Hadronic light-by-light contribution to the muon $g-2$.

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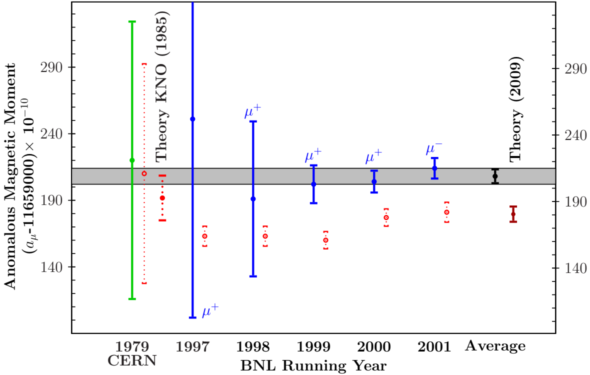
Outline

Purpose

- The main purpose of this work is to reduce the theoretical uncertainty in the computation of the a_μ, in which the main source of uncertainty comes from the hadronic contributions. This is why we decided to analyze the hadronic light-by-light contribution using RχT.



Magnetic moment $\overrightarrow{\mu}$

- Classically, the magnetic moment of a particle is defined as

$$\overrightarrow{\mu} = \frac{q}{2m}\overrightarrow{L}$$

- From Stern-Gerlach experiments we learned that particles have intrinsic angular momentum or spin.

- So that particles coupling to the photon, with $m \neq 0$ have an intrinsic magnetic moment

$$\overrightarrow{\mu} = g\frac{Q}{2m}\overrightarrow{s}$$

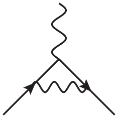
where $g_\ell = 2$ is LO prediction in QED with a classic EM field.

Anomalous magnetic moment a_ℓ

- However, from hyperfine splitting of the ground state of hydrogen and deuterium in 1947, Nafe *et al.* measured[1]

$$\delta\mu/\mu = 0.00126 \pm 0.00019$$

- Which came to be consistent with Schwinger's[2] prediction of a deviation from $g = 2$, defined as the anomalous magnetic moment a_μ.



$$a_\ell := \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2).$$

[1] J. E. Nafe et al., Phys.Rev. 71 (1947)
[2] J. S. Schwinger, Phys.Rev. 73 (1948)

Why $\ell = \mu$?

- Ever since, there has been more precise measurements and computations of the a_ℓ, making it feasible to search for physics Beyond Standard Model (BSM) in a_ℓ.

- On other hand, angular momentum conservation shows that in $\ell \to \gamma\ell$ processes, ℓ must flip its spin. Only for massive particles, spin flips are allowed $\Rightarrow$ the amplitude must be proportional to the mass m_ℓ.

- Therefore, contributions Beyond Standard Model (BSM) to the a_ℓ, like chiral d=5 operator $\frac{g}{\Lambda}\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi$ must be suppressed by a factor $\sim \frac{gm_\ell}{\Lambda^2}$.

- If current discrepancy is from BSM contribution to a_μ,

$$\Lambda \approx \sqrt{g}\ 100\ \text{TeV}$$

Why not $\ell = \tau$?

- Since transition probability is squared modulus of the amplitude, BSM effects will be easier to detect with $\ell = \mu$

$$\left(\frac{m_\mu}{m_e}\right)^2 \sim 4 \times 10^4$$

- Therefore, BSM effects should be larger on a_τ. Nevertheless, τ_τ is so small that experimental results[3] are still compatible with $a_\tau = 0$.

$$\tau_\mu = 2.197 \times 10^{-6} s, \quad \tau_\tau = 2.906 \times 10^{-13} s \quad \Rightarrow \quad \frac{\tau_\tau}{\tau_\mu} \sim 10^{-7}$$

[3] K. Ackerstaff *et al.*, [OPAL Collab.] Phys.Lett.B431(1998)
M. Acciarri *et al.*, [L3 Collab.] Phys.Lett.B434(1998)
W. Lohmann, Nucl.Phys.B144(2005)

$\ell = \mu$

- Even though measurements of a_e are 2250 times more precise[4] a_μ is

$$\frac{1}{2250}\left(\frac{m_\mu}{m_e}\right)^2 \sim 19$$

  times more sensitive to BSM contributions.

- Therefore, it would be more plausible to find such a deviation in the a_μ.

[4] R.S. Van Dyck *et al.*, PRL59(1987);
P.J. Mohr *et al.*, Rev.Mod.Phys.72(2000)

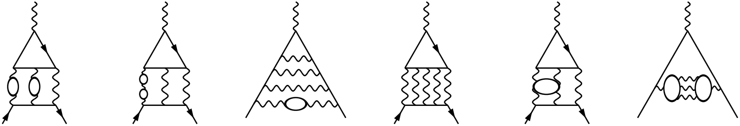
Contributions to a_μ

- The computation of a_μ can be splitted in different contributions, whose values can be found in PDG[5]

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had}$$

- a_μ^{QED} are all corrections[6] that might come from QED

$$a_\mu^{QED} = 116584718.95(0.08) \times 10^{-11} + \mathcal{O}\left(\frac{\alpha}{\pi}\right)^6$$



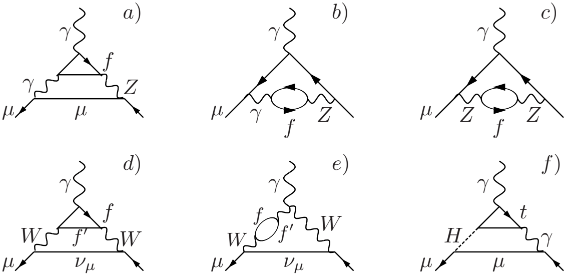
(18) (18) (2072) (120) (18) (2)

[5]K.A. Olive *et al.* (Particle Data Group), Chin.Phys.C38(2014)
[6]T. Aoyama *et al.* PRL 109(2012)

a_μ^{EW}

- a_μ^{EW} are Electroweak contribution that are not a_μ^{QED} ($W^\pm, Z, H$) at two loops[7]. Three loops contribution is negligible ($\lesssim 0.4 \times 10^{-11}$).

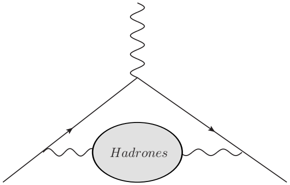
$$a_\mu^{EW} = 153.6(1.0) \times 10^{-11}$$



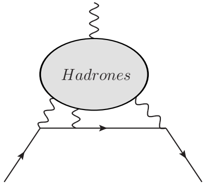
[7] C. Gnendiger *et al.*, Phys.Rev.D88 (2013)

Hadronic contributions

- a_μ^{Had} can be separated into two contributions, the PDG values are the following.[8]



Hadronic Vacuum Polarization (HVP) contribution.
$a_\mu^{HVP} = 6923(42)(3) \times 10^{-11}$



Hadronic light-by-light (HLbL) contribution. $a_\mu^{HLbL} = 105(26) \times 10^{-11}$

[8] K.A. Olive *et al.*, (Particle Data Group), Chin. Phys.C38(2014)
For HVP, M. Davier *et al.* Eur.Phys.J. C71 (2011)
For HLbL J. Prades *et al.* Advanced series on directions in HEP Vol20.

Hadronic contributions to a_μ

- All the contributions and their uncertainties are shown in the next table.

Contribution	$\times 10^{11}$	Uncertainty$\times 10^{11}$
QED	116 584 718.95	0.08
EW	153.6	1.0
Had	7 028	$(42)_{\text{Vac. Pol.}}$ $(26)_{\text{Light-by-Light}}$
Total	116 591 803	(1)(42)(26)
Exp	116 592 091	(54)(33)

- Clearly, the largest uncertainty comes from the hadronic contribution.
- With these values there is a discrepancy

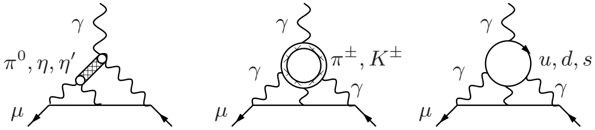
$$a_\mu^{exp} - a_\mu^{SM} = 288(63)(49) \times 10^{-11} \quad \sim 3.5\sigma$$

Hadronic contribution a_μ

- The main uncertainty comes from these contributions[9] $\sim 5 \times 10^{-10}$, in which[10] HLbL uncertainty is $\sim 3 \times 10^{-10}$.

- The current experimental[11] error is 6.3×10^{-10}.

- Being that Fermilab & J-Parc are planning to lower[12] their error in their a_μ measurements to 1.6×10^{-10}, it becomes mandatory to reduce theoretical uncertainty.



(a) [L.D.] (b) [L.D.] (c) [S.D.]

[9] M. Davier *et al., Eur.Phys.J.C71(2011)*

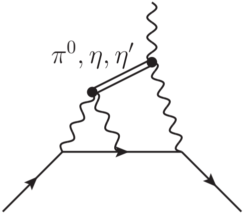
[10] J. Prades *et al.*, Advanced series on directions in high energy physics 20

[11] G. W. Bennet *et al.*, [Muon g-2 Collab.],PRD73(2006)

[12] S. Maxfield & K. Ishida talks at PHIPSI13, Rome, September 2013

$F_{\pi\gamma\gamma}$

- Our contribution to a_μ comes from diagram (a)



- Cancellation between loop diagrams (b) and (c) give[13] a contribution $\sim 1/10$ smaller than that of (a).
- We use Resonance Chiral Theory[14] (RχT) to compute the pion transition form factor $F_{\pi\gamma^*\gamma^*}$.

[13] F. Jegerlehner & A. Nyffeler, Phys.Rep.477(2009)

[14] G. Ecker, J. Gasser A. Pich & E. De Rafael Nucl.Phys. B321(1989)
P.D. Ruíz-Femenía *et al.*, JHEP 0307 (2003)

$F_{\pi\gamma\gamma}$

- Using RχT we find[15]

$$F_{\pi\gamma^*\gamma^*}(p^2,q^2,r^2) = \frac{2r^2}{3F}\left[-\frac{N_C}{8\pi^2 r^2} + 4F_V^2\frac{d_3(p^2+q^2)}{(M_V^2-p^2)(M_V^2-q^2)r^2}\right.$$
$$+\frac{4F_V^2 d_{123}}{(M_V^2-p^2)(M_V^2-q^2)} + \frac{16F_V^2 P_3}{(M_V^2-p^2)(M_V^2-q^2)(M_P^2-r^2)}$$
$$\left.-\frac{2\sqrt{2}}{M_V^2-p^2}\left(\frac{F_V}{M_V}\frac{r^2 c_{1235} - p^2 c_{1256} + q^2 c_{125}}{r^2} + \frac{8P_2F_V}{(M_P^2-r^2)}\right) + (q^2 \leftrightarrow p^2)\right]$$

- p^2, q^2 and r^2 are the γ's and π squared momenta. P_2 and P_3 comes from couplings with pseudoscalar resonances. All parameters can be obtained[16] from QCD asymptotic behavior for VVP Green functions.

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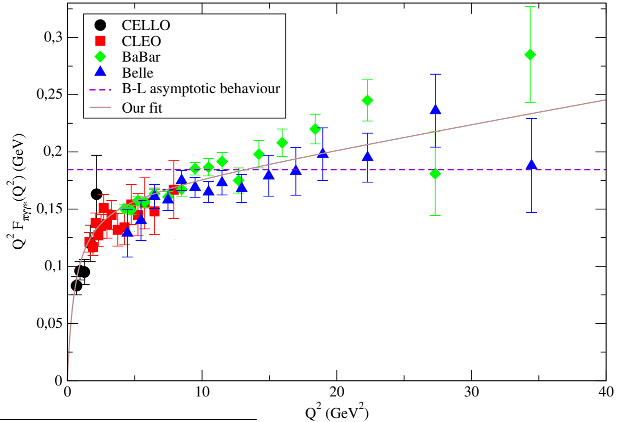
[15] K. Kampf & J. Novotný, PRD84 (2011)
P. Roig, AG & G. López Castro, PRD89 (2014)

[16] J. Sanz-Cillero and P. Roig, Phys.Rev.Lett. B733 (2014)

$F_{\pi\gamma\gamma}$ parameters

- BaBar[17] showed a different behavior, therefore we[18] decided to fit P_2 using these and Belle[19] data.



[17] P. del Amo Sanchez *et al.*, [BaBar Collab.], PRD84(2011)

[18] P. Roig, AG, G. López Castro, PRD89 (2014)

[19] C. P. Shen *et al.*, [Belle Collab.], PRD88(2013)

$F_{\pi\gamma\gamma}$ parameters

- Determination of F_V adjusting BaBar[20] data of $\tau \to \nu_\tau 3\pi$ varies 5%, so we vary it 10% around the asymptotic value.

- There is a very good agreement with data due to the value of P_2, from fit of $e^+e^- \to e^+e^-\pi^0$ data (previous slide), getting

$$P_2 = -(1.13 \pm 0.12)10^{-3} \text{ GeV}$$

- And from $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$ decays, we get

$$P_3 = -(1.2 \pm 0.3)10^{-2} \text{ GeV}^2$$

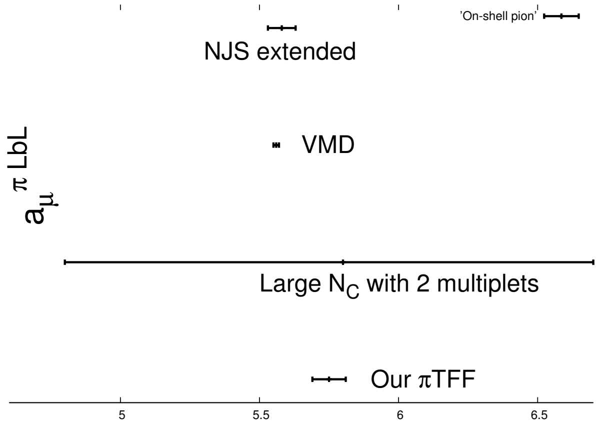
- Thus, we get $a_\mu^{\pi^0 LbL} = 6.66 \pm 0.21 \times 10^{-10}$ which compares well with previous calculation[21].

[20] O. Shekhovtsova *et al.* PRD 88 (2013)

[21] K. Kampf y J. Novotný, PRD84 (2011), who have an error of F_V^{Kampf} $\sim$ 8.4% (within $\pm 0.1 F_V$) and do not use Belle data(2013).

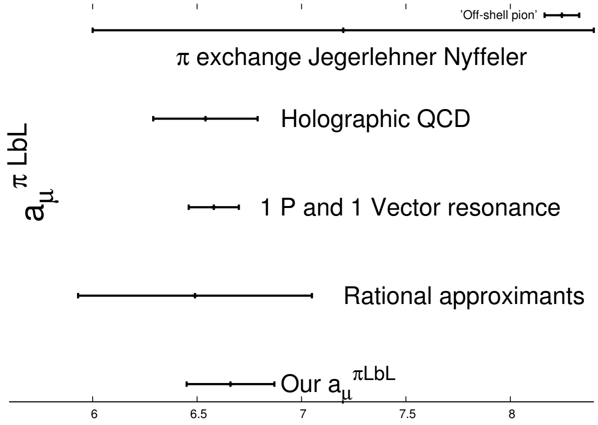
$a_\mu^{\pi^0 LbL}$

- Comparing our on-shell result with other works



$a_\mu^{\pi^0 LbL}$

- And our off-shell result with other works



η y η'

- Including η y η', parametrized consistently with $1/N_C$ small

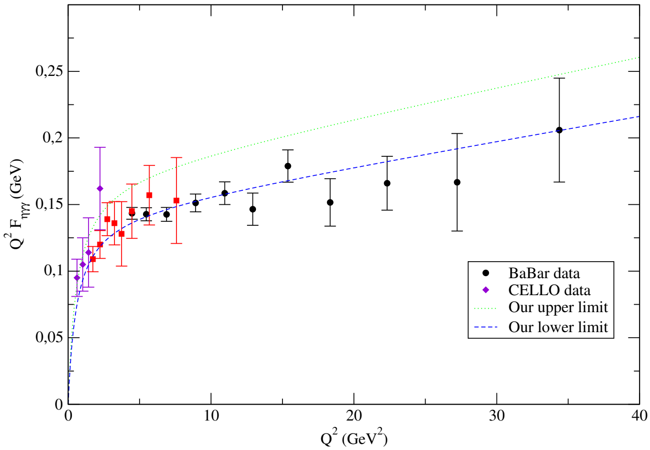
$$diag(U) = \left(\frac{\pi^0 + C_q\eta + C_{q'}\eta'}{\sqrt{2}}, \frac{-\pi^0 + C_q\eta + C_{q'}\eta'}{\sqrt{2}}, -C_s\eta + C_{s'}\eta'\right)$$

- Therefore, the Form Factors change only by a factor.

$$F_{\eta^{(\prime)}\gamma\gamma}(p^2, q^2, r^2) = \left(\frac{5}{3}C_{q^{(\prime)}} \mp \frac{\sqrt{2}}{3}C_{s^{(\prime)}}\right) F_{\pi\gamma\gamma}(p^2, q^2, r^2)$$

η y η'

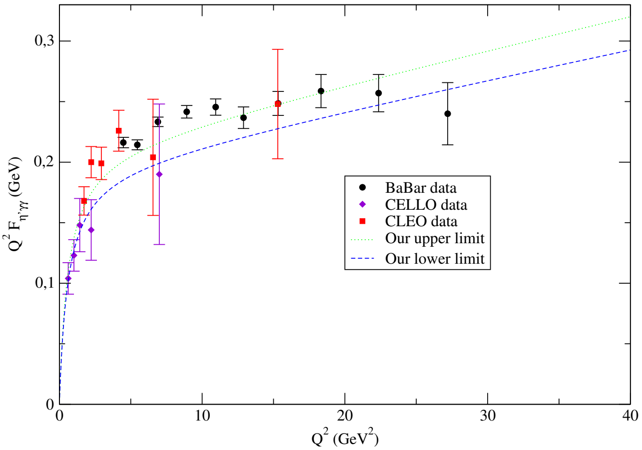
- So we get the following prediction[22] for η



[22] P. Roig, AG, G. López Castro, PRD89 (2014)

η y η'

- and[23] for η'



[23] P. Roig, AG, G. López Castro, PRD89 (2014)

a_μ^{PLbL}

- We get for η

$$a_\mu^{\eta LbL} = 2.04 \pm 0.44 \times 10^{-10}$$

- While for η'

$$a_\mu^{\eta' LbL} = 1.77 \pm 0.23 \times 10^{-10}$$

- Getting a total pseudoscalar exchange contribution of

$$a_\mu^{PLbL} = 10.47 \pm 0.54 \times 10^{-10}$$

- Adding contributions from K, π and heavy quarks loops, scalar and axial resonances[24], we get

$$\boxed{a_\mu^{HLbL} = 11.8 \pm 2.0 \times 10^{-10}}$$

[24] F. Jegerlehner and A. Nyffeler, Physics Reports 477(2009)

a_μ^{HLbL}

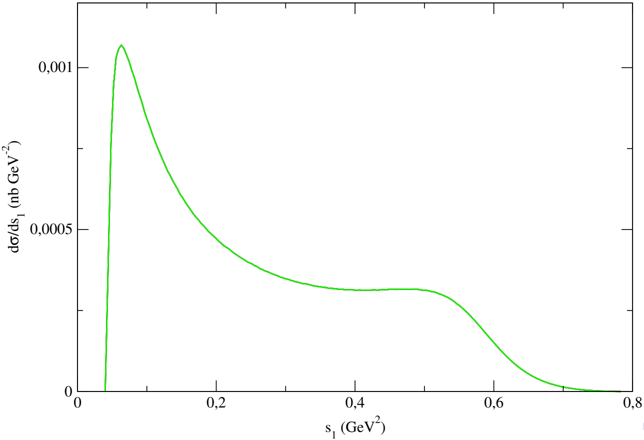
- Now we can compare our results with earlier results.

$a_\mu^{HLbL} \cdot 10^{10}$	Contribution
11.6 ± 4.0	F. Jegerlehner and A. Nyffeler Phys.Rep 477(2009)
10.5 ± 2.6	Prades, De Rafael and Vainshtein[25]
	Advanced series on directions in high energy physics. Vol. 20
11.8 ± 2.0	Our contribution

[25] Prades *et al.* only include the *charm* loop in the heavy quark loop evaluation.

Proposal of new observable

- This led us to propose the measurement of $\frac{d}{ds_1}\sigma(e^+e^- \to \mu^+\mu^-\pi^0)$ @$(1.02\text{ GeV})^2$(KLOE-2) as a new form of measuring $F_{\pi\gamma\gamma}$, being s_1 the dilepton $\mu^+\mu^-$ invariant mass.



Proposal of new observable

- This new observable would give complementary information about πTFF for using it directly in a_μ^{HLbL} calculation.

- Taking into account the factors for $\eta^{(\prime)}$, an analogous observable can be obtained for these particles too.

- With $d\sigma/ds_1$ information about $P2$, $P3$, $C_{q^{(\prime)}}$ and $C_{s^{(\prime)}}$ could be measured to improve even more the theoretical prediction of a_μ^{HLbL}.

Conclusions

- We found a new contribution to a_μ^{HLbL} consistent with other theoretical models and with a larger precision.
- We have found that the pion pole approximation underestimates in about 14% the transition form factor in agreement with earlier results[26].
- We obtained the first prediction for $\sigma(e^+e^- \to \mu^+\mu^-\pi^0)$ to be measured at KLOE-2. For η and η' at Novosibirsk experiments.

[26] F Jegerlehner y A. Nyffeler, Phys.Rep.477(2009)

Back up

a_μ in different frameworks

$a_\mu^{\pi^0 LbL} \cdot 10^{10}$	Model and Reference
5.58 ± 0.05	Nambu-Jona-Lasinio extended (Bijnens et al. 1995)
5.56 ± 0.01	VMD (Hayakawa et al. 1995)
5.8 ± 1.0	Large N_C 2 vector meson π-pole (Knecht and Nyffeler 2002)
7.2 ± 1.2	π exchange (Jegerlehner and Nyffeler 2009)
6.54 ± 0.25	Holographic QCD (Cappiello et al. 2011)
6.58 ± 0.12	A pseudoscalar and a vector meson (Kampf, Novotny 2011)
6.49 ± 0.56	Rational aproximants (Masjuan and Vanderhaeghen 2012)
5.0 ± 0.4	Non-local Chiral Quark model (Dorokhov et al 2012)
5.75 ± 0.06	our result with real π
6.66 ± 0.21	Our result (2014)

Resonance Chiral Theory RχT

- The relevant degrees of freedom are[27] the octet of the lightest pseudoscalar (π, K, η and η').

- The expansion parameter in this theory is $1/N_C$, and in large N_C the $U(1)_A$ broken symmetry is restored, that is the reason for taking η' at the same level as the other resonances.

[27] G. Ecker, J. Gasser A. Pich y E. De Rafael Nucl.Phys. B321(1989)

RχT

- Thus, being $U(3)_V$ the underlying symmetry, the interaction terms between resonances, external currents and $\{\pi, K, \eta, \eta'\}$ are

$$\mathcal{L}^V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$

$$\mathcal{L}_{VJP} = \sum_i^7 \frac{c_i}{M_V}\mathcal{O}^i_{VJP}; \qquad \mathcal{L}_{VVP} = \sum_i^4 \frac{d_i}{M_V}\mathcal{O}^i_{VVP}$$

- These are examples of such operators[28]

$$\mathcal{O}^2_{VJP} = \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\rangle$$

$$\mathcal{O}^1_{VVP} = \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle$$

[28] P.D. Ruíz-Femenía *et al.*, JHEP 0307 (2003)

$F_{\pi\gamma\gamma}$ parameters

- RχT parameters can be found using short distance behavior of QCD, which predicts an asymptotic behavior of s^{-1} for this process.
- Thus, short distance relationships[29] ensure a convergent behavior

$$d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} - \frac{4\sqrt{2}P_2}{F_V}; \qquad c_{125} = 0; \qquad d_{123} = \frac{1}{24};$$

$$F_V = \sqrt{3}F; \qquad c_{125} = 0; \qquad c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V}$$

[29] J. Sanz-Cillero and P. Roig, Phys.Rev.Lett.B733(2014)

Restored $U(1)_A$

- Within t'Hooft's large N_C, the anomaly term is suppressed by a factor $1/N_C$ with respecto to the rest of the QCD lagrangian

$$\frac{g^2}{8\pi^2}\frac{\theta}{N_C}TrF^{\mu\nu}\tilde{F}_{\mu\nu},$$

- Therefore in the limit $N_C \to \infty$ the $U(1)_A$ symmetry is restored.

Wess-Zumino-Witten

- A fundamental part of the analysis is the *WZW* term, wich is order p^4 in the chiral counting and describe intrinsic odd interactions [30].

$$Z[U,l,r] = -\frac{iN_C}{240\pi^2}\int_{M^5} d^5x\varepsilon^{ijklm}\langle\Sigma_i^L\Sigma_j^L\Sigma_k^L\Sigma_l^L\Sigma_m^L\rangle$$
$$-\frac{iN_C}{48\pi^2}\int d^4x\varepsilon_{\mu\nu\rho\sigma}(W(U,l,r)^{\mu\nu\rho\sigma} - W(\mathbf{1},l,r)^{\mu\nu\rho\sigma})$$
$$W(U,l,r)_{\mu\nu\rho\sigma} = \langle U\ell_\mu\ell_\nu\ell_\rho U^\dagger r_\sigma + \frac{1}{4}U\ell_\mu U^\dagger r_\nu U\ell_\rho U^\dagger r_\sigma + iU\partial_\mu\ell_\nu\ell_\rho U^\dagger r_\sigma$$
$$+ i\partial_\mu r_\nu U\ell_\rho U^\dagger r_\sigma - i\Sigma_\mu^L\ell_\nu U^\dagger r_\rho U\ell_\sigma + \Sigma_\mu^L U^\dagger\partial_\nu r_\rho U\ell_\sigma$$
$$- \Sigma_\mu^L\Sigma_\nu^L U^\dagger r_\rho U\ell_\sigma + \Sigma_\mu^L\ell_\nu\partial_\rho\ell_\sigma + \Sigma_\mu^L\partial_\nu\ell_\rho\ell_\sigma - i\Sigma_\mu^L\ell_\nu\ell_\rho\ell_\sigma$$
$$+ \frac{1}{2}\Sigma_\mu^L\ell_\nu\Sigma_\rho^L\ell_\sigma - i\Sigma_\mu^L\Sigma_\nu^L\Sigma_\rho^L\ell_\sigma - (L\leftrightarrow R)\rangle,$$
$$\Sigma_\mu^L = U^\dagger\partial_\mu U, \Sigma_\mu^R = U\partial_\mu U^\dagger, \tag{1}$$

[30] J. Wess and B. Zumino Phys.Lett.37B(1971)
E. Witten, Nucl. Phys. B223 (1983)

Contribución de resonancias a las LEC de χPT a $\mathcal{O}(p^4)$

- El lagrangiano de interacción de las resonancias vectoriales es

$$\mathcal{L}(V) = \langle V_{\mu\nu} J^{\mu\nu} \rangle; \qquad J^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + i \frac{G_V}{2\sqrt{2}} [u^\mu, u^\nu]$$

- Con $f^\mu \nu_\pm = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$, donde

$$F_{R,L}^{\mu\nu} = \partial^\mu (r,\ell)^\nu - \partial^\nu (r,\ell)^\mu - i\left[(r,\ell)^\mu, (r,\ell)^\nu\right]$$

- siendo r y ℓ las corrientes vectoriales y axiales externas, respectivamente.
- y $u^\mu = i\left[u^\dagger \left(\partial^\mu - i r^\mu\right) u - u \left(\partial^\mu - i\ell^\mu\right) u^\dagger\right] = i u^\dagger D_\mu U u^\dagger$
- F_V y G_V son parámetros reales.

- Así, se encuentra que V debe cumplir una ecuación de constricción

$$\nabla^\alpha \nabla_\rho V^{\alpha\beta} - \nabla^\beta \nabla_\rho V^{\rho\alpha} + M_V^2 V^{\alpha\beta} = -2J^{\alpha\beta}$$

- Donde $\nabla_\mu R = \partial_\mu R + [\Gamma_\alpha, R]$ y

$$\Gamma_\alpha = \frac{1}{2}[u^\dagger(\partial_\alpha - ir_\alpha)u + u(\partial_\alpha - i\ell_\alpha)u^\dagger].$$

Al sustituir V y a órden p^4 se tiene que

$$L_1^V = \frac{G_V^2}{8M_V^2} \qquad L_2^V = 2L_1^V \qquad L_3^V = -6L_1^V$$

$$L_9^V = \frac{F_V G_V}{2M_V^2} \qquad L_{10}^V = -\frac{F_V^2}{4M_V^2}$$

- y de igual forma para las demás resonancias.