XV MEXICAN WORKSHOP ON PARTICLES AND FIELDS

Charmonia and Contact Interaction

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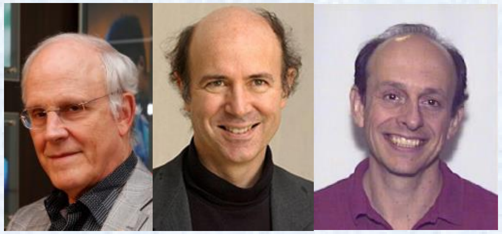
QCD and Hadron Physics

- QCD is the theory of quarks, gluons and their interactions.
- QCD is a powerful tool in the description of large momentum transfer experiments due to asymptotic freedom.

David Gross and Frank Wilczek Phys. Rev. D 8, 3633(1973)
David Politzer Phys. Rev. Lett. 30, 1346 (1973)



- The properties of confinement and chiral symmetry breaking in the non perturbative sector of QCD are not obvious from the QCD Lagrangian: these emerging phenomena are extremely important to study the spectrum of the observed hadrons, as well as their static and dynamic properties, starting from quarks and gluons.

Schwinger-Dyson Equations (SDE)

- Schwinger-Dyson equations (SDE) are the equations of motion of a quantum field theory.

F. J. Dyson The S-Matrix in Quantum Electrodynamics
Phys. Rev. 75, 1736(1949)

J. Schwinger, On Green's Functions of Quantized Fields I, II,
PNAS 37 452–459 (1951)

- They provide a generating tool for perturbation theory.
- They are non pertubative in nature and can be used to study:
  - Confinement of quark and gluons.
  - Dynamical chiral symmetry breaking.
  - Hadrons as bound states.

Schwinger-Dyson Equations (SDE)

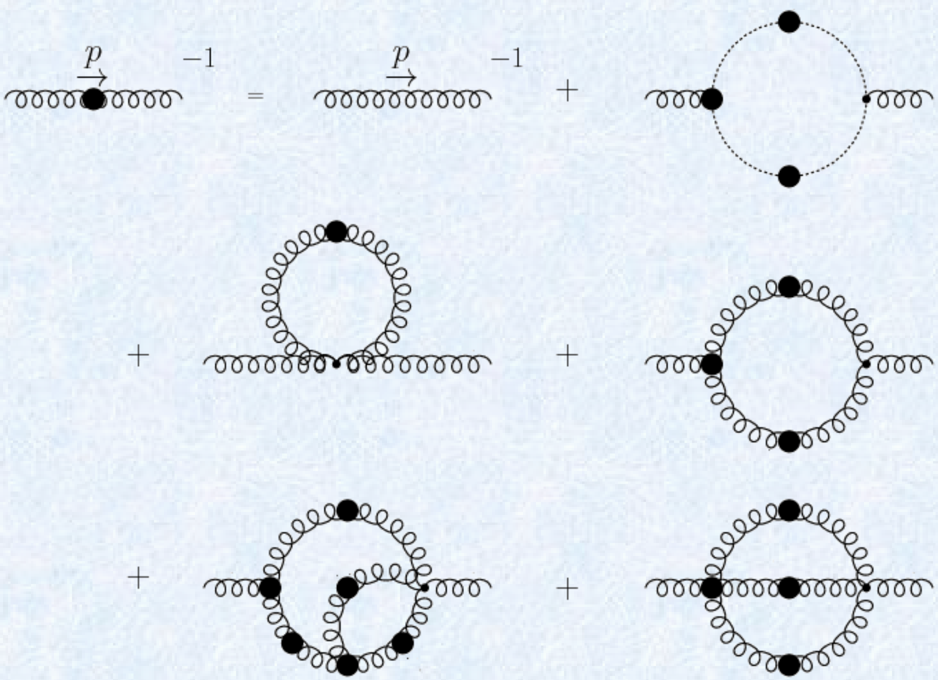
SDE are an infinite set of coupled nonlinear integral equations for the n-point Green functions.

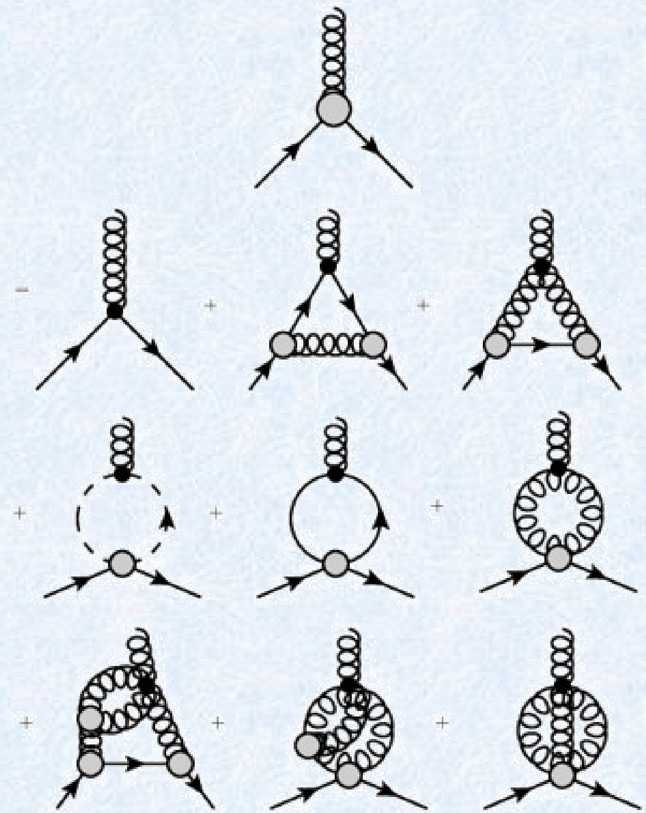
The structure of these equations is such that they relate n-point Green functions to n+1-point Green functions.

Their derivation does not require the coupling strength to be small. Therefore, they are ideally suited to combine infrared and ultraviolet properties of QCD.

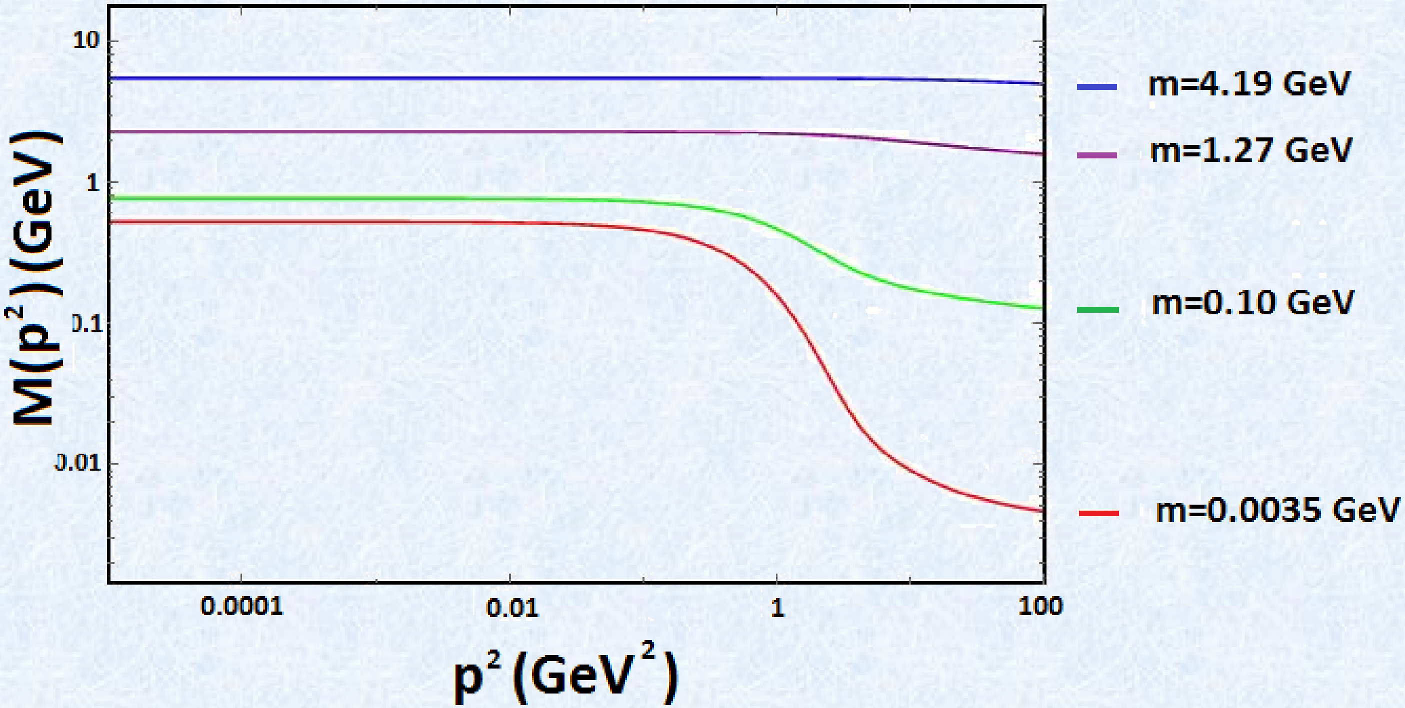
However, we need to introduce a truncation scheme for the complete tower of these equations, while faithfully maintaining the fundamental QCD properties.

Schwinger-Dyson Equations (SDE)





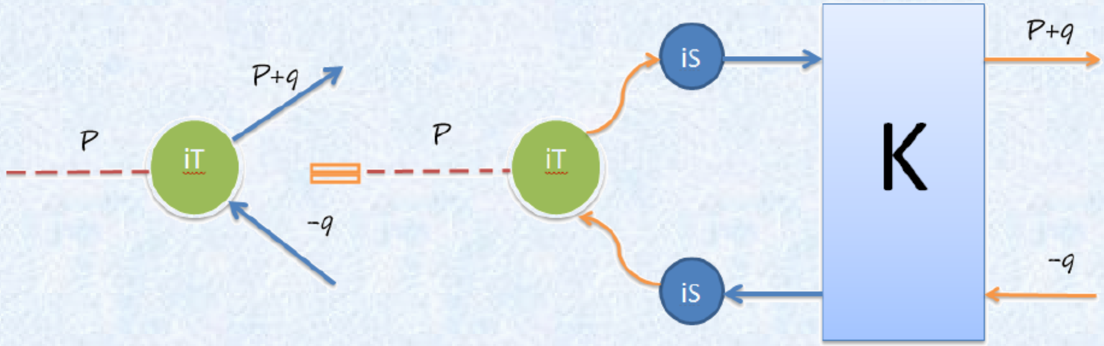
Schwinger-Dyson Equations (SDE)



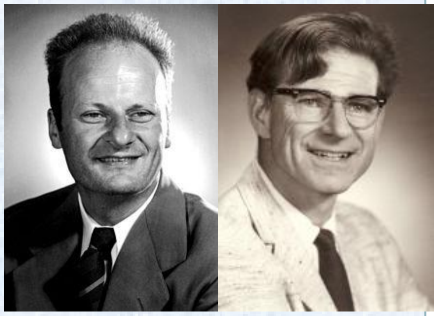
With a phenomenology based ***ansatz*** for the kernel of the gap equation, we can study the running mass function of quarks.

Bethe-Salpeter Equation

Bound states correspond to poles in n-point functions. A meson appears as a pole in the two-quark, two antiquark Green function → Bethe-Salpeter Equation.



$$[\Gamma_H(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4}\, \chi(q;P)_{sr} K^{rs}_{tu}(q,k;P)$$



E. E. Salpeter Phys. Rev. 84, 1226 (1951)

E. E. Salpeter and H. A. Bethe Phys. Rev. 84, 1232 (1951)

Axial-Vector Ward-Takahashi Identity

Axial vector Ward-Takahashi identity in the chiral limit

$$-\,\mathrm{i}P_\mu\Gamma_{5\mu}(k;P)=S^{-1}(k_+)\gamma_5+\gamma_5 S^{-1}(k_-),\qquad k_{\mp}=k\mp P$$

encodes the chiral symmetry properties of QCD & relates the kernel in the meson BSE to that in the quark SDE.

$$\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,K_{tu;rs}(k,q;P)\,[\gamma_5 S(q_-)+S(q_+)\gamma_5]_{sr}=[\Sigma(k_+)\gamma_5+\gamma_5\Sigma(k_-)]_{tu}$$

This guarantees a massless pion in the chiral limit when chiral symmetry is broken dynamically.

Contact Interaction

- We use a contact interaction model mediated by a vector-vector interaction employed in:

  L. Xiomara Gutiérrez, et. al., **Phys. Rev. C81, 065202 (2010);**
  Phys. Rev. C82, 065202 (2010);
  Phys. Rev. C83, 065206 (2011).

- This model provides a simple scheme to exploratory studies of the spontaneous chiral symmetry breaking and its consequences like:
  - Dynamical mass generation.
  - Quark condensate.
  - Goldstone bosons in chiral limit.
  - Confinement.

Contact Interaction

L. Xiomara Gutiérrez, et. al., Phys. Rev. C81, 065202 (2010);

H.L.L. Roberts, et. al., Phys. Rev. C82, 065202 (2010);
Phys. Rev. C83, 065206 (2011).

Few Body Syst. 51, 1 (2011)

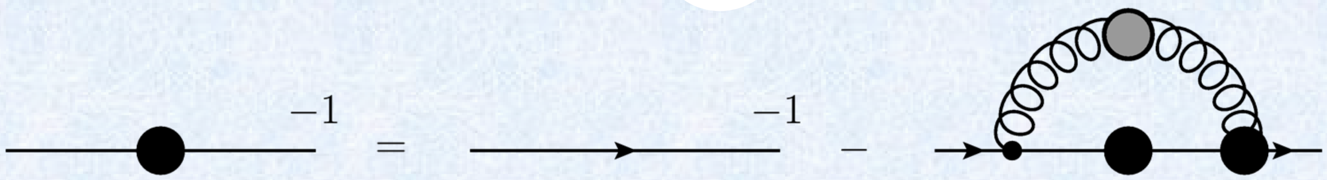
C. Chen, et. al., Few Body Syst. 53, 293 (2012)

"Elastic and Transition Form Factors of the $\Delta(1232)$",
J. Segovia, C. Chen, I.C. Cloet, C.D, Roberts, S.M. Schmidt, S. Wan,
Few Body Sys. 55, 1 (2014).

"Insights into the $\gamma^*N \to \Delta$ Transition",
J. Segovia, C. Chen, C.D, Roberts, S. Wan,
Phys. Rev. C88. 3, 032201 (2014).

"Nucleon and Roper Electromagnetic Elastic and Transition Form Factors",
D.J. Wilson, I.C. Cloet, L. Chang, C.D, Roberts,
Phys. Rev. C85. 3, 025205 (2012).

Contact Interaction



Quark SDE:

$$S_f^{-1}(p) = i\gamma \cdot p + m_f + \Sigma_f(p)$$

$$\Sigma_f(p) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \Gamma^a_\nu(p,q)$$

Gluon propagator:

$$g^2 \Delta_{\mu\nu}(k) = \delta_{\mu\nu} \frac{4\pi\alpha_{\mathrm{IR}}}{m_g^2} \equiv \delta_{\mu\nu} \frac{1}{m_G^2}\,,$$

$$m_g = 800 \text{ MeV}$$
$$\alpha_{\mathrm{IR}} = 0.93\pi$$

P. Boucaud et. al., Few Body Syst. 53, 387 (2012).

Quark-gluon vertex:

$$\Gamma^a_\mu(p,q) = \frac{\lambda^a}{2}\gamma_\mu$$

Contact Interaction

Gap equation: $$S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{4}{3}\frac{1}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu$$

General form of the solution: $$S_f^{-1}(p) = \mathrm{i}\gamma \cdot p + M_f$$

Solution is: $$M_f = m_f + \frac{M_f}{3\pi^2 m_G^2}\int_0^\infty \mathrm{d}s\, s\, \frac{1}{s + M_f^2}$$

Proper time regularization:

$$\frac{1}{s+M^2} = \int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{-\tau(s+M^2)} \to \int_{\tau_{\mathrm{UV}}^2}^{\tau_{\mathrm{IR}^2}} \mathrm{d}\tau\, \mathrm{e}^{-\tau(s+M^2)}$$

$$= \frac{\mathrm{e}^{-\tau_{UV}^2(s+M^2)} - \mathrm{e}^{-\tau_{IR}^2(s+M^2)}}{s+M^2}$$

Contact Interaction

Solution: $$M_f = m_f + \frac{M_f}{3\pi^2 m_G^2}\mathcal{C}_{01}(M_f^2;\tau_{\rm IR},\tau_{\rm UV})$$

$$\mathcal{C}_{\alpha\beta}(M^2;\tau_{\rm IR},\tau_{\rm UV}) = \frac{\left(M^2\right)^\nu}{\Gamma(\beta)}\Gamma(\beta-2,\tau_{\rm UV}^2 M^2,\tau_{\rm IR}^2 M^2)$$

with: $\nu = \alpha - (\beta - 2)$ and $\Gamma(a, z_1, z_2)$

is a generalized incomplete Gamma function.

Flavour	m_f (GeV)	M_{CI} (GeV)	M_{MT} (GeV)
Up/Down	0.007	0.358	0.502
Strange	0.17	0.533	0.776
Charm	1.275	1.337	1.968
Bottom	4.18	4.18	4.584

Bethe-Salpeter Amplitudes within a CI

For our contact interaction model:

$$K(p,q;P)_{tu;rs} = -\frac{1}{m_G^2}\delta_{\mu\nu}\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{ts}\left[\frac{\lambda^a}{2}\gamma_\nu\right]_{ru}$$

Thus the BSE for a meson is:

$$\Gamma_H(k;P) = -\frac{4}{3}\frac{1}{m_G^2}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\gamma_\mu S_f(q+P)\Gamma_H(q;P)S_g(q)\gamma_\mu$$

What are the BSA for different mesons?

Bethe-Salpeter Amplitudes within a CI

Classification of mesons:

L	J^{PC}	Type	L	J^{PC}	Type
0	0^{-+}	Pseudoscalars	1	0^{++}	Scalars
0	1^{--}	Vectors	1	1^{++}, 1^{+-}	Axial Vectors

BS-amplitudes:

$$\begin{aligned}
\Gamma_{0^{-+}}(P) &= \gamma_5\left[\mathrm{i}E_{0^{-+}} + \frac{1}{2M}\gamma\cdot P F_{0^{-+}}\right] \\
\Gamma_{0^{++}}(P) &= \mathbb{1}E_{0^{-+}} \\
\Gamma_{1^{--}}(P) &= \gamma_\mu^T E_{1^{--}} + \frac{1}{2M}\sigma_{\mu\nu}P_\nu F_{1^{--}} \\
\Gamma_{1^{++}\mu}(P) &= \gamma_5\left[\gamma_\mu^T E_{1^{++}} + \frac{1}{2M}\sigma_{\mu\nu}P_\nu F_{1^{++}}\right]
\end{aligned}$$

Charmonia

m_g=0.8	Λ_{UV}=0.905	Λ_{IR}=0.24	m_c=1.578	α_{IR}=0.93π
Masses are in GeV	$m_{\eta c}$(1S)	$m_{J/\Psi}$ (1S)	$m_{\chi c0}$(1P)	$m_{\chi c1}$(1P)
PDG(2010)	2.983	3.096	3.414	3.510
Contact Interaction	2.98	2.994	3.419	3.442
Munczek (1993)	2.821	3.1	3.605	-----
Souchlas (2010)	3.02	3.19		
Souchlas (2010)	3.04	3.24		
Krassnigg (2011)	2.928	3.111	3.321	3.437
El-Bennich (2014)	3.065			
El-Bennich (2014)	3.210			
Decay constants (GeV) (g_{so}=0.24)				
PDG(2010)	0.361	0.416		
Contact Interaction	0.0838	0.0796		
Krassnigg (2011)	0.399	0.448		
Souchlas (2010)	0.239	0.198		
Souchlas (2010)	0.387	0.415		

Charmonia

- The leptonic decay constant is highly influenced by the high momentum tail of the quark mass function.
- This high momentum region probes the wavefunction at the origin
- CI yields constant dressing functions with no perturbative tail.
- By increasing the mass of heavy quarks, charmonuim becomes increasingly point like—and the closer the quarks get the smaller the interaction between them.
- We need to reduce the effective interaction strength for the CI to extend to the heavy quarks sector.
- A reduction in the strength of the kernel has to be compensated by an increased ultraviolet cutoff.

Charmonia

m_g=0.8	Λ_{UV}=2.778	Λ_{IR}=0.24	m_c=0.956	α_{IR}=0.146
Masses are in GeV	$m_{\eta c}$(1S)	$m_{J/\Psi}$ (1S)	$m_{\chi c0}$(1P)	$m_{\chi c1}$(1P)
PDG(2010)	2.983	3.096	3.414	3.510
Contact Interaction	2.949	3.128	3.327	3.355
Munczek (1993)	2.821	3.1	3.605	-----
Souchlas (2010)	3.02	3.19		
Souchlas (2010)	3.04	3.24		
Krassnigg (2011)	2.928	3.111	3.321	3.437
El-Bennich (2014)	3.065			
El-Bennich (2014)	3.210			
Decay constants (GeV) (g_{so}=0.08)				
PDG(2010)	0.361	0.416		
Contact Interaction	0.305	0.217		
Krassnigg (2011)	0.399	0.448		
Souchlas (2010)	0.239	0.198		
Souchlas (2010)	0.387	0.415		

Charmonia

We can also measure the charge radius of $\eta_c(1S)$. Our results are in good agreement with earlier SDE as well as lattice studies.

Charge Radius $m_{\eta c}(1S)$ – $r^2=-6\, \partial F(Q^2)/\partial Q^2 \vert_{Q^2=0}$		
m_g=0.8 GeV, Λ_{UV}=2.778 GeV, Λ_{IR}=0.24 GeV, m_c=1.578 GeV, α_{IR}=0.146		
Schwinger-Dyson Equations	**Lattice**	**Contact Interaction**
Bhagwat (2007)	**Dudek (2007)**	**Our work**
0.219 fm	**0.25 fm**	**0.21 fm**

Final Remarks

- The CI has proven to be an unifying calculational tool for static and dynamic properties of light-ground-and-excited mesons and baryons.
- The decay constant for η_c is now in reasonable agreement with experiment and earlier SDE calculations.
- The decay constant for J/Ψ is in agreement with recent SDE calculations.
- Charge radius of η_c form factor has fairly reasonable agreement with phenomenological results
- Further results will include.
  - Calculation of charge radii and J/Ψ form factors.
  - Calculation of the radiative transitions $\eta_c \to \gamma\gamma$ and $J/\Psi \to \eta_c \gamma$.