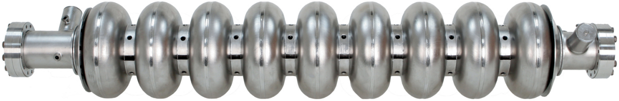
RF Cavities I

M. Sc. Salvador Sosa Güitrón

Old Dominion University

Mexican Particle Accelerator School 2015

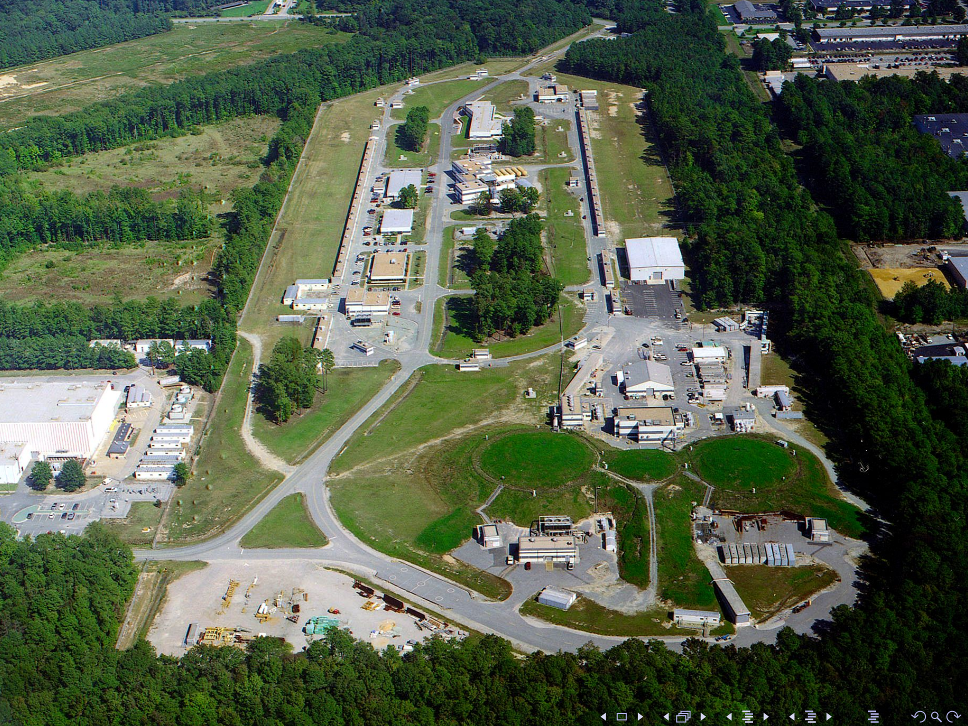
Guanajuato, Mexico



Overview

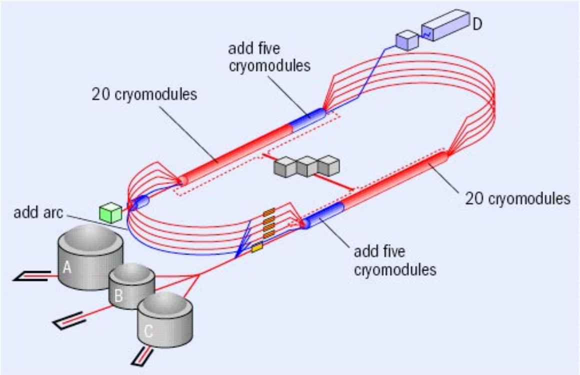
1. RF Cavities
   - Figures of Merit
   - Coupled Cavity
   - Pill-box Cavity
2. Designing Real Cavities
   - Multicell Cavities
   - Mechanical Analysis
   - HOM
   - Frequency Control
3. Cavity Examples


INPUT
SL17
SL16



CEBAF 12 GeV Upgrade

- Exceptional performance of the 5-cell SRF cavities used in CEBAF.
- 10 new C-100 cryomodules using 7-cell Low-Loss SRF cavities and overall having the same length as their predecessors.
- Double the He refrigeration capacity.
- Beam permit status during Fall 2015!



RF Cavities

- Cavity enclosed by conducting walls, can sustain resonant electromagnetic modes.
- Electromagnetic cavity resonating at MW frequencies (50-3000 MHz).
- Shape optimized so that a particular mode can efficiently transfer energy to the beam
- Cavity considerations depend on particular application.

Lorentz Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

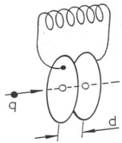
- Electric field must be longitudinal to the particle velocity
- Magnetic field exerts deflection, but no acceleration

DC vs RF

- DC accelerators are limited by breakdown voltage, only a few MeV
- High energy accesible by using traveling waves
- Energy transfer from a wave to a particle is more efficient if they propagate at the same velocity

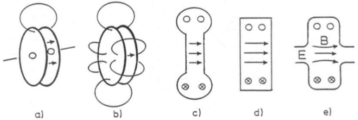
Equivalent Circuit for an rf Cavity

Simple LC circuit representing an accelerating resonator



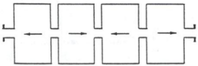
Simple lumped L-C circuit repesenting an accelerating resonator.
$\omega_o^2 = 1/LC$

Metamorphosis of the LC circuit into an accelerating cavity



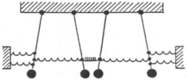
Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical β between 0.5 and 1.0). Fig. 5c resembles a low β version of the pillbox variety ($0.2<\beta<0.5$).

Chain of weakly coupled pillbox cavities representing an accelerating module



Chain of weakly-coupled pillbox cavities representing an accelerating module

Chain of coupled pendula as its mechanical analogue



Chain of coupled pendula as a mechanical analogue to Fig. 6a

Electromagnetic Modes

Electromagnetic modes satisfy Maxwell wave equations:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$
$$\nabla^2 \mathbf{H} - \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

The cavity walls are included through the boundary conditions:

- No tangential electric field, $\hat{n} \times \mathbf{E} = 0$
- No normal magnetic field, $\hat{n} \cdot \mathbf{H} = 0$

Electromagnetic Modes Solutions

Assume harmonic solutions proportional to $\exp(-i\omega t)$:

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = 0$$
$$\nabla^2 \mathbf{H} + \frac{\omega^2}{c^2}\mathbf{H} = 0$$

- Infinite solutions...
- For acceleration, choose a mode with:
  - Electric field along the particle trajectory
  - Null magnetic field along the particle trajectory
  - Field propagation matched to particle velocity

Accelerating Field E_{acc}

- Time of particle spent in the cavity:

$$t = \frac{d}{c} = \frac{1}{2} T_{RF} \tag{1}$$

- Accelerating voltage:

$$V_{acc} = \left| \int_{z=0}^{z=d} E_{el} dz \right| \tag{2}$$

- Average Accelerating Field:

$$\boxed{E_{acc} = \frac{V_{acc}}{d}} \tag{3}$$

Energy in the Cavity

Energy density in the electromagnetic field:

$$u = \frac{1}{2}(\epsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{B}^2) \tag{4}$$

- Harmonic dependence means the energy oscillates from the electric to the magnetic field
- Energy stored in E field same as energy stored in H field
- Total energy in the cavity

$$\boxed{U = \frac{1}{2}\mu_0 \int_v |\mathbf{H}|^2 dv = \frac{1}{2}\epsilon_0 \int_v |\mathbf{E}|^2 dv} \tag{5}$$

Power Dissipation

- Currents flow in a thin layer near the surface of the cavity
- Power Dissipation per unit area, due to Joule Heating:

$$P_a = \frac{1}{2} R_s \mathbf{H}^2 \tag{6}$$

- Total power dissipated in the cavity walls

$$\boxed{P_c = \frac{1}{2} R_s \oint_s |\mathbf{H}|^2 ds} \tag{7}$$

Scaling of the Power Dissipated

- Normal Conductors, $R_s \propto \omega^{1/2}$

$$P/A \propto \omega^{1/2} \tag{8}$$

- Superconductors, $R_s \propto \omega^2$

$$P/A \propto \omega^2 \tag{9}$$

Quality Factor Q_0

- Definition:

$$Q_0 = \omega \frac{\text{Energy Stored in the Cavity}}{\text{Power Dissipated in the Cavity Walls}} = \frac{\omega U}{P_c} \tag{10}$$

- In terms of the fields:

$$\boxed{Q_0 = \frac{\omega \mu_0 \int_v |\mathbf{H}|^2 dv}{Rs \oint_s |\mathbf{H}|^2 ds}} \tag{11}$$

G factor

- Geometric Factor, $G = Q_0 R_s$:
- Depends on cavity shape and electromagnetic mode
- Independent of cavity size and material

$$G = Q_0 R_s = \omega\mu_0 \frac{\int_v |\mathbf{H}|^2 dv}{\oint_s |\mathbf{H}|^2 ds} \tag{12}$$

Shunt Impedance, R/Q

Shunt impedance R_{sh}:

$$R_{sh} \equiv \frac{V_c^2}{P_{diss}} \quad \text{in } \Omega$$

V_c = accelerating voltage

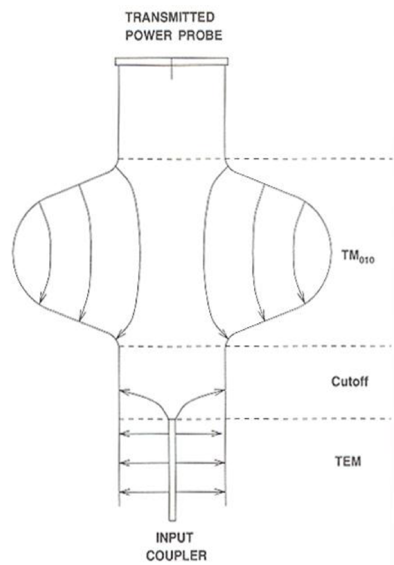
Note: Sometimes the shunt impedance is defined as $\frac{V_c^2}{2P_{diss}}$
or quoted as impedance per unit length (ohm/m)

R/Q (in Ω)

$$\frac{R}{Q} = \frac{V^2}{P}\frac{P}{\omega U} = \frac{E^2}{U}\frac{L^2}{\omega}$$

External Coupling

- Input coupler, power from RF source transported to the cavity through coaxial cable
- Waveguides: electromagnetic mode conversion
- Coupling strength adjusted by changing the penetration of the antenna
- Output coupler picks up power transmitted through the cavity, weakly coupled



Cavity with external coupling

- Energized RF cavity and turn off the RF:

$$\frac{dU}{dt} = -P_{total} \tag{13}$$

- $P_{total} = P_c + P_e + P_t$
  - P_e is the power dissipated in the input power coupler
  - P_t is the power going into transmitted power coupler (weakly coupled)
- $P_{total} \approx P_c + P_e$

Loaded Quality Factor Q_L

Define, analogous to Q_0:

$$\boxed{Q_L = \frac{\omega_0 U}{P_{total}}} \tag{14}$$

Then,

$$\frac{dU}{dt} = -\frac{\omega_0 U}{Q_L} \tag{15}$$

has solution

$$U = U_0 \exp\left(-\frac{\omega_0 t}{Q_L}\right) \tag{16}$$

The energy decays exponentially with characteristic decay time

$$\boxed{\tau_L = \frac{Q_L}{\omega_0}} \tag{17}$$

Q external

From the last slide:

$$\frac{P_{total}}{\omega_0 U} = \frac{P_c + P_e}{\omega_0 U} \tag{18}$$

introduce

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e} \tag{19}$$

Definition of external Q:

$$\boxed{Q_e = \frac{\omega_0 U}{P_e}} \tag{20}$$

Typical values for CEBAF 7-cell cavities:

- $Q_0 = 1 \times 10^{10}$
- $Q_e = 2 \times 10^{7}$

Coupling Coefficient

- Coupling coefficient measures interaction of the coupler with the cavity

Define

$$\beta \equiv \frac{Q_0}{Q_e} \tag{21}$$

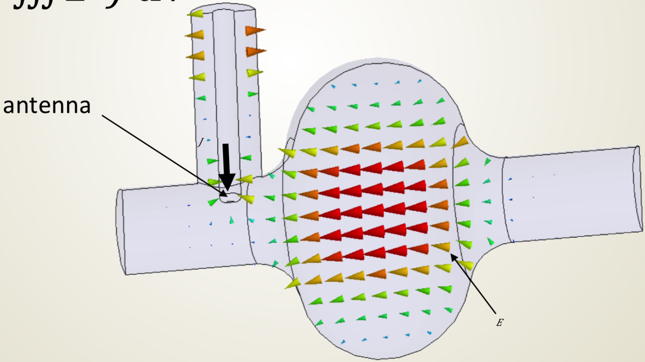
then

$$\frac{1}{Q_L} = \frac{1+\beta}{Q_0} \tag{22}$$

$$\boxed{\beta = \frac{P_e}{P_c}} \tag{23}$$

Electric (antenna) coupling

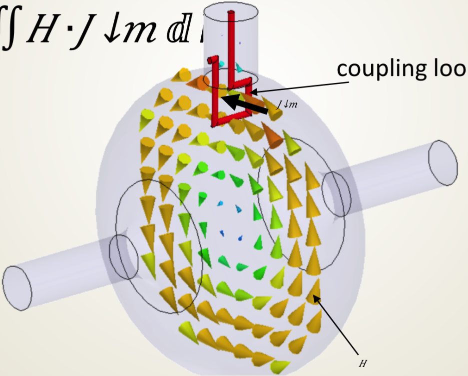
- The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.
- The coupling can be adjusted by varying the penetration.
- Coupling $\propto \iiint E \cdot J \, dV$



courtesy: David Alesini/INFN

Magnetic (loop) coupling

- The magnetic field of the cavity main mode is intercepted by a coupling loop
- The coupling can be adjusted by changing the size or the orientation of the loop.
- Coupling: $\propto \iiint H \cdot J \downarrow m \, d$



courtesy: David Alesini/INFN

Several Loss Mechanisms

- Wall losses
- Power absorbed by beam
- Coupling to outside world

$$P = \sum_k P_k \tag{24}$$

Associate a Q to each loss mechanism

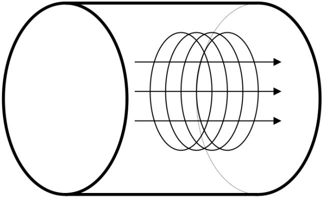
$$\frac{1}{Q_L} = \frac{\sum_k P_k}{\omega U}, \qquad \beta_k = \frac{Q_0}{Q_k} = \frac{P_k}{P_0} \tag{25}$$

Pill Box Cavity

Hollow right cylindrical enclosure

Operated in the TM_{010} mode $H_z = 0$

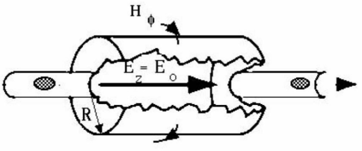
TM_{010} mode



$$\frac{\partial^2 E_z}{\partial^2 r} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial^2 t} \qquad \omega_0 = \frac{2.405c}{R}$$

$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$



Modes in Pill Box Cavity

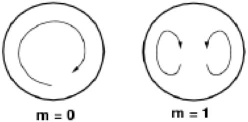
- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length
- TM_{0mn}
  - Monopoles modes that can couple to the beam and exchange energy
- TM_{1mn}
  - Dipole modes that can deflect the beam
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam

TM mode indices *mnp*

The modes are described by three indices.

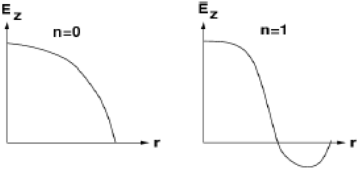
m is the number of variation of field of the azimuthal variable ϕ:

m = 0, 1, 2, ...



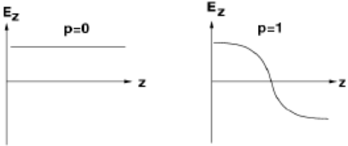
n is the number of nulls in E_z along the radial direction

n = 1, 2, 3, ...



p is the number of nodes of E_z along the z-axis.

p = 0, 1, 2, ...



TM_{010} Mode in a Pill Box Cavity

Energy content

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

Power dissipation

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R+L)R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

Geometrical factor

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R+L)}$$

TM010 Mode in a Pill Box Cavity

Energy Gain

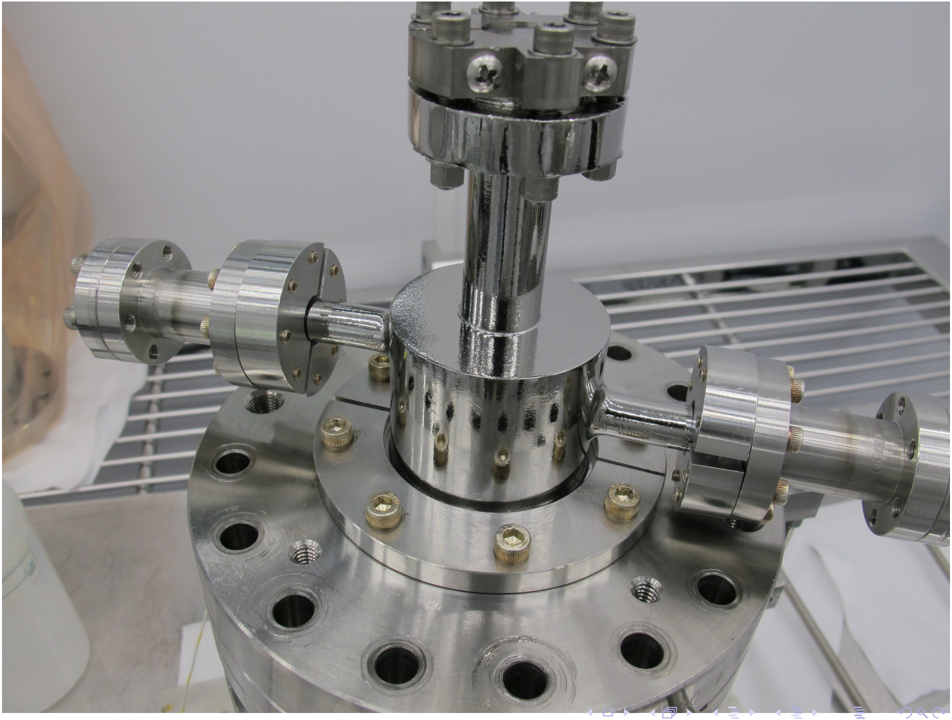
$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

Gradient

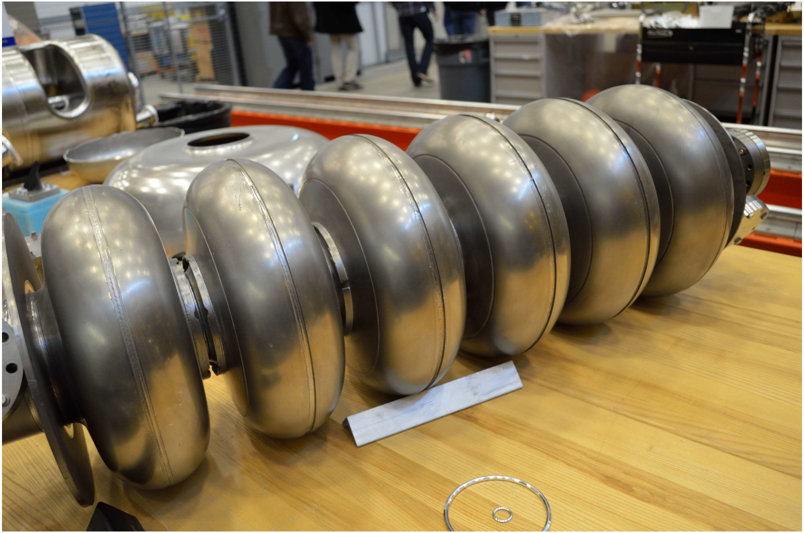
$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

Shunt impedance

$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2 \left(\frac{\pi L}{\lambda} \right)$$



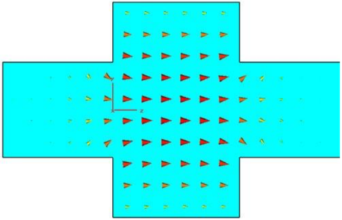
Elliptical Cavities

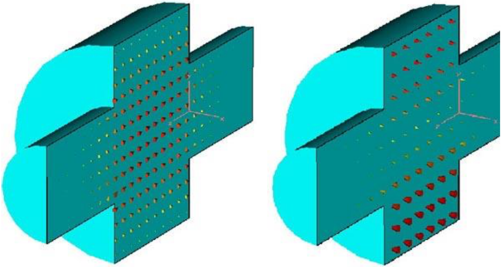


Real Cavities

Beam tubes reduce the electric field on axis

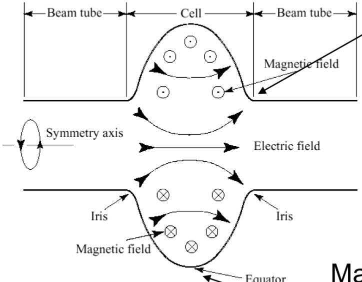
- Gradient decreases
- Peak fields increase
- R/Q decreases

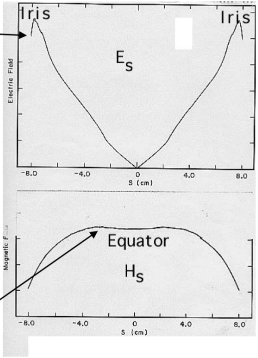




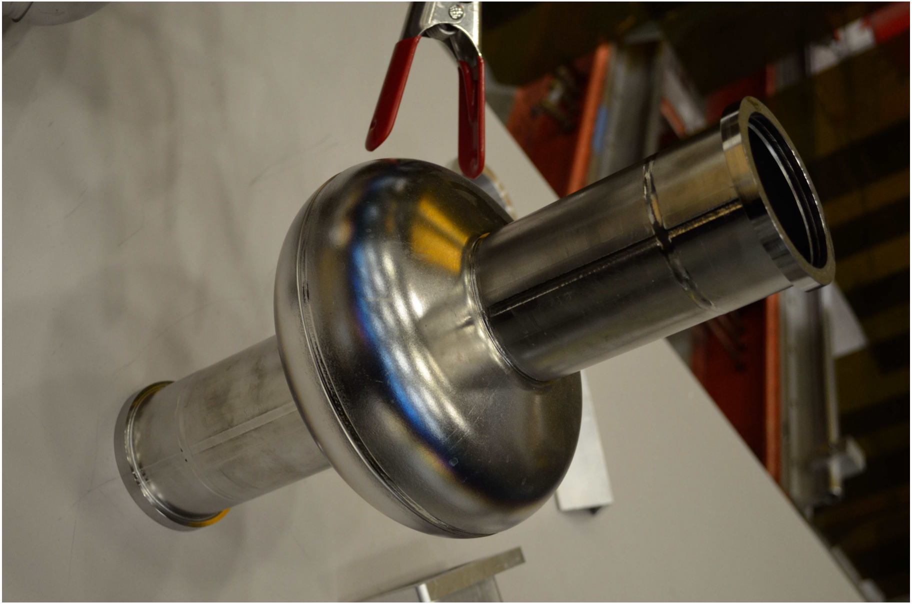
Single Cell Cavities

Electric field high at iris





Magnetic field high at equator



Cell Shape

- There is no perfect shape cavity
- Cavity depends on the specific application

Design Considerations

$\frac{H_{s,\max}}{E_{acc}}$	minimum	critical field
$\frac{E_{s,\max}}{E_{acc}}$	minimum	field emission
$\frac{< H_s^2 >}{E_{acc}^2}$	minimum	shunt impedance, current losses
$\frac{< E_s^2 >}{E_{acc}^2}$	minimum	dielectric losses
$\frac{U}{E_{acc}^2}$	minimum maximum	control of microphonics voltage drop for high charge per bunch

TM Design Considerations

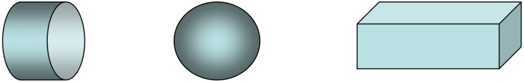
- Field emission is not a hard limitation on superconducting cavities if the surface preparation is done the right way
- Magnetic flux on the wall limits performance of superconducting cavities: Q_0 decrease and quench.

Minimize B_{peak}/E_{acc}:

- Cavities operate at higher gradients
- Cavities operate at lower cryogenic load

RF Simulation Codes for Cavity Design

The solution to 2D (or 3D) Helmholtz equation can be analytically found only for very few geometries (pillbox, spherical resonators or rectangular resonator).



We need numerical methods:

$$\underbrace{(\nabla^2+\omega^2\varepsilon\mu)}\underbrace{A}=0$$

Approximating operator
(Finite Difference Methods)

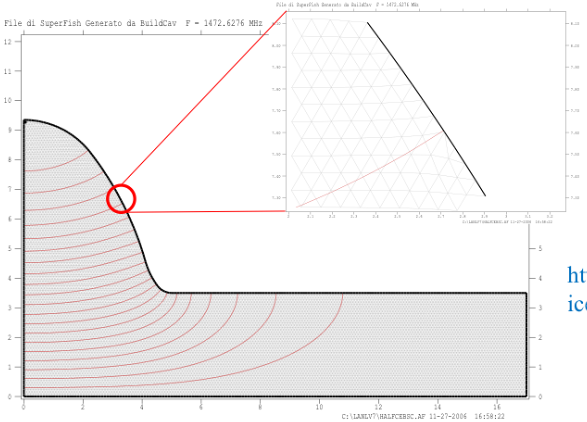
Approximating function
(Finite Element Methods)

• 2D is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.

• 3D is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers and if needed to model fabrication errors. Also coupling strength for FPC and damping of HOMs can be modeled only 3D.

SUPERFISH

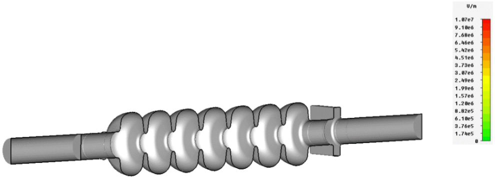
- Free, 2D finite-difference code to design cylindrically symmetric structures (monopole modes only)
- Use symmetry planes to reduce number of mesh points



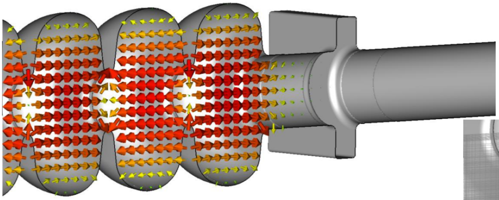
http://laacg1.lanl.gov/laacg/services/download_sf.phtml

CST Microwave Studio

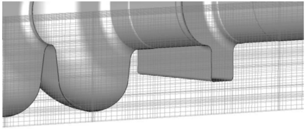
- Expensive, 3D finite-element code, used to design complex RF structure. http://www.cst.com/Content/Products/MWS/Overview.aspx



- Runs on PC
- Perfect Boundary Approximation



Hexahedral mesh

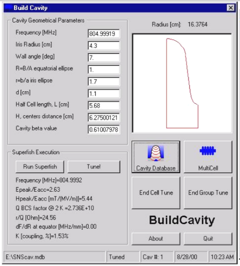


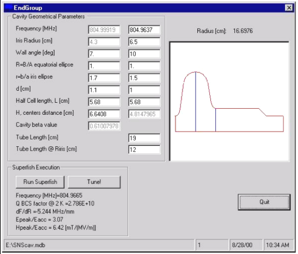
Tools used for the parametrization

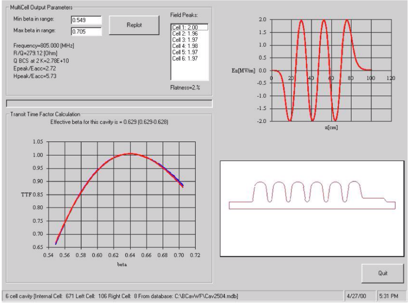
BuildCavity: parametric tool for the analysis of the cavity shape on the EM parameters:

- All RF computations are handled by SUPERFISH
- Inner cell tuning is performed through the cell diameter, all the characteristic cell parameters stay constant: R, r, α, d, L, R_{iris}
- End cell tuning is performed through the wall angle inclination, α, or distance, d.

  R, L and R_{iris} are independently settable.
- Multicell cavity is then built to minimize the field unflatness, compute the effective β and the final cavity performances.
- A proper file to transfer the cavity geometry to ANSYS is then generated







Inner cell data

L = 56.8 mm
R = 1
r = 1.7
α = 7°
d = 11 mm
R_{iris} = 43 mm

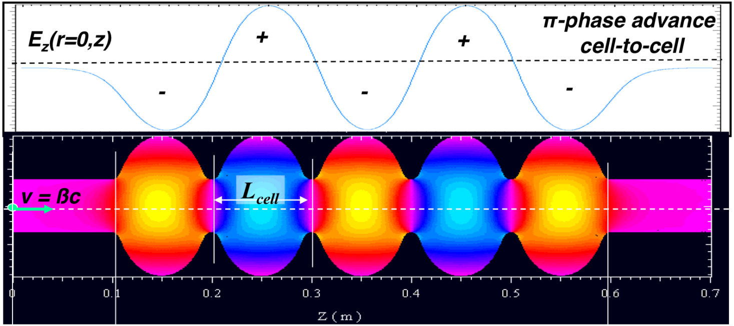
Defining Cavity Parameters

- Cavity Frequency
- Diameter of the Equatior
- Cell Length
- Radius of iris

Beam Acceleration: cell length

Accelerating mode in a multi-cell structure

Cell length, $L = \lambda\beta/2$



s L_{active}

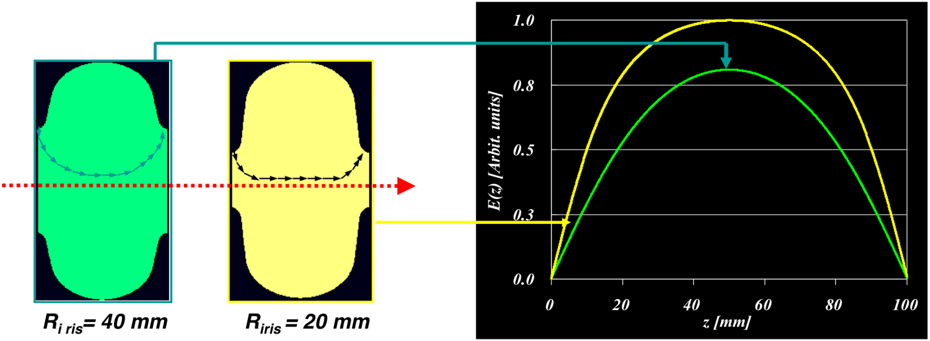
Synchronic acceleration and max of $(R/Q)_{acc} \leftrightarrow L_{active} = NL_{cell} = Nc\beta/(2f)$ and the injection takes place at an optimum phase φ_{opt} which ensures that particles will arrive at the mid-plane of the first cell when E_{acc} reaches its maximum (+q passing to the right) or minimum (-q passing to the right).

One Big "Knob": R_{iris}

Why for a smaller aperture (R_{iris})?

- (R/Q) is bigger
- E_{peak}/E_{acc} , B_{peak}/E_{acc} is lower

E_{acc} is higher at the same stored energy in the cell



E_z (z) for small and big iris radius

More on R_{iris}

We know that a smaller aperture makes:

- *(R/Q) higher*
- B_{peak}/E_{acc} , E_{peak}/E_{acc} *lower*

(+)

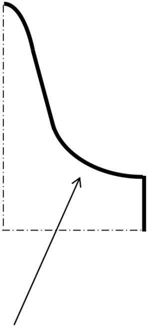
but unfortunately a smaller aperture makes:

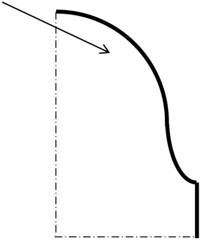
- *HOMs impedances* ($k_\perp$, $k_\parallel$) *higher*
- *cell-to-cell coupling* (k_{cc}) *weaker*

(-)

“Rule of thumb” for Optimizing Peak Surface Fields

Add “*magnetic volume*” at the equator to reduce B_{peak}

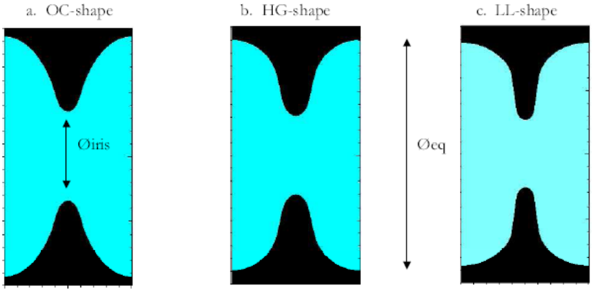




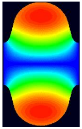
Add “*electric volume*” at the iris to reduce E_{peak}

Example: CEBAF Upgrade

- “High Gradient” shape: lowest E_p/E_{acc}
- “Low Loss” shape: lowest cryogenic losses, maximize G(R/Q)



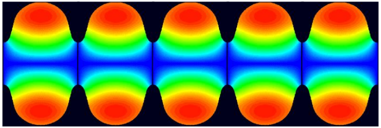
Multicell Cavities



Single-cell is attractive from the RF-point of view:

- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation

- ***But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.***



A multi-cell structure is less expensive and offers higher real-estate gradient but:

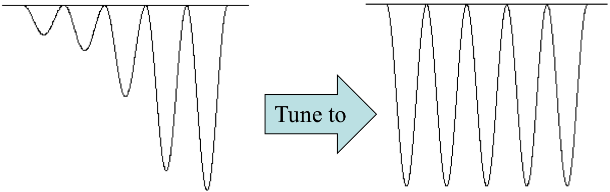
Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells

- ***Other problems arise: HOM trapping…***

Field Flatness

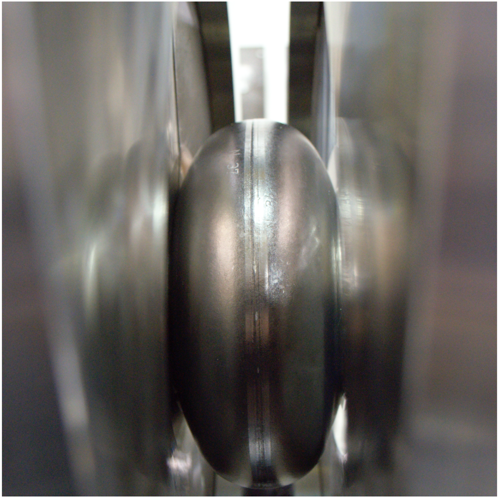
Geometrical differences between cells causes a mixing of the eigenmodes

Sensitivity to mechanical deformation depends on mode spacing



Field flatness factor for elliptical cavities with arbitrary ß=v/c $a_{ff} = \frac{N^2}{k_{cc} \cdot \beta}$

Tuning Cells

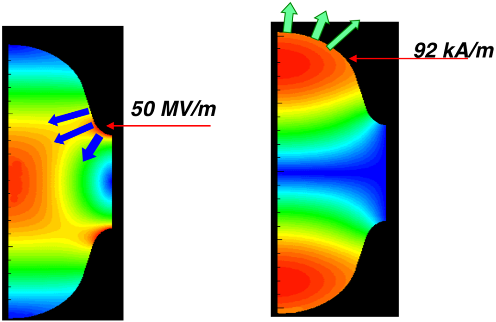


Mechanical Analysis

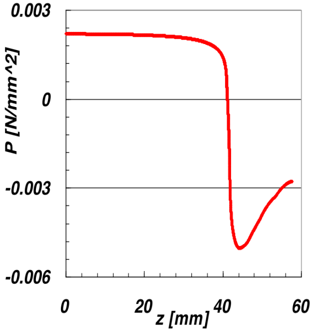
The mechanical design of a cavity follows its RF design:

- Lorentz Force Detuning
- Mechanical Resonances
- Structural stability under different load conditions

Lorentz Force Detuning



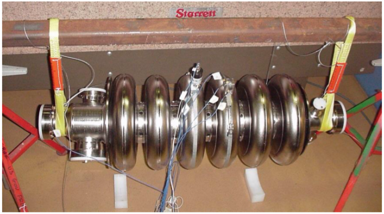
$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$

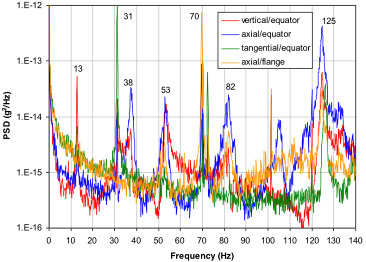


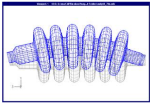
E and H at E_{acc} = 25 MV/m in TESLA inner-cup

Mode Analysis

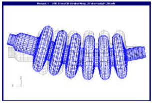
Calculate mechanical resonances of a multi-cell cavity as they modulate frequency of the accelerating mode. Sources of their excitation: vacuum pumps, ground vibrations…



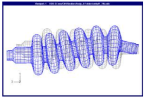




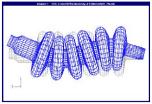
Mode 1 – 14 Hz



Mode 2 – 26 Hz



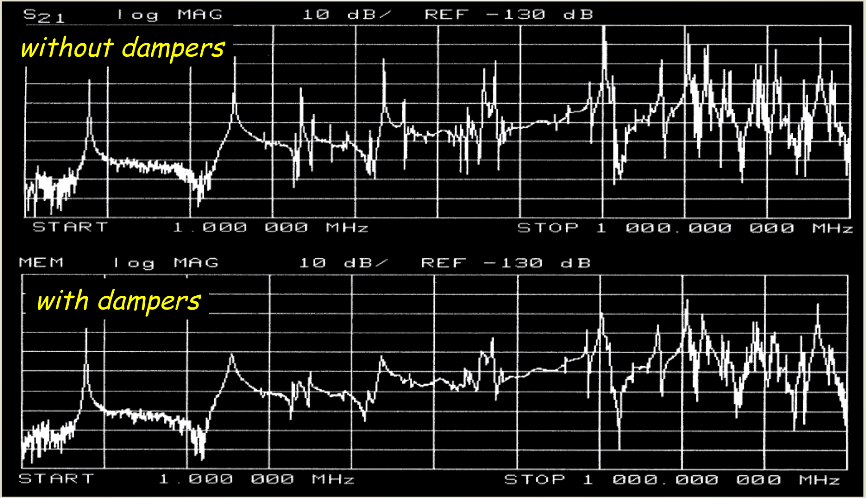
Mode 3 – 40 Hz



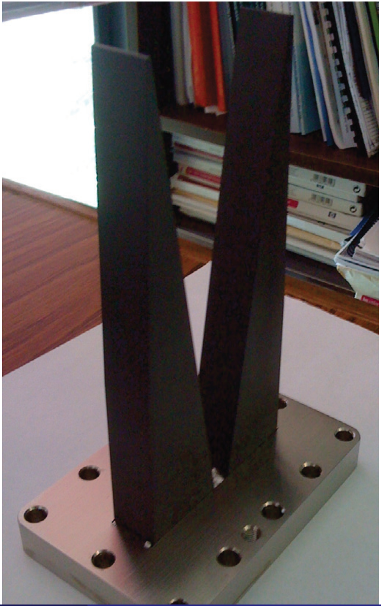
Mode 5 – 72 Hz

	Natural Frequency (Hz)	
Mode	**Test Data**	**FE Analysis**
1	13	14
2	31	26
3	38	40
4	53	48
5	70	72
6	82	83
7	125	124

Higher order modes (measured spectrum)



HOM absorber



Frequency Control

Energy gain $$W = qV\cos\phi$$

Energy gain error $$\frac{\delta W}{W} = \frac{\delta V}{V} - \delta\phi\tan\phi$$

The fluctuations in cavity field amplitude and phase come mostly from the fluctuations in cavity frequency

Need for fast frequency control

Minimization of rf power requires matching of average cavity frequency to reference frequency

Need for slow frequency tuners

Some Definitions

- Ponderomotive effects: changes in frequency caused by the electromagnetic field (radiation pressure)
  - Static Lorentz detuning (cw operation)
  - Dynamic Lorentz detuning (pulsed operation)

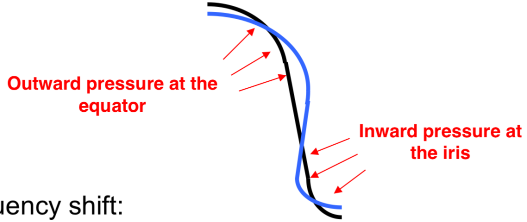
- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

Note: The two are not completely independent.

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

Lorentz Detuning

Pressure deforms the cavity wall:



RF power produces radiation pressure:

$$P = \frac{\mu_0 H^2 - \varepsilon_0 E^2}{4}$$

Deformation produces a frequency shift:

$$\Delta f = -k_L E_{acc}^2$$

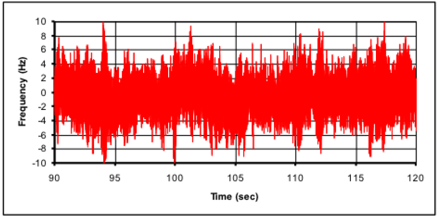
Microphonics

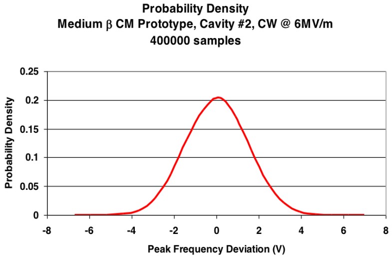
- **Total detuning**

$$\delta\omega_0 + \delta\omega_m$$

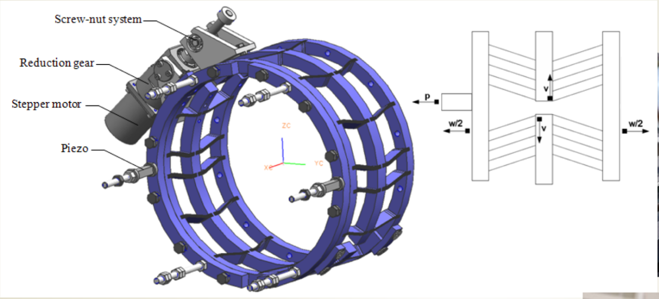
where $\delta\omega_0$ is the static detuning (controllable)

and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)



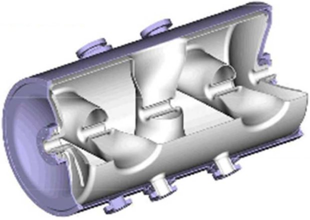


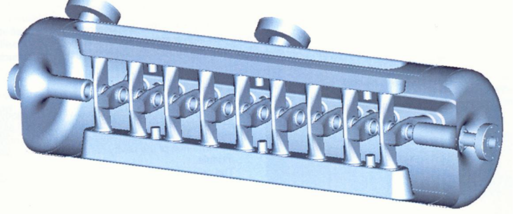
Blade tuner

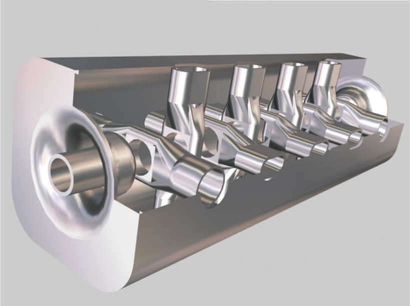


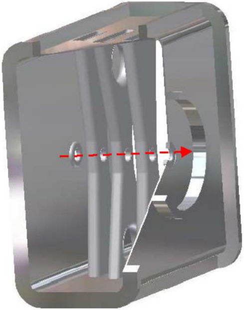
- Developed by INFN Milan
- Azimuthal motion transferred to longitudinal strain
- Zero backlash
- CuBe threaded shaft used for a screw nut system
- Stepping motor and gear combination driver
- Two piezo actuators for fast action
- All components in cold location

courtesy: Eiji Kako/KEK









Slides credit

- Jean Delayen
- Gianluigi Ciovatti
- Erk Jensen
- Frank Marhauser

References

- H. Padamsee, J. Knobloch & T. Hays, *RF Superconductivity for Accelerators*, Wiley Series in Beam Physics and Accelerator Technology, 1988.
- M. Kelley *Superconducting RF Cavities*, Reviews of Accelerator Science and Technology Vol.5 , 2012.
- H. Padamsee,*SRF Technology*, Wiley Series in Beam Physics and Accelerator Technology.
- Lectures from USPAS SRF Winter 2015 and Summer 2015.