Exploring the transition between two mesons and a tetraquark

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arXiv:1509.07563 [hep-ph]

XV Mexican Workshop on particles and Fields, Mazatlán 2015

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Motivation

Multi-quark systems can have important implications in the phenomena we observe in nature, from an enhanced spectroscopy to quark recombination effects.

Recent experimental results provide strong evidence on the formation of four quark states.

LHCb Collab. (R. Aaij et al.), Phys. Rev. Lett. 112, 222002 (2014).

$$e^+e^- \to (D^*\bar{D}^*)^{\pm}\pi^{\mp} \qquad D^{*+}\bar{D}^{*0} \qquad Z_c^{\pm}(4025)$$

S. Choi et al. (Belle Collaboration), Phys. Rev. Lett. 100, 142001 (2008)

Z.Q. Liu et al. (Belle Collaboration), Phys. Rev. Lett. 110, 252002 (2013)

M. Ablikim et al. (BES III Collaboration), Phys. Rev. Lett. 110, 252001 (2013).

$$e^+e^- \to \pi^+\pi^- J/\psi \qquad \pi^{\pm} J/\psi$$

Zc(3900)

Motivation

Since the early years of the quark model, theoretical studies have been performed to inquiry on the existence and stability of the tetraquark as an isolated object

How its mixing with a meson state can help us to understand the observed spectroscopy of states like the σ meson.

Less attention has been paid to the features of the tetraquark formation as two mesons are forced to approach each other as it could happen in a meson-meson collision

or, when the four quarks are produced very close in space as in the WW decay, which eventually freeze out to two mesons.

At which stage they turn into a tetraquark or mixed state?

How their properties reflect such modification?

In the present work we address these questions by considering a system composed of two identical quarks (qq) and two identical anti-quarks (QQ)

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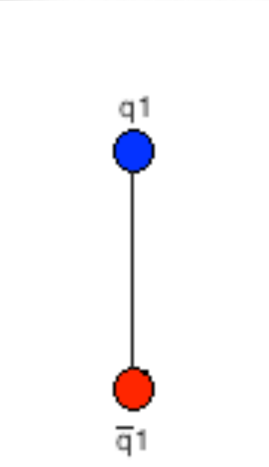
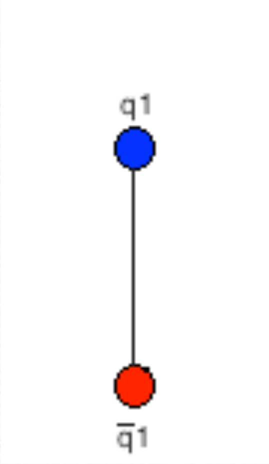
Two Non-interacting Mesons



Meson: quark-antiquark state
interacting via a linear potential



$$\left[\frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + kr\right] \Psi\left(\vec{r}_1, \vec{r}_2\right) = E\ \Psi\left(\vec{r}_1, \vec{r}_2\right).$$

Exact solution

$$\rho_n(r) = \frac{1}{r} \mathrm{Ai}\left(r\left[2\mu k\right]^{1/3} - \xi_n\right).$$

$$E_n = \left[\frac{k^2}{2\mu}\right]^{1/3} |\xi_n|$$

Variational approach

$$F_\lambda(r) = \sqrt{\frac{3\lambda^2}{2\pi}} e^{-\lambda r^{3/2}}.$$

$$\lambda_0 = \frac{2\sqrt{k\mu}}{3},$$

$$E_0 = \frac{3^{5/3} k \Gamma\left(\frac{8}{3}\right)}{2^{7/3}(k\mu)^{1/3}}.$$

Meson Mean square radius

$$\langle r_M^2 \rangle \equiv \left\langle \sum_{i=1}^{2} \left(\vec{r}_i - \vec{R} \right)^2 \right\rangle$$

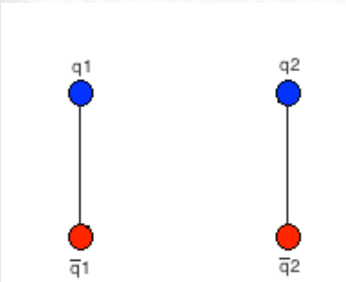
$$\langle r_M^2 \rangle_0 = \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \frac{3^{4/3}}{4} \Gamma\left(\frac{10}{3}\right),$$

m_2/m_1	λ_0	E_0	E_{exact}	$\langle r_M^2 \rangle_0$
1	0.4714	2.3472	2.33811	1.50255
1.44643	0.5125	2.2197	2.21106	1.38885
4.6131	0.6043	1.9889	1.98118	1.52605
14.0774	0.6441	1.9061	1.8987	1.73656

Meson optimal variational parameter and energy, the exact energy and mean square radius for different mass ratios respect to the lightest u mass

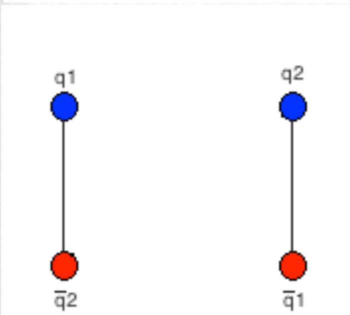
Four-body system

Many-body potential

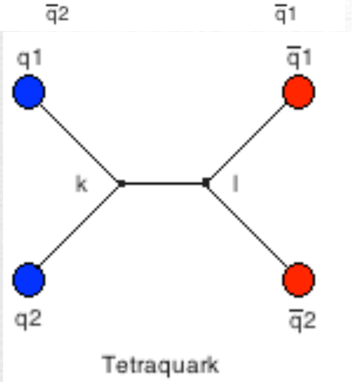


$$V_{m1} = V(\vec{r}_1, \vec{r}_3) + V(\vec{r}_2, \vec{r}_4)$$

Quark exchange among mesons



$$V_{m2} = V(\vec{r}_1, \vec{r}_4) + V(\vec{r}_2, \vec{r}_3).$$



$$V_{4Q} = \sum_{i=1}^{2} V(\vec{k}, \vec{r}_i) + \sum_{j=3}^{4} V(\vec{l}, \vec{r}_j) + V(\vec{k}, \vec{l}).$$

Tetraquark configuration

k and **l** vectors minimize the total length to connect the four particles. Numerical determination is invoked after any single particle changes position.

$$\boxed{V = min(V_{m1}, V_{m2}, V_{4Q})}$$

F. Lenz et al., Annals of Phys. 170 65(1986).

Variational approach

Variational wave function

Variational parameter

$$\Psi_\lambda = \Phi_{FG}\left(e^{-\lambda Q}\right),$$

FG: Fermi correlations among identical quarks

J. Carlson and V. R. Pandharipande Phys. Rev. D 43 1652(1991).

J. Vijande, A. Valcarce, J. M. Richard, Phys. Rev. D 76 114013 (2007); ibid 87 034040(2013).

$$Q_{m_1} = r_{13}^{3/2} + r_{24}^{3/2}, \qquad Q_{m_2} = r_{14}^{3/2} + r_{23}^{3/2} \qquad Q_{4Q} = r_{1k}^{3/2} + r_{2k}^{3/2} + r_{kl}^{3/2} + r_{3l}^{3/2} + r_{4l}^{3/2},$$

Energy calculation

$$H = \sum_{i=1}^{4} \frac{P_i^2}{2m_i} + V.$$

$$\langle H \rangle_\lambda = T_{FG} + \langle W \rangle_\lambda + \langle V \rangle_\lambda\,, \qquad W = \sum_{i=1}^{4} \frac{\lambda^2}{2m_i} \sum_j \left[\partial_j Q\right]^2,$$

$$\frac{\partial \langle H \rangle_\lambda}{\partial \lambda} = 0$$

C. J. Horowitz, E. J. Moniz and John W. Negele, Phys. Rev. D 31 1689(1985).
M. Oka and C. J. Horowitz, Phys. Rev. D 31 2773(1985) G. Toledo Sanchez and J. Piekarewicz, Phys. Rev. C 65, 045208 (2001); ibid 70, 035206 (2004).

Results

We performed a MC simulation to study three cases:

Two mesons interacting via quark exchange

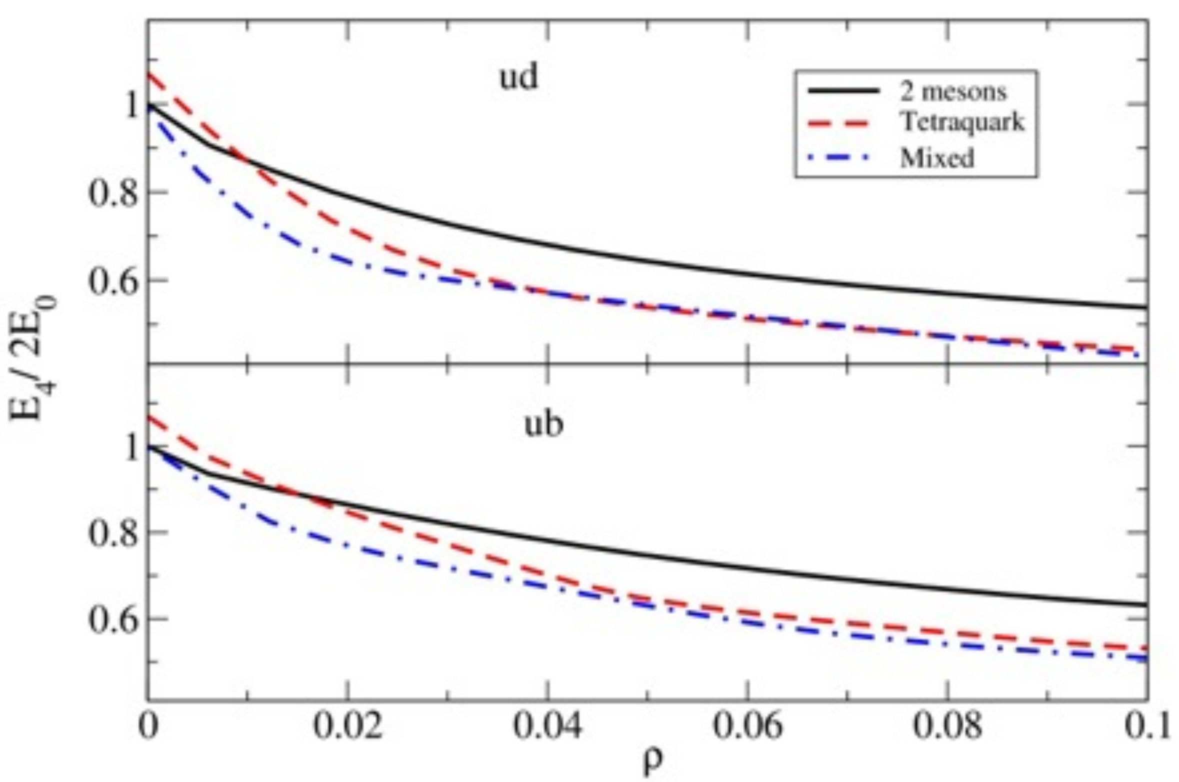
Tetraquark

Mixed

We define a particle density parameter as a measure of the inter-particle separation: $\varrho = N/V = 4/L^3$

N: number of particles, V: box volume.

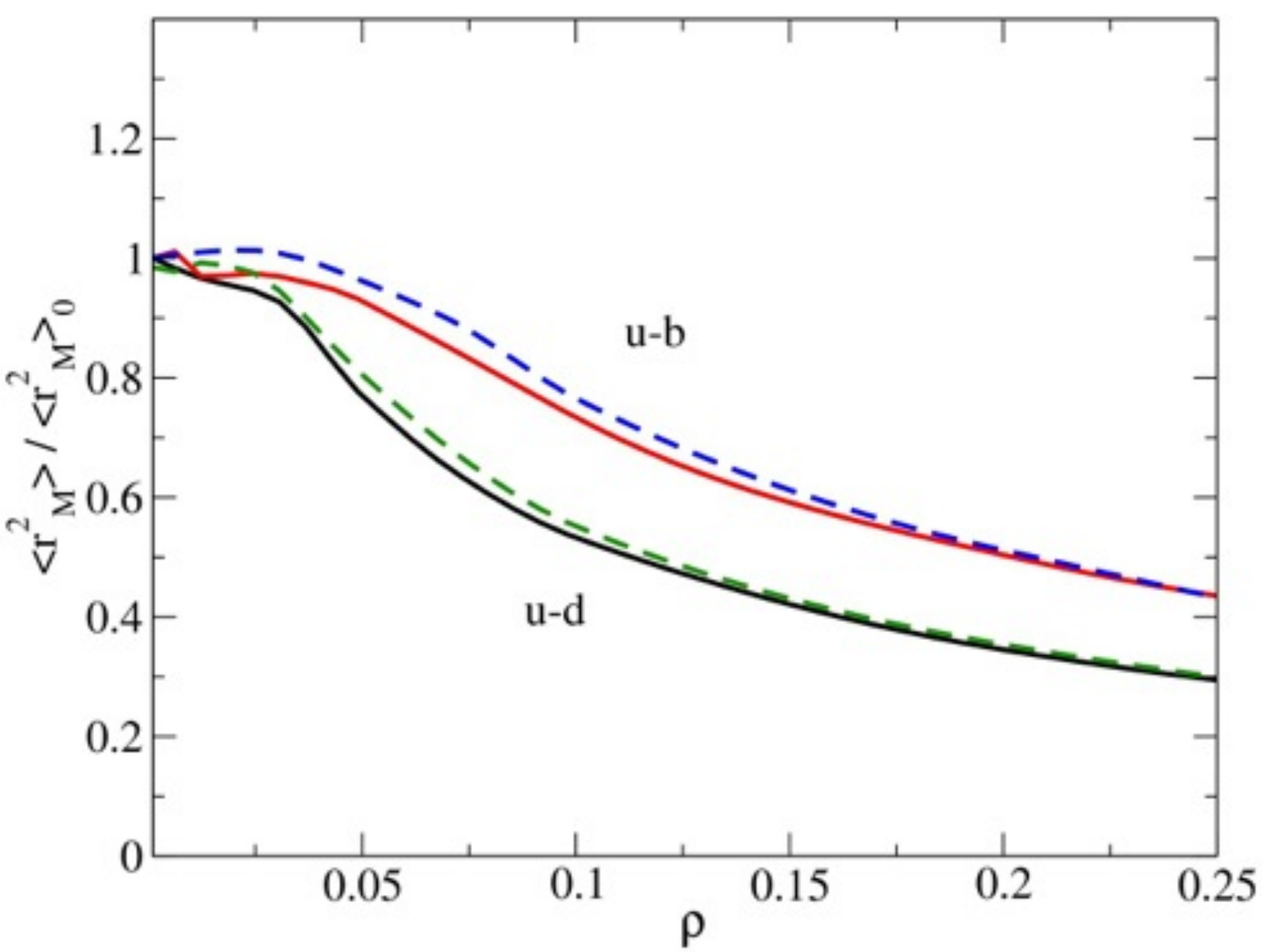
Energy evolution

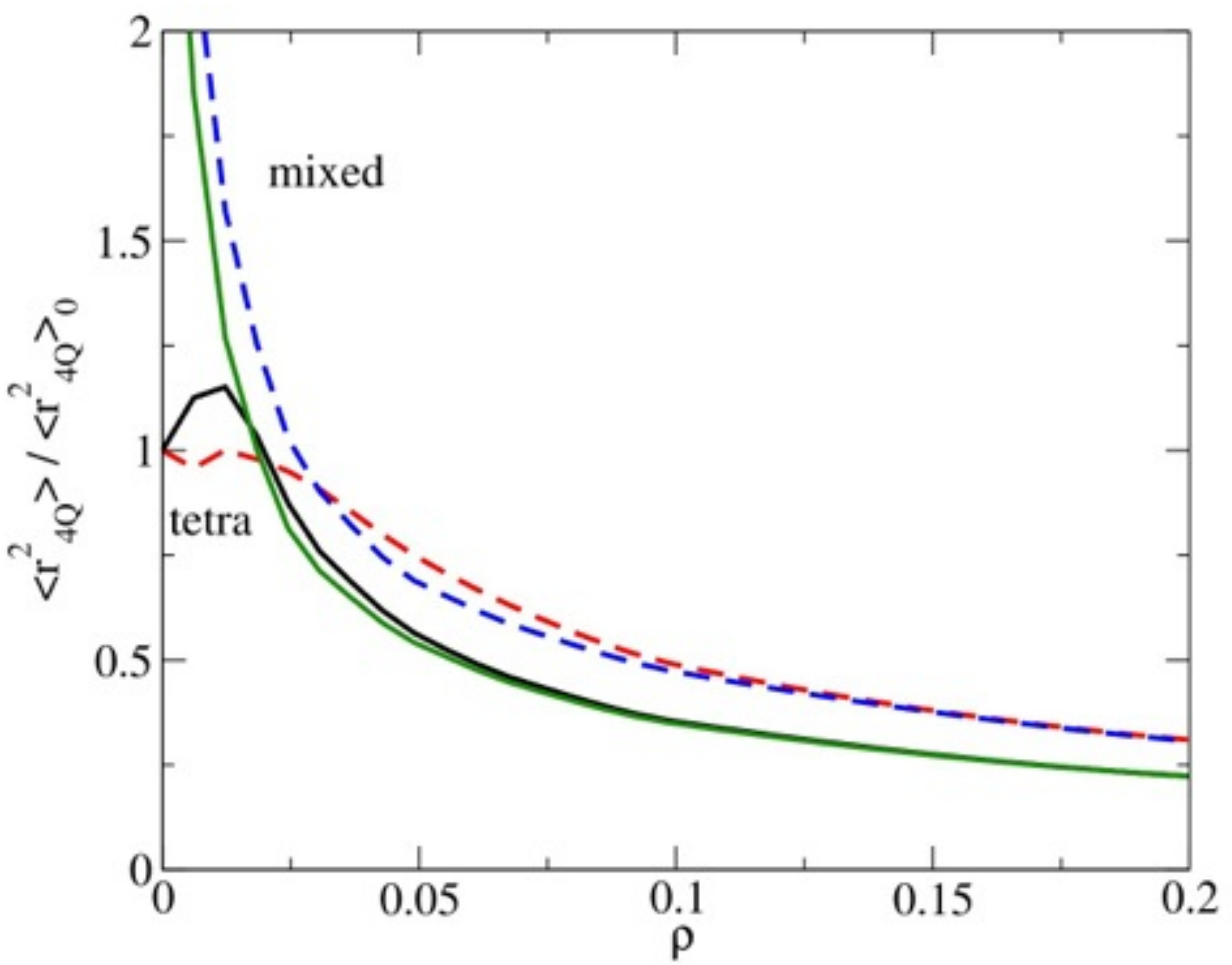


m_2/m_1	$2E_0$	E_{4Q_0}	$\langle r^2_{4Q} \rangle_0$
1	4.6944	5.02 ± 0.02	13.9 ± 0.1
1.44643	4.4394	4.77 ± 0.02	13.2 ± 0.2
4.6131	3.9778	4.24 ± 0.02	12.5 ± 0.3
14.0774	3.8122	4.07 ± 0.02	13.1 ± 0.2

Tetraquark case. Zero density limit. Energy and Mean Square radius, compared to two mesons for several mass ratios

Mean square radius





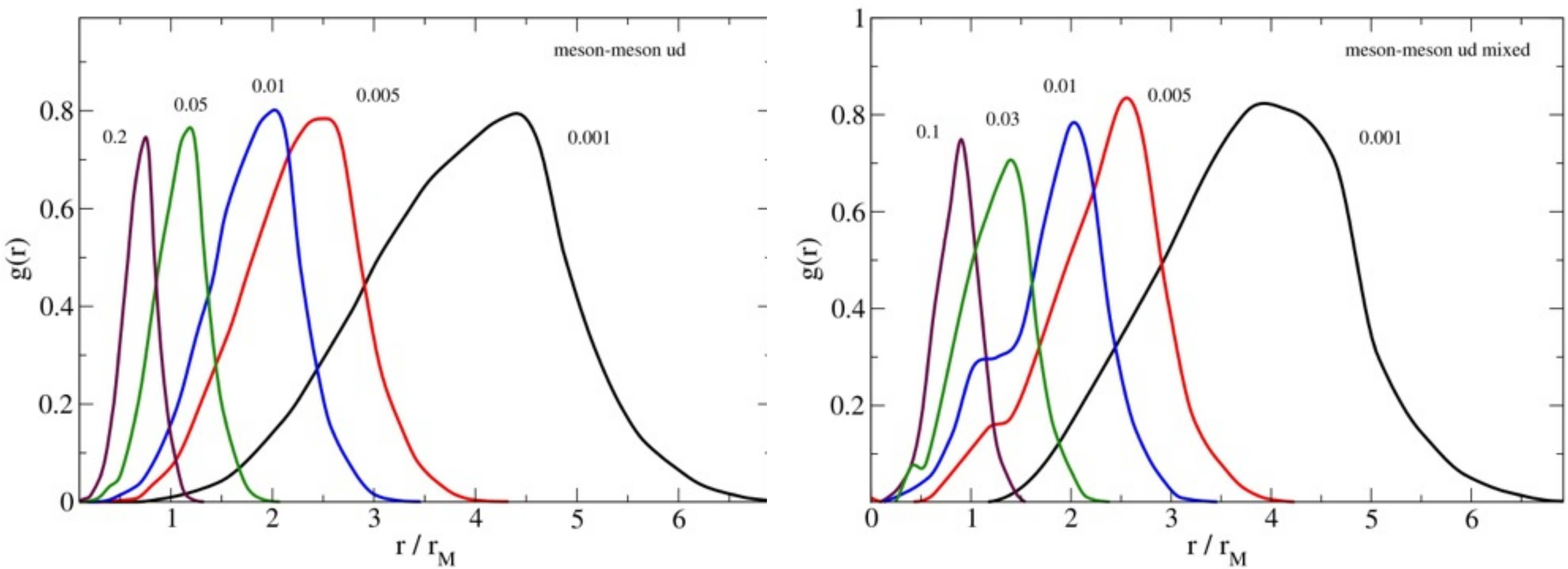
Meson

Tetraquark

Normalized to the zero density limit

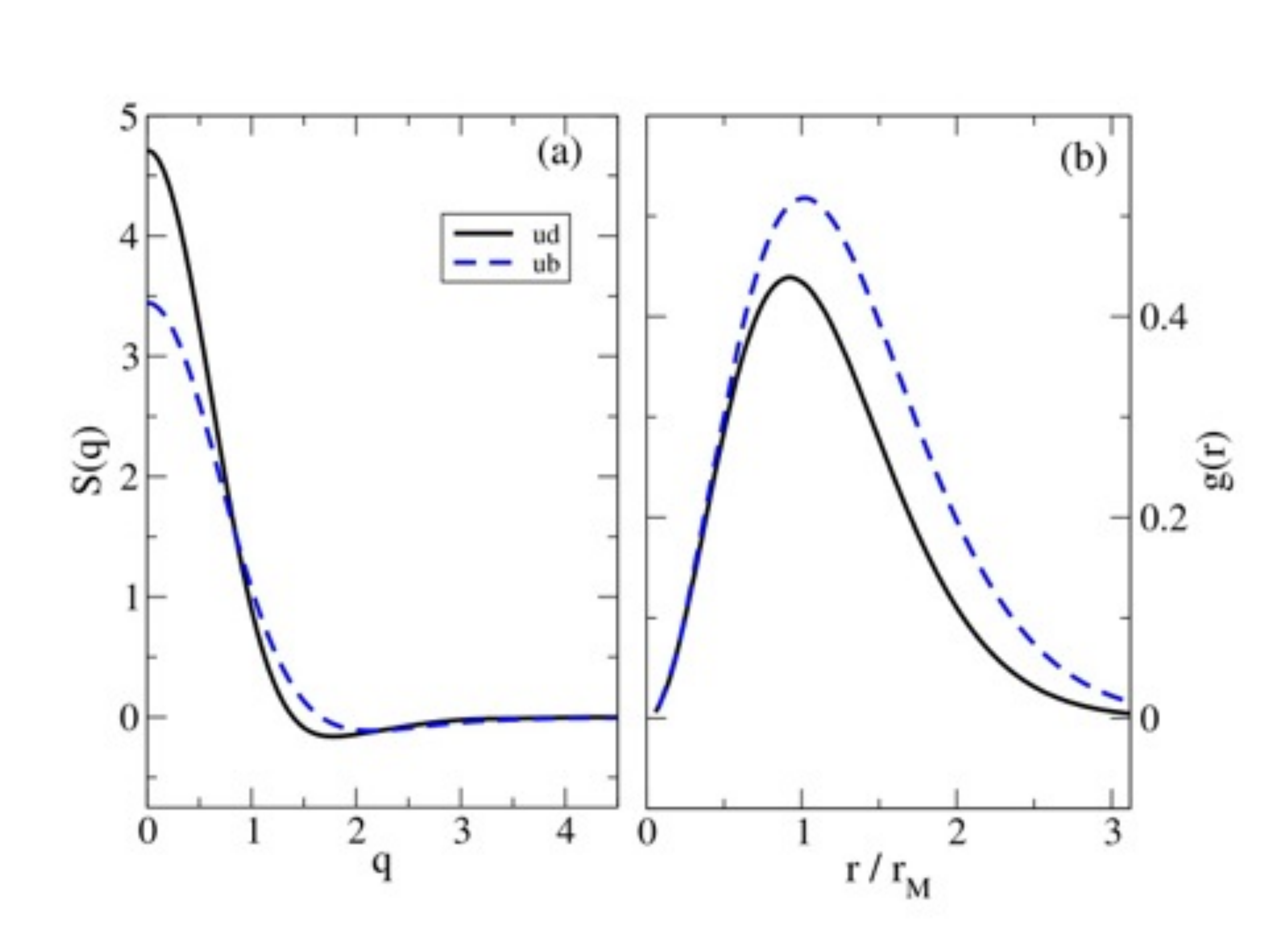
Two particle correlation function

$$g(r) \equiv \frac{V}{4\pi r^2 N^2} \left\langle \sum_{i<j=1}^{N} \delta\left(\vec{r} - \vec{r}_{ij}\right) \right\rangle$$



Meson- Meson correlation function is modified by the tetraquark presence

Di-quark



Di-quark correlation function parametrization

$$g(r)_{q-q} = A_0 r^2 e^{-r^{A_2}/A_1^2}$$

A0=0.64, A1=1.24, A2=1.51

Samuel H. Blitz and Richard F. Lebed, Phys. Rev. D 91 094025(2015). Pedro Bicudo and Marc Wagner, Phys. Rev. D 87, 114511 (2013).

The static structure factor S(q) can be obtained as the Fourier transform of the correlation function g(r)

$$S(\mathbf{q}) = 1 + \frac{N}{V} \int d^3r \; g(\mathbf{r}) \; e^{-i\mathbf{q}\cdot\mathbf{r}}.$$

Dynamical recombination

We define the recombination probability for the two mesons case by

$$Pr_{2m} = \frac{N(V_{m1} \leftrightarrow V_{m2})}{N(V_{m1}) + N(V_{m2})}$$

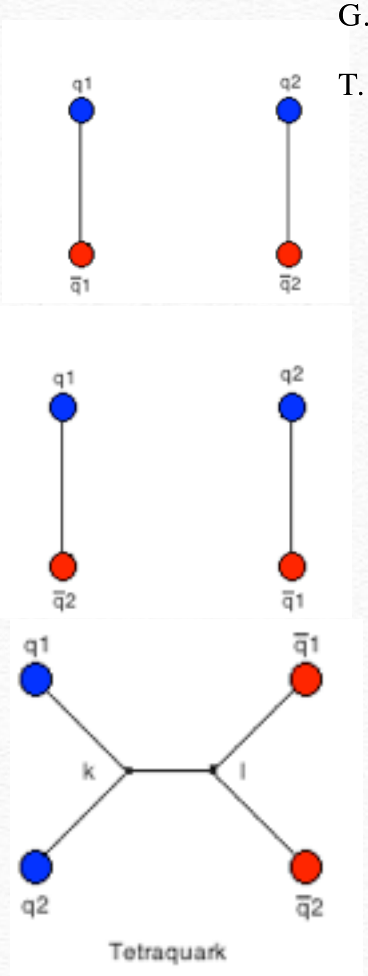
where $N(V_i \rightarrow V_j)$ denotes the number of flippings from the i to the j configuration, and $N(V_i)$ is the number of times the system visits the i configuration.

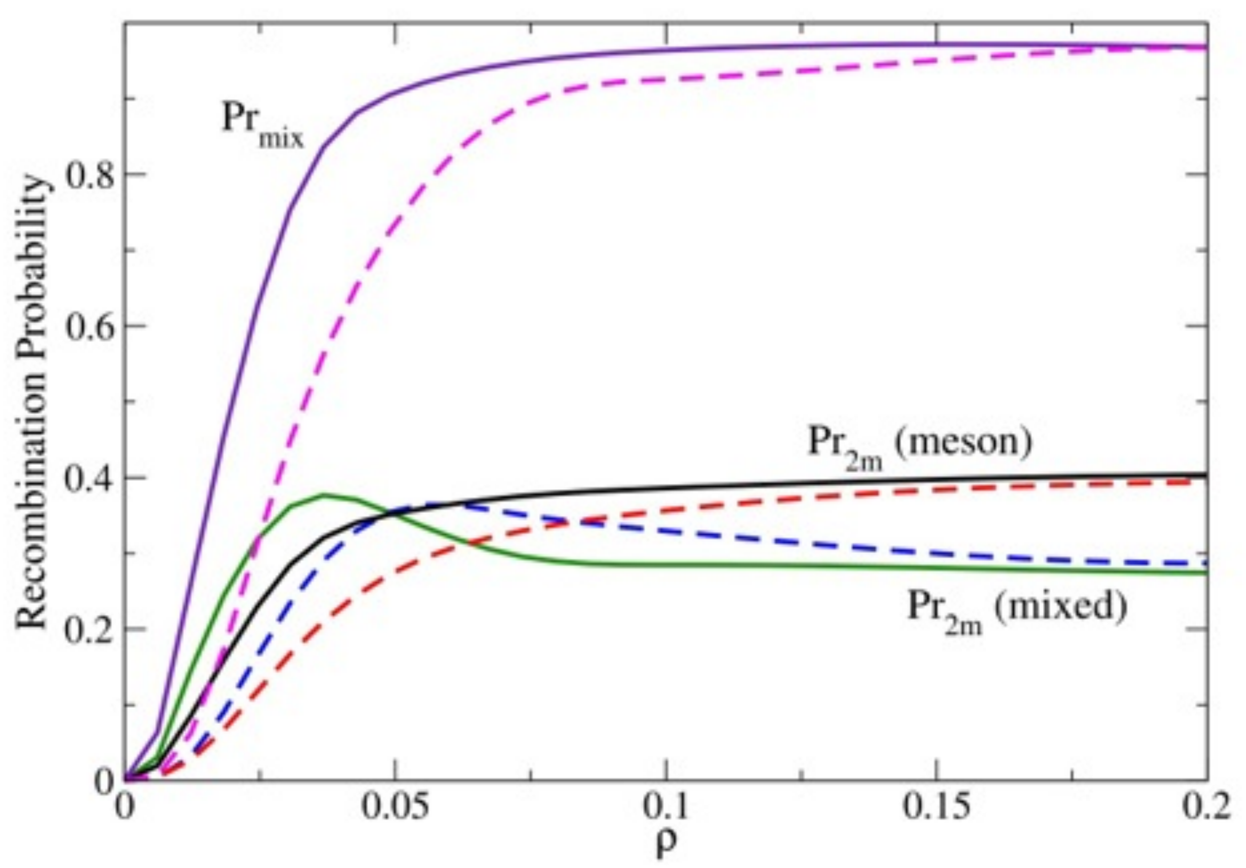
If we consider the tetraquark configuration as another possible configuration contributing to the recombination, the probability becomes:

$$Pr_{mix} = [N(V_{m1} \leftrightarrow V_{m2}) + N(V_{m1} \leftrightarrow V_{4Q}) + N(V_{m2} \leftrightarrow V_{4Q}) + N(V_{4Q} \rightarrow V_{4Q})] \,/\, [N(V_{m1}) + N(V_{m2}) + N(V_{4Q})]$$

G. Abbiendi et al, (The OPAL Collaboration), Eur. Phys. J. C. 45 307(2006).

T. Sjostrand and V. A. Khoze, Z. Phys.C 62 281(1994); Phys. Rev. Lett. 72 28(1994).





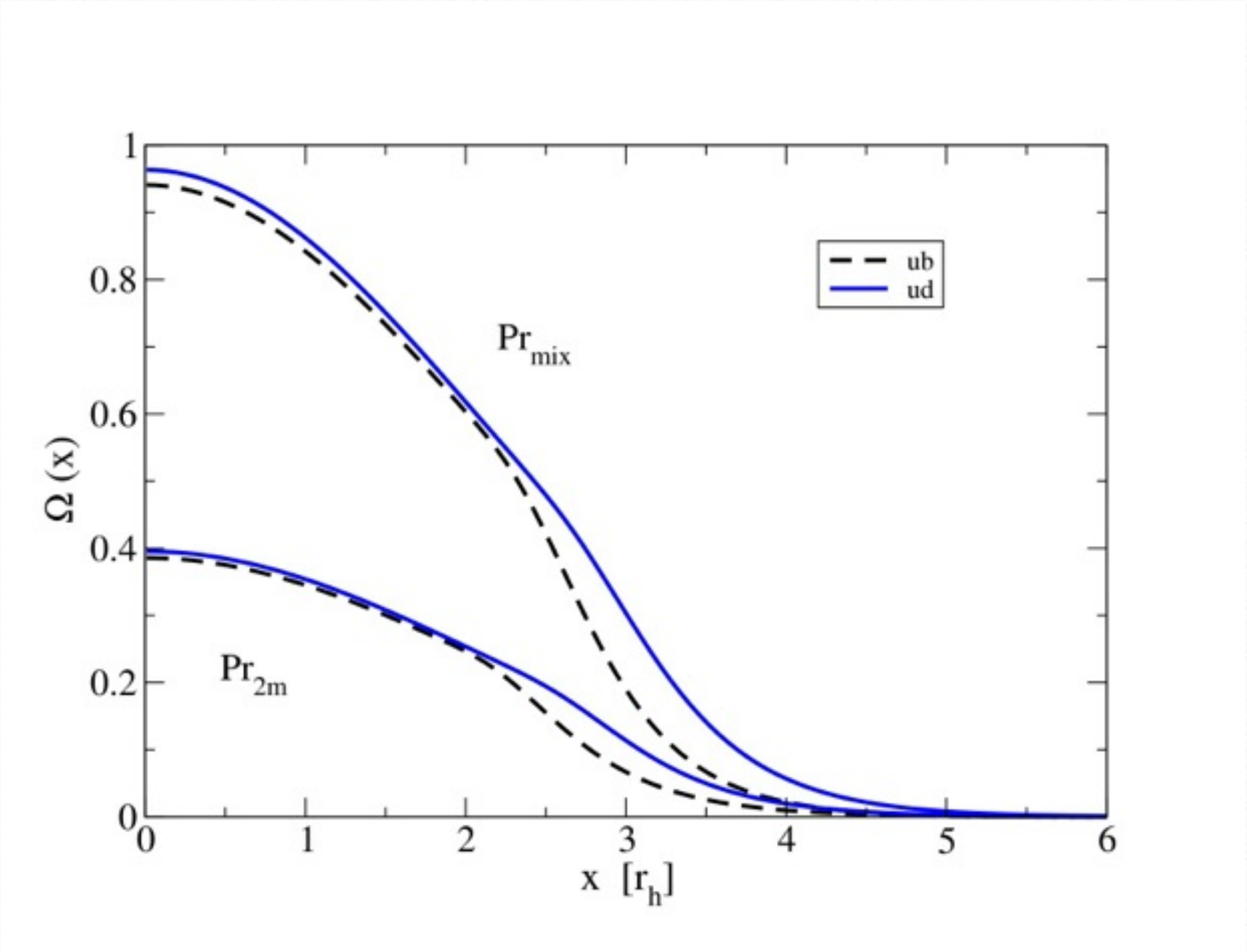
Dynamical Recombination

A qualitative estimate of this effect can be exhibited considering a simple expansion model, such that we can evolve the system along the density profile. We can define the color strength function

$$\Omega(x) \equiv P_{frag}(t) Pr(x)$$

where $P_{frag}(t)$ is the probability that the system has not yet fragmented, with $\tau_{frag} \approx 3 r_h$ the proper lifetime, taken as three times the meson radius r_h.

$$P_{frag}(t) = exp(-t^2/\tau_{frag}^2)$$

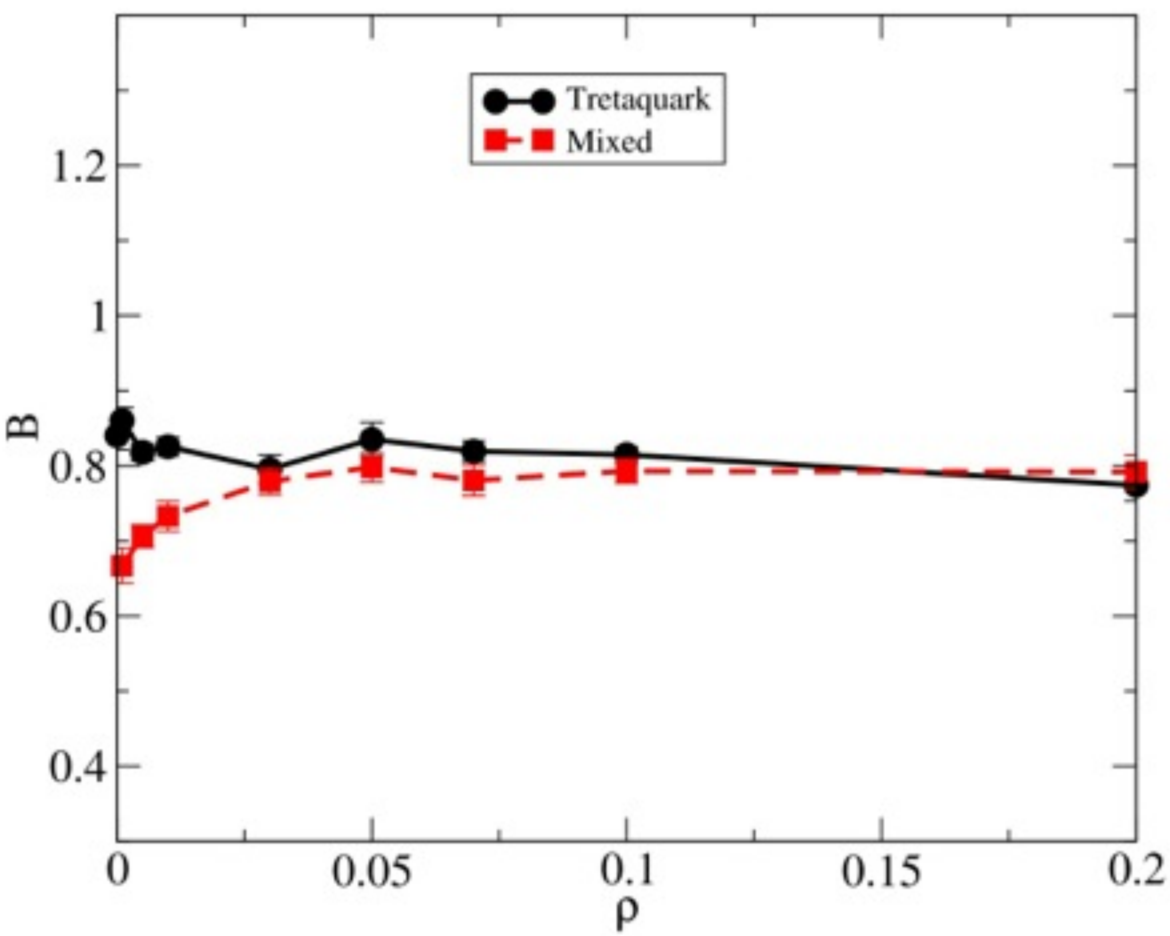


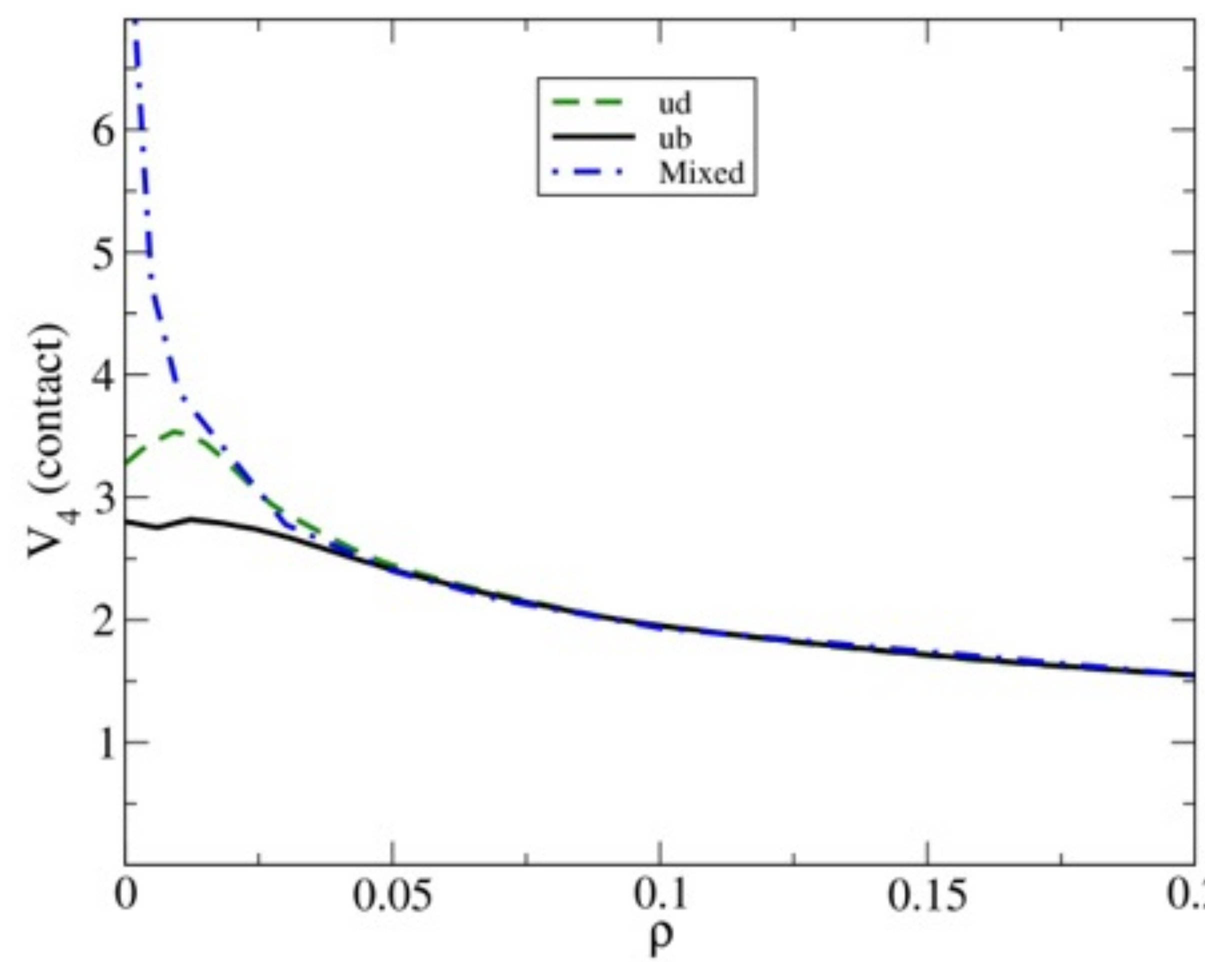
Tetraquark Potential

$$V(R) = R_0 + BR \qquad R \equiv \sqrt{\sum r_{ij}^2}$$

Linear behaviour

Effective Contact





Conclusions

We have performed a MC simulation considering three possible structures: two mesons, tetraquark and mixed configurations

We determined wether it is energetically more favorable to form a tetraquark or two mesons and the mixing among them, as a function of the particle density.

We have shown that there is a modification in the meson-meson correlation function by the presence of the tetraquark state at intermediate densities

A parameterization was found for the diquark , which is useful to compute additional static properties, in particular we computed the diquark static structure factor.

We did track the dynamical flipping among configurations and determined the recombination probability evolution as a function of the particle density. We have shown that the probability is largely affected when considering the tetraquark as an intermediate recombination state.

The linear behavior of the four-body potential on the invariant length linking the quarks was analyzed and found that the presence of a mixed state is reflected in the decreasing strength of the slope.

Thanks !

Variational parameter