Pion Transition Form Factor and DSEs

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Outline

- Introduction
- The tools: DSEs
- Pion transition form factor
- Conclusions

Introduction

- Understanding strong interactions is still being a challenge for physicists; although scientists have developed the powerful theory of quarks and gluons, namely, Quantum Chromodynamics (QCD).
- Quarks and gluons are the fundamental degrees of freedom of such theory; however, they are not found free. Instead, they form color-singlet states known as hadrons.
- Hadron form factors are intimately related to their internal structure. But, due to the non perturbative nature of QCD, unraveling hadron form factors from first principles is an outstanding problem.
- Dyson-Schwinger equations (DSEs) are the equations of motion of QCD and they combine the IR and UV behavior of the theory at once, therefore, DSEs are an ideal platform to study quarks and hadrons.

Pion and DSEs studies

- **Elastic and transition form factors:**

[1] Phys.Rev. C65 045211 (2002) by P. Maris et al. (DSE).

[2] Phys. Rev. D80 052002 (2009) BaBar (Experiment).

[3] Phys.Rev. C82 065202 (2010) by H.L.L. Roberts et al. (DSE).

[4] Phys. Rev. D86 092007 (2012) Belle (Experiment).

[5] Phys.Rev.Lett. 111 14 141802 (2013) by L. Chang et al. (DSE)

- **PDAs and PDFs:**

[6] Phys.Rev. D68 034025 (2003) by W. Detmold et al. (Lattice).

[7] Phys.Rev.Lett. 110 13, 132001 (2013) by L. Chang et al. (DSE).

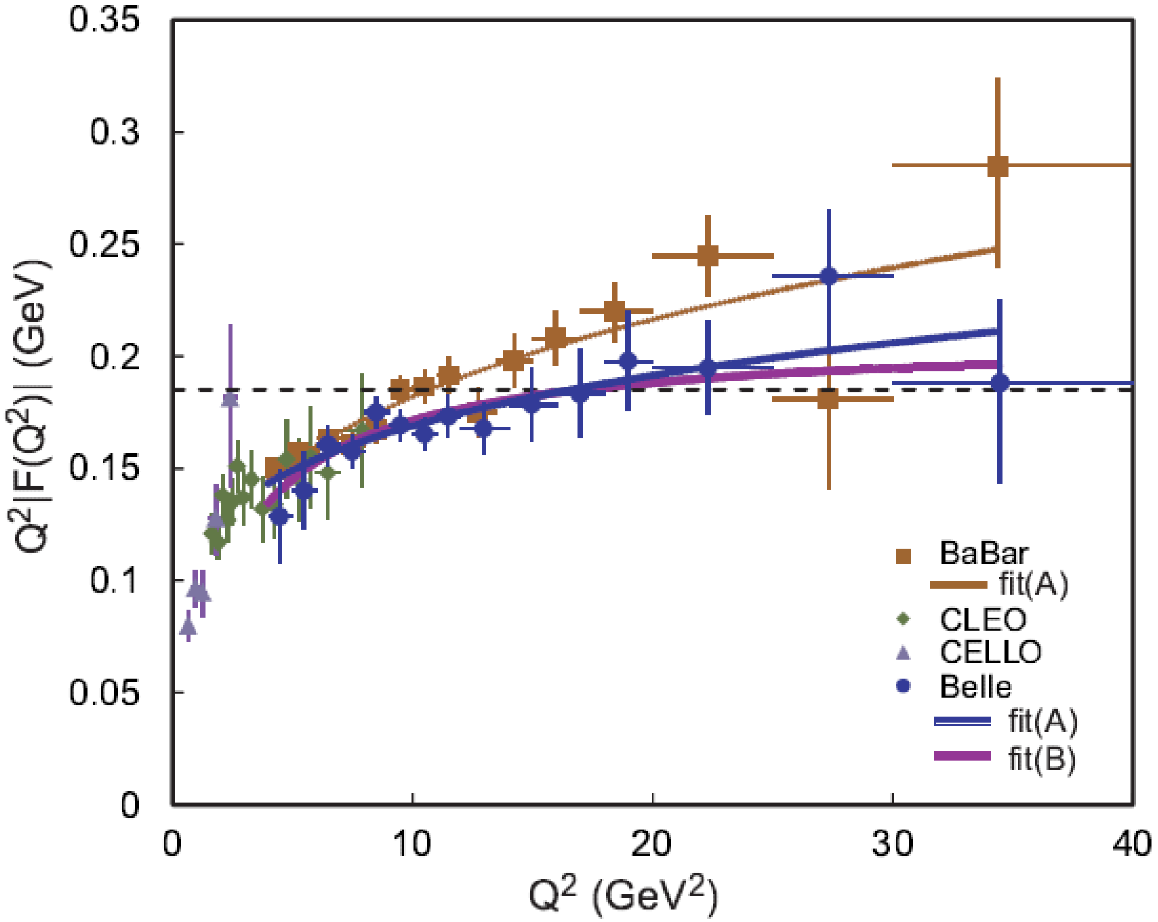
[8] Phys.Lett. B749 (2015) 547-550 by L. Chang and A.W. Thomas (DSE).

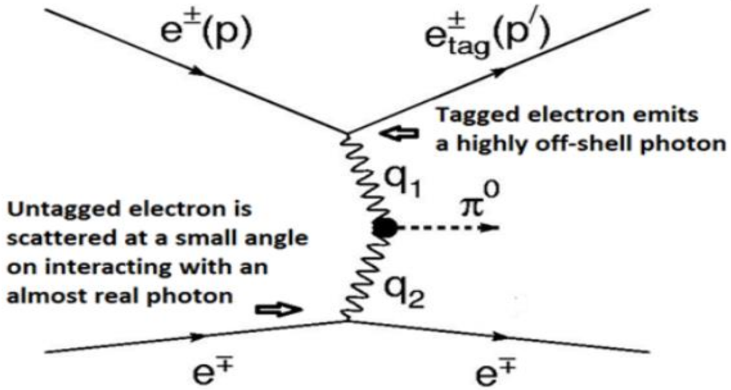
- **Valence quark distributions:**

[9] Phys.Rev. C83 062201 (2011) by T. Nguyen et al. (DSE).

- Static properties as masses, interaction radii and decay constants have been also studied for pion and other hadrons (such as Kaon) through DSEs. **DSEs are a very powerful tool.**

Pion transition form factor - $\gamma\gamma^* \to \pi^0$

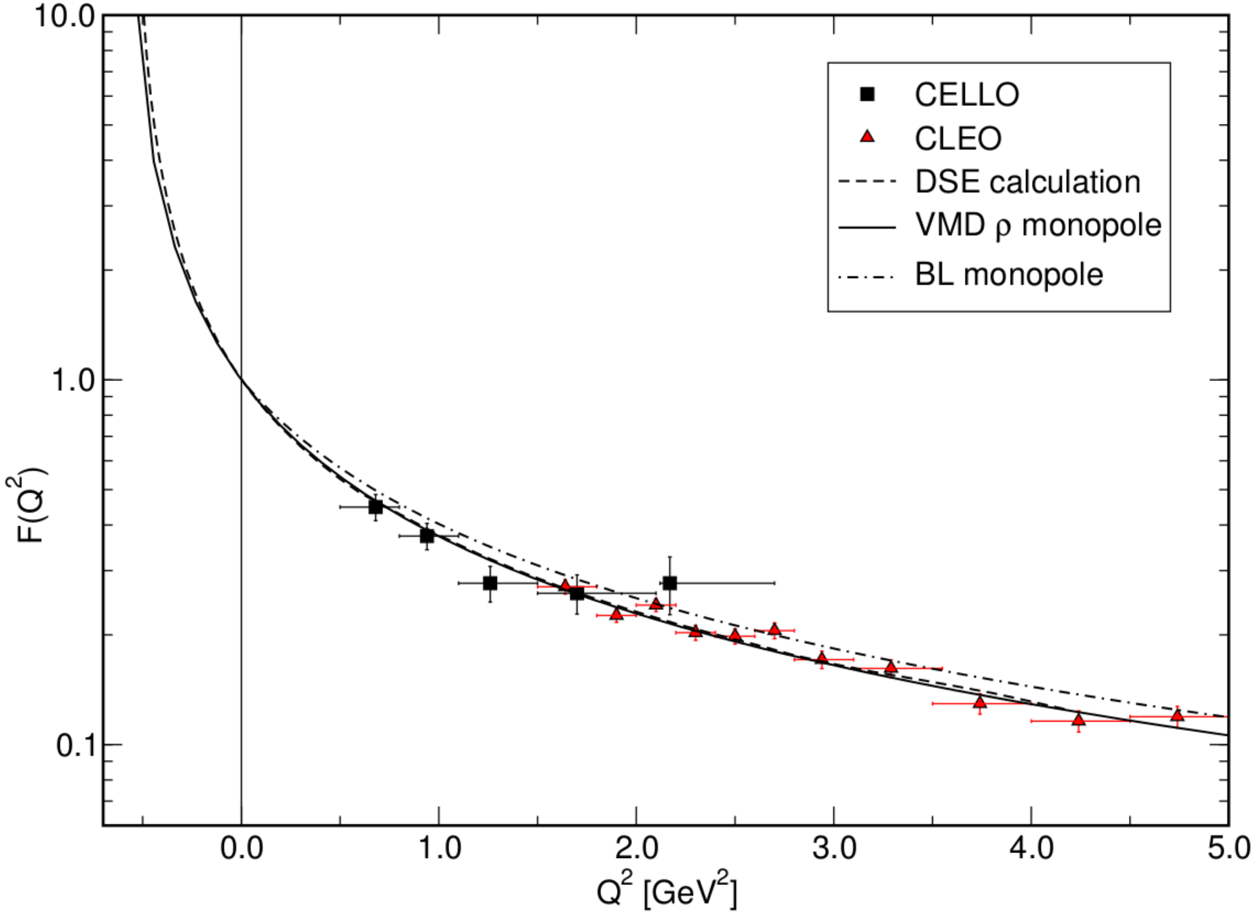




- Many experiments have been done so far; but, at large Q^2, there is no agreement between the only available data (BaBar and Belle).
- This needs to be explained, and we need to have predictions before future experiments of Belle II.

- [Dashed]: Well known asymptotic/conformal limit, $2f_\pi$. **G.P. Lepage, S.J. Brodsky, Phys.Rev. D22, 2157 (1980) [10].**

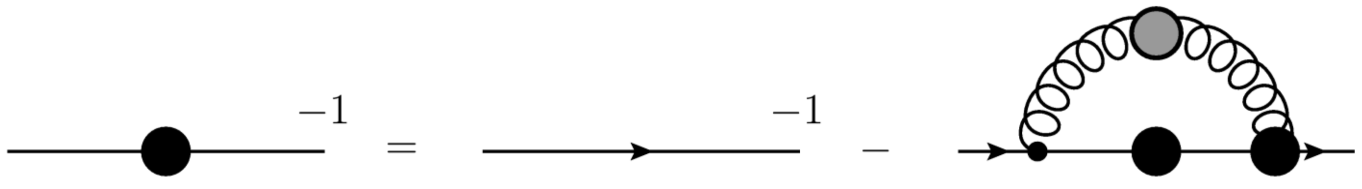
Pion transition form factor



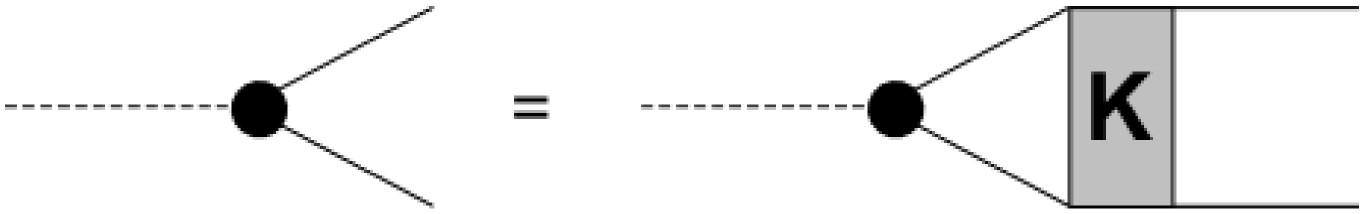
- **Previous DSE results:** P. Maris and P. Tandy, 2002 (ref. [1]). The methods developed back then did not allow to reach arbitrarily large momentum.

The tools: DSEs

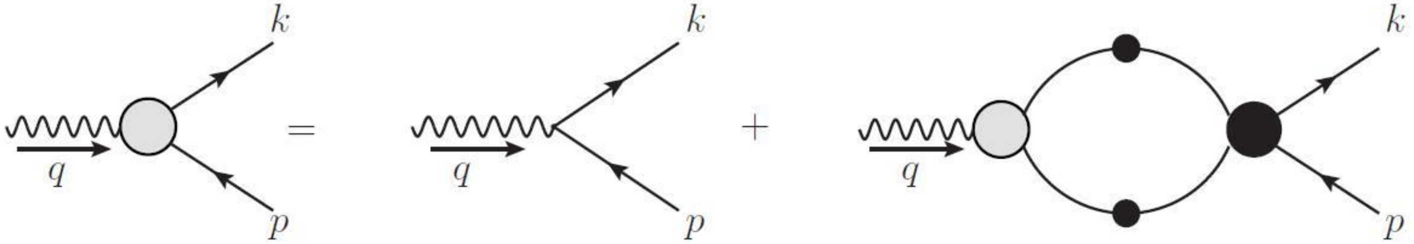
1. Quark Propagator

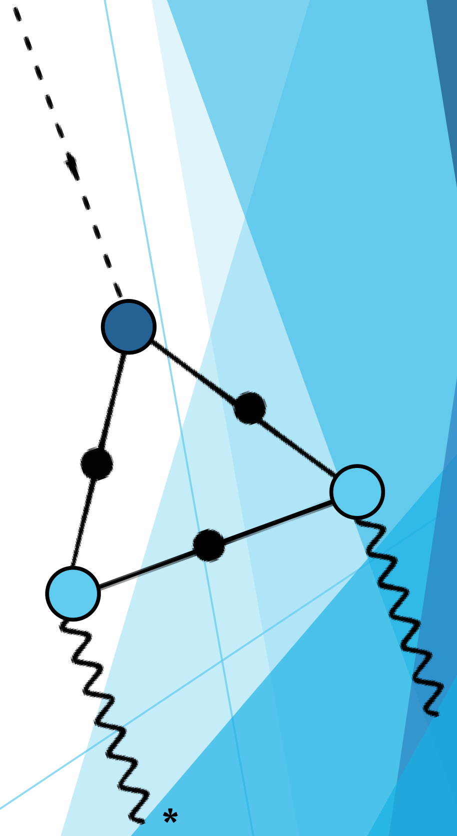


2. Bethe-Salpeter Amplitudes

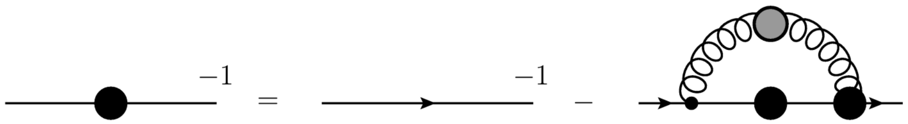


3. Quark-Photon Vertex





The tools: Quark propagator



- The renormalized DSE for the quark propagator is written as:

$$S^{-1}(p,\mu) = Z_{2F}(i\gamma\cdot p) + Z_4 m(\mu) + Z_{1F}\int_q^\Lambda g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\Gamma^a_\nu(p,q,\mu)\ .$$

- In the rainbow truncation:

$$S^{-1}(p,\mu) = Z_{2F}(i\gamma\cdot p) + Z_4 m(\mu) + Z_{1F}\int_q^\Lambda G(p-q) D^0_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\frac{\lambda^a}{2}\gamma_\nu\ .$$

- Where G(p-q) is an effective coupling, we model it as in **Phys. Rev. C84, 042202(R) (2011)** [11] by S.-x. Qin, L. Chang et al..

The tools: N-ccp parametrization

- The quark propagator is written as:

$$S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)\,.$$

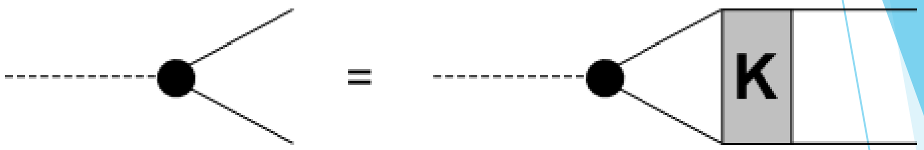
- It can be written in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N}\left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}}\right)\,,\quad \sigma_s(q) = \sum_{k=1}^{N}\left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}}\right)\,.$$

- Constrained to the UV conditions of the free propagator form.

N. Souchlas (adv. P. Tandy), (2009) [12]. *Quark Dynamics and Constituent Masses in Heavy Quarks Systems*. **PhD. Thesis.**

The tools: Bethe-Salpeter equation



- The Bethe-Salpeter equation (BSE) is written as:

$$\Gamma_M^{ab}(p;P) = \int_q^{\Lambda} K(p,q;P) S^a(q+\eta P) \Gamma_M^{ab}(q;P) S^b(q-(1-\eta)P)$$

- Where the Bethe-Salpeter amplitude (BSA) for the pion is:

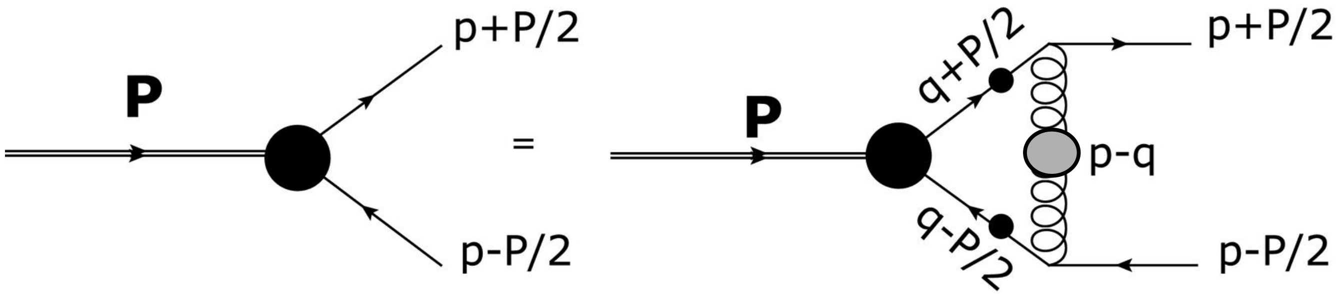
$$\Gamma_\pi^{qq}(p;P) = i\gamma_5\, E_\pi(p;P) + \gamma_5 \gamma\cdot P\; F_\pi(p;P) \\ +\gamma_5 (\gamma\cdot p)(p\cdot P)\; G_\pi(p;P) + \gamma_5\, p_\alpha \sigma_{\alpha\beta} P_\beta\; H_\pi(q;P)$$

The tools: Bethe-Salpeter equation

- In the rainbow-ladder truncation (RL):

$$\Gamma_M^{ab}(p;P) = -\int_q^{\Lambda} \frac{G(k^2)}{k^2} D^0_{\mu\nu}(k) \frac{\lambda^c}{2} \gamma_\mu S^a(q+\eta P) \Gamma_M^{ab}(q;P_i) S^b(q-(1-\eta)P) \frac{\lambda^c}{2} \gamma_\nu \; .$$

- Which corresponds to:



when the quarks share equal amount of momentum.

The tools: Nakanishi representation

- We parametrize the BSA using a Nakanishi-like representation, **Phys.Rev. 130 1230-1235 (1963) [13].** Which consists in splitting the BSA into IR and UV parts and writing them as follows:

$$A(q,P) = \int_{-1}^{1} dz \int_{0}^{\infty} d\Lambda \left[\frac{\rho^{i}(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{m+n}} + \frac{\rho^{u}(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{n}} \right] .$$

- Where the spectral density is written as:

$$\rho^{i,u}(z,\Lambda) = \rho_1(z)\delta(\Lambda - \Lambda^{i,u}) + \cdots$$

- Our form of A(q,P) is slightly different. First, we choose:

$$\rho_1(z) = \rho_\nu(z) \sim (1 - z^2)^{\nu} .$$

The tools: Nakanishi representation

- Then we choose the parametrization explained by Chang et al. (2013) in ref. [7]:

$$A^i(k,P) = c_A^i \int_{-1}^{1} dz \rho_{\nu_A^i}(z)[b_A \hat{\Delta}^4_{\Lambda_A^i}(k_z^2) + \bar{b}_A \hat{\Delta}^5_{\Lambda_A^i}(k_z^2)] \; . \quad E^u(k;P) = c_E^u \int_{-1}^{1} dz\, \rho_{\nu_E^u}(z) \hat{\Delta}^{1+\alpha}_{\Lambda_E^u}(k_z^2)$$

$$F^u(k;P) = c_F^u \int_{-1}^{1} dz\, \rho_{\nu_F^u}(z) k^2 \Lambda_F^u \Delta^2_{\Lambda_F^u}(k_z^2) \qquad G^u(k;P) = c_G^u \int_{-1}^{1} dz\, \rho_{\nu_G^u}(z) \Lambda_G^u \Delta^2_{\Lambda_G^u}(k_z^2)$$

- A stands for amplitude (E,F,G); *i*, *u* for IR and UV. H(k,P) is negligible. Λ, ν, c, b are parameters fitted to the numerical data.

- With the following definitions:

$$\hat{\Delta}_\Lambda(s) = \Lambda\, \Delta_\Lambda(s)\; ,\; \Delta_\Lambda(s) = (s+\Lambda^2)^{-1}\; ,\; k_z^2 = k^2 + z\, k\cdot P\; .$$

The tools: Quark-Photon vertex.

- We employ unamputated vertex ansatz:

$$S\Gamma_\mu S \rightarrow \chi(k_f, k_i) = \sum_{i=1}^{3} T_{\mu i} X_i(k_f, k_i)$$

- Where the tensor structures are:

$$T_{1\mu} = \gamma_\mu$$
$$T_{2\mu} = \beta\gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{\beta}\gamma \cdot k_i \gamma_\mu \gamma \cdot k_f$$
$$T_{3\mu} = i\,\beta(\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + i\bar{\beta}(\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f)$$

$$\beta = 1 + \alpha(Q^2)$$
$$\bar{\beta} = 1 - \beta$$

- And, the dressing functions:

$$X_1(k_f, k_i) = \Delta_{k^2\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_2(k_f, k_i) = \Delta_{\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_3(k_f, k_i) = \Delta_{\sigma_S}(k_f^2, k_i^2)\,.$$

$$\Delta_F(k_f, k_i) = \frac{F(k_f) - F(k_i)}{k_f - k_i}$$

The tools: Quark-Photon vertex.

- Owing too the Abelian anomaly, one finds that it is impossible to simultaneously conserve the vector and axial-vector currents (S. L. Adler, **Phys.Rev. 177 (1969) 2426-2438 [14]**).

- We then introduce a non zero $\alpha(Q^2)$, which ensures that G(0) = ½. This is equivalent to a shift of variables. In particular, we use:

$$\alpha(Q^2) = \alpha_0 \mathrm{Exp}[-\mathcal{E}_\pi/M], \mathcal{E}_\pi = Q/2 \text{ (Energy in Breit Frame)}.$$

- Where $M^2=M^2(p^2)=p^2$ is the *euclidean* constituent quark mass.

- This vertex satisfies the WTI and it is free of kinematic singularities (R. Delbourgo, P. C. West, **J. Phys. A 10 (1977) 1049 [15]**). Clearly, when Q^2 grows, we recover the free-field limit.

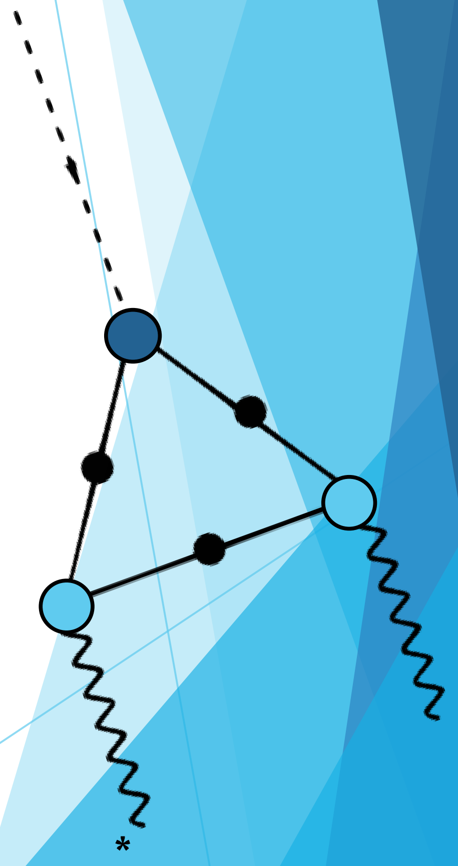
Pion Transition Form Factor

- The pion transition form factor (TFF) is written as:

$$\mathcal{T}_{\mu\nu}(k_1, k_2) = T_{\mu\nu}(k_1, k_2) + T_{\nu\mu}(k_2, k_1) \ ,$$

$$T_{\mu\nu}(k_1, k_2) = \frac{\alpha_{em}}{\pi f_\pi} \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G(k_1^2, k_2^2, k_1 \cdot k_2)$$

$$= \mathrm{tr} \int \frac{d^4 l}{(2\pi)^4} \chi_\pi(l_1, l_2) i\Gamma_\mu(l_2, l_{12}) S(l_{12}) i\Gamma_\nu(l_{12}, l_1) \ .$$



- The parametrizations of quark propagator and BSA allows us to solve analytically the integrations over momentum after Feynman Parametrization.

- And then, numerically integrate over Feynman Parameters.

TFF: Asymptotic limit

- According to ref. [1], the asymptotic limit is:

$$G(Q_1^2, Q_2^2; \mu) \rightarrow f_\pi \left\{ \frac{J_\omega(\mu)}{Q_1^2 + Q_2^2} + O\left(\frac{\alpha_s}{\pi}, \frac{1}{(Q_1^2 + Q_2^2)^2} \right) \right\} ,$$

where:

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} , \quad J_\omega(\mu) = \frac{4}{3} \int_0^1 dx \frac{\phi_\pi(x, \mu)}{1 + \omega^2 (2x - 1)} .$$

- $G(Q^2)$ has been divided by $2\pi^2 f_\pi$, in order to follow the experimental convention.

- For asymptotic QCD, we have: $\phi_\pi^{asym}(x, \mu \rightarrow \infty) = 6x(1 - x)$.

Therefore, we arrive at the *conformal limit* (ref. [9]): $Q^2 G(Q^2) \rightarrow 2 f_\pi$.

TFF: Asymptotic limit

- At fixed finite renormalization point (μ = 2 GeV), we have instead (ref. [7]):

$$\phi_\pi(x,\mu) = 1.71[x(1-x)]^{\alpha_-}\left[1 + a_2 C_2^{\alpha_+}(2x-1)\right] .$$

Then, one should expect something almost twice bigger:

$$Q^2 G(Q^2) \to f_\pi J_1(\mu = 2\ \mathrm{GeV}) = 3.62\ f_\pi\ .$$

- **What is happening?** On one hand, the scale in which the asymptotic PDA is approached is uncertain, but it should be at *very large* momentum. On the other hand, pion's PDA should evolve with resolution scale $\mu^2=Q^2$: this relates the PDA at some fixed renormalization point with the asymptotic PDA, allowing us to arrive at the conformal limit as Q^2 grows.

Pion Distribution Amplitude

- To fully understand pion, one must extract information from its PDA, the projection of the BSA onto the lightcone:

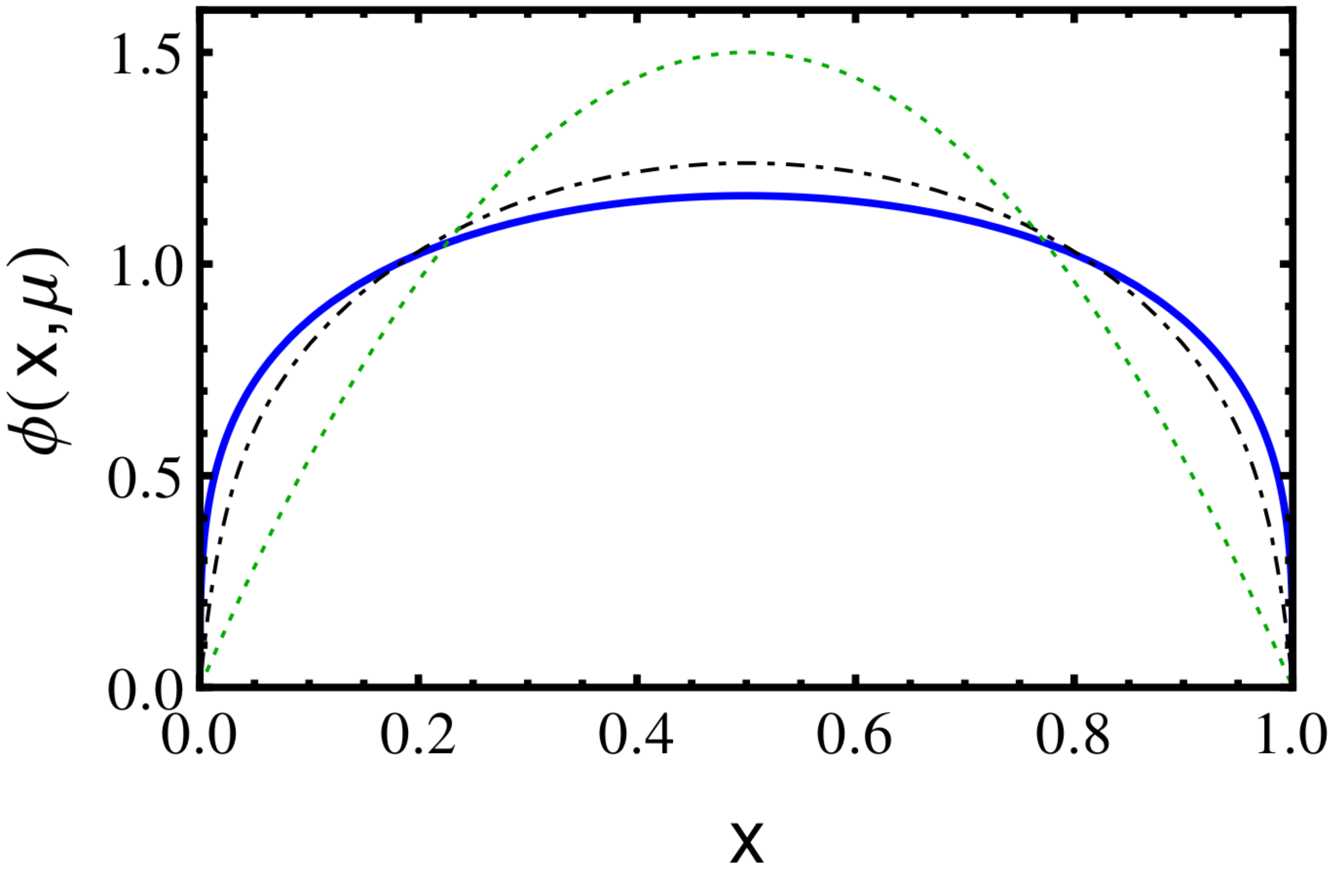
$$f_\pi \phi_\pi(x,\mu) = \mathrm{tr} \int \frac{d^4 q}{(2\pi)^4} \delta(n\cdot q^+ - xn\cdot P)\gamma_5\gamma\cdot n\chi_\pi(q;P)$$

- It should evolve with the resolution scale $\mu^2=Q^2$ through the ERBL evolution equations, **Phys.Rev.Lett. 11 092001 (2013) [16]**.

- Evolution enables the dressed-quark and -antiquark degrees-of-freedom, to split into less well-dressed partons via the addition of gluons and sea quarks in the manner prescribed by QCD dynamics. This can be read from the leading twist expansion (for example):

$$G(Q^2) = 4\pi^2 f_\pi \int_0^1 dx T_H(x,Q^2,\alpha(\mu);\mu)\phi_\pi(x,\mu)$$

Pion Distribution Amplitude



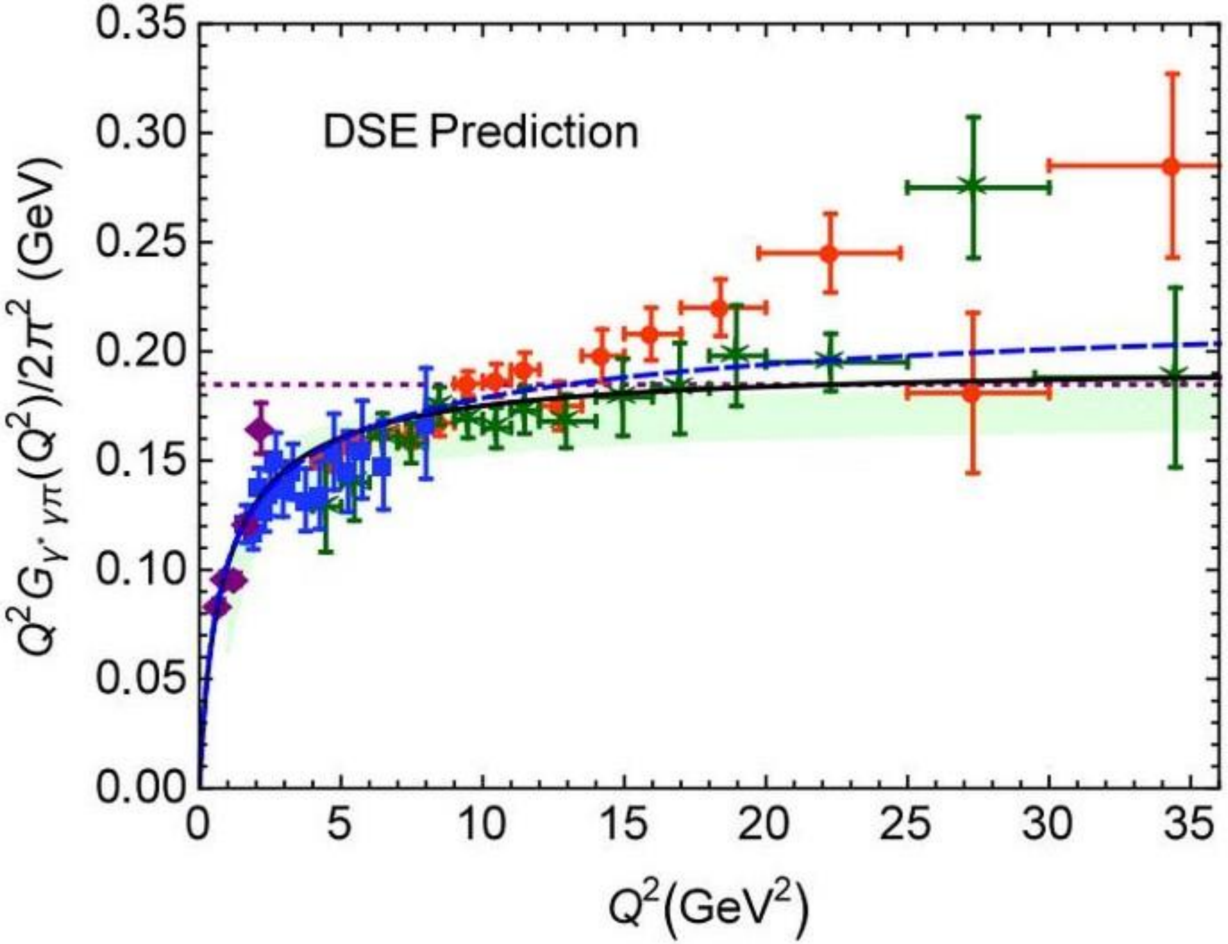
- **PDA at different scales:** [Green, dashed] Asymptotic PDA, 6x(1-x). [Blue, solid] μ=2 GeV from reference [7]. [Black, dot-dashed] Evolution from μ=2 GeV to μ=10 GeV.

TFF: The tools

We have gathered the following tools:

- Nccp parametrization for quark propagator.
- Nakanishi-like representation for BSA.
- Quark-photon vertex (WTI solution, gauge technique).
- PDA and its evolution.

DSE Prediction



- **Transition Form Factor:** [**Black**, solid] DSE Prediction (G(0)=½, r=0.68 fm). [Blue, dashed] Unevolved DSE Prediction (μ = 2 GeV). [Green, band] BMS model (A.P. Bakulev et al., **Phys.Rev. D84, 034015 (2011) [17]**).

DSE Prediction - Q^2 Evolution

- As we have said before, we expect to arrive at the conformal limit as Q^2 grows; and, we foresee that such limit would be reached from below. However, there is a logarithmic missmatch:

1. $G(Q^2)$ reaches a little above the asymptotic limit. The growth is *logarithmically slow*, however; and whilst the curve remains a line-width above the asymptotic limit on a large domain, logarithmic growth eventually becomes suppression and the curve thereafter proceeds towards the QCD asymptotic limit from above.

2. This discrepance originates in the failure of RL truncation to incorporate of gluon and quark splitting effects contained in QCD, and hence its failure to fully express interferences between the anomalous dimensions.

Conclusions

arXiv: 1510.02799 [nucl-th]. **Submitted to PLB**

- We described a computation of the pion transition form factor, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations, obtained in the rainbow-ladder truncation.
- We have unified the description and explanation of this transition with the charged pion electromagnetic form factor [5] and its valence-quark distribution amplitude [7].
- The novel analysis techniques we employed made it possible to compute $G(Q^2)$, on the entire domain of space-like momenta, for the first time in a framework with a direct connection to QCD.
- This enabled us to demonstrate that a fully self-contained and consistent treatment can readily connect a pion PDA that is a broad, concave function at the hadronic scale with the perturbative QCD prediction for the transition form factor in the hard photon limit.