PROBING GLUON SATURATION THROUGH DI-HADRON AND TRI-HADRON CORRELATIONS AT A FUTURE ELECTRON ION COLLIDER

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IN COLLABORATION WITH
A. AYALA, J. JALILIAN-MARIAN, M.E. TEJEDA YEOMANS,

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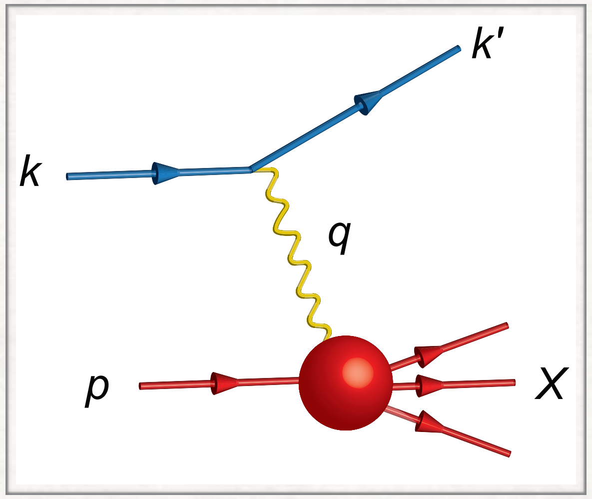
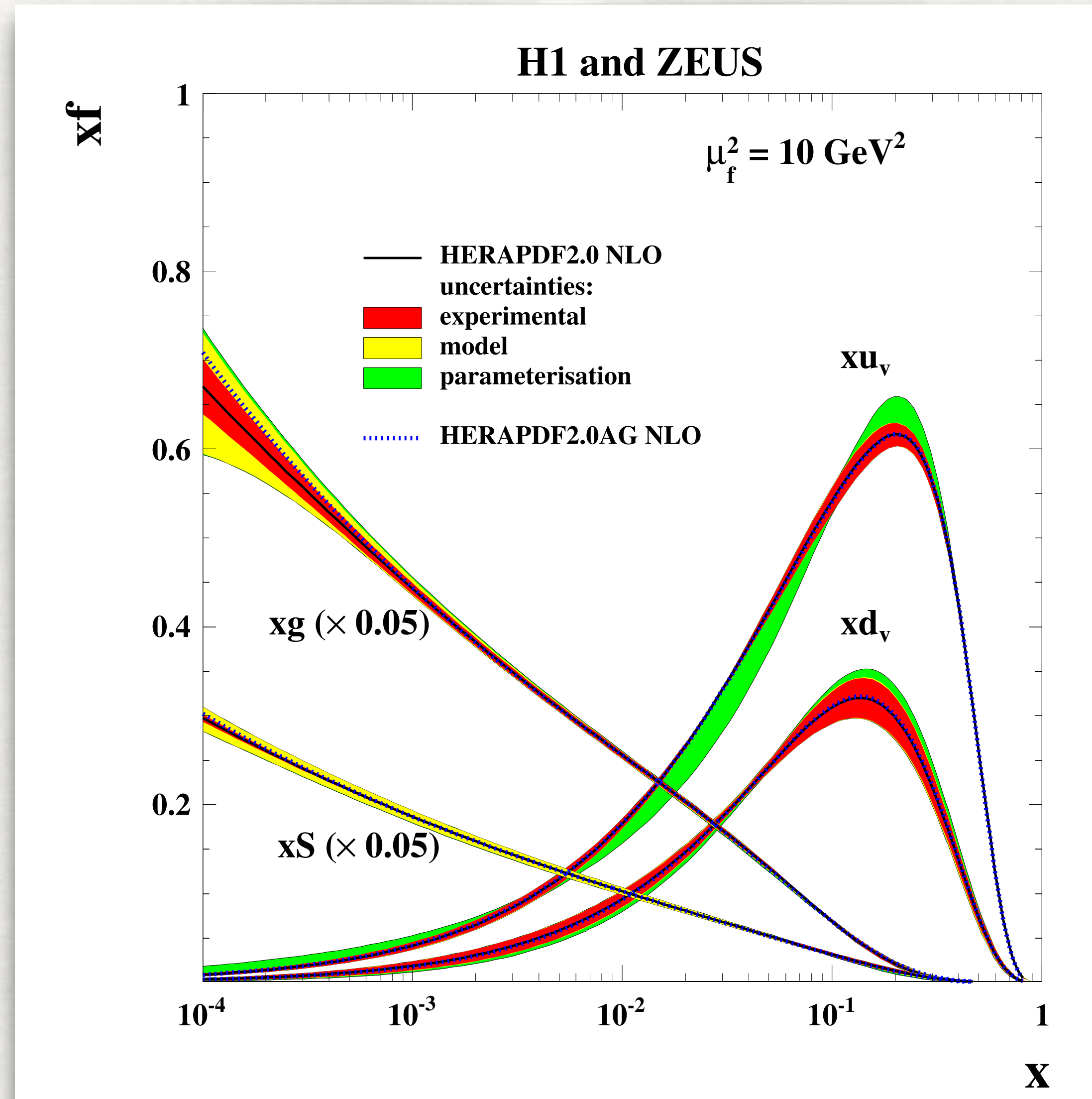
(02.-06. NOV. 2015)

DIS AT HERA: PARTON DISTRIBUTION FUNCTIONS



HERA collider (92-07): Deep Inelastic Scattering (DIS) of of electrons on protons

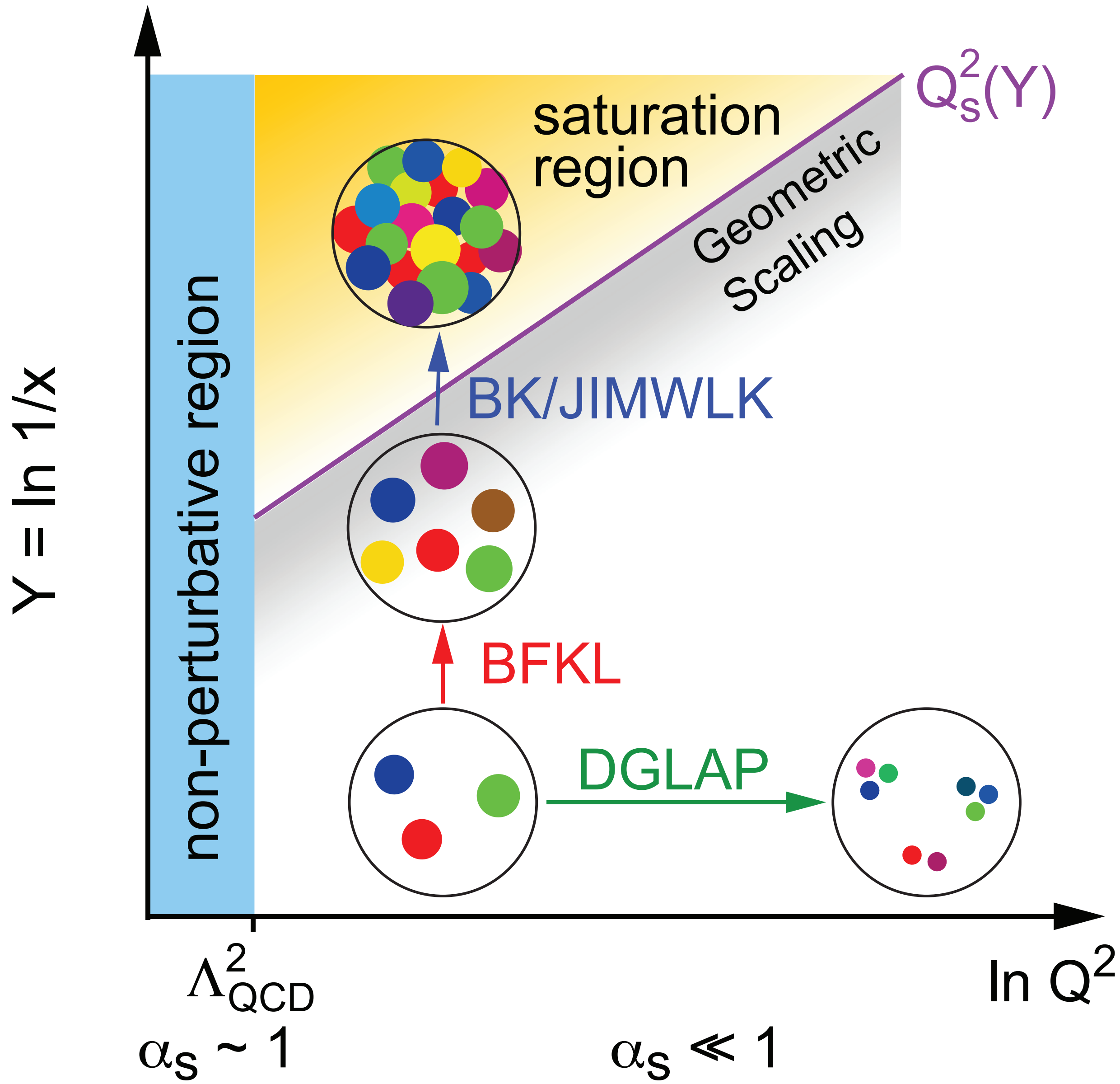
Photon virtuality
$Q^2 = -q^2$



- observation: gluon g(x) and sea-quark s(x) parton distribution functions grow like powers for x→0 with $x=Q^2/2p{\cdot}q \in[0,1]$
- parton distribution functions f(x): probability to find a quark, gluon with proton momentum fraction x in proton
- power like growth
  →integral over x does not convergent at x=0
  → invalidates probability interpretation
  at some x, new QCD dynamics must set in

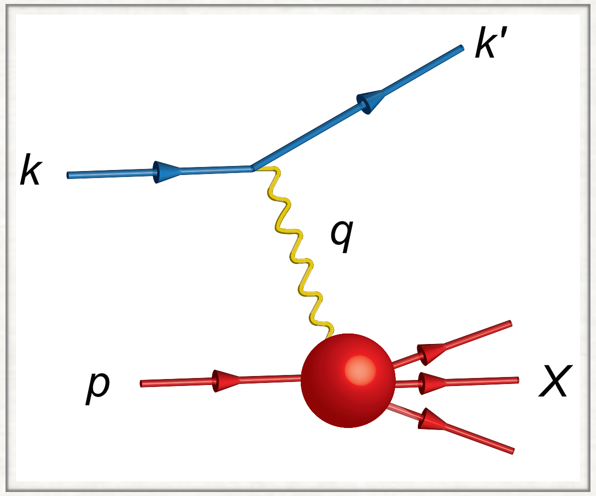
The proton at high energies: saturation

theory considerations:



- effective finite size $1/Q$ of partons at finite Q^2
- at some $x \ll 1$, partons 'overlap' $=$ recominbation effects
- turning it around: system is characterized by <u>saturation scale</u> Q_s
- grows with energy $Q_s \sim x^{-\Delta}$, $\Delta > 0$ & can reach in principle perturbative values $Q_s > 1\text{GeV}$

THEORY PREDICTIONS FOR HIGH & SATURATED GLUON DENSITIES



$x = Q^2/2p \cdot q \to 0$ limit corresponds to perturbative

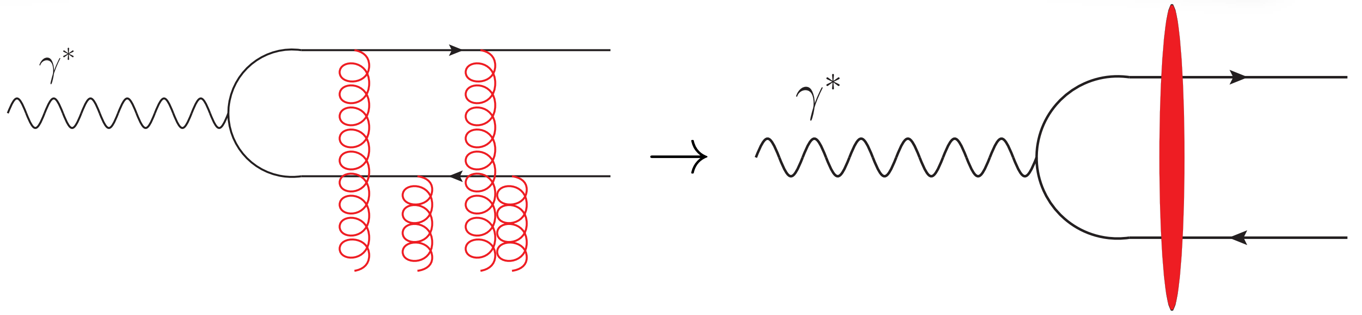
high energy limit $2p \cdot q \to \infty$ for fixed resolution Q^2

- make use of factorisation of cross-sections in the high energy limit
- allows to resum interaction of quarks & gluons with strong gluon field to all orders in the strong coupling → resummation of finite density effects
- DIS X-sec. as convolution of "photon wave function" (process-dependent) and "color dipole factor" (universal, resums ln1/x)

$$\sigma_{L,T}^{\gamma^* A}(x,Q^2) = 2\sum_f \int d^2\boldsymbol{b}\, d^2\boldsymbol{r} \int_0^1 dz \left|\psi_{L,T}^{(f)}(r,z;Q^2)\right|^2 \mathcal{N}(x,\boldsymbol{r},\boldsymbol{b})$$

- physical picture: virtual photon splits into color dipole (quark-antiquark pair) which interacts with Lorentz contracted target field

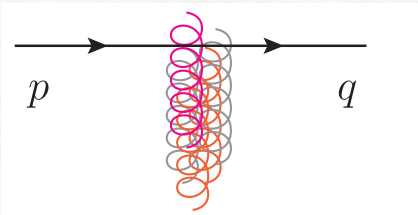
$$A^{+,a}(z^-,\boldsymbol{z}) = \alpha^a(\boldsymbol{z})\delta(z^-)$$



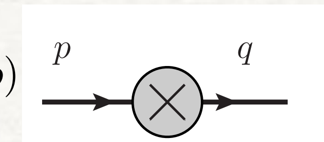
$x \to 0$: a single interaction with a strong & Lorentz contracted gluon field

PROPAGATORS IN THE PRESENCE OF A STRONG BACKGROUND FIELD

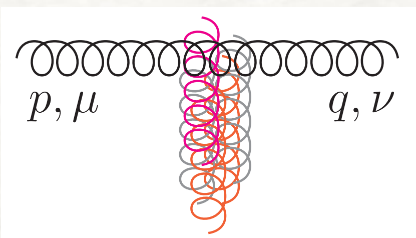
use light-cone gauge, with $k^- = n^- \cdot k$, $(n^-)^2 = 0$, $n^- \sim$ target momentum



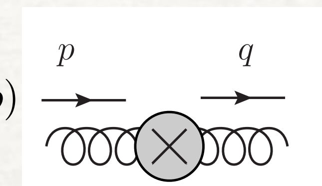
$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$



$$\tilde{S}_F^{(0)}(q)$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



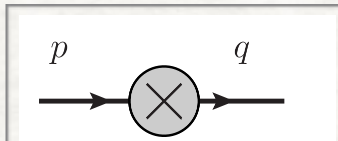
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

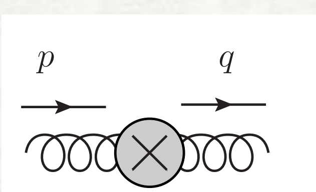
$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:



$$= 2\pi\delta(p^- - q^-)\not{n}^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \cdot \left\{\theta(p^-)[V(\boldsymbol{z}) - 1] - \theta(-p^-)[V^\dagger(\boldsymbol{z}) - 1]\right\}$$



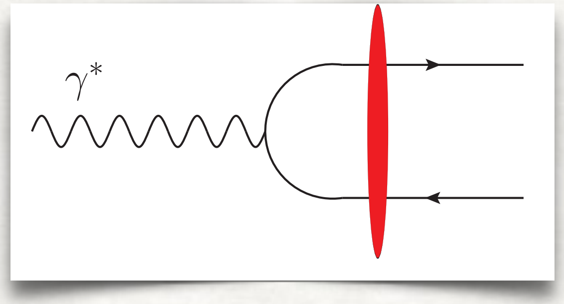
$$= -2\pi\delta(p^- - q^-)2p^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \cdot \left\{\theta(p^-)[U(\boldsymbol{z}) - 1] - \theta(-p^-)[U^\dagger(\boldsymbol{z}) - 1]\right\}$$

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) T^c$$

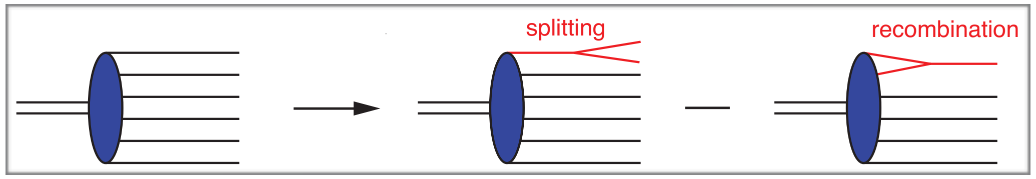
strong background field resummed into path ordered exponentials (Wilson lines)

PHENOMENOLOGY: DIS AT HERA

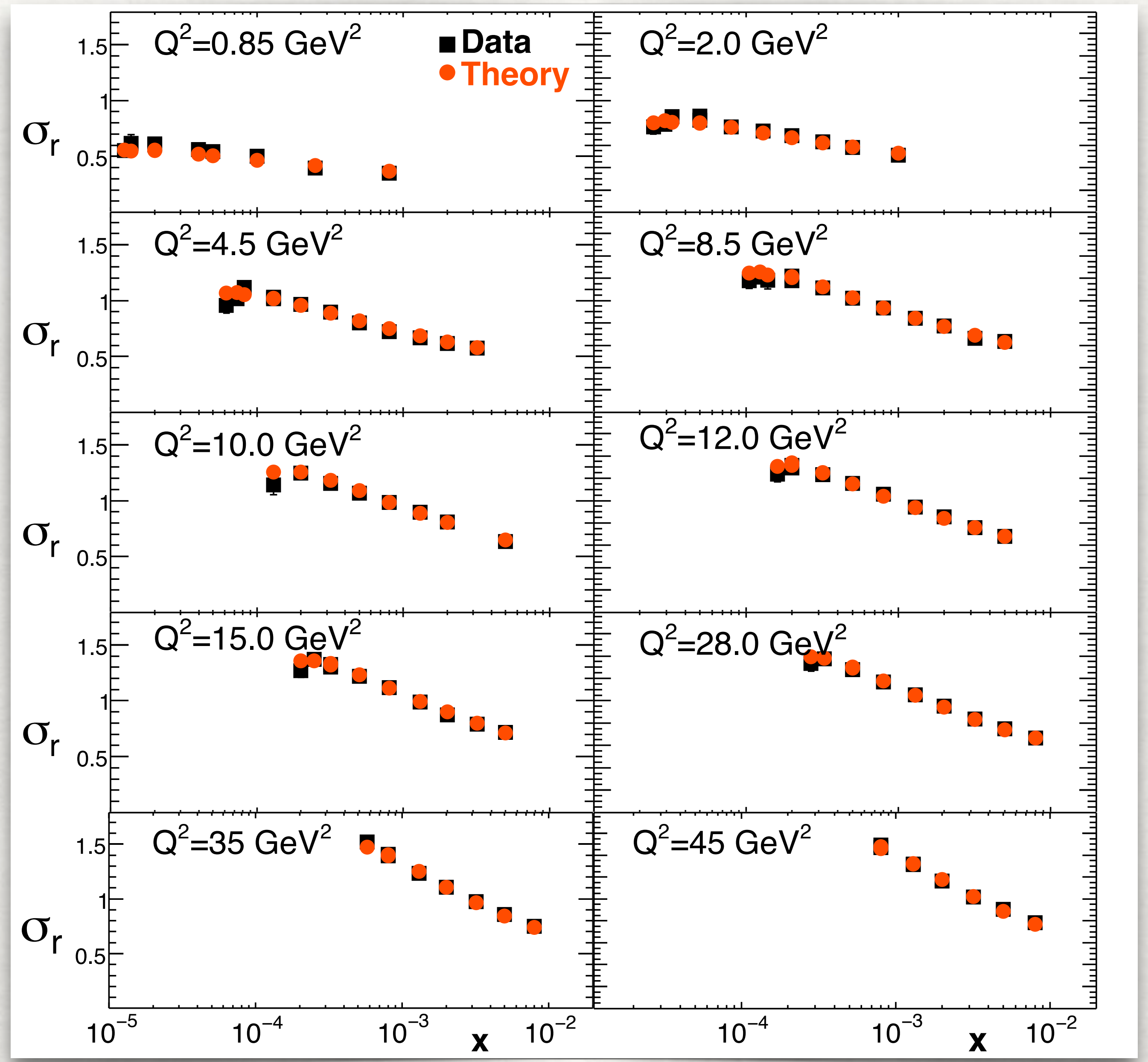


$$\sigma_{L,T}^{\gamma^* A}(x,Q^2) = 2\sum_f \int d^2\boldsymbol{b} d^2\boldsymbol{r} \int_0^1 dz \left|\psi_{L,T}^{(f)}(r,z;Q^2)\right|^2 \mathcal{N}(x,\boldsymbol{r},\boldsymbol{b})$$

- DIS cross-section as convolution of photon wave function and dipole density
- color dipole follows non-linear JIMWLK or BK evolution equation in ln(1/x)



- fixing initial conditions through fit allows description of combined HERA data, but also (dilute!) DGLAP describes data
- saturation at the edge Q_s~1-2GeV^2



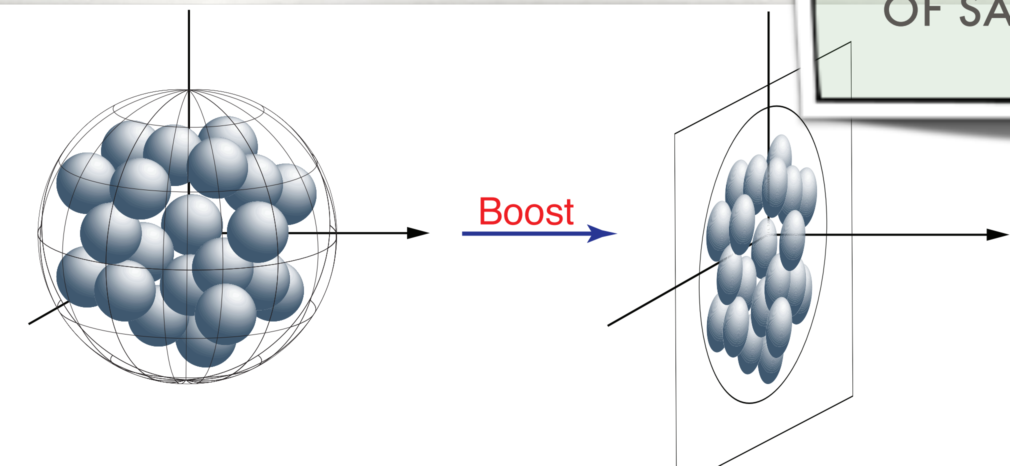
[Albacete, Armesto, Milhano,Quiroga, Salgado,EPJ C71 (2011) 1705]

PHENOMENOLOGY IN COLLISIONS WITH HEAVY NUCLEI

instead of going to higher energies (expensive), possible to study large nuclei ….

COLOR GLASS CONDENSATE (CGC)= BUZZWORD WHICH REFERS TO THE PHYSICS OF SATURATION AND IN PARTICULAR THE DEVELOPED THEORY

Expect those effects to be even more enhanced in boosted nuclei:

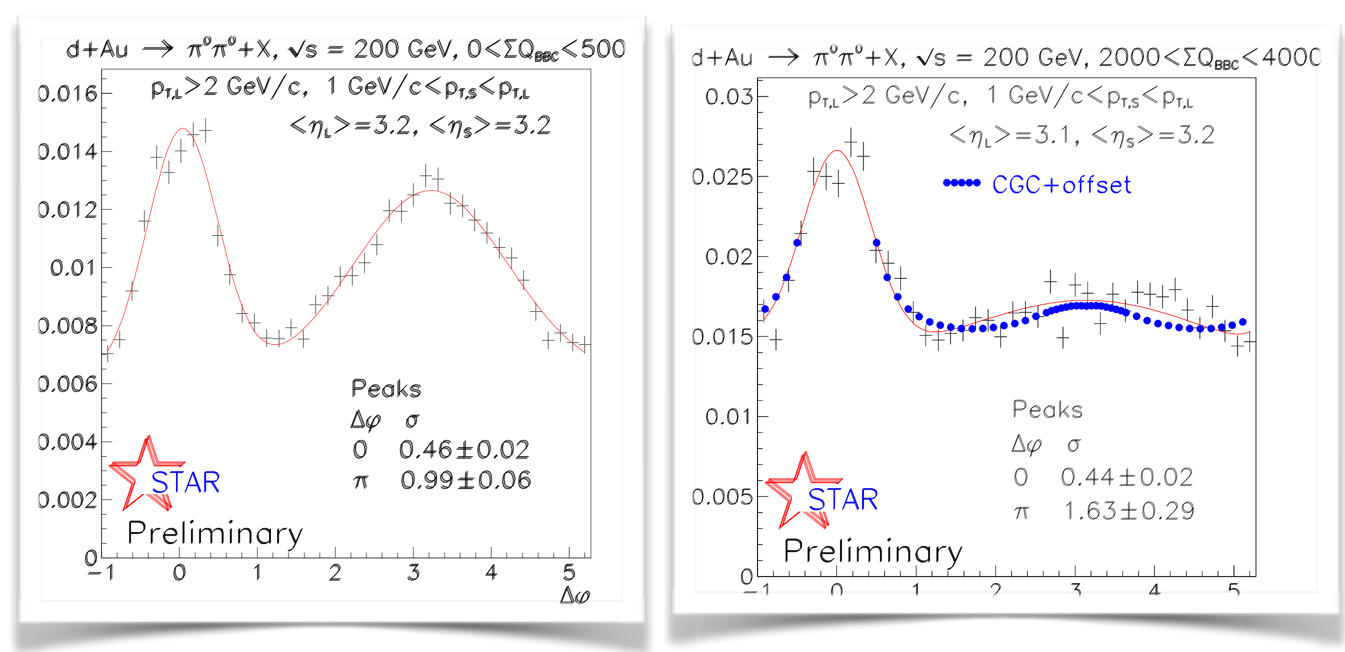


$$Q_s^2 \sim \#\ \textbf{gluons/unit transverse area} \sim A^{1/3}$$

d-Au collisions at RHIC: depletion of away side peak in central collisions described by CGC

many more studies at RHIC, LHC in pp, pA, AA collisions

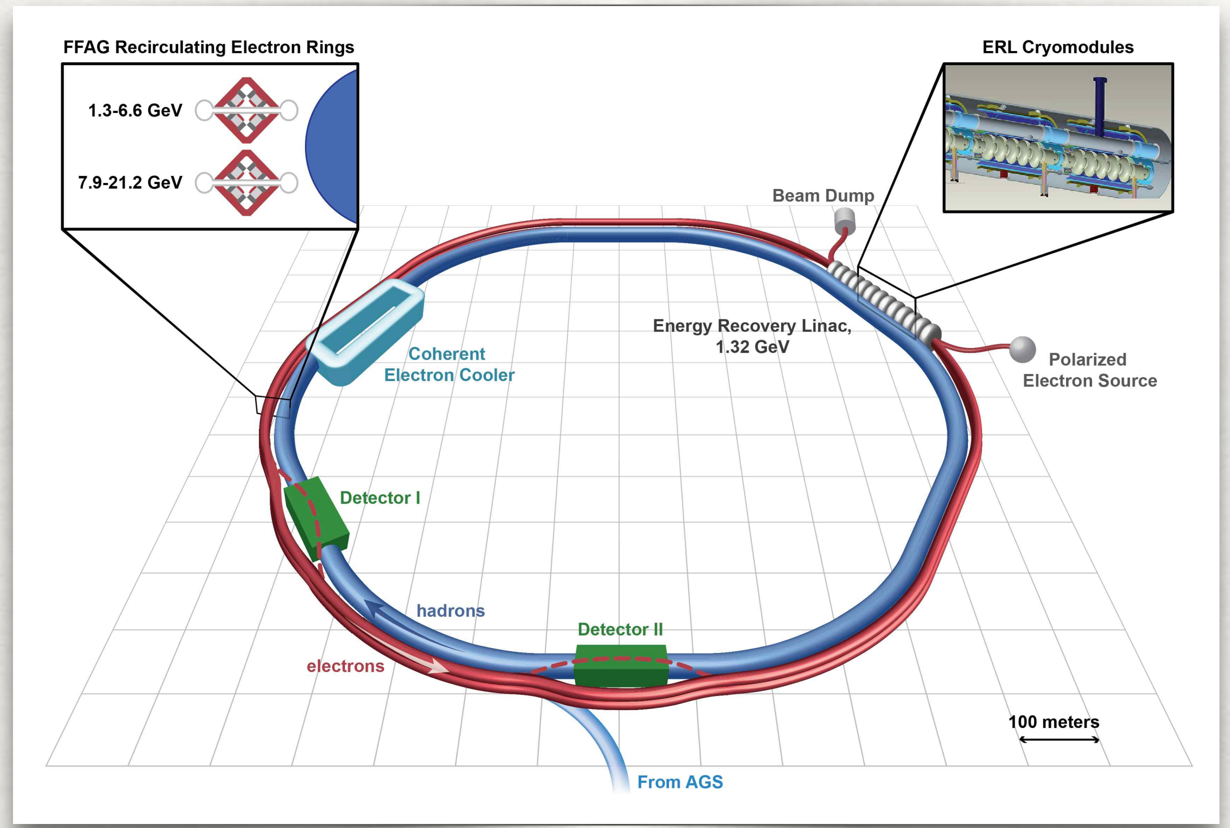
plethora of interesting phenomena, but also subject to large theory uncertainties due to uncontrolled re-scatterings→ no ultimate proof

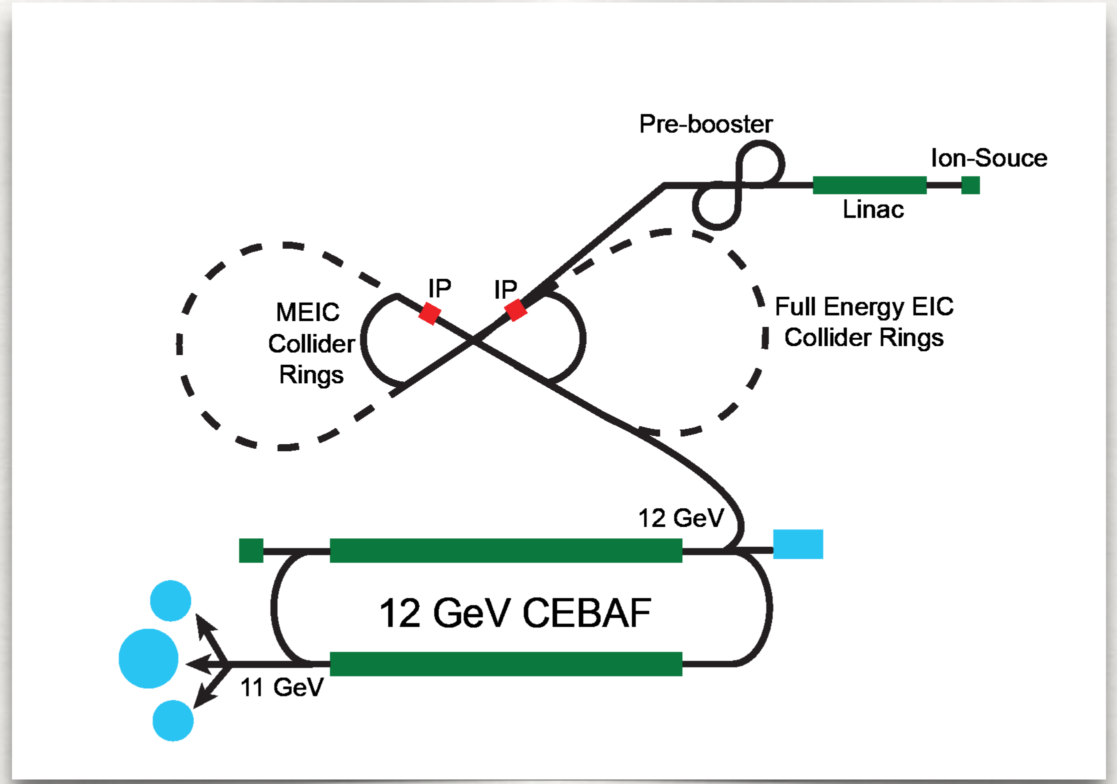


A COLLIDER TO SEARCH FOR A DEFINITE ANSWER:

THE ELECTRON ION COLLIDER PROJECT

the world's first eA collider: will allow to probe heavy nuclei at small x (using 16GeV electrons on 100GeV/u ions)



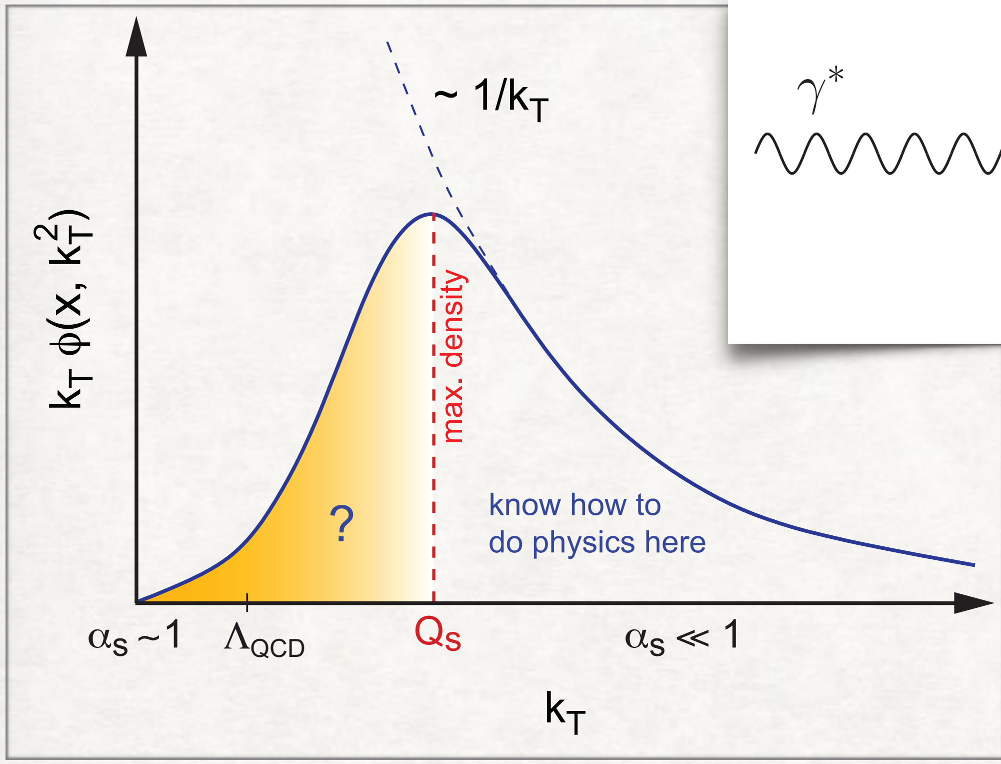


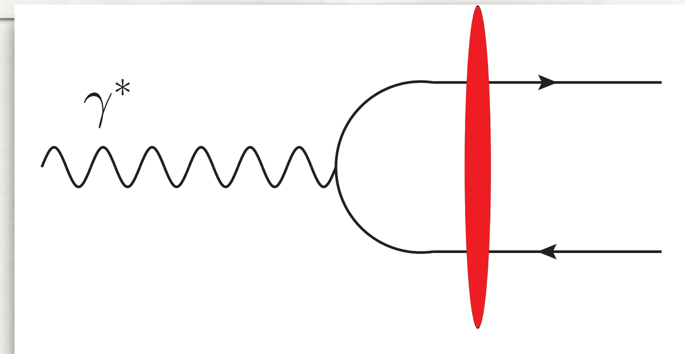
Brookhaven National Laboratory: supplement RHIC with Electron Recovery Linac (eRHIC)

Jefferson Lab: supplement CEBAF with hadron accelerator (MEIC)

2015: ENDORSED BY NUCLEAR SCIENCE ADVISORY COMMITTEE (NSAC) AS HIGHEST PRIORITY FOR NEW FACILITY CONSTRUCTION IN US NUCLEAR SCIENCE LONG RANGE PLAN

AN EIC OBSERVABLE TO SEARCH FOR SATURATION EFFECTS: DI-HADRON DE-CORRELATION IN DIS

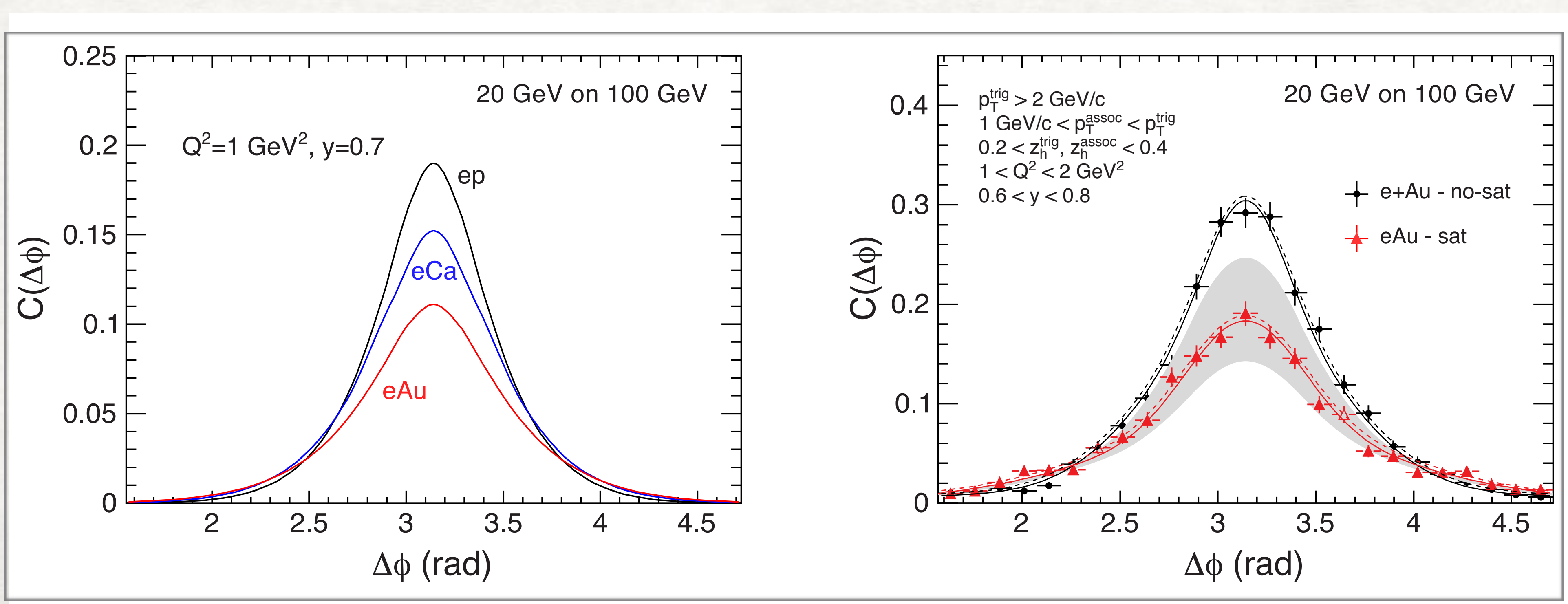




measure azimuthal angle of di-hadron final state

collinear factorization (dilute pQCD): gluon kT peaked at kT=0 - expect dihadrons back-to-back

Saturation (CGC): gluon kT peaked at saturation scale - expect de-correlated di-hadrons

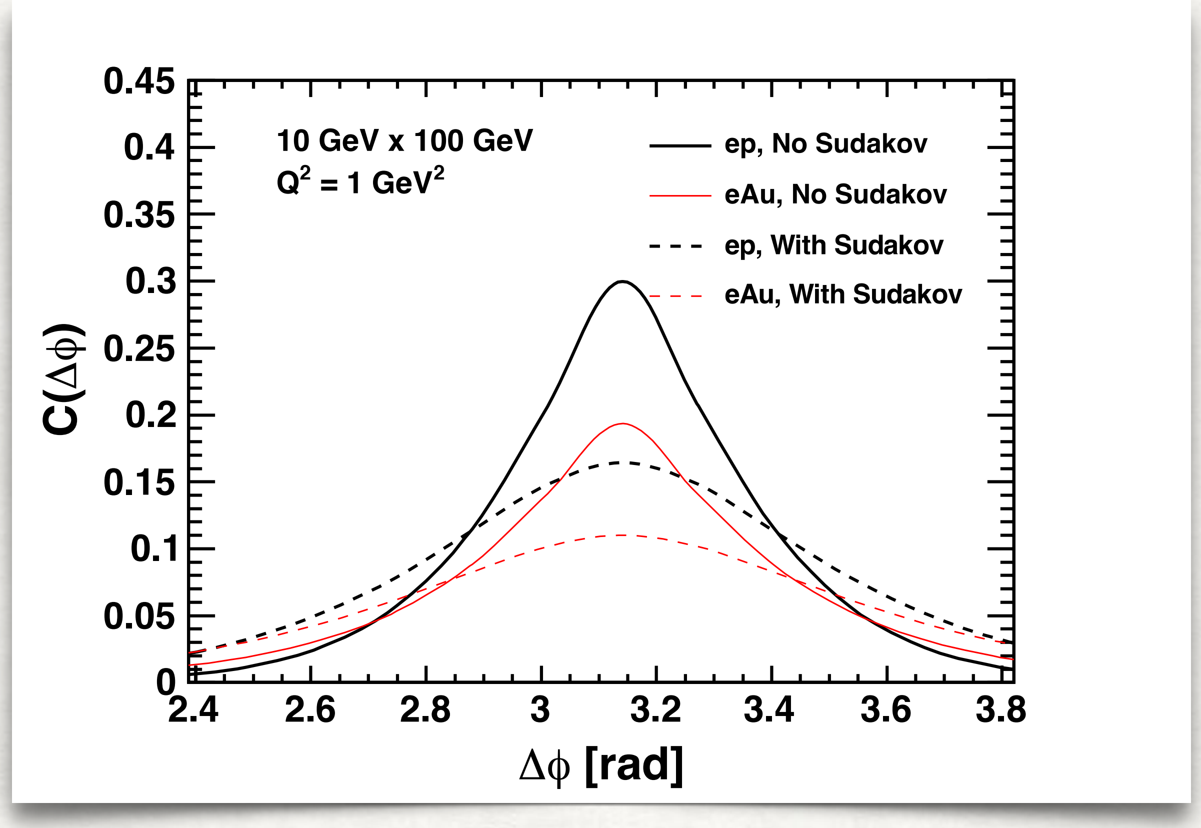


PRECISION EXPERIMENTS REQUIRE THEORY PRECISION

- current studies: LO accuracy + Sudakov resummation of soft logarithms

expect also (large?) collinear logs
+ scale setting uncertainties

→ higher order correction can lead to large effects



[Zheng,Aschenauer, Lee, Xiao, PRD89 (2014)7, 074037]

☑ evolution of dipole etc. densities & higher correlators know up to NLO

[Balitsky, Chirilli; PRD 88 (2013) 111501, PRD 77 (2008) 014019]; [Kovner,Lublinsky, Mulian; PRD 89 (2014) 6, 061704]

☑ instabilities get addressed

[Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos; PLB 744 (2015) 293]

☐ photon wave function: only inclusive (on the level of correlation functions)

[Balitsky, Chirilli; PRD 87 (2013) 1, 014013], [Beuf; PRD 85 (2012) 034039]

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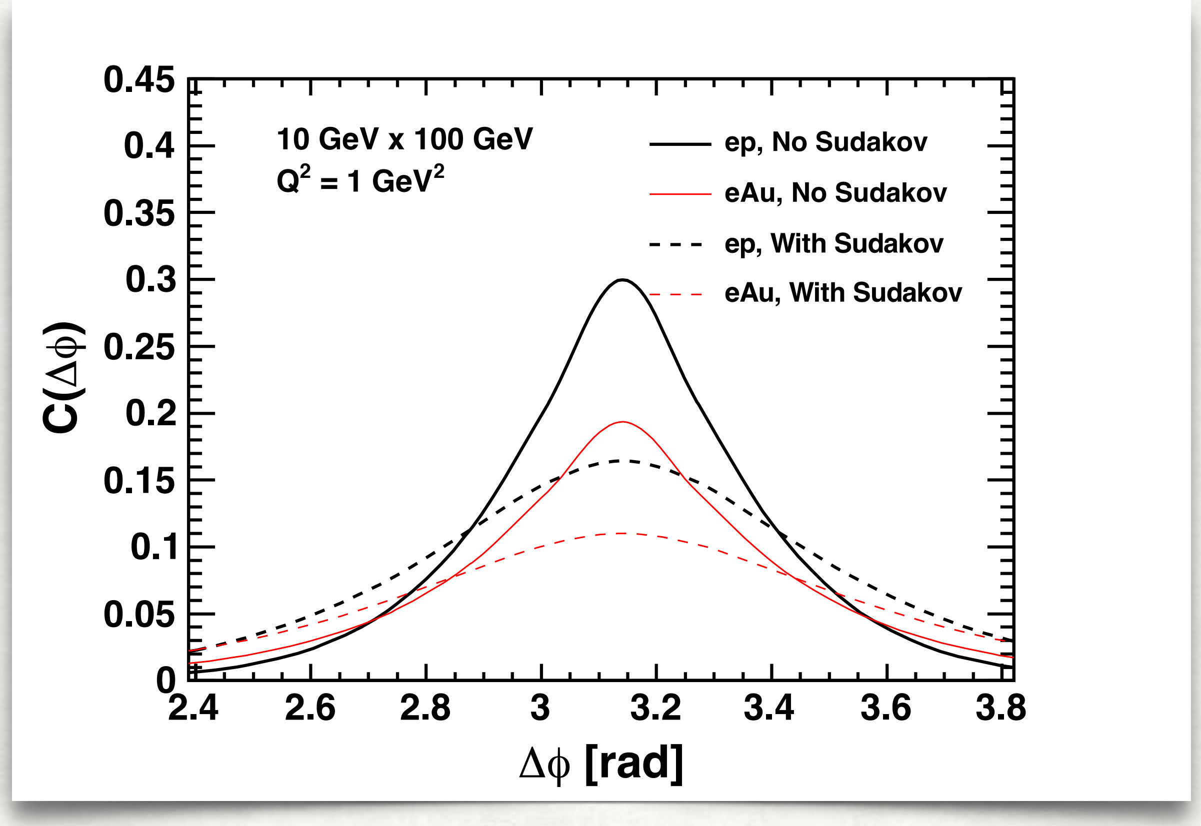


[Zheng,Aschenauer, Lee, Xiao, PRD89 (2014)7, 074037]

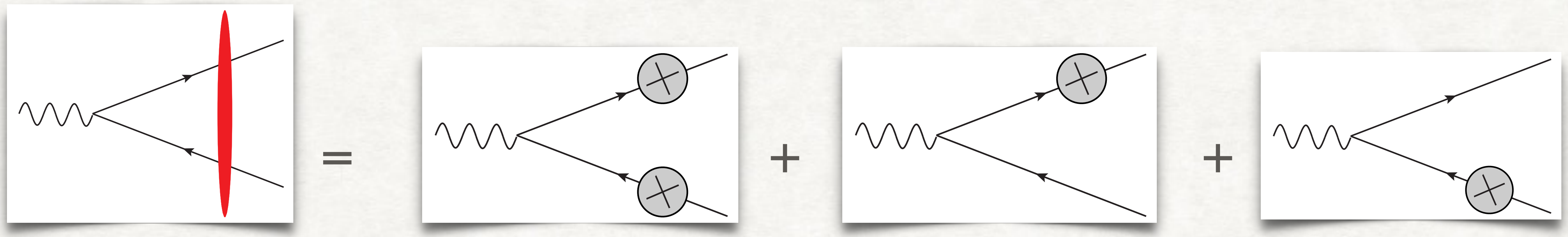
our project: calculate (NEW: NLO from momentum space)

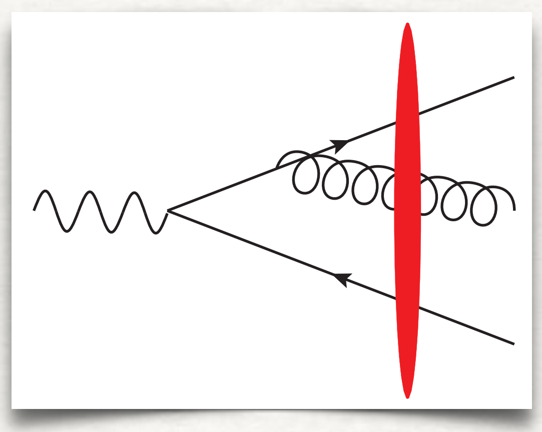
A. tri-hadron production at LO (new observable!)
   expect more stringent tests of CGC through more complex final state

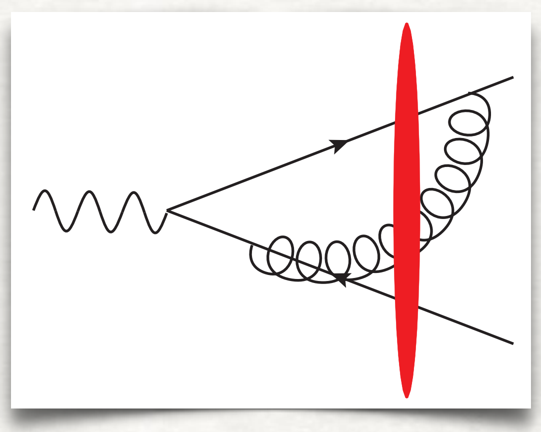
B. di-hadron production at NLO (3 partons a subset!)
   reduce uncertainties + possibly identify overlap region between collinear factorisation and saturation physics

1 EXTRA HADRON CAN CAUSE A LOT OF WORK!

di-hadrons at LO: paper & pencil calculation *e.g.* [Gelis, Jalilian-Marian,PRD67, 074019 (2003)]







each line & each final state splits into two terms (free + interaction)

→ real NLO: 16 diagrams (amp. level)

→ virtual NLO: 32 diagrams (amp. level)

on X-sec. level: up to 16 Gamma matrices in a single Dirac trace

→ 15! = 1307674368000 individual terms (not all non-zero though)

- necessary to achieve (potential) cancelations of diagrams BEFORE evaluation
- require automatization of calculation (= use of Computer algebra codes)

REDUCE # OF DIAGRAMS

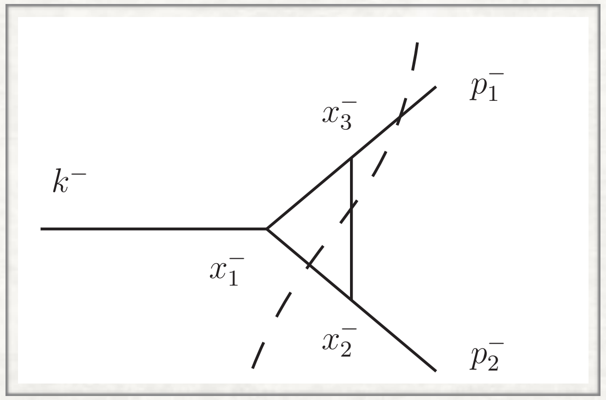
CONFIGURATION SPACE: CUTS AT $X^-=0$

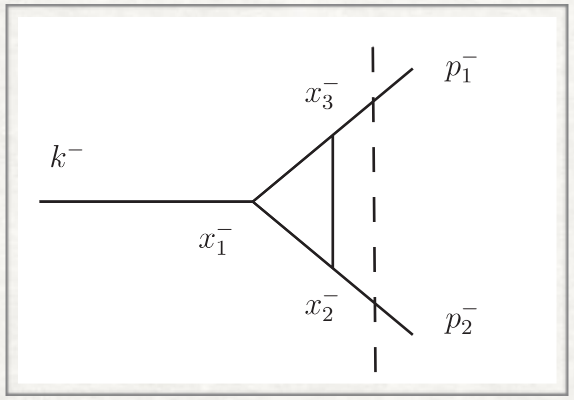
- diagrams to configuration space → momentum delta function as integral at each vertex + four momentum integral at each internal internal line
- Feynman propagator in configuration space

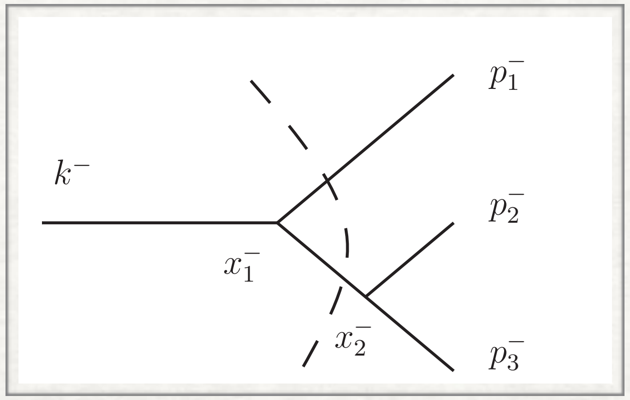
$$\Delta_F^{(0)}(x) = \int \frac{d^dp}{(2\pi)^d} \frac{i \cdot e^{-ip\cdot x}}{p^2 - m^2 + i0} = \int \frac{dp^+}{(2\pi)} \int \frac{dp^- d^{d-2}\boldsymbol{p}}{(2\pi)^{d-1}} \frac{e^{-ip^-x^+ + i\boldsymbol{p}\cdot\boldsymbol{x}}}{2p^-} \cdot \frac{i \cdot e^{-ip^+x^-}}{p^+ - \frac{\boldsymbol{p}^2+m^2-i0}{2p^-}}$$

$$= \int \frac{dp^- d^{d-2}\boldsymbol{p}}{(2\pi)^{d-1}} \frac{e^{-ipx}}{2p^-} \left[\theta(p^-)\theta(x^-) - \theta(-p^-)\theta(-x^-)\right]_{p^+ = \frac{\boldsymbol{p}^2+m^2}{2p^-}}$$

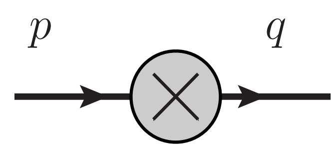
- divide x_i^- integral $\int_{-\infty}^{\infty} dx_i^- \to \int_{-\infty}^{0} dx_i^- + \int_0^{\infty} dx_i^-$ → each of our diagrams cut by a line separating positive & negative light-cone time
- s-channel kinematics [$k^-=p_1^- + p_2^- + \ldots$, all positive] → only s-channel type cuts possible (~vertical cuts)







CONFIGURATION SPACE CAN HELP

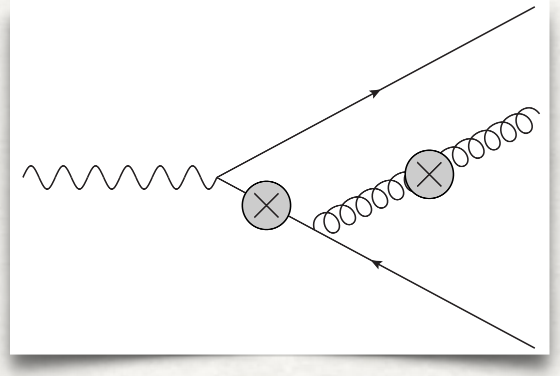
- recall: 

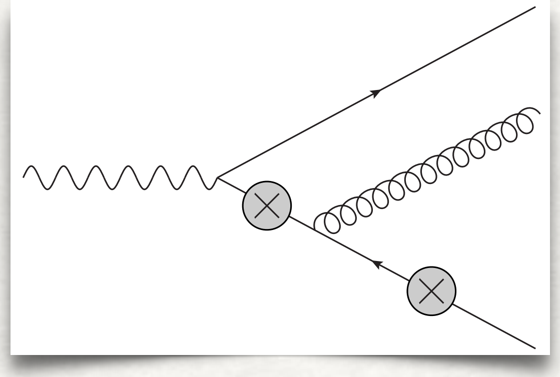
$\propto \delta(p^- - q^-)$ *i.e.* minus momentum flow not altered through interaction

- recall: interaction placed at slice z^-=0 $A^{+,a}(z^-, \boldsymbol{z}) = \alpha^a(\boldsymbol{z})\delta(z^-)$

  → interaction must be always placed at a z^-=0 cut of the diagram.
  Note: this applies equally to configuration and momentum space

- evaluates already a large fraction of diagrams (~50%) to zero





forbidden configurations: cannot be accommodated by vertical (s-channel type) cut

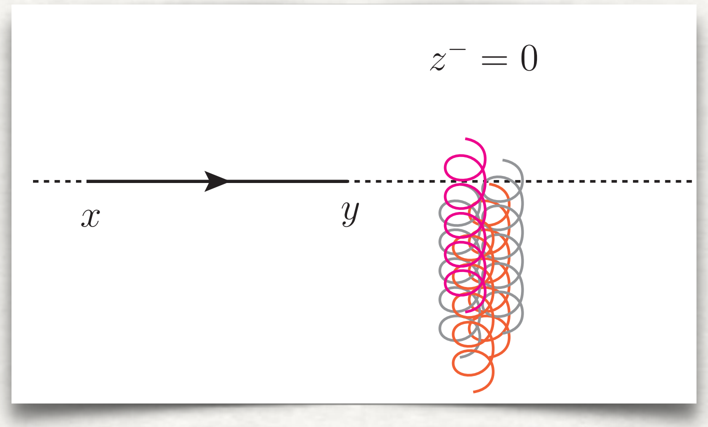
CAN WE DO BETTER? .... MORE CONSTRAINTS

consider complete configuration space propagator (free + interacting part)

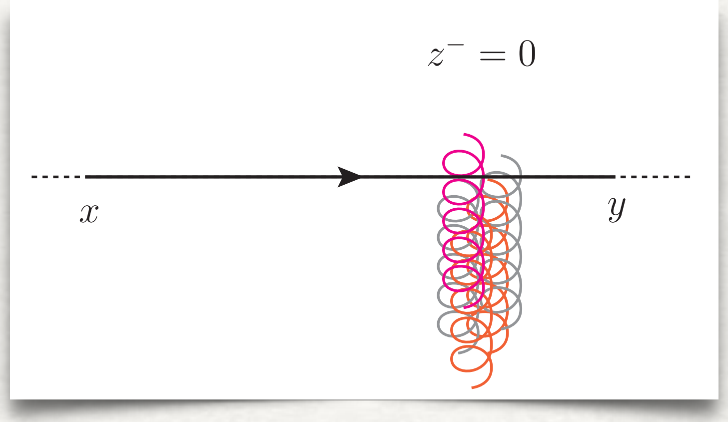
$$S_F(x,y) = \int \frac{d^dp}{(2\pi)^d}\frac{d^dq}{(2\pi)^d} e^{-ipx}\left[\tilde{S}_F^{(0)}(p)(2\pi)^d\delta^{(d)}(p-q) + \tilde{S}_F^{(0)}(p)\tau_F(p,q)\tilde{S}_F^{(0)}(q)\right]e^{iqy}$$

obtain free propagation for

- $x^-,y^-<0$ ("before interaction")
- $x^-,y^->0$ ("after interaction")

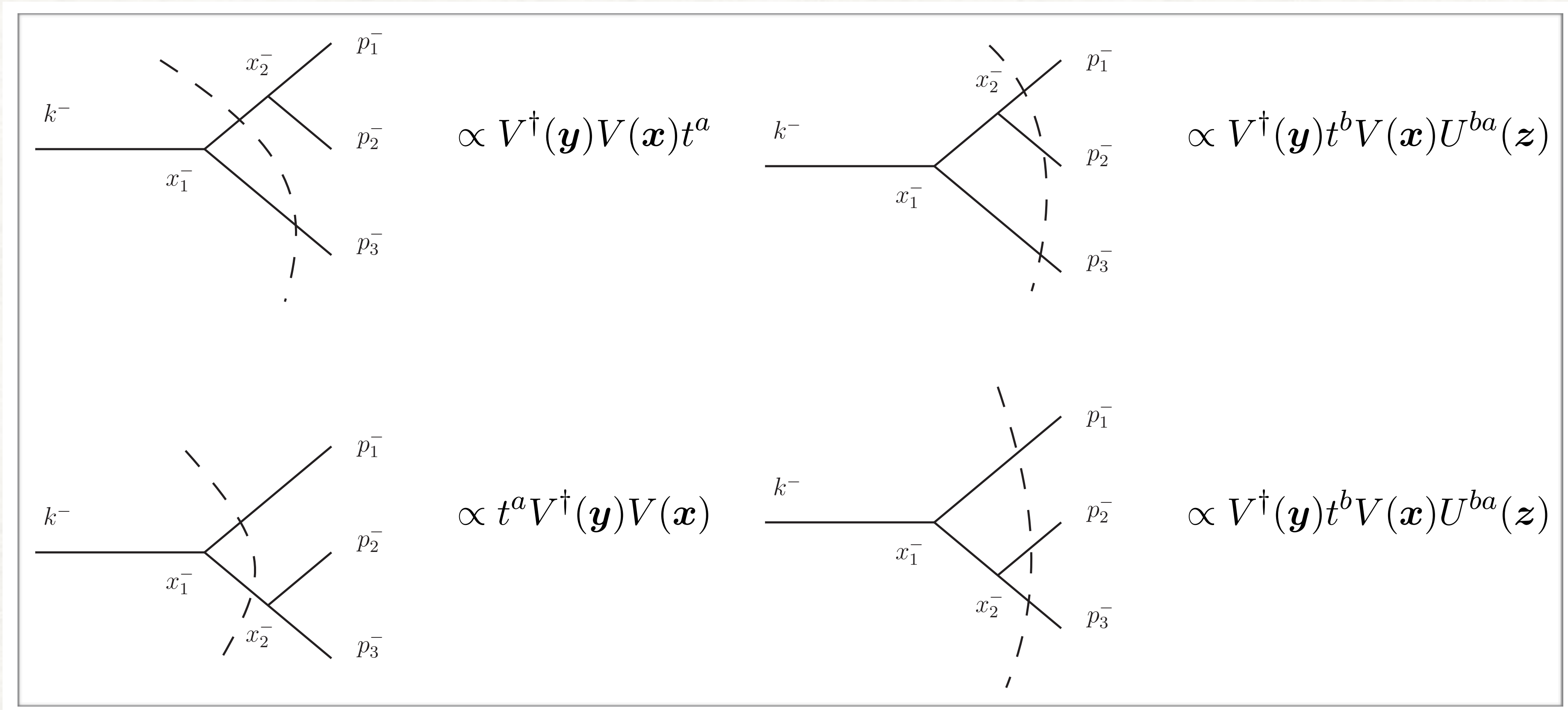


propagator proportional to complete Wilson line V (fermion) or U (gluon) if we cross cut at light-cone time 0



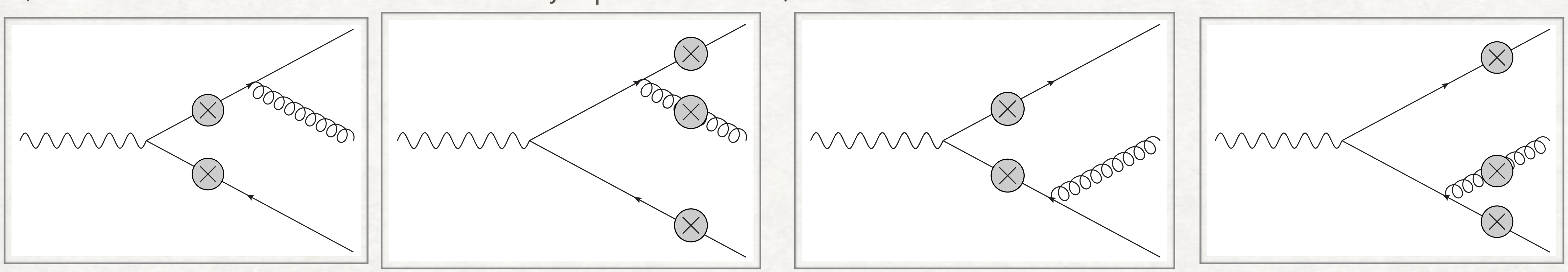
- no direct translation to momentum space
  adding free propagation & interaction→ mixing of different mom. space diagrams
- but strong constraints on the structure of the full result

CONFIGURATION SPACE PREDICTS WHICH OPERATORS HAVE NON-ZERO COEFFICIENTS



momentum space: necessary coefficients from only 4 (instead of 16) diagrams

(cancelation of all other contributions verified by explicit calculations)



virtual corrections: similar result,

necessary coefficients from 8 (instead of 32) diagrams

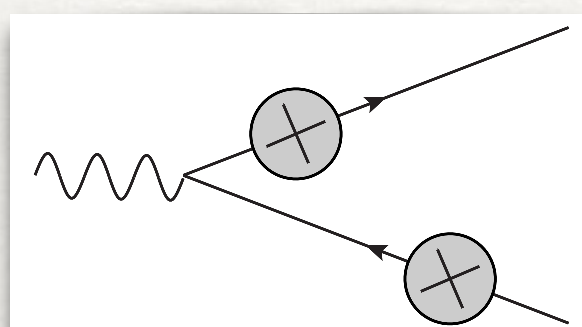
LOOP INTEGRALS

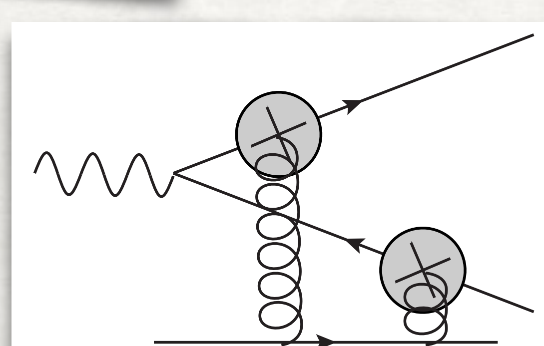
something slightly strange:

LOOP INTEGRALS ALSO FOR REAL CORRECTIONS

technical reason:

- momentum space amplitudes obtained from field correlators during LSZ reduction procedure
- integration over coordinates at vertices yields delta functions which evaluate momentum integrals trivially
- here: coordinate dependence of background field → delta functions absent

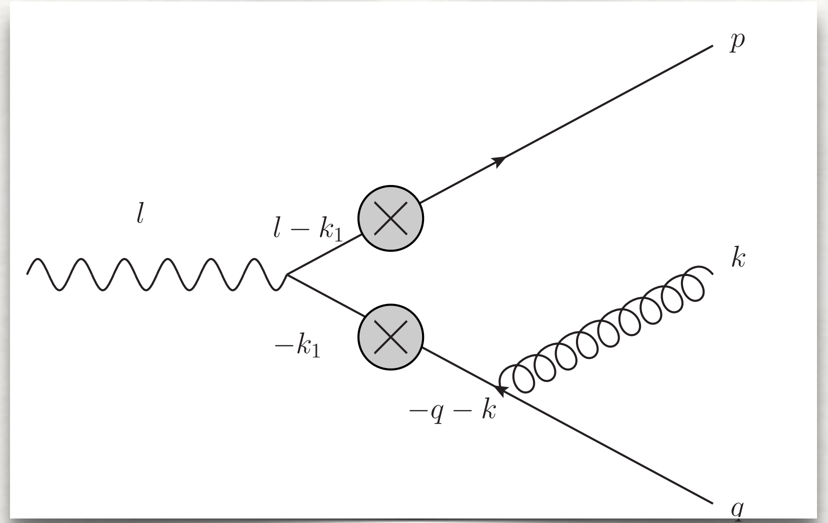


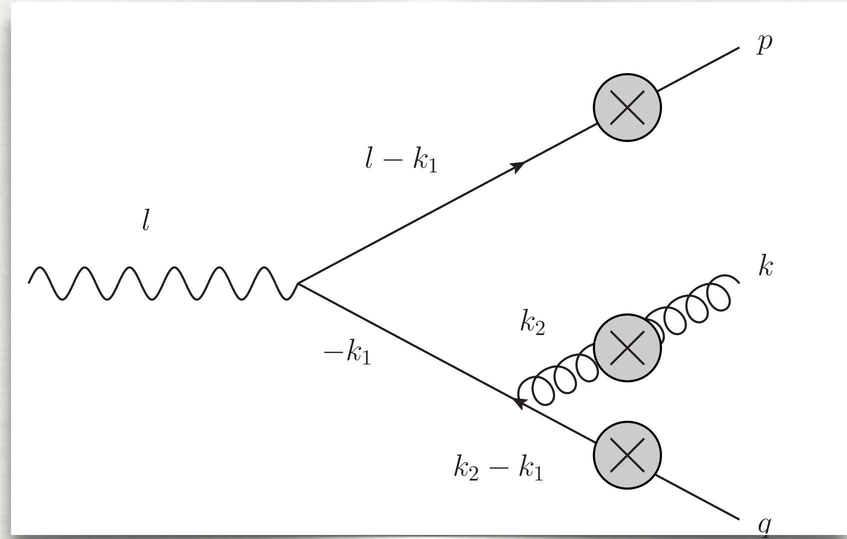


intuitive picture:
background field = t-channel gluons interacting with the target→ naturally provide a loop which is factorized & (partially) absorbed into the projectile in the high energy limit

for the rest of the talk: **focus on real corrections/3 partons**

a 1-loop and a 2-loop topology





k_1 and k_2 are loop momenta
new complication: exponentials/Fourier factors

conventional: *e.g.* k_1^+ integration by taking residues, then transverse integrals
particular for 2 loop case: complicated transverse integrals

developed a new technique

- ⋆ complete exponential factors to 4 d
- ⋆ evaluate integral using "standard" momentum space techniques

A 1-LOOP EXPAMPLE:

$$I(p_1,p_2) = \int \frac{d^dk_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l-k_1)^2]} e^{ix_t(\cdot k_{1,t}-p_{1,t})} e^{-iy_t\cdot(k_{1,t}+p_{2,t})} (2\pi)^2 \delta(p_1^- - k_1^-)\delta(l^- - k_1^- - p_2^-)$$

start with integral which contains

- delta functions
- transverse exponential factors

$$I(p_1,p_2) = 2\pi\delta(l^- - p_1^- - p_2^-) e^{-iy_t\cdot(p_{1,t}+p_{2,t})} \int dr^+ \int dr^- \delta(r^+) \int \frac{d^dk_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l-k_1)^2]} e^{ir\cdot k_1}$$

- introduce relative coordinate r=x-y
- represent delta function by integral
- introduce dummy integral over r^+

➜ obtain 4 (d) dimensional integral

next step:

$$\left(\frac{i}{k^2-m^2+i0}\right)^\lambda = \frac{1}{\Gamma(\lambda)}\int_0^\infty d\alpha\,\alpha^{\lambda-1} e^{i\alpha(k^2-m^2+i0)}$$

- Schwinger-/α-parameters
- complete square in exponent, Wick rotation, Gauss integration, etc.
- reconstruct delta function to evaluate (some) integrals over α-parameters

to facilitate these steps for 2, 3 loops (virtual!): "developed" Mathematica package ARepCGC; implements necessary text-book methods **[V. Smirnov, Springer 2006]**

INTEGRALS FOR REAL CORRECTIONS

- 1-loop: in terms of modified Bessel function

$$I(p_1,p_2) = 8\pi^2\delta(l^- - p_1^- - p_2^-)\frac{e^{-ix_t\cdot p_{1,t}}}{l^-}e^{-iy_t\cdot p_{2,t}}K_0\left(\sqrt{\alpha(1-\alpha)Q^2(\boldsymbol{x}-\boldsymbol{y})^2}\right), \quad \alpha = p_1^-/l^-$$

- 2-loop: one remaining integration (at first)

$$\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}\frac{(2\pi)^3\delta(k_1^- - k_3^- - p_1^-)\delta(l^- - k_1^- - p_2^-)\delta(k_3^- - p_3^-)}{[k_1^2][(l-k_1)^2][(k_1-k_3)^2][k_3^2]}$$
$$e^{ix_t\cdot(k_{1,t}-k_{3,t}-p_{1,t})}e^{iy_t\cdot(l_t-k_{1,t}-p_{2,t})}e^{iz_t\cdot(k_{3,t}-p_{3,t})}$$

$$\propto e^{-ix_t\cdot p_{1,t}}e^{-iy_t\cdot p_{2,t}}e^{-iz_t\cdot p_{3,t}}\int_0^{\rho_3^{\max}}\frac{d\rho_3}{\rho_3}K_0\left[\sqrt{\frac{\rho_1(1-\rho_1)Q^2((\boldsymbol{x}-\boldsymbol{y})^2+\rho_3(\boldsymbol{x}(1-\xi)-\boldsymbol{y}+\xi\boldsymbol{z})^2)}{\rho_3}}\right]$$

ξ, ρ_1, $\rho_3^{\max}$ in terms of external momenta

- 2-loop integral: evaluated into infinite sum over Bessel functions; numerics: keeping integral might be most stable
- tensor integrals from differentiation w.r.t. external coordinates inclusive: obtain (unexpected) endpoint contributions

FROM GAMMA MATRICES TO CROSS-SECTIONS

FORM EVALUATES DIRAC TRACES

- possible to express elements of Dirac trace to two general tensor integrals
- Evaluation using FORM
  [Vermaseren, math-ph/0010025]
- result lengthy, but in principle usable (~23 pages)
- currently working on further simplification through reduction of tensor integrals
  (work in progress)

```
A1squared =
   + qminus * ( DENn(k)*dot(p,k)*IntR1(nminus,nminus,nminus,1,1,1,p)*
     IntR1c(muc1,muc1,nminus,1,1,1,p) + DENn(k)*dot(p,k)*IntR1(nminus,
     nminus,nminus,1,1,1,p)*IntR1c(muc2,muc2,nminus,1,1,1,p) - DENn(k)*
     dot(p,k)*IntR1(nminus,mu2,nminus,1,1,1,p)*IntR1c(nminus,mu2,nminus,1,
     1,1,p) - DENn(k)*dot(p,k)*IntR1(nminus,muc2,nminus,1,1,1,p)*IntR1c(
     nminus,muc2,nminus,1,1,1,p) - DENn(k)*dot(p,k)*IntR1(mu1,nminus,
     nminus,1,1,1,p)*IntR1c(mu1,nminus,nminus,1,1,1,p) + DENn(k)*dot(p,k)*
     IntR1(mu1,mu1,nminus,1,1,1,p)*IntR1c(nminus,nminus,nminus,1,1,1,p) +
     DENn(k)*dot(p,k)*IntR1(mu2,mu2,nminus,1,1,1,p)*IntR1c(nminus,nminus,
     nminus,1,1,1,p) - DENn(k)*dot(p,k)*IntR1(muc1,nminus,nminus,1,1,1,p)*
     IntR1c(muc1,nminus,nminus,1,1,1,p) - IntR1(nminus,nminus,p,1,1,1,p)*
     IntR1c(muc1,muc1,nminus,1,1,1,p) + IntR1(nminus,nminus,p,1,1,1,p)*
     IntR1c(muc2,muc2,nminus,1,1,1,p) + IntR1(nminus,mu2,p,1,1,1,p)*
     IntR1c(nminus,mu2,nminus,1,1,1,p) - IntR1(nminus,muc2,p,1,1,1,p)*
     IntR1c(nminus,muc2,nminus,1,1,1,p) + IntR1(mu1,p,mu1,1,1,1,p)*IntR1c(
     nminus,nminus,nminus,1,1,1,p) - IntR1(mu1,nminus,p,1,1,1,p)*IntR1c(
     mu1,nminus,nminus,1,1,1,p) - IntR1(mu1,nminus,mu1,1,1,1,p)*IntR1c(p,
     nminus,nminus,1,1,1,p) + IntR1(mu1,mu1,p,1,1,1,p)*IntR1c(nminus,
     nminus,nminus,1,1,1,p) - IntR1(mu2,mu2,p,1,1,1,p)*IntR1c(nminus,
     nminus,nminus,1,1,1,p) - IntR1(mu3,p,mu3,1,1,1,p)*IntR1c(nminus,
     nminus,nminus,1,1,1,p) + IntR1(mu3,nminus,mu3,1,1,1,p)*IntR1c(p,
     nminus,nminus,1,1,1,p) + IntR1(muc1,nminus,p,1,1,1,p)*IntR1c(muc1,
     nminus,nminus,1,1,1,p) )

   + pminus*qminus * (  - DENn(k)*IntR1(k,nminus,nminus,1,1,1,p)*IntR1c(
     muc3,nminus,muc3,1,1,1,p) + DENn(k)*IntR1(k,nminus,mu3,1,1,1,p)*
     IntR1c(mu3,nminus,nminus,1,1,1,p) - DENn(k)*IntR1(k,mu3,mu3,1,1,1,p)*
     IntR1c(nminus,nminus,nminus,1,1,1,p) + DENn(k)*IntR1(k,muc3,nminus,1,
     1,1,p)*IntR1c(nminus,nminus,muc3,1,1,1,p) + DENn(k)*IntR1(nminus,k,
     nminus,1,1,1,p)*IntR1c(nminus,muc3,muc3,1,1,1,p) - DENn(k)*IntR1(
     nminus,k,mu3,1,1,1,p)*IntR1c(nminus,mu3,nminus,1,1,1,p) + DENn(k)*
     IntR1(nminus,nminus,k,1,1,1,p)*IntR1c(muc1,muc1,nminus,1,1,1,p) -
     DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(k,muc3,muc3,1,1,1,
     p) + DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(muc2,muc2,k,1
     ,1,1,p) + DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(muc3,k,
     muc3,1,1,1,p) + DENn(k)*IntR1(nminus,nminus,mu3,1,1,1,p)*IntR1c(k,mu3
     ,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,nminus,mu3,1,1,1,p)*IntR1c(
     mu3,k,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,mu2,k,1,1,1,p)*IntR1c(
     nminus,mu2,nminus,1,1,1,p) + DENn(k)*IntR1(nminus,mu3,mu3,1,1,1,p)*
     IntR1c(nminus,k,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,muc2,nminus,1,
     1,1,p)*IntR1c(nminus,muc2,k,1,1,1,p) - DENn(k)*IntR1(nminus,muc3,
     nminus,1,1,1,p)*IntR1c(nminus,k,muc3,1,1,1,p) - DENn(k)*IntR1(mu1,
     nminus,nminus,1,1,1,p)*IntR1c(mu1,nminus,k,1,1,1,p) + DENn(k)*IntR1(
```

- precision experiments (future EIC) require theory precision - we're working on it

- developed techniques (reduction, integrals) - might have been available before, but never been exploited in a systematic way for this kind of calculation

- proof of concept for NLO momentum space calculation
  advantage: benefit from standard techniques for higher orders in QCD (important: soft- and collinear singularities, ….)

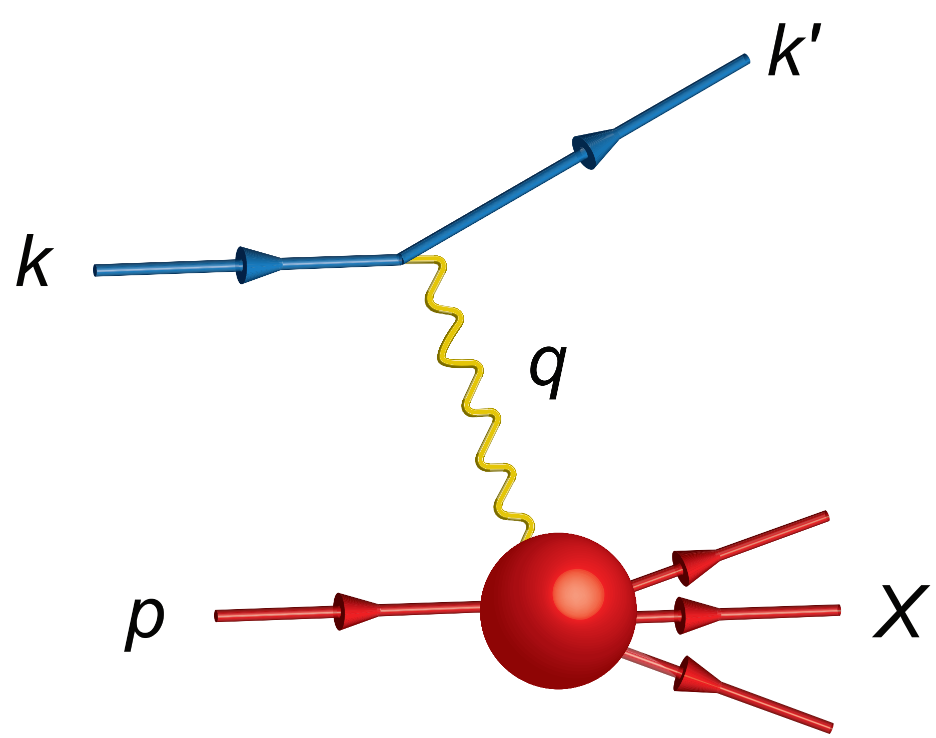
- the results provides not only a (hopefully) important contribution to future EIC studies, but the developed techniques should also allow to evaluate NLO correction for saturation/CGC observables in *e.g.* pA at RHIC/LHC

- A result of few lines can explode, if extended to extra final state or next-to-leading order - requires a systematic approach

DANKESCHÖN!

Electron-nucleus/-on scattering

- knowldege of scattering enery + nucleon mass
  + measure scattered electron ➡ control kinematics



Photon virtuality
$Q^2 = -q^2$

Resolution
$\lambda \sim \frac{1}{Q}$

Mass of system X
$$W = (p+q)^2$$
$$= M_N^2 + 2p \cdot q - Q^2$$

Bjorken $\boldsymbol{x}$ $= \dfrac{Q^2}{2p \cdot q}$

Inelasticity $y = \dfrac{2p \cdot q}{2p \cdot k}$

PDF'S, COLLINEAR FACTORISATION AND ALL THAT

- collinear factorisation = factorisation in the limit of infinite virtuality Q:

  proton structure functions = convolution of

$$F_2(x, Q^2) = \sum_{k=q,g} \hat{C}_{2,k} \otimes \hat{f}_k$$

parton distribution functions (pdfs):
= probability to find parton (quark, gluon) which carries the fraction x of the proton momentum (non-perturbative→ from fits to data)

and **coefficents**
$C_{2q} = 1 + \alpha_s C_{2q}^{(1)} + \ldots$, $C_{2g} = \alpha_s C_{2g}^{(1)} + \ldots$ (calculated in perturbation theory)

<u>exact theory statement up to terms suppressed by 1/Q!</u>

- essential for pQCD predictions and pQCD success story in $\gamma^* p$, pp, ...

THE PROTON AT SMALL X: THE HERA LEGACY

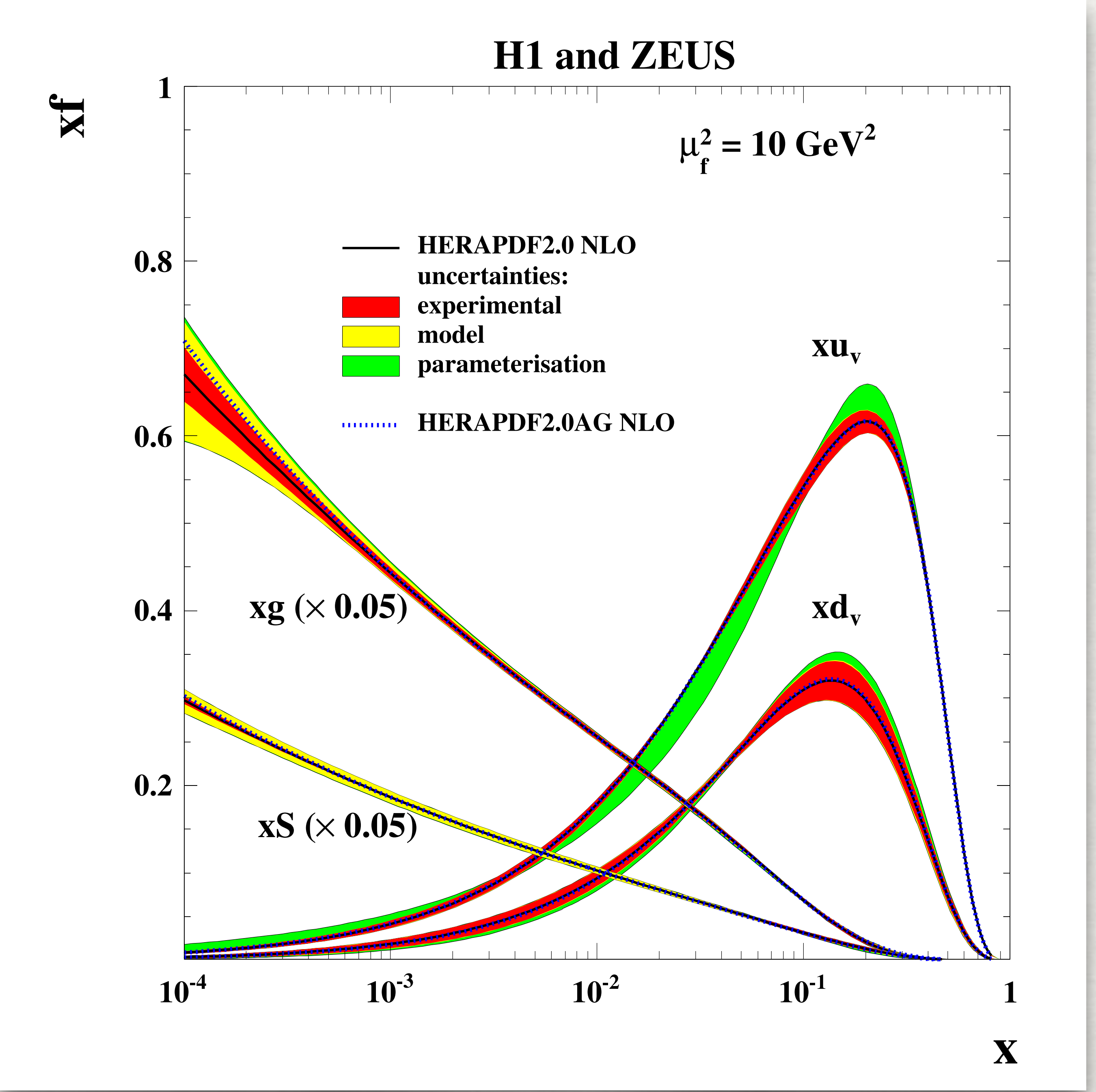
HERA@DESY (1992-2007): at the first time DIS on a proton at a Collider → access to small x region [large c.o.m. energy at fixed resolution Q]

important HERA result:

proton at small x dominated by gluons and seaquarks (qqbar pairs from gluon)

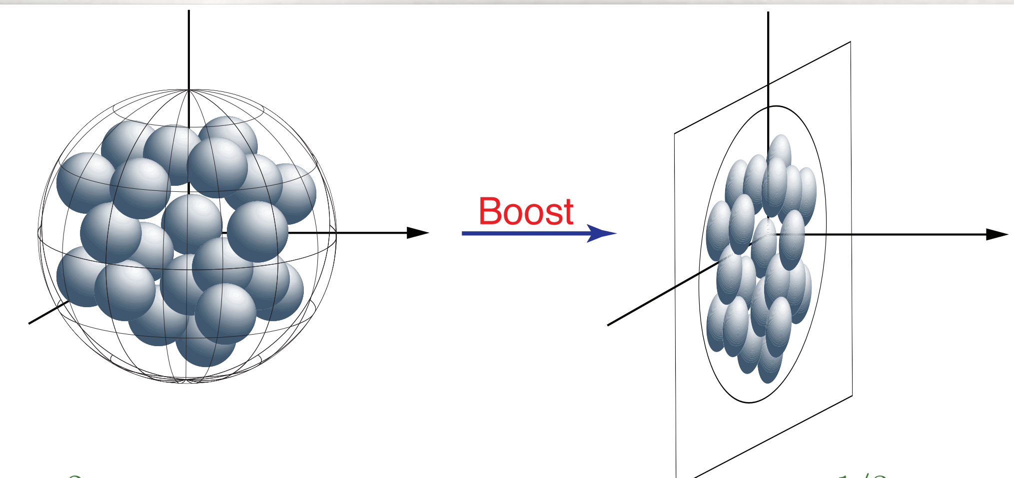
powerlike rise of gluon distribution at small x

BUT: rise cannot continue forever (probability distribution!)

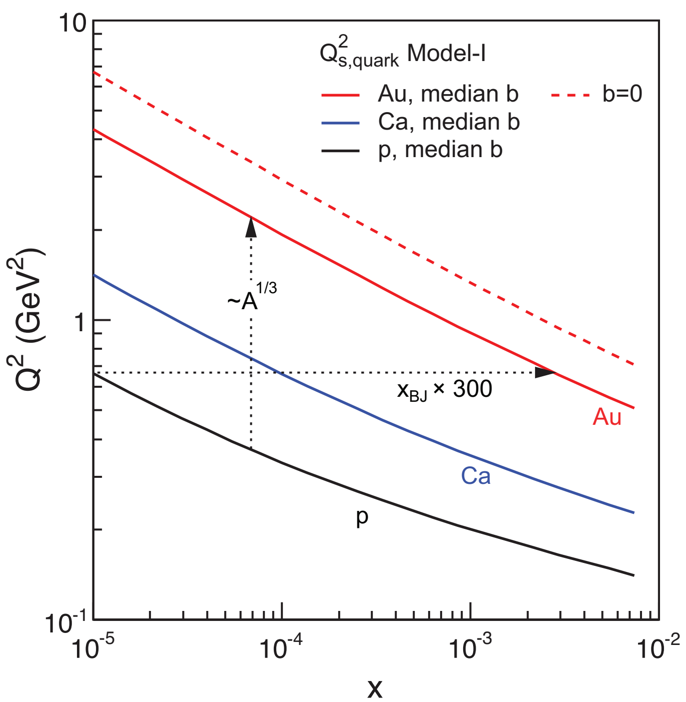


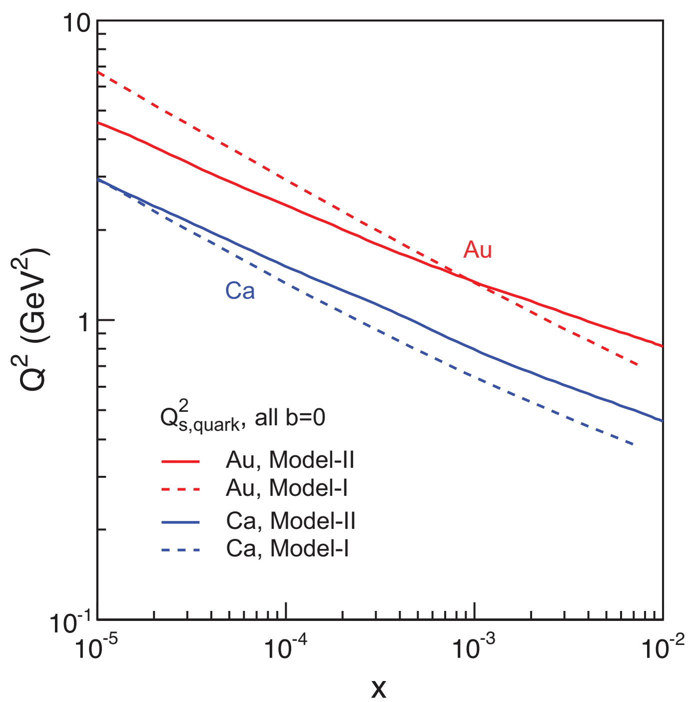
DONEC QUIS NUNC

Expect those effects to be even more enhanced in boosted nuclei:



$Q_s^2 \sim \#$ **gluons/unit transverse area** $\sim A^{1/3}$





MOMENTUM VS. CONFIGURATION SPACE

	conventional pQCD (make use of know techniques)	inclusion of finite masses (charm mass!)	intuition: interaction at t=0 with Lorentz contracted target
momentum space	well explored	complication, but doable	lose intuitive picture at first -> large # of cancelations
configuration space	poorly explored	very difficult	many diagrams automatically zero

our approach:
work in momentum space, but exploit relation to configuration space to set a large fraction of all diagrams to zero

THE LC-TIME SLICE $X^-=0$: 'CUTS' THROUGH DIAGRAMS

Determine Fourier transform of "background field vertex" for propagator

$$\int \frac{d^4q}{(2\pi)^4}\frac{d^4q}{(2\pi)^4} e^{-ip\cdot x}\tilde{\Delta}_F(p)\tau(p,q)\tilde{\Delta}_F(q)e^{iq\cdot y}$$

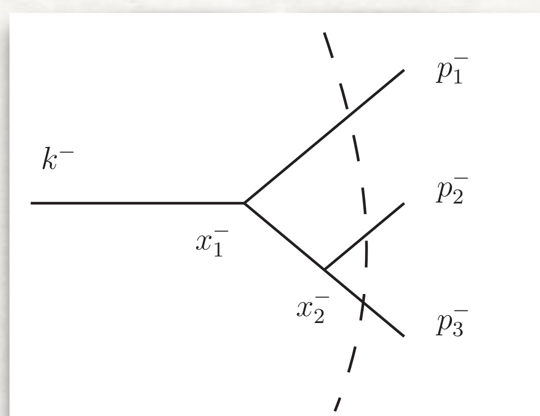
and final state

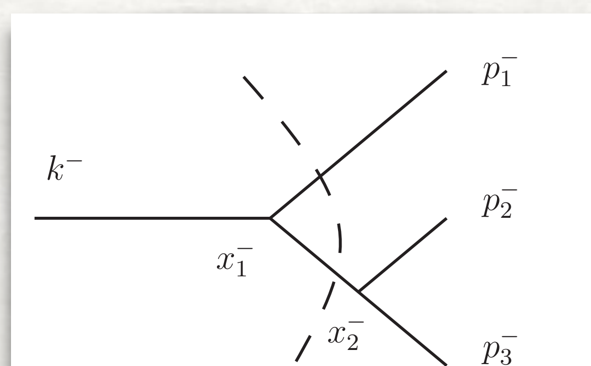
$$\int \frac{d^4q}{(2\pi)^4}\tau(p,q)\tilde{\Delta}_F(q)e^{iq\cdot y}$$

Find light-cone time constraints $y^- > 0 > x^-$ for $p^- > 0$ and $x^- > 0 > y^-$ for $p^- < 0$

reason: conservation of light-cone momentum at vertex $\boldsymbol{\tau}$

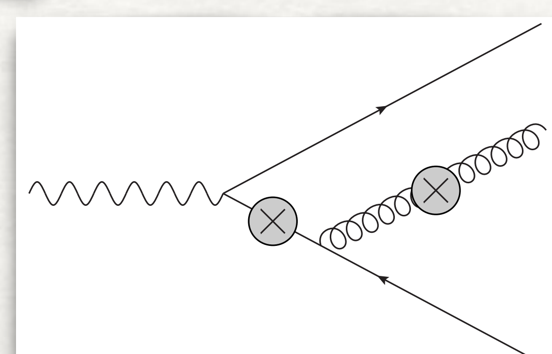
important consequence: interaction for each diagram only allowed along a certain time-slice =cut of diagrams

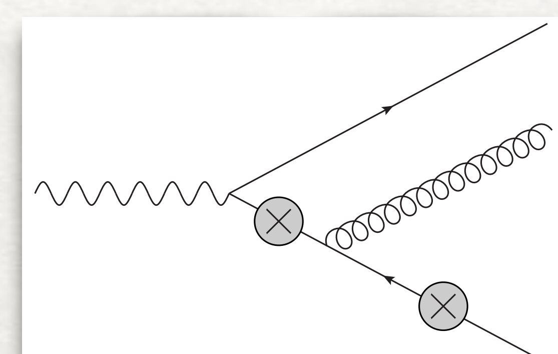




example: 3 partons (real NLO): interaction term $\boldsymbol{\tau}$ only allowed if the regarding line is "cut"

examples of excluded configurations

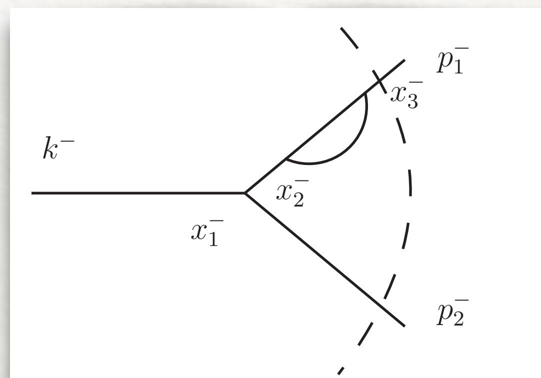


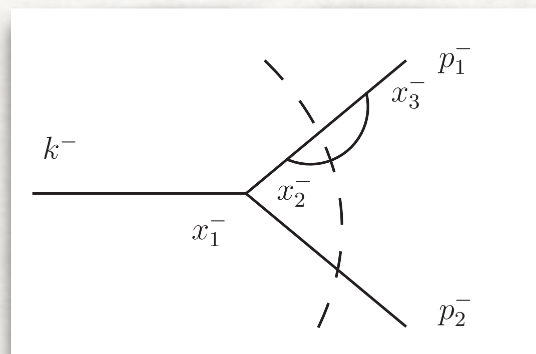


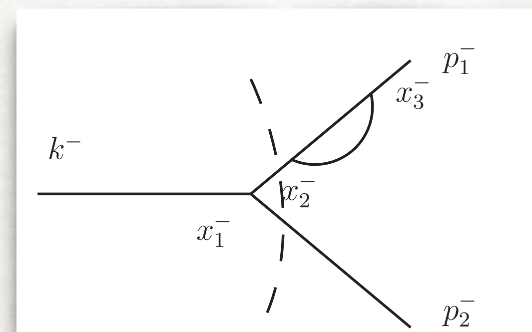
THE LC-TIME SLICE X^-=0: ‘CUTS’ THROUGH DIAGRAMS

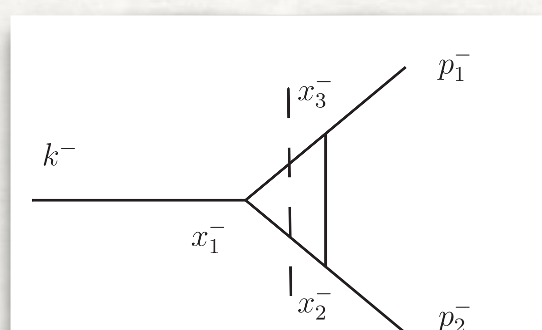
applies also to virtual diagrams: organized into ‘cut’ configurations

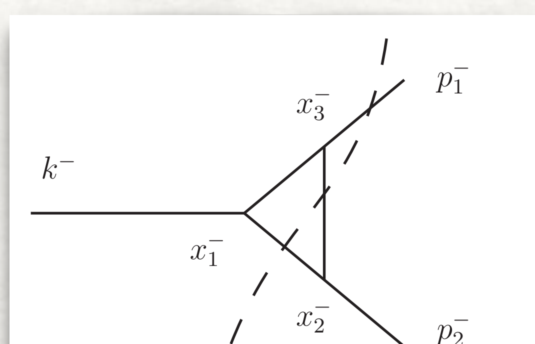
Note: different cuts can contain the same diagram

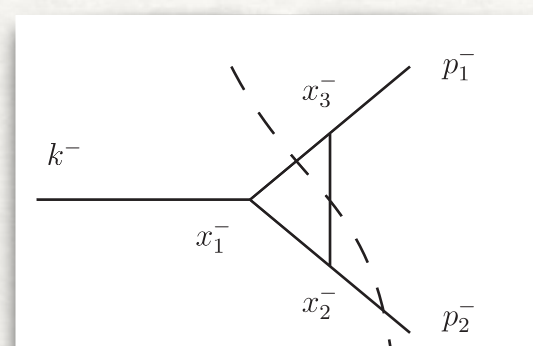


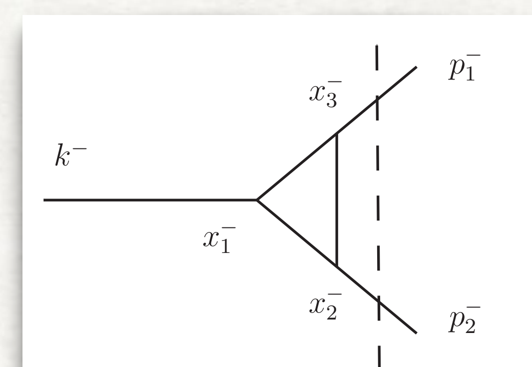












EVALUATING THE LORENTZ- AND DIRAC STRUCTURE

A. Dirac trace through 2 most general structures, closely related to loop integraexpressls

$$I_1^{\mu_1\mu_2\mu_3}(p_1,p_2) = \int \frac{d^dk_1}{i\pi^{d/2}} \frac{k_1^{\mu_1}(l-k_1)^{\mu_2}p_1^{\mu_3}}{[k_1^2][(l-k_1)^2][p_1^2]} e^{ix_t\cdot(k_{1,t}-p_{1,t})} e^{iy_t\cdot(-k1,t-p_{2,t})}$$
$$(2\pi)^2\delta(p_1^- - k_1^-)\delta(l^- - k_1^- - p_2^-)$$

$$I_{R_2}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3) =$$
$$\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} \frac{k_1^{\mu_1}(l-k_1)^{\mu_2}(k_1-k_3)^{\mu_3}k_3^{\mu_4}}{[k_1^2-m^2][(l-k_1)^2-m^2][(k_1-k_3)^2-m^2][k_3^2]}$$
$$e^{ix_t\cdot(k_{1,t}-k_{3,t}-p_{1,t})} e^{iy_t\cdot(l_t-k_{1,t}-p_{2,t})} e^{iz_t\cdot(k_{3,t}-p_{3,t})}$$
$$(2\pi)^3\delta(k_1^- - k_3^- - p_1^-)\delta(l^- - k_1^- - p_2^-)\delta(k_3^- - p_3^-)$$