# Long distance weak annihilation contribution to $B^{ \pm} \rightarrow(\pi / K)^{ \pm} \ell^{+} \ell^{-}$ 

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(1) Motivation
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## Introduction

In the SM mixings between quarks occur in the CC

$$
\begin{equation*}
\mathcal{L}_{C C} \sim-\frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{U}_{L} V_{C K M} \gamma^{\mu} D_{L}+\text { h.c. } \tag{1}
\end{equation*}
$$

which are well described by the CKM matrix ${ }^{1}$

$$
\begin{aligned}
V_{\mathrm{CKM}} \equiv V_{L}^{u} V_{L}^{d \dagger}= & \left(\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)=\left(\begin{array}{ccc}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23}-c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23}-s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
s_{12} s_{23}-c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23}-s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{array}\right) \\
& =\left(\begin{array}{ccc}
1-\lambda^{2} / 2 & \lambda & A \lambda^{3}(\rho-i \eta) \\
-\lambda & 1-\lambda^{2} / 2 & A \lambda^{2} \\
A \lambda^{3}(1-\rho-i \eta) & -A \lambda^{2} & 1
\end{array}\right)+\mathcal{O}\left(\lambda^{4}\right) .
\end{aligned}
$$

where ${ }^{2}\left|V_{u s}\right| \sim \lambda \sim 0.2255$, and

$$
\begin{gathered}
s_{12}=\lambda=\frac{\left|V_{u s}\right|}{\sqrt{\left|V_{u d}\right|^{2}+\left|V_{u s}\right|^{2}}}, \quad s_{23}=A \lambda^{2}=\lambda\left|\frac{V_{c b}}{V_{u s}}\right|, \\
s_{13} e^{i \delta}=V_{u b}^{*}=A \lambda^{3}(\rho+i \eta)=\frac{A \lambda^{3}(\bar{\rho}+i \bar{\eta}) \sqrt{1-A^{2} \lambda^{4}}}{\sqrt{1-\lambda^{2}}\left[1-A^{2} \lambda^{4}(\bar{\rho}+i \bar{\eta})\right]} .
\end{gathered}
$$

${ }^{1}$ M. Kobayashi and T. Maskawa, PTP 49, 652 (1973).
${ }^{2}$ Wolfenstein, 1983.

## Current status ${ }^{3}$



Experiments expect to reach the \% level in the near future.
${ }^{3}$ K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014).

## Flavor Changing Neutral Current

In the SM FCNC are supressed at the tree level.


The CMS and LHCb collaboration, Nature 522, 6892 (2015).
They are induced by loops and are sensitive to new physics (NP) effects.

## FCNC experimental results

- Branching Ratios

$S M^{a}$
LHCb and CMS:

$$
\begin{gathered}
\operatorname{Br}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)=\left(2.8_{-0.6}^{+0.7}\right) \times 10^{-9} \\
\operatorname{Br}\left(B^{0} \rightarrow \mu^{+} \mu^{-}\right)=\left(3.9_{-1.4}^{+1.6}\right) \times 10^{-10}
\end{gathered}
$$

$$
\operatorname{Br}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)=(3.66 \pm 0.23) \times 10^{-9}
$$

$$
\operatorname{Br}\left(B^{0} \rightarrow \mu^{+} \mu^{-}\right)=(1.06 \pm 0.09) \times 10^{-10}
$$

${ }^{a}$ Bobeth et.al. PRL 112 (2014) 101801.
Agree with SM at $1.2 \sigma$ and $2.2 \sigma$, respectively.

- Angular Observables ${ }^{4}$ in $B \rightarrow K^{*} \ell^{+} \ell^{-}$


Figure 2 - The angular observables $F_{\mathrm{L}}, A_{\mathrm{FB}}, S_{5}$ and $P_{5}^{\prime}$ overlaid with SM predictions from (purple) Ref. ${ }^{3,23}$ and (orange) Ref. ${ }^{24}$.
${ }^{4}$ LHCb coll., PRL 111 (2013) 191801. Straub et. al., Eur.Phys.J. C75 (2015) 8, 382. Mathias et.al., JHEP 1412 (2014) 125 and 1305 (2013) 137.

## Tests of lepton universality

- Ratios ${ }^{5}$

$$
\begin{equation*}
R_{D *}=\frac{\operatorname{Br}\left(B^{0} \rightarrow D^{*+} \tau^{-} \nu_{\tau}\right)}{\operatorname{Br}\left(B^{0} \rightarrow D^{*+} \ell^{-} \nu_{\ell}\right)} \tag{2}
\end{equation*}
$$

- $R\left(D^{*}\right)$ also measured by B factories
- All current measurements based on events with leptonic tau decays
- Belle, Babar results statistics dominated

${ }^{5}$ Kamenik, talk given at Jozef Stefan Institute. LHCb, PRL 115 (2015) 111803.

- $S M$ prediction for $R(D)$ based on $L Q C D$

$$
\mathrm{R}(D)_{\mathrm{SM}}=0.297(17)
$$

J.F.K. \& Mescia, 0802.3790 see also
Bailey et al., 1206.4992

## We are interested in

$\Rightarrow$ the ratio ${ }^{6}$ :

where

$$
R_{K}=\frac{\int_{q_{\min }^{2}}^{q_{m}^{2}} \frac{\mathrm{~d} \mathrm{\Gamma}\left[B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}\right]}{\mathrm{d} q^{2}} \mathrm{~d} q^{2}}{\int_{q_{\min }^{2} \max \left[\left[B^{+} \rightarrow K^{+}+e^{+} e^{-}\right]\right.}^{\mathrm{d} q^{2}} \mathrm{~d} q^{2}},=0.745_{-0.074}^{+0.090} \text { (stat.) } \pm 0.036 \text { (syst.) }
$$

${ }^{6}$ LHCb, PRL 113 (2014) 151601. Babar, PRD 86 (2012)031012. Belle, PRL 103 (2009)

## If tensions are due to NP...

Tree-level new physics in $b \rightarrow \boldsymbol{s} \mu^{+} \mu^{-}$
s-channel: $Z^{\prime}$ boson

[Altmannshofer and DS 1308.1501, Gauld
et al. 1308.1959, Buras and Girrbach
1309.2466, Gauld et al. 1310.1082 , Buras et al.
1311.6729, Altmannshofer et al.
1403.1269, Buras et al. 1409.4557, Glashow
et al. 1411.0565, Crivellin et al.
1501.00993, Altmannshofer and DS
1411.3161, Grivellin et al. 1503.03477]
t-channel: scalar or vector leptoquark

[Hiller and Schmaltz 1408.1627, Biswas et al.
1409.0882, Buras et al. 1409.4557, Sahoo and

Mohanta 1501.05193, Hiller and Schmaltz
1411.4773]

Neutrinos: See Prof. Valle talk.
${ }^{7}$ D. Straub, talk given at Rencontres de Moriond (2015).

## This talk

## Could this anomaly be caused by a missing process ?

## Weak efective hamiltonian for FCNC

For $B^{+} \rightarrow(K, \pi)^{+} \ell^{+} \ell^{-}$:



## Weak efective hamiltonian for FCNC

$$
\mathcal{H}_{\mathrm{eff}}=-\frac{4 G_{F}}{\sqrt{2}}\left(\lambda_{t} \mathcal{H}_{\mathrm{eff}}^{(t)}+\lambda_{u} \mathcal{H}_{\mathrm{eff}}^{(u)}\right)
$$

with the CKM combination $\lambda_{i}=V_{i b} V_{i s}^{*}$ and

$$
\begin{array}{rlrl} 
& \mathcal{H}_{\mathrm{eff}}^{(t)}=C_{1} \mathcal{O}_{1}^{c}+C_{2} \mathcal{O}_{2}^{c}+\sum_{i=3}^{6} C_{i} \mathcal{O}_{i}+ & \sum_{i=7,8,9,10, P, S}\left(C_{i} \mathcal{O}_{i}+C_{i}^{\prime} \mathcal{O}_{i}^{\prime}\right), \\
& \mathcal{H}_{\mathrm{eff}}^{(u)}=C_{1}\left(\mathcal{O}_{1}^{c}-\mathcal{O}_{1}^{u}\right)+C_{2}\left(\mathcal{O}_{2}^{c}-\mathcal{O}_{2}^{u}\right) . \\
\mathcal{O}_{7}= & \frac{e}{g^{2}} m_{b}\left(\bar{s} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu}, & \mathcal{O}_{7}^{\prime} & =\frac{e}{g^{2}} m_{b}\left(\bar{s} \sigma_{\mu \nu} P_{L} b\right) F^{\mu \nu}, \\
\mathcal{O}_{8}= & \frac{1}{g} m_{b}\left(\bar{s} \sigma_{\mu \nu} T^{a} P_{R} b\right) G^{\mu \nu a}, & \mathcal{O}_{8}^{\prime} & =\frac{1}{g} m_{b}\left(\bar{s} \sigma_{\mu \nu} T^{a} P_{L} b\right) G^{\mu \nu a}, \\
\mathcal{O}_{9}= & \frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\mu} \gamma^{\mu} \mu\right), & \mathcal{O}_{9}^{\prime}=\frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\mu} \gamma^{\mu} \mu\right), \\
\mathcal{O}_{10}= & \frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\mu} \gamma^{\mu} \gamma_{5} \mu\right), & \mathcal{O}_{10}^{\prime}=\frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\mu} \gamma^{\mu} \gamma_{5} \mu\right), \\
\mathcal{O}_{S}= & \frac{e^{2}}{16 \pi^{2}} m_{b}\left(\bar{s} P_{R} b\right)(\bar{\mu} \mu), & \mathcal{O}_{S}^{\prime}=\frac{e^{2}}{16 \pi^{2}} m_{b}\left(\bar{s} P_{L} b\right)(\bar{\mu} \mu), \\
\mathcal{O}_{P}= & \frac{e^{2}}{16 \pi^{2}} m_{b}\left(\bar{s} P_{R} b\right)\left(\bar{\mu} \gamma_{5} \mu\right), & \mathcal{O}_{P}^{\prime}=\frac{e^{2}}{16 \pi^{2}} m_{b}\left(\bar{s} P_{L} b\right)\left(\bar{\mu} \gamma_{5} \mu\right),
\end{array}
$$

## Four quark operators



Fig. 1. Leading contributions to $\left\langle\gamma^{*} \bar{K}^{*}\right| H_{\text {eff }}|\bar{B}\rangle$. The circled cross marks the possible insertions of the virtual photon line.

$$
\begin{aligned}
& P_{1}=\left(\bar{s}_{L} \gamma_{\mu} T^{a} c_{L}\right)\left(\bar{c}_{L} \gamma^{\mu} T^{a} b_{L}\right) \\
& P_{2}=\left(\bar{s}_{L} \gamma_{\mu} c_{L}\right)\left(\bar{c}_{L} \gamma^{\mu} b_{L}\right) \\
& P_{3}=\left(\bar{s}_{L} \gamma_{\mu} b_{L}\right) \sum_{q}\left(\bar{q} \gamma^{\mu} q\right) \\
& P_{4}=\left(\bar{s}_{L} \gamma_{\mu} T^{a} b_{L}\right) \sum_{q}\left(\bar{q} \gamma^{\mu} T^{a} q\right) \\
& P_{5}=\left(\bar{s}_{L} \gamma_{\mu_{1}} \gamma_{\mu_{2}} \gamma_{\mu_{3}} b_{L}\right) \sum_{q}\left(\bar{q} \gamma^{\mu_{1}} \gamma^{\mu_{2}} \gamma^{\mu_{3}} q\right) \\
& P_{6}=\left(\bar{s}_{L} \gamma_{\mu_{1}} \gamma_{\mu_{2}} \gamma_{\mu_{3}} T^{a} b_{L}\right) \sum_{q}\left(\bar{q} \gamma^{\mu_{1}} \gamma^{\mu_{2}} \gamma^{\mu_{3}} T^{a} q\right)
\end{aligned}
$$

## Weak efective hamiltonian for FCNC

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}}= & -\frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{\alpha_{e}}{\pi} V_{t b} V_{t s}^{*} \\
& \times\left(\mathcal{C}_{7} \frac{m_{b}}{e}\left[\bar{s} \sigma_{\mu \nu} P_{R} b\right] F^{\mu \nu}+\mathcal{C}_{9}\left[\bar{s} \gamma_{\mu} P_{L} b\right]\left[\bar{\ell} \gamma^{\mu} \ell\right]+\mathcal{C}_{10}\left[\bar{s} \gamma_{\mu} P_{L} b\right]\left[\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right]\right)+\text { h.c. } .
\end{aligned}
$$

with the matrix elements

$$
\begin{aligned}
\left\langle K\left(p_{K}\right)\right| \bar{s} \gamma_{\mu} b\left|\bar{B}\left(p_{B}\right)\right\rangle & =\left(2 p_{B}-q\right)_{\mu} f_{+}\left(q^{2}\right)+\frac{M_{B}^{2}-M_{K}^{2}}{q^{2}} q_{\mu}\left[f_{0}\left(q^{2}\right)-f_{+}\left(q^{2}\right)\right], \\
\left\langle K\left(p_{K}\right)\right| \bar{s} i \sigma_{\mu \nu} q^{\nu} b\left|\bar{B}\left(p_{B}\right)\right\rangle & =-\left[\left(2 p_{B}-q\right)_{\mu} q^{2}-\left(M_{B}^{2}-M_{K}^{2}\right) q_{\mu}\right] \frac{f_{T}\left(q^{2}\right)}{M_{B}+M_{K}} .
\end{aligned}
$$

similar for $B \rightarrow \pi$. All of them related to a single form factor $f_{+}\left(q^{2}\right) \equiv \xi_{P}\left(q^{2}\right)$ in the limit $m_{b} \rightarrow \infty$ (Isgur-Wise).

$$
\begin{aligned}
\mathcal{M}[\bar{B} \rightarrow K \bar{l} l]=i \frac{G_{F} \alpha_{e}}{\sqrt{2} \pi} V_{t b} V_{t s}^{*} \xi_{P}\left(q^{2}\right)( & \left.F_{V} p_{B}^{\mu}\left[\bar{l} \gamma_{\mu} l\right]+F_{A} p_{B}^{\mu}\left[\bar{l} \gamma_{\mu} \gamma_{5}\right]\right] \\
& \left.+\left(F_{S}+\cos \theta F_{T}\right)[\bar{l} l]+\left(F_{P}+\cos \theta F_{T 5}\right)\left[\bar{l} \bar{\gamma}_{5} l\right]\right)
\end{aligned}
$$

the leading contributions comes from

$$
\begin{equation*}
F_{V}=C_{9}+\frac{2 m_{b}}{M_{B}} \frac{\mathcal{T}_{P}\left(q^{2}\right)}{\xi\left(q^{2}\right)}, F_{A}=C_{10} \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathcal{T}_{P}\left(q^{2}\right)=\xi\left(q^{2}\right)\left[C_{7}^{\text {eff }(0)}+\frac{M_{B}}{m_{b}} Y^{(0)}\left(q^{2}\right)\right] \tag{4}
\end{equation*}
$$

$C_{9}=4.214$ and $C_{10}=-4.312$ have been computed at $\mathrm{NNLL}^{8}$ and $C_{7}^{\text {eff }}$ at $\mathrm{NLO}^{9}$. WA included in $Y^{(0)}$.

[^0]
## Form Factors

The form factors are taken in the form

$$
\begin{aligned}
\xi_{\pi}\left(q^{2}\right) & =\frac{0.918}{1-q^{2} /(5.32 \mathrm{GeV})^{2}}-\frac{0.675}{1-q^{2} /(6.18 \mathrm{GeV})^{2}}+\mathcal{P}_{\pi}\left(q^{2}\right) \\
\xi_{K}\left(q^{2}\right) & =\frac{0.0541}{1-q^{2} /(5.41 \mathrm{GeV})^{2}}+\frac{0.2166}{\left[1-q^{2} /(5.41 \mathrm{GeV})^{2}\right]^{2}}+\mathcal{P}_{K}\left(q^{2}\right)
\end{aligned}
$$

where $\mathcal{P}_{P}\left(q^{2}\right)$ is a polynomial ${ }^{10}$.
${ }^{10}$ Ball, Zwicky, PRD 71 (2005) 014015.

## SM prediction

$$
\begin{aligned}
& \Gamma_{l}^{\mathrm{SM}}=\frac{\Gamma_{0}}{3} \int_{q_{\min }^{2}}^{q_{\max }^{2}} d q^{2} \xi_{P}^{2}\left(q^{2}\right) \sqrt{\lambda}^{3}\left(\left|F_{A}\right|^{2}+\left|F_{V}\right|^{2}\right) \\
& \quad \times\left\{1+\mathcal{O}\left(\frac{m_{l}^{4}}{q^{4}}\right)+\frac{m_{l}^{2}}{M_{B}^{2}} \times \mathcal{O}\left(\alpha_{s}, \frac{q^{2}}{M_{B}^{2}} \sqrt{\frac{\Lambda_{\mathrm{QCD}}}{E}}\right)\right\} \\
& \lambda=M_{B}^{4}+M_{K}^{4}+q^{4}-2\left(M_{B}^{2} M_{K}^{2}+M_{B}^{2} q^{2}+M_{K}^{2} q^{2}\right), \quad \Gamma_{0}=\frac{G_{F}^{2} \alpha_{e}^{2}\left|V_{t b} V_{t s}^{*}\right|^{2}}{512 \pi^{5} M_{B}^{3}}
\end{aligned}
$$

with:

|  | $B^{-} \rightarrow K^{-} \bar{l} l$ |  |  |
| :---: | :---: | :---: | :---: |
|  | SM value | $\xi_{P}[\%]$ | $\mu_{b}[\%]$ |
| $\mathcal{B}_{\mu}$ | $1.60_{-0.46}^{+0.51}$ | ${ }_{-27.0}^{+29.9}$ | +2.0 |
|  | $1.27_{-0.36}^{+0.40}$ | +29.4 | +2.2 |
| $\left[10^{-7}\right]$ | $1.91_{-0.54}^{+0.59}$ | -26.6 | -2.1 |
|  | $1.59_{-0.44}^{+0.48}$ | -26.2 | +2.2 |
|  |  | -26.0 | -2.2 |
|  |  |  | +2.4 |

for $\left(q_{\min }^{2}, q_{\text {max }}^{2}\right)=(1,6)(2,6)(1,7)(2,7) \mathrm{GeV}^{2}$, from top to bottom ${ }^{11}$.
${ }^{11}$ Bobeth et.al., JHEP 12 (2007) 040.

## An interesting observable

$$
\begin{aligned}
& R_{K} \equiv \frac{\Gamma_{\mu}}{\Gamma_{e}}=\int_{q_{\min }^{2}}^{q_{\max }^{2}} d q^{2} \frac{d \Gamma_{\mu}}{d q^{2}} / \int_{q_{\min }^{2}}^{q_{\max }^{2}} d q^{2} \frac{d \Gamma_{e}}{d q^{2}} \\
& =0.745_{-0.074}^{+0.090}(\text { stat. }) \pm 0.036(\text { syst. })(\mathrm{LHCb})
\end{aligned}
$$

## Long distance WA contribution to $B^{ \pm} \rightarrow(\pi / K)^{ \pm} \ell^{+} \ell^{-}$

## LD WA contribution to $K^{ \pm}$

has been previously considered by Ecker, Pich and Rafael ${ }^{12}$.

Penguin type (when quark structure is resolved)


Fig. 2. One-loop diagrams for $K \rightarrow \pi^{\prime} \gamma^{\prime}$ which can lead to terms in the amplitude proportional to $q^{2}\left(p+p^{\prime}\right)_{\mu}$. The notation for vertices is as in fig. 1.
${ }^{12}$ Ecker, Pich and Rafael, Nucl. Phys. B . 291 (1987) 692.

The decay amplitude corresponding to WA

$$
\begin{equation*}
\mathcal{M}_{L D}^{W A}=\frac{e^{2}}{q^{2}} \bar{\ell} \gamma^{\mu} \ell \mathcal{M}_{\mu}^{W A} \tag{5}
\end{equation*}
$$

where $\mathcal{M}_{\mu}^{W A}$ denotes the effective hadronic electromagnetic current coupled to the leptonic current. Conservation of the electromagnetic current demands

$$
\begin{equation*}
\mathcal{M}_{\mu}^{W A}=\left[\left(p_{B}+p_{P}\right)_{\mu}-\frac{m_{B}^{2}-m_{P}^{2}}{q^{2}} q_{\mu}\right] F\left(q^{2}\right) \tag{6}
\end{equation*}
$$

where only the first term within square brackets gives a non-vanishing contribution.

$$
q^{\mu} \bar{\ell} \gamma_{\mu} \ell=0, \quad \text { we can also replace }\left(p_{B}+p_{P}\right)_{\mu} \rightarrow 2 p_{B \mu}
$$

LONG-DISTANCE WEAK ANNIHILATION CONTRIBUTION .


$$
B^{ \pm} \rightarrow P^{ \pm} \ell^{+} \ell^{-}
$$

(d)

(f)



subleading in the $1 / N_{C}$

The leading order one-photon exchange (WA) amplitude corresponding to the diagrams ( i ) and ( j ) can be computed taking into account that

$$
\begin{equation*}
\langle 0| \bar{u} \gamma^{\mu}\left(1-\gamma_{5}\right) b\left|B^{-}\right\rangle=-i f_{B} p_{B}^{\mu}, \quad\left\langle P^{-}\right| \bar{D} \gamma_{\mu}\left(1-\gamma_{5}\right) u|0\rangle=i f_{P} p_{P \mu}, \tag{7}
\end{equation*}
$$

and is given by

$$
\begin{align*}
\mathcal{M}_{L D, W A}= & \sqrt{2} G_{F}(4 \pi \alpha) V_{u b} V_{u D}^{*} f_{B} f_{P} \frac{1}{q^{2}\left(m_{B}^{2}-m_{P}^{2}\right)} \\
& \times\left[m_{B}^{2}\left(F_{P}\left(q^{2}\right)-1\right)-m_{P}^{2}\left(F_{B}\left(q^{2}\right)-1\right)\right] p_{B}^{\mu} \bar{\ell} \gamma_{\mu} \ell \tag{8}
\end{align*}
$$

where $f_{X}$ denotes the decay constant of the pseudoscalar meson $X$ according to the PDG ${ }^{13}$ conventions for $f_{K, \pi, B}$ and $F_{X}\left(q^{2}\right)$ is the electromagnetic form factor of the corresponding meson.


## Long distance WA contribution to $C_{9}$

Due to the vector nature of the one-photon exchange contribution, its amplitude can be absorbed into the contribution of the $O_{9}$ operator in the SM amplitude under the replacement

$$
\begin{equation*}
\xi_{P}\left(q^{2}\right) F_{V} \longrightarrow \xi_{P}\left(q^{2}\right) F_{V}+\kappa_{P} m_{B}^{2}\left[\frac{F_{P}\left(q^{2}\right)-1}{q^{2}}\right] \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\kappa_{P}=-8 \pi^{2} \frac{V_{u b} V_{u D}^{*}}{V_{t b} V_{t D}^{*}} \frac{f_{B} f_{P}}{m_{B}^{2}-m_{P}^{2}} . \tag{10}
\end{equation*}
$$

Note that $\kappa_{P} \sim \mathcal{O}\left(10^{-2}\right) \times \frac{V_{u b} V_{u p}^{*}}{V_{t b} V_{t D}^{*}}$ so that its influence is governed by the ratio of CKM factors which is $\sim \mathcal{O}\left(\lambda^{0}\right)$ for $P=\pi$ and $\mathcal{O}\left(\lambda^{2}\right)$ for $P=K$.

This suggests a larger effect for $B^{-} \rightarrow \pi^{-} \ell^{+} \ell^{-}$transitions but a detailed analysis of the electromagnetic meson form factors is needed to confirm these expectations.

## Form Factors

They are important in the 1 GeV region, where theory is better controlled

## Two different approaches are used

- Resonance Chiral Theory (Ecker et.al. Nucl. Phys. B . 321 (1989) 311)
- Gounaris- Sakurai parametrizations (PRL 21 (1968) 244)


## $\pi$ electromagnetic form factor



## $K$ electromagnetic form factor



- There is a huge breaking of lepton universality.
- It is important only for $q^{2} \leq 0.3 \mathrm{GeV}^{2}$.
- Its effect is always smaller than the SD contribution

$$
\begin{aligned}
R_{K}^{S M} & =1+(3 \pm 1) \times 10^{-4} \\
R_{\pi}^{S M} & =1+(6 \pm 1) \times 10^{-4} \\
R_{P}^{L D}-1 & =\mathcal{O}\left(10^{-5}\right)(P=K, \pi)
\end{aligned}
$$

## Matching of the $\mathrm{R} \chi \mathrm{T}$ and SD descriptions of the WA contributions

For $q^{2} \leq q_{\text {match }}^{2}$

$$
P=(K, \pi)
$$


while for $q^{2} \geq q_{\text {match }}^{2}$


## Smooth matching between the LD and SD WA contributions



## Smooth matching between the LD and SD WA contributions



## Results

| $B^{-} \rightarrow \pi^{-} \ell^{+} \ell^{-}$ |  |  | $B^{-} \rightarrow K^{-} \ell^{+} \ell^{-}$ |
| :--- | :--- | :--- | :--- |
|  | $0.05 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ | $1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ | $1 \leq q^{2} \leq 6 \mathrm{GeV}^{2}$ |
| LD | $(9.06 \pm 0.15) \cdot 10^{-9}$ | $(4.74 \pm 0.05) \cdot 10^{-10}$ | $(1.70 \pm 0.21) \cdot 10^{-9}$ |
| interf. | $(-2.57 \pm 0.13) \cdot 10^{-9}$ | $\left(-2_{-1}^{+2}\right) \cdot 10^{-10}$ | $(-6 \pm 2) \cdot 10^{-11}$ |
| SD | $\left(9.57_{-1.01}^{+1.45}\right) \cdot 10^{-9}$ | $\left(8.43_{-0.87}^{+1.31}\right) \cdot 10^{-9}$ | $\left(1.90_{-0.41}^{+0.69}\right) \times 10^{-7}$ |
| Total | $\left(1.61_{-0.11}^{+0.15}\right) \cdot 10^{-8}$ | $\left(8.69_{-0.87}^{+1.31}\right) \cdot 10^{-9}$ | $\left(1.92_{-0.41}^{+0.69}\right) \times 10^{-7}$ |

## Results

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| :--- | :--- | :--- | :--- |
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| interf. | $(-2.57 \pm 0.13) \cdot 10^{-9}$ | $\left(-2_{-1}^{+2}\right) \cdot 10^{-10}$ | $(-6 \pm 2) \cdot 10^{-11}$ |
| SD | $\left(9.57_{-1.01}^{+1.45}\right) \cdot 10^{-9}$ | $\left(8.43_{-0.87}^{+1.31}\right) \cdot 10^{-9}$ | $\left(1.90_{-0.41}^{+0.69}\right) \times 10^{-7}$ |
| Total | $\left(1.61_{-0.11}^{+0.15}\right) \cdot 10^{-8}$ | $\left(8.69_{-0.87}^{+1.31}\right) \cdot 10^{-9}$ | $\left(1.92_{-0.41}^{+0.69}\right) \times 10^{-7}$ |
|  | $68 \%$ | $3 \%$ | $1 \%$ |

- Current accuracy is sensitive to it! Still ideal place to look for NP.
- It can be controlled, so it is a good place to search for NP!
- Very large hadronic contamination. It is better to take $q^{2} \geq 1 \mathrm{GeV}^{2}$.

For the fully integrated rate, the LHCb ${ }^{14} 15$ measurements

$$
\begin{gather*}
\operatorname{Br}\left(B^{-} \rightarrow \pi^{-} \ell^{+} \ell^{-}\right)=(2.3 \pm 0.6 \pm 0.1) \times 10^{-8} \\
\operatorname{Br}\left(B^{-} \rightarrow \pi^{-} \ell^{+} \ell^{-}\right)=(1.83 \pm 0.24 \pm 0.05) \times 10^{-8} \\
B r_{S D+L D}^{S M}\left(B^{-} \rightarrow \pi^{-} \ell^{+} \ell^{-}\right)=\left(2.6_{-0.3}^{+0.4}\right) \times 10^{-8} \tag{11}
\end{gather*}
$$

${ }^{14}$ Ali et.al., JHEP12(2012)125.
${ }^{15}$ arXiv:1509.00414

## CP violation

A direct CP assymetry can be generated

$$
\begin{equation*}
A_{C P}(P)=\frac{\Gamma\left(B^{+} \rightarrow P^{+} \ell^{+} \ell^{-}\right)-\Gamma\left(B^{-} \rightarrow P^{-} \ell^{+} \ell^{-}\right)}{\Gamma\left(B^{+} \rightarrow P^{+} \ell^{+} \ell^{-}\right)+\Gamma\left(B^{-} \rightarrow P^{-} \ell^{+} \ell^{-}\right)}, \tag{12}
\end{equation*}
$$

from the interference of the SD and LD diagrams, such that

$$
\begin{aligned}
\Delta_{C P}= & \Gamma\left(B^{+} \rightarrow P^{+} \ell^{+} \ell^{-}\right)-\Gamma\left(B^{-} \rightarrow P^{-} \ell^{+} \ell^{-}\right) \\
= & -32 \alpha^{2} G_{F}^{2} f_{P} f_{B} \operatorname{Im}\left\{V_{t b} V_{t D}^{*} V_{u b}^{*} V_{u D}\right\} \\
& \times \int d q^{2} \int d s_{12} \frac{1}{q^{2}\left(M_{B}^{2}-m_{P}^{2}\right)}\left[2\left(P_{B} \cdot P_{+}\right)\left(P_{B} \cdot P_{-}\right)-\frac{M_{B}^{2} q^{2}}{2}\right](13) \\
& \times \operatorname{Im}\left\{\xi_{P}\left(q^{2}\right) F_{V}\left(q^{2}\right)\left[M_{B}^{2}\left(F_{P}\left(q^{2}\right)-1\right)-m_{P}^{2}\left(F_{B}\left(q^{2}\right)-1\right)\right]\right\},
\end{aligned}
$$

where $s_{12}=\left(p_{K}+p_{+}\right)^{2}$.

$$
A_{C P}(P)=\left\{\begin{array}{l}
(16.1 \pm 1.9) \%, \quad \text { for } P=\pi, 0.05 \leq q^{2} \leq 8 \mathrm{GeV}^{2},  \tag{14}\\
(7.8 \pm 2.9) \%, \quad \text { for } P=\pi, 1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}, \\
(-1.0 \pm 0.3) \%, \quad \text { for } P=K, 1 \leq q^{2} \leq 6 \mathrm{GeV}^{2} .
\end{array}\right.
$$

| $\left(q_{\text {min }}^{2}, q_{\text {max }}^{2}\right)$ | Hou $^{16}[\%]$ | Our results $^{17}[\%]$ | Khodjamiriam $^{18}[\%]$ |
| :---: | :---: | :---: | :---: |
| $(1,8) \mathrm{GeV}^{2}$ | $13 \pm 2$ | $7.8 \pm 2.9$ | - |
| $(1,6) \mathrm{GeV}^{2}$ | $16 \pm 2$ | $9.2 \pm 1.7$ | $\left(14.3_{-2.9}^{+3.5}\right)$ |
| $(2,6) \mathrm{GeV}^{2}$ | $13_{-3}^{+2}$ | $7.7 \pm 0.5$ | - |

Recently, for $P=\pi\left(\mathrm{LHCb}^{19}\right)$

$$
\begin{equation*}
A_{C P}=0.11 \pm 0.12 \pm 0.01,(1,6) \mathrm{GeV}^{2} \tag{15}
\end{equation*}
$$

${ }^{16}$ Hout et.al. PRD 90013002 (2014)
${ }^{17}$ A. Guevara, G. López-Castro, P. Roig and ST, PRD 92 (2015) 054035.
${ }^{18}$ Khodjamiriam et.al. 1506.07760.
${ }^{19}$ R. Aaij et.al. arXiv:1509.00414.

## Conclusions

One-photon exchange contribution to the rare $B^{ \pm} \rightarrow P^{ \pm} \ell^{+} \ell^{-}$decays, with $P=\pi$ or $K$.

- Its effects in BR of the $P=K$ case $\sim 1 \%$ with respect to the (top quark loop dominated) SD contribution for $1 \leq q^{2} \leq 6 \mathrm{GeV}^{2}$.
$\Rightarrow$ The corresponding effect in $P=\pi$ turns out to be significant in integrated observables starting close to the threshold.
- We suggest to take the range $1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ for precision measurements.
- More refined measurements of the fully integrated branching fraction for this decay could be sensitive to our contribution.
- CP asymmetry is large in the case of a pion in the final state for $0.05 \leq q^{2} \leq 8$ $\mathrm{GeV}^{2}$, but also sizable and worth measuring in the $1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ interval.
$\Rightarrow$ Our CP violation results are smaller than those obtained from SD because of the different description (origin) of the WA amplitudes at low energies.


[^0]:    ${ }^{8}$ Beneke et.al. NPB 612 (2001)25.
    ${ }^{9}$ Buras et. al., Rev. Mod. Phys. 68, 1125 (1996)

