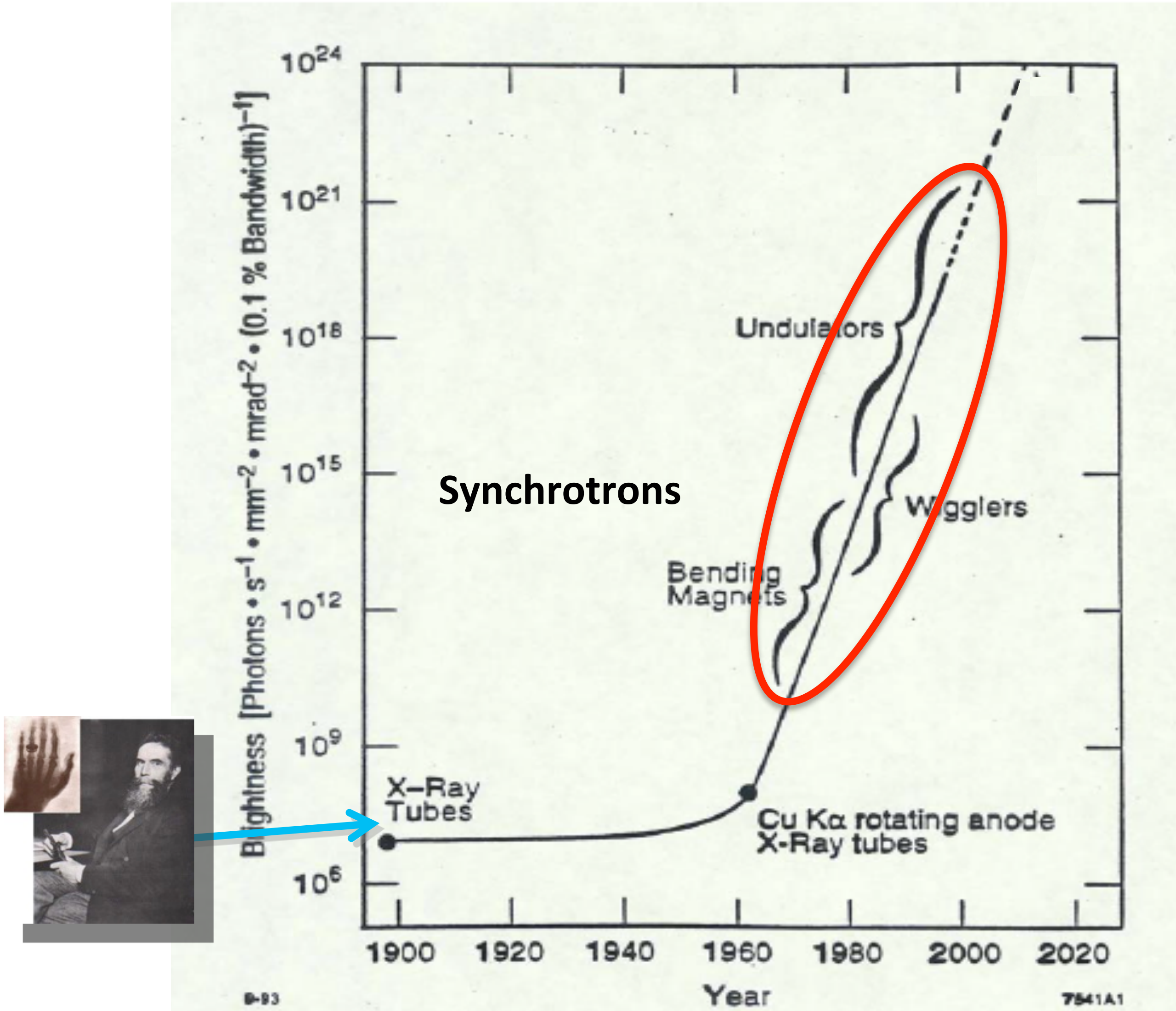
$$sin\theta = \frac{sin\theta^*}{\gamma(1+\beta cos\theta^*)}$$

for $\beta \approx 1$ and $-\pi/2 < \theta^* < \pi/2$

$$\boxed{sin\theta \approx \theta \leq \pm\frac{1}{\gamma}}$$

$$\boxed{\Delta\theta = \frac{2}{\gamma}}$$

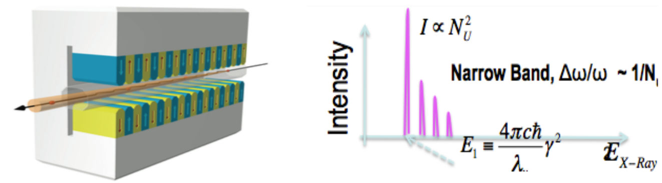
X-Ray brightness



Herman Winick March 26, 2007

4th Generation

approaching diffraction-limit with undulators

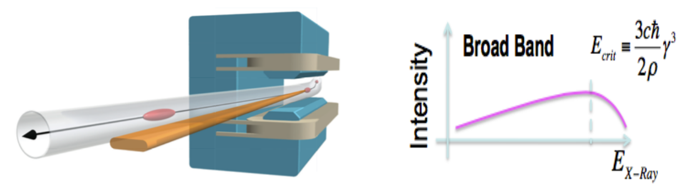


3rd Generation

optimized for brightness, wigglers and undulators

2nd Generation

dedicated sources, bending magnets



1st Generation

parasitic operation, bending magnets

Synchrotron radiation source metrics

- Flux

$$F = \frac{d^2 N_{photons}}{dt\ (d\lambda/\lambda)}$$

$$Flux\ units = \frac{number\ of\ photons\ in\ (\Delta\lambda/\lambda = 0.1\%)}{s}$$

- Brightness

$$B = \frac{d^4 F}{dx\ dy\ dx'\ dy'}$$

$$Bright.\ units = \frac{number\ of\ photons\ in\ (\Delta\lambda/\lambda = 0.1\%)}{s\ mm^2\ mrad^2}$$

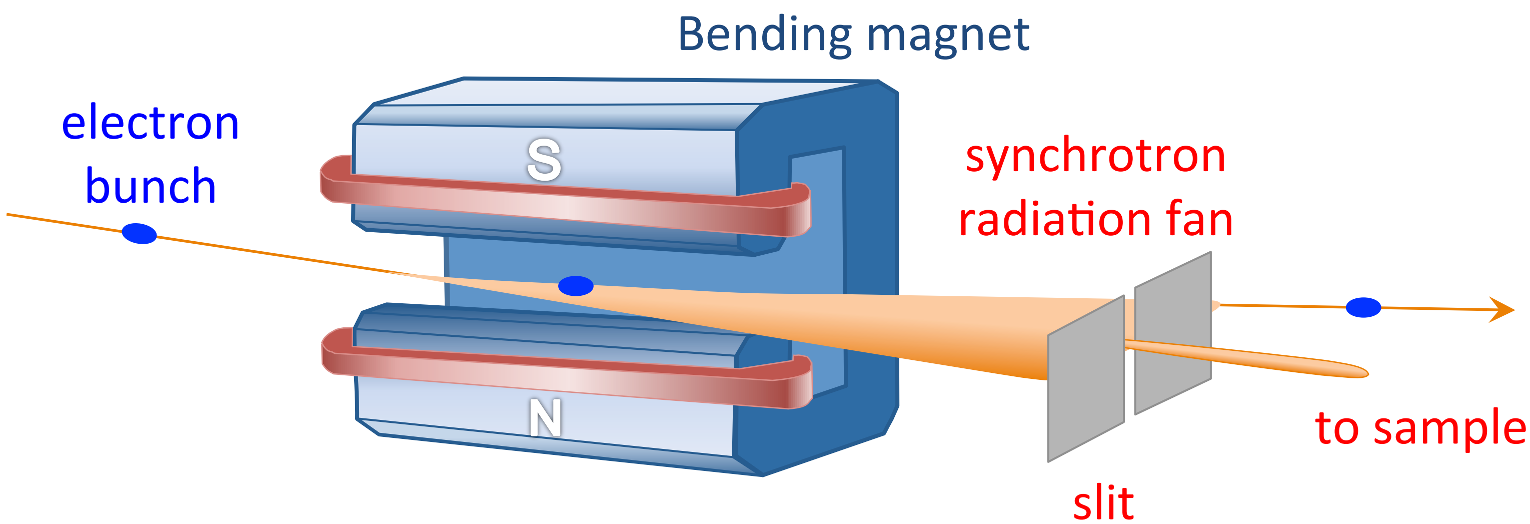
- Coherent flux

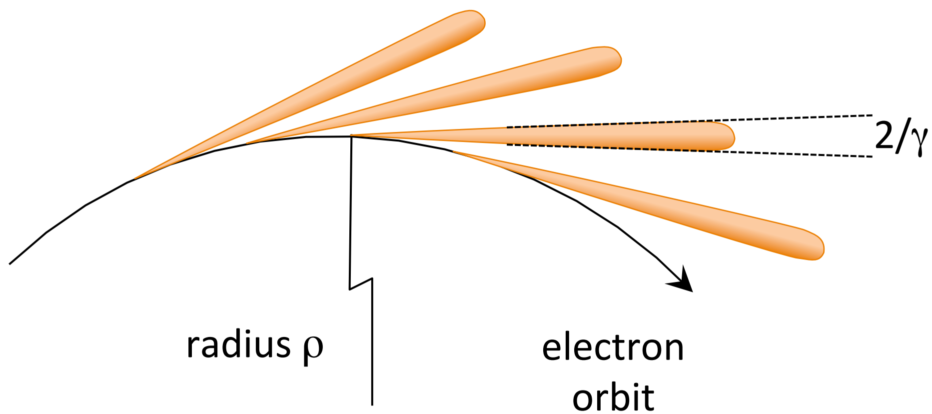
$$F_{coherent} = B \left(\frac{\lambda}{2}\right)^2$$

diffraction limited photon phase-space area

$$2\pi\sigma_r\sigma_{r'} = \frac{\lambda}{2}$$

Synchrotron radiation from dipoles

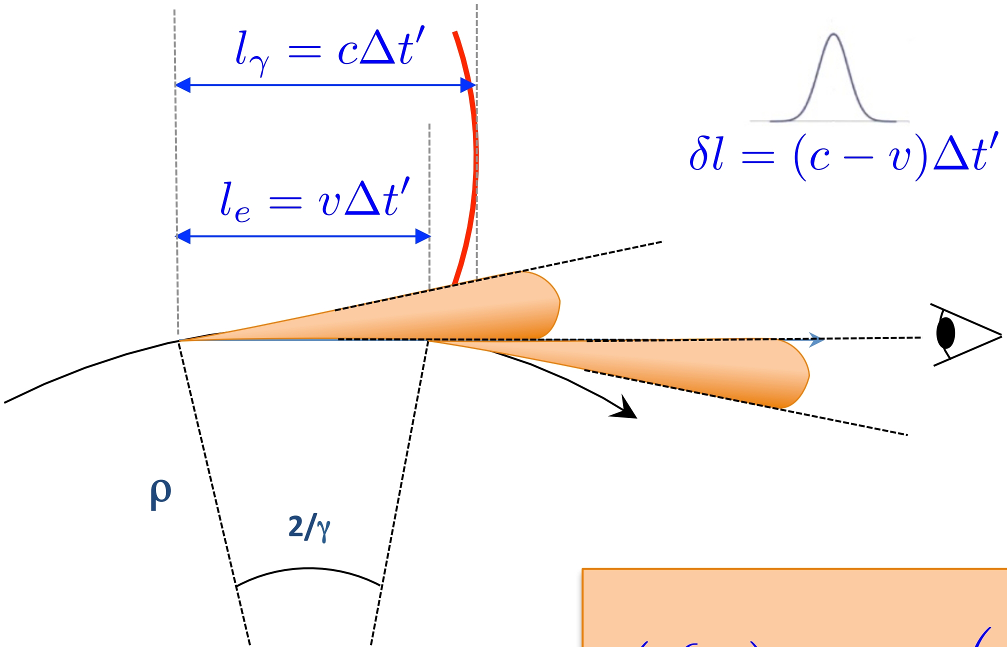




$$E = 3\ GeV$$
$$\gamma \approx 6000$$
$$\Delta\theta = \frac{2}{\gamma} \approx 0.34\ mrad$$

Synchrotron radiation from dipoles: critical energy

Calculation of the pulse duration: critical photon energy/frequency



$$\Delta t = (1-\beta)\Delta t' = (1-\beta)\frac{2\rho/\gamma}{\beta c} \approx \frac{\rho}{2c\gamma^3}$$

$$f_{typical} \approx \frac{1}{\Delta t} \approx \frac{2c\gamma^3}{\rho}$$

$$\omega_c = \frac{3c\gamma^3}{2\rho} \qquad \left(\frac{\epsilon_c}{keV}\right) = 2.2\frac{\left(\frac{E}{GeV}\right)^3}{\left(\frac{\rho}{m}\right)}$$

$$\boxed{\left(\frac{\epsilon_c}{keV}\right) = 0.665\left(\frac{E}{GeV}\right)^2\left(\frac{B}{T}\right)}$$

Critical photon energy

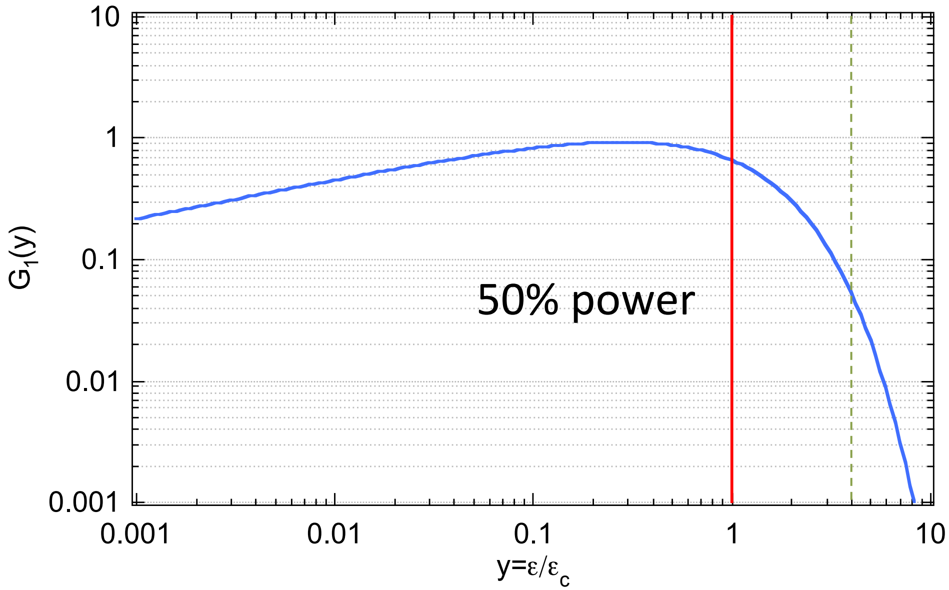
Synchrotron radiation from dipoles: spectral flux

Spectral flux distribution from dipole integrated over vertical angle:

$$\frac{dF}{d\theta} = 2.46 \times 10^{13} \left(\frac{E}{GeV}\right)\left(\frac{I}{A}\right) G_1(y) \qquad y = \frac{\epsilon}{\epsilon_c}$$

θ: horizontal observation angle

The spectral flux distribution, if normalized to the critical frequency, does not depend on the particle energy.



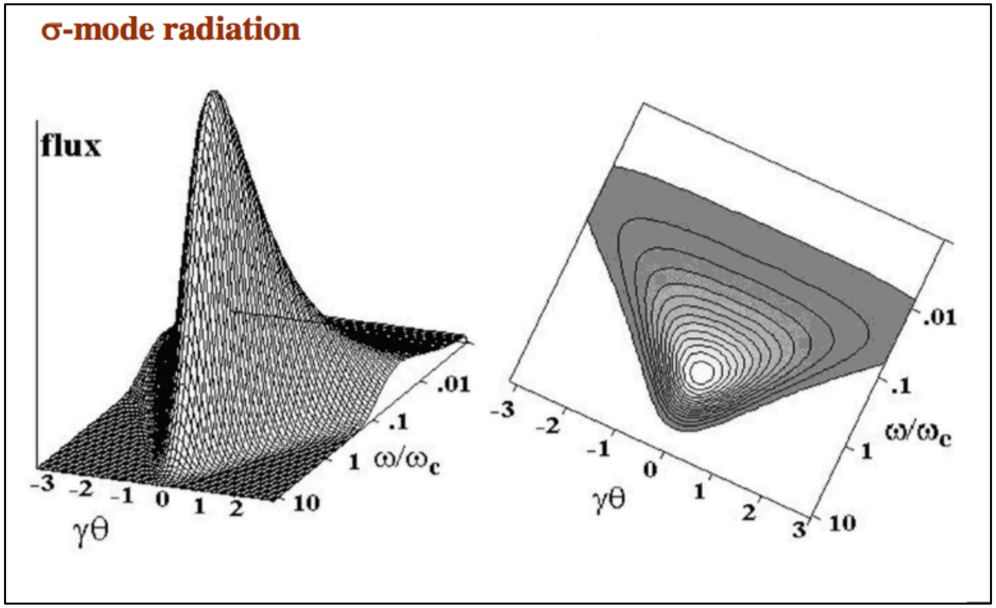
Universal function

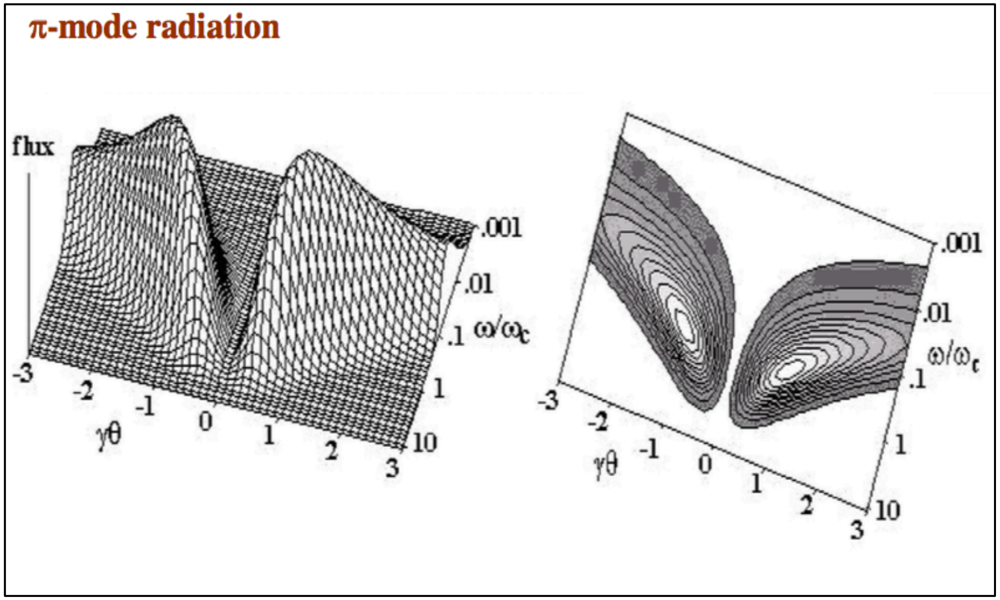
$$G_1(\epsilon/\epsilon_c) = \frac{9\sqrt{3}}{8\pi}\frac{\epsilon}{\epsilon_c}\int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(x)\ dx$$

- Critical energy divides spectrum in 2 parts of equal radiated power.
- Note the slow increase in radiation intensity at low frequencies and the exponential decay above the critical frequency.
- At $\varepsilon \approx 4\ \varepsilon_c$, the intensity decays by one order of magnitude wrt intensity at ε_c

Polarization

- **Linear** in the plane of the ring, **elliptical** out of the plane.
- The sense of the electric field rotation reverses as the vertical observation angle changes from positive to negative.





H. Wiedemann

Synchrotron radiation from insertion devices

- Insertion devices (ID) do not change the shape of the storage ring!

$$\int_{-\infty}^{+\infty} B_y(y=0,z)dz = 0$$

first field integral

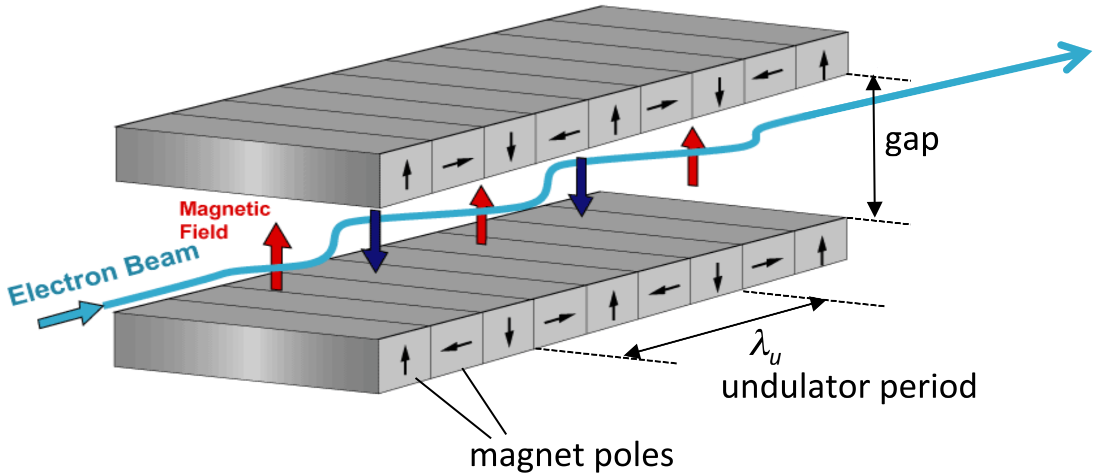
$$\int_{-\infty}^{+\infty} \int_{-\infty}^{z'} B_y(y=0,z)\ dz\ dz' = 0$$

second field integral

- IDs – wigglers and undulators – consist of a series of alternating magnet poles that deflect the beam periodically in opposite directions.
- Purpose:
  - harden radiation
  - increase intensity
  - high brightness monochromatic radiation
  - elliptically polarized radiation
- IDs are the main sources of radiation in 3rd and 4th generation storage rings.

Synchrotron radiation from insertion devices

periodic magnetic fields



magnetic field

$$B_y(z) = B_0\ cos\,(k_u z) \qquad k_u = \frac{2\pi}{\lambda_u}$$

electron path inside ID

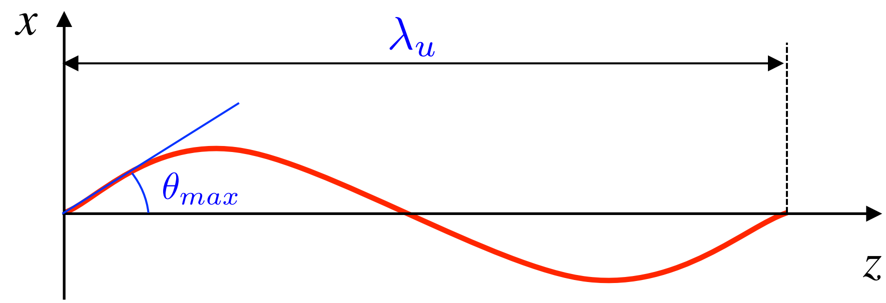
$$x = \frac{eB_0}{\gamma m_0 c k_u^2} cos(k_u z)$$

$$\theta = \frac{dx}{dz} = -\frac{eB_0}{\gamma m_0 c k_u} sin(k_u z)$$

$$K = \frac{\theta_{max}}{1/\gamma} = \frac{eB_0}{m_0 c k_u}$$

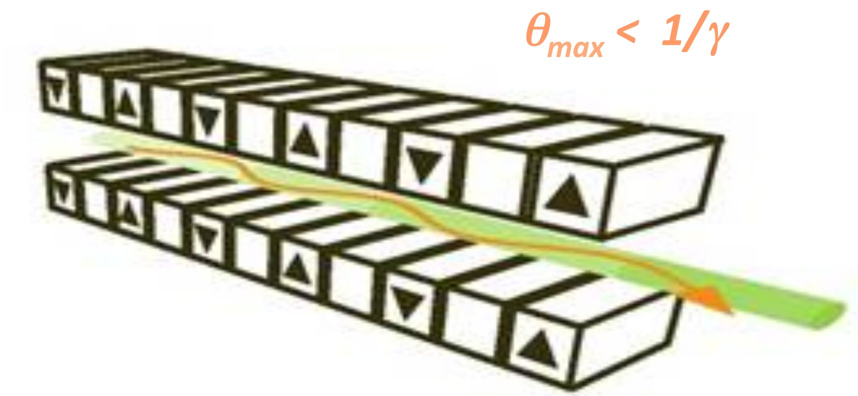
$$\boxed{K = 0.934\ \lambda_u(cm) B_0(T)}$$

$$x_{max} = \frac{K}{\gamma k_u}$$



Wigglers and undulators

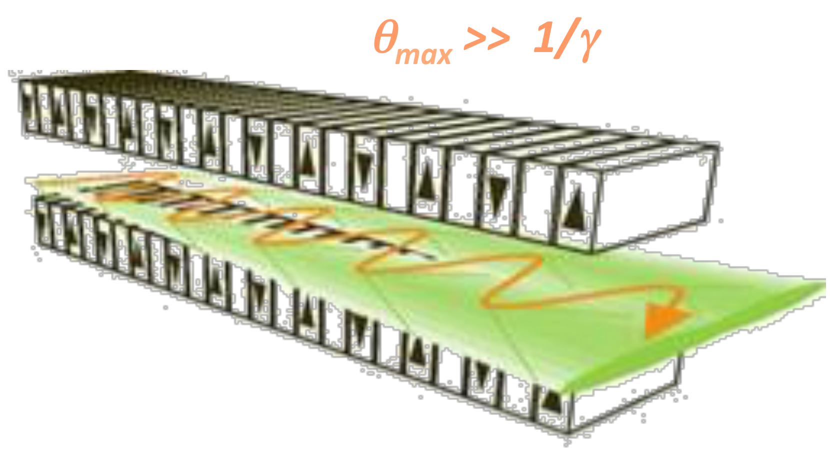
Undulator K < 1



If K<1, the electron trajectory will overlap with the emitted radiation fan and interference effects will occur.

The flux from an undulator has sharp peaks arising from interference of radiation from different poles.

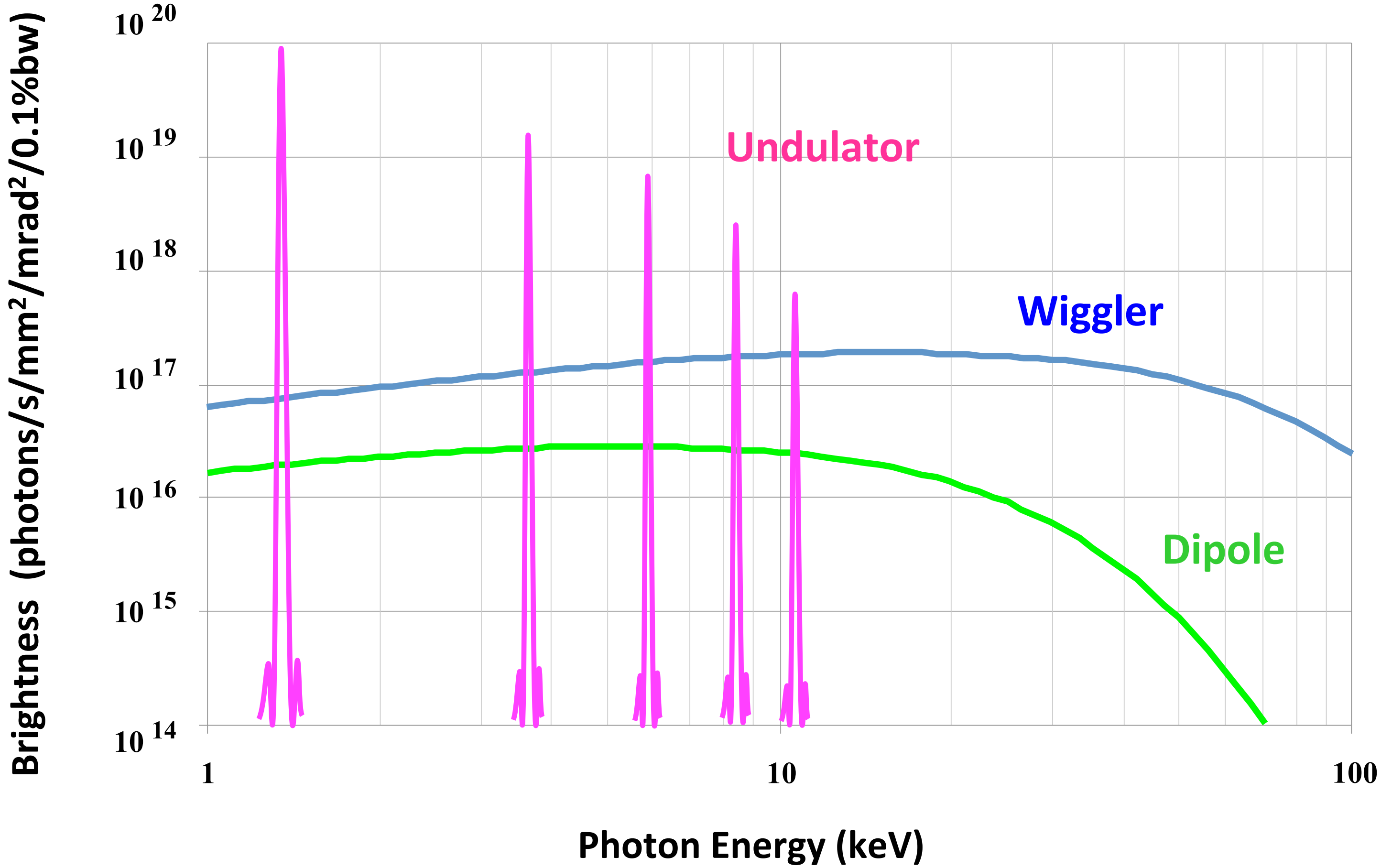
Wiggler K >> 1



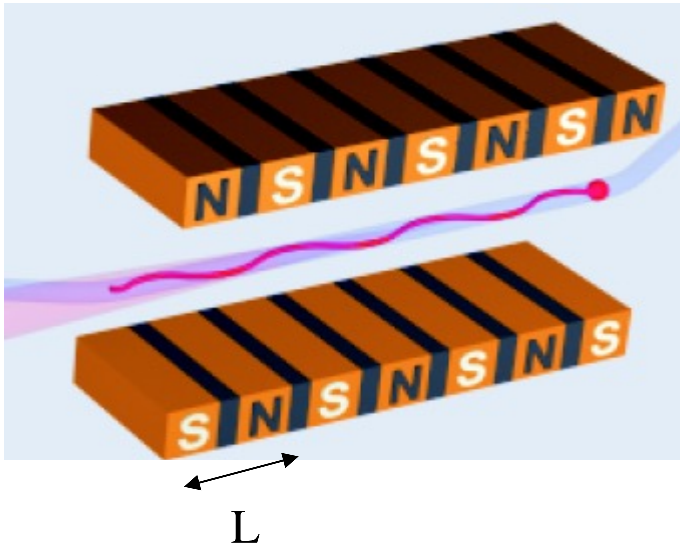
If K >> 1, there will be little overlap and the source points can be treated as independent and bending-magnet like.

The flux from a wiggler is the sum of the flux from N poles.

Wigglers and undulators

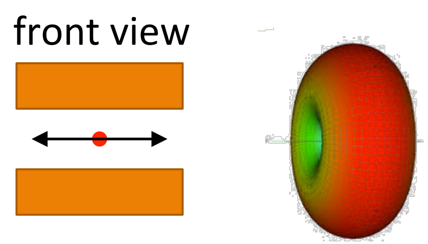


Undulator radiation – first approximation



In the electron frame:

- Periodic B field → periodic B and E-fields moving at speed ≈ c
- Lorentz contraction: L → L/γ
- Emission of waves with wavelength L/γ



In the laboratory frame:

- Doppler effect → wavelength further reduced by a factor of ≈ 2γ, changing from L/γ to L/2γ^2

- **Overall effect: L → L/2γ^2**
- **Centimeters → 0.1 – 1000 Å (x-rays, UV)**

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}$$

Fundamental undulator wavelength

E = 3 GeV
γ = 5871
λ_u = 20 mm
λ_1 = 3 Å

Undulator radiation: corrections to 1st approximation

- The Doppler effect changes with the emission angle θ. The correct Doppler multiplication factor is $\gamma(1+\beta\cos\theta^*)$. Assuming small angles and $\beta \approx 1$:

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$

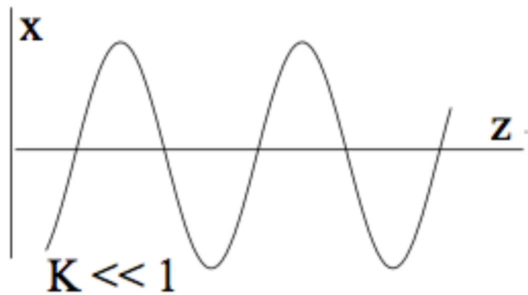
- The undulation of the electron with respect to the z axis decrease its average velocity component

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1+\gamma^2\theta^2+\frac{K^2}{2}\right)$$

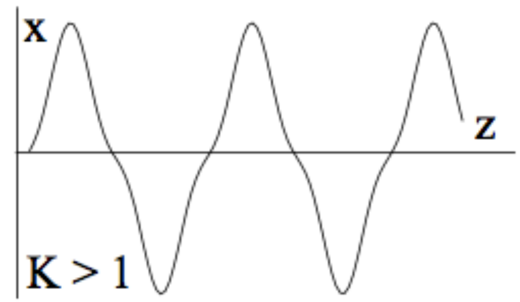
K can be used to tune the undulator frequency. In permanent magnet undulators, B_0 can be varied by changing the undulator gap.

Stronger undulator field

- What happens if we increase the field strength ?
  1) Higher odd harmonics
  2) Even harmonics



transverse motion
completely
non-relativistic



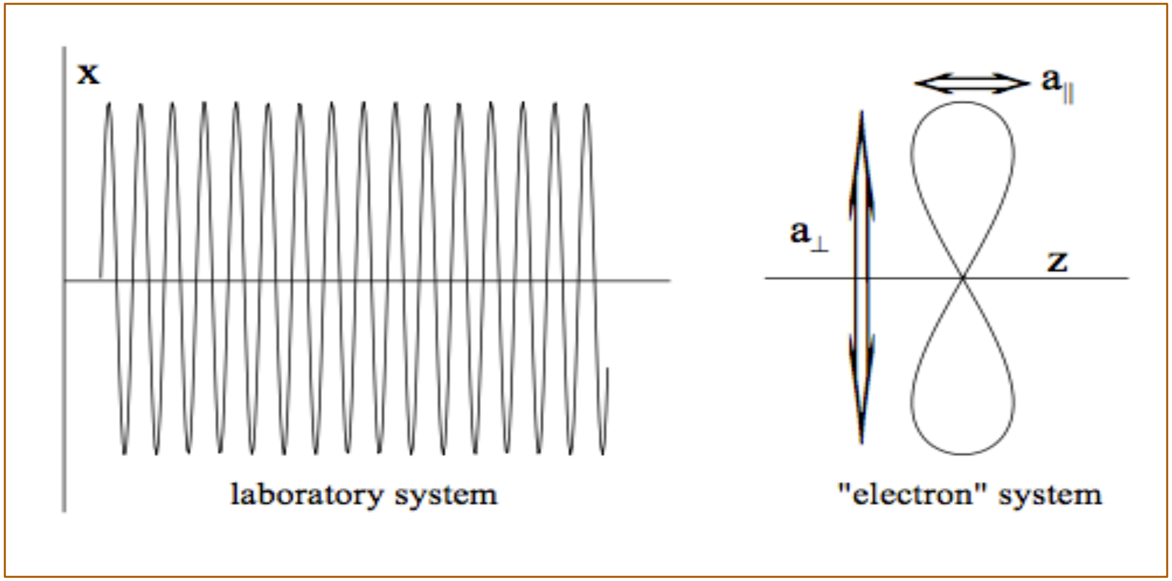
relativistic effect on
transverse motion

H. Wiedemann

The perturbation is symmetric about the cusps and valleys causing appearance of **higher odd-harmonics (3rd, 5th, 7th, …)** of the fundamental oscillation.

Stronger undulator field

- What happens if we increase the field strength ?
    1) Higher odd harmonics
    2) Even harmonics



$$x(t) = a\ cos(k_u c\bar{\beta} t)$$

$$z(t) = c\bar{\beta}t + \frac{1}{8}k_u a^2 sin(2k_u c\bar{\beta}t)$$

$$\bar{\beta} = \beta\left(1 - \frac{K^2}{4\gamma^2}\right)$$

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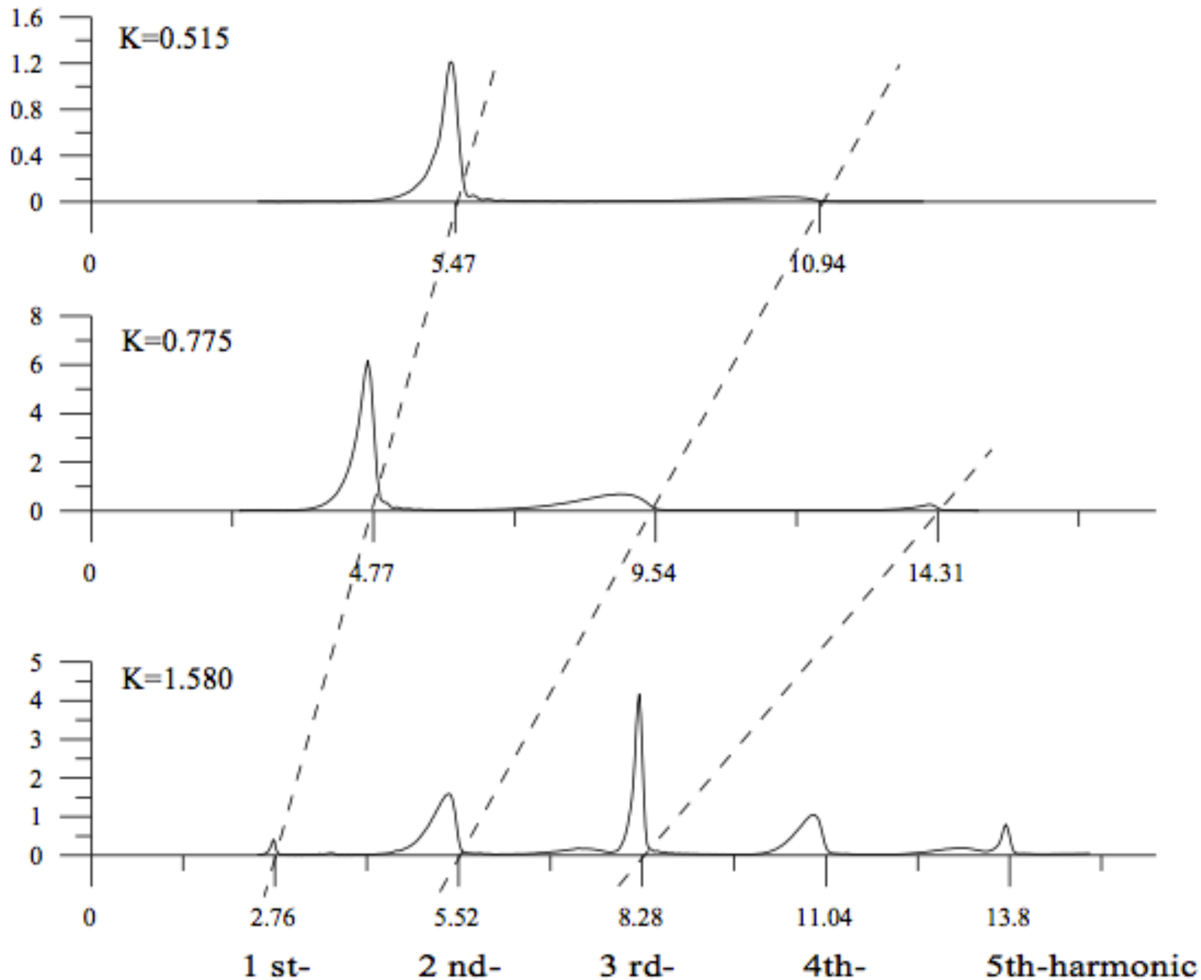
Longitudinal oscillation generates **even-harmonics (2nd, 4th, 6th, …)** of the fundamental oscillation.

PEP-undulator, 77 mm, 27 periods, 7.1 GeV



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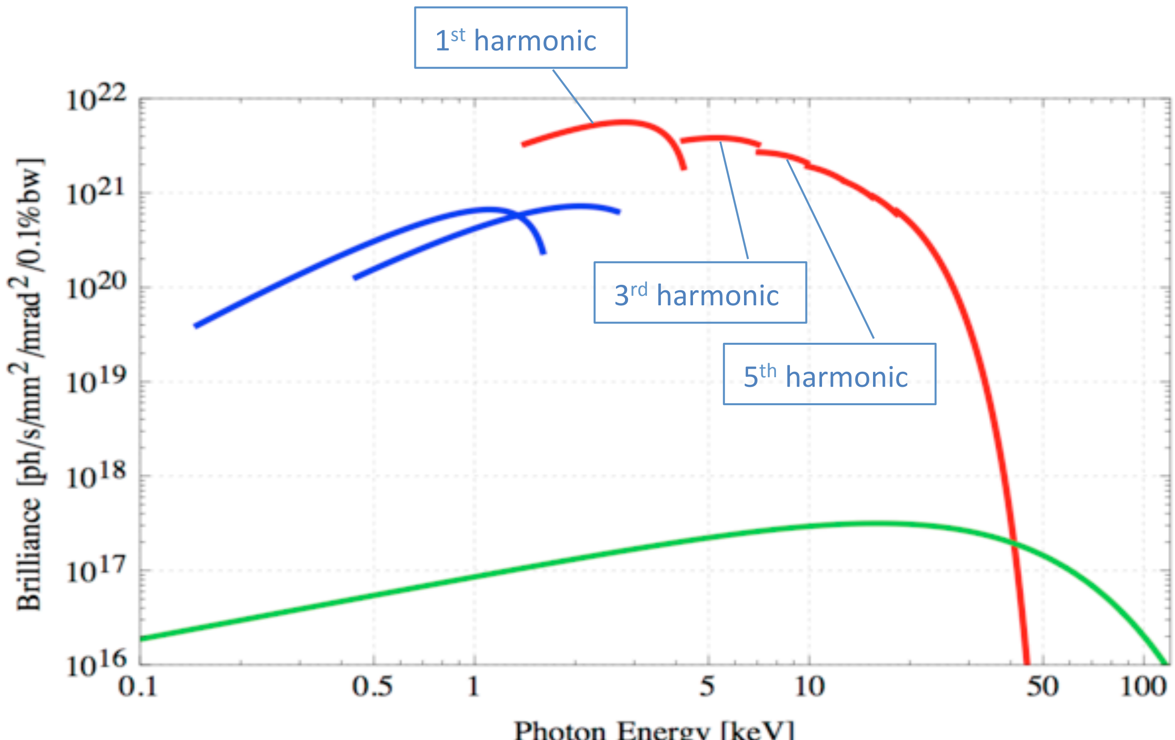
Tuning the peaks of undulator radiation

The undulator strength K can be adjusted by changing the gap: $K = 0.934\ \lambda_u(cm) B_0(T)$

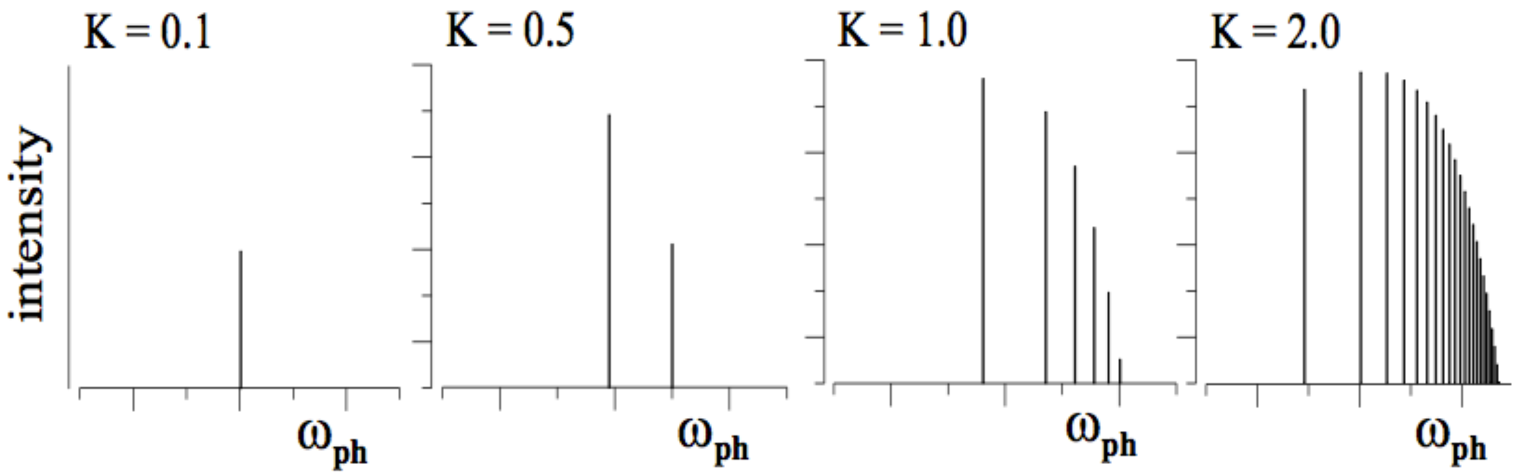
The radiated harmonics will change according to: $\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1 + \gamma^2\theta^2 + \frac{K^2}{2}\right)$

The lowest energy (largest λ) is fixed by the minimum gap (largest K). As the gap is opened, the magnetic field decreases and the X-ray energy increases. The intensity of the radiation decreases.



Red curve: tuning range for Sirius IVU19 undulator. λ_u=19 mm, minimum gap=4.5 mm (K=2.3)

Transition from undulator to wiggler



- Critical photon energy from wiggler magnet at angle θ with axis:

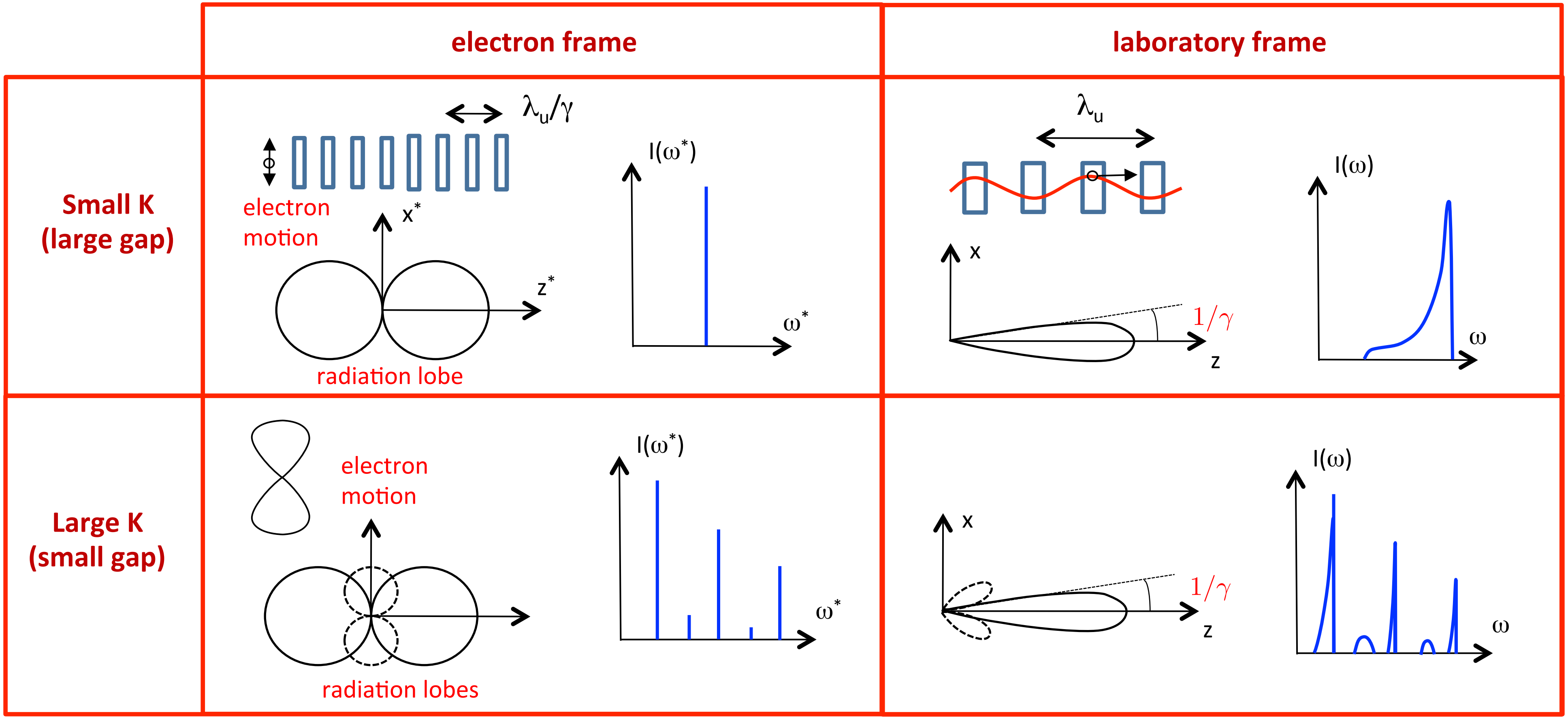
$$\epsilon_c(\theta) = \epsilon_c(0)\sqrt{1 - \left(\frac{\gamma\theta}{K}\right)^2}$$

- 2 source points separated by

$$x_0 = \frac{K}{\gamma}\frac{\lambda_w}{2\pi}$$

- Polarization remains linear out of the plane of the orbit.

Undulator radiation



bandwidth

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{nN}$$

angular spread

$$\theta_{fwhm} \approx \frac{1}{\gamma}\sqrt{\frac{1+K^2/2}{nN}}$$

The more periods, the more monochromatic and collimated the radiation is.

Types of undulators

- Short period undulators: increase photon energy.
  - In-vacuum: elimination of the vacuum chamber allows smaller gaps.
  - Cryogenic PM: improve magnetic properties of PM materials.
  - Superconducting: use SC wire to increase magnetic field.

- Non-planar undulators: produce circular polarization with crossed magnetic fields, B_x and B_y.
  - Apple-II, Delta
  - Helical
- Quasi-periodic undulators: suppress higher harmonics
- Revolver undulators
- Canted undulators


ESRF undulator

NSLS-II 3 m in vacuum undulator

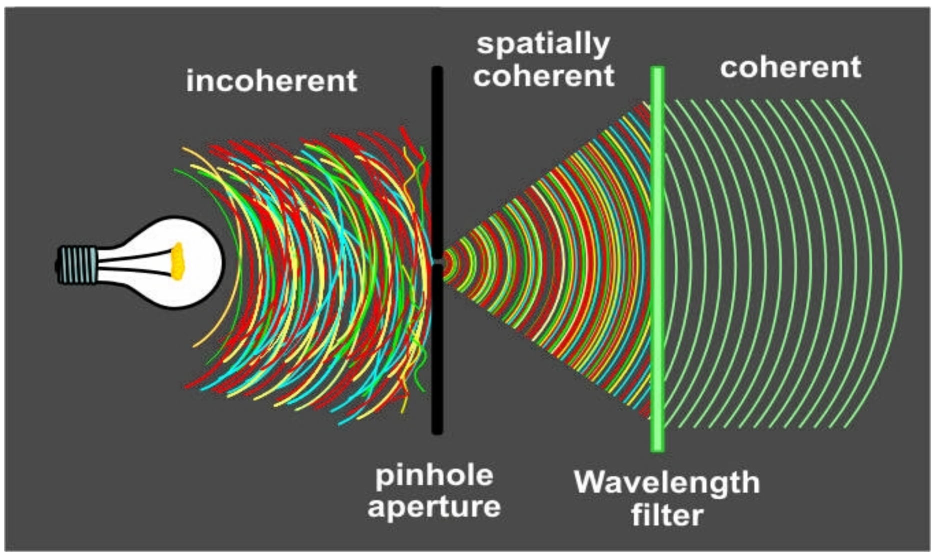

APS SC undulator

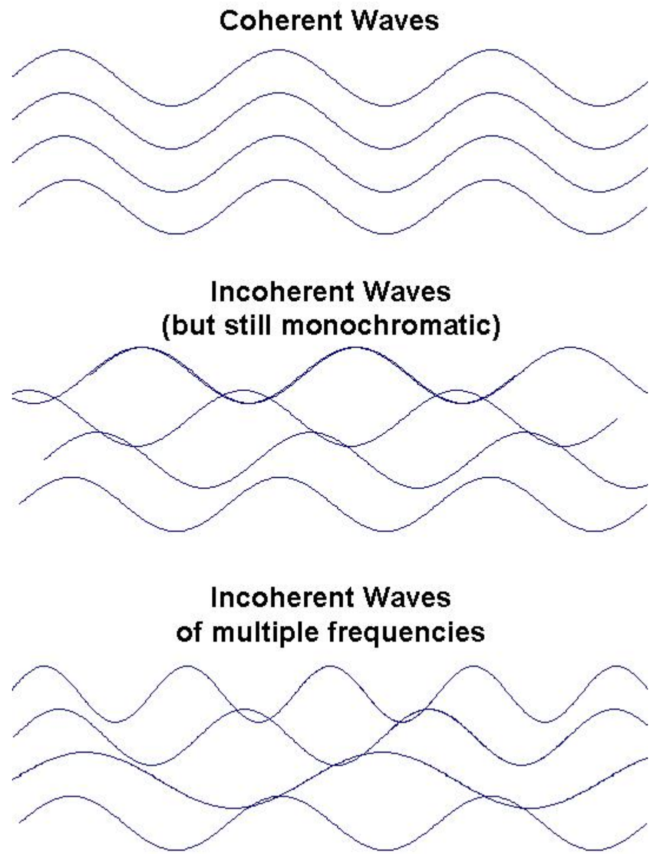
LNLS APPLE-II undulator

Coherence

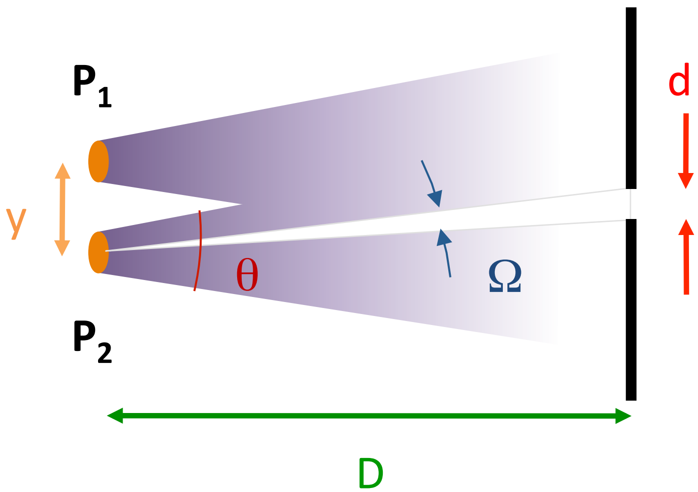
A coherent wave is a wave that can produce observable interference and diffraction phenomena.

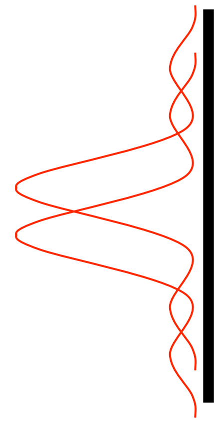
Coherent sources of visible light are available for a long time. Coherent sources of x-rays, on the contrary, were not available until recently.





Transverse (space) coherence





central diffraction maximum

$$\alpha_0(P_1) = 0$$
$$\alpha_0(P_2) = y/D$$

Pattern can still be detected if

$$\alpha_0(P_2) < \alpha_1(P_1)$$
$$or$$
$$y\Omega < 2\lambda$$

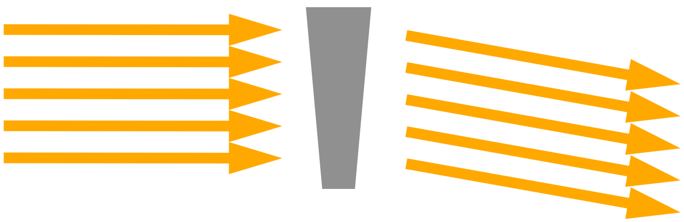
- If the emission occurs over θ, only a fraction ***$\Omega/\theta = 2\lambda/y\theta$*** produces diffraction. This defines a coherence power

$$\left(\frac{\Omega}{\theta}\right)_x \left(\frac{\Omega}{\theta}\right)_y = \frac{4\lambda^2}{y\theta_y \; x\theta_x}$$

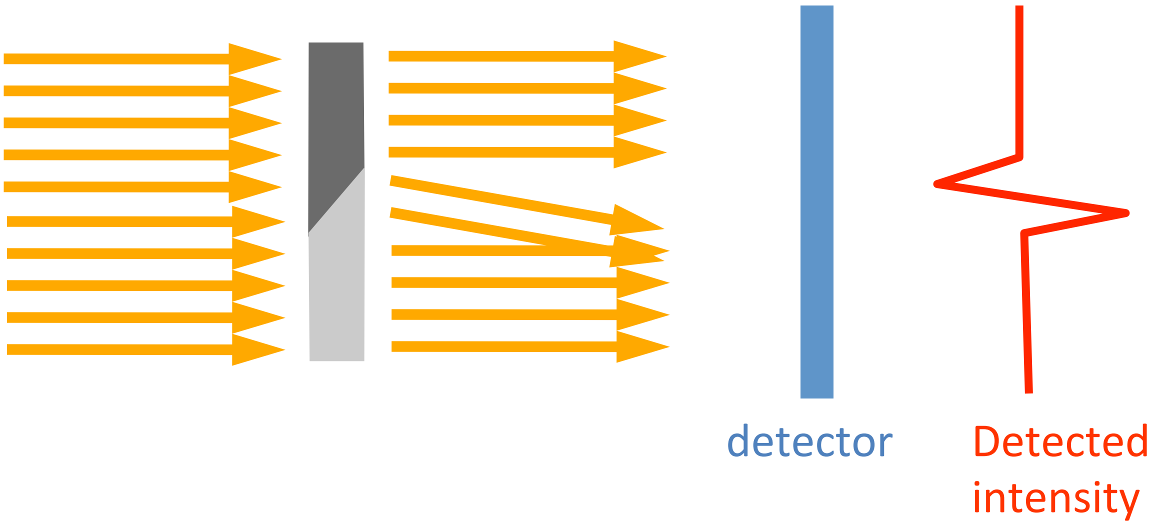
  - Proportional to λ^2
  - Improvement in beam size and divergence increases both Brightness and Transverse Coherence.
- Full transverse coherence → diffraction limit

Example: X-ray imaging with coherent light

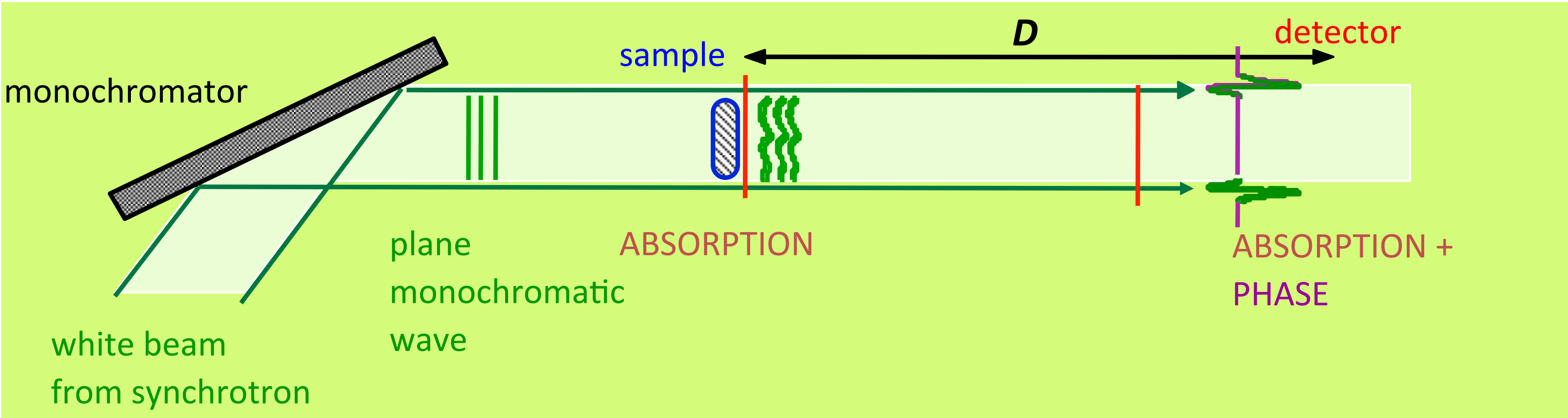
Absorption – described by the absorption coefficient



Refraction (and diffraction/ interference) – described by the refractive index

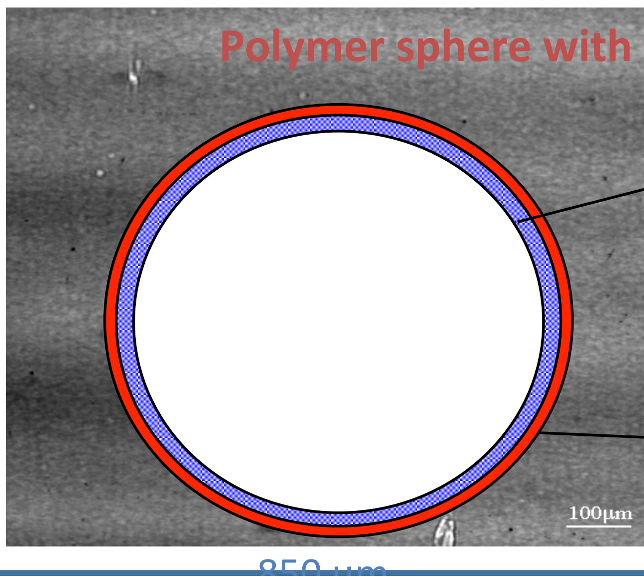


Phase propagation contrast



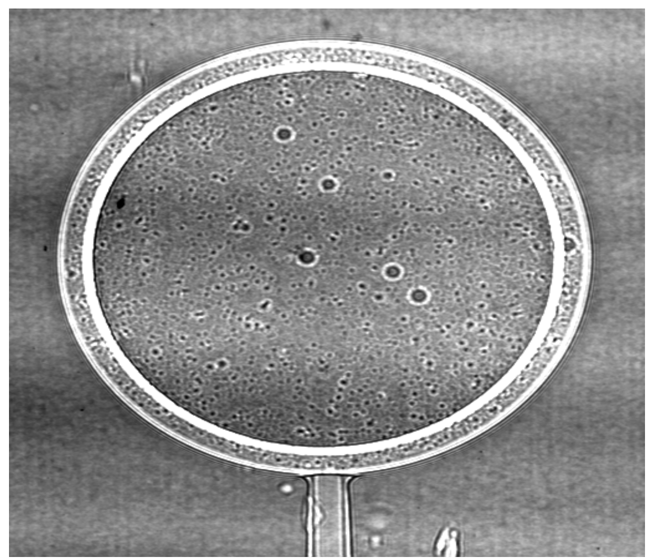
Absorption

Propagation



inner layer
polystyrene
thickness 30 µm

outer layer
parylene
thickness 15 µm



Muchas Gracias!

References

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