The Gatto-Sartori-Tonin relation: A symmetrical origin

U. J. Saldaña Salazar
(CINVESTAV)

XV-MWPF
Nov 3, 2015
Mazatlan, Mexico

CONACyT-166639

Cinvestav

The Gatto-Sartori-Tonin relation: A symmetrical origin

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

ArXiv: 1509.08877
UJSS

U. J. Saldaña Salazar
(CINVESTAV)

XV-MWPF
Nov 3, 2015
Mazatlan, Mexico

CONACyT-166639

Cinvestav

Outline

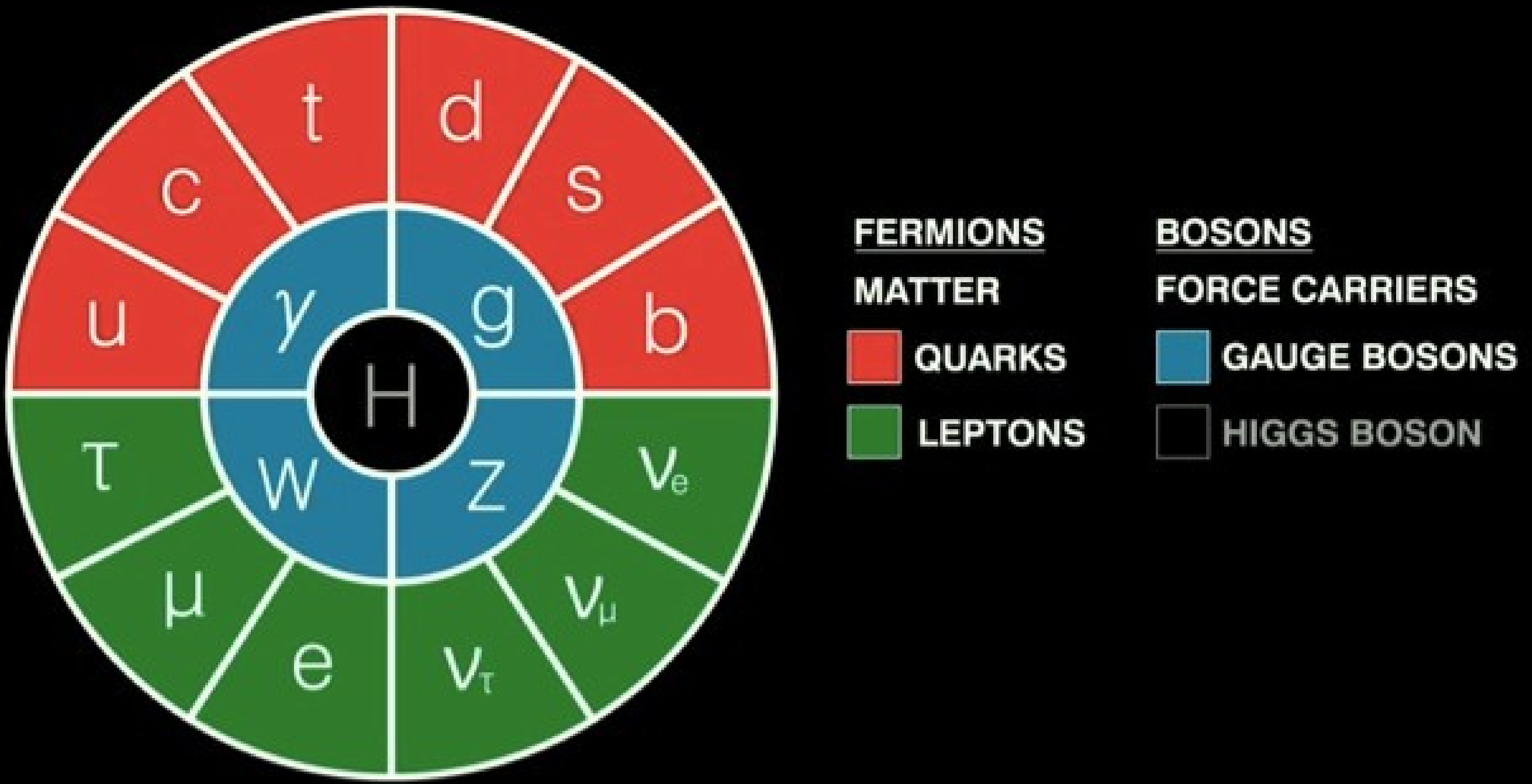
The four mass ratios parametrization

The Gatto-Sartori-Tonin condition

Conclusions

The four mass ratios parametrization

Standard Model parameters



$g, \ g', \ \alpha_s$

$\lambda, \ v,$

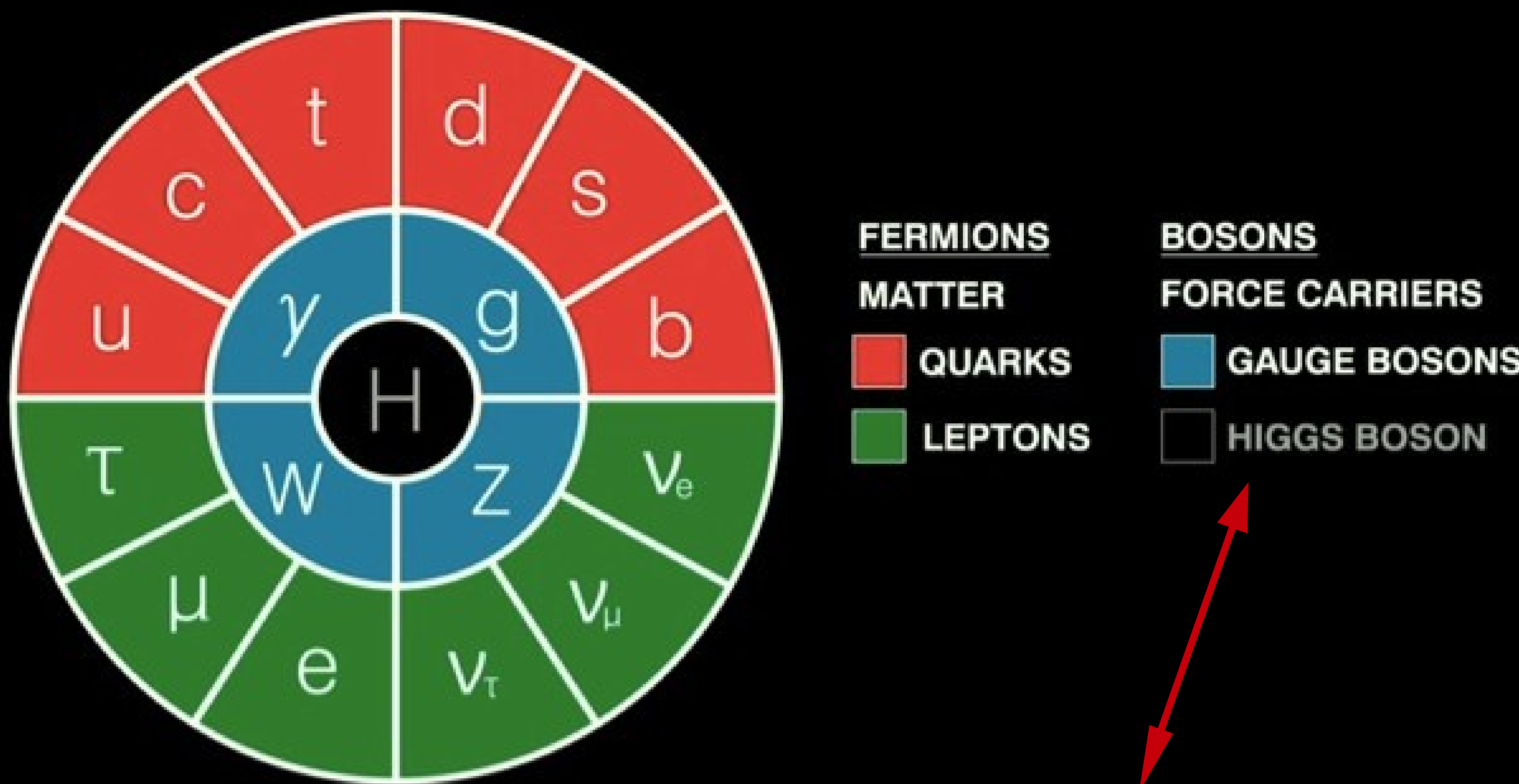
$m_u, \ m_c, \ m_t,$
$m_d, \ m_s, \ m_b,$

$m_e, \ m_\mu, \ m_\tau,$
$m_{\nu 1}, \ m_{\nu 2}, \ m_{\nu 3},$

$\theta^q_{12}, \ \theta^q_{13}, \ \theta^q_{23}, \delta^q_{cp}$

$\theta^\ell_{12}, \ \theta^\ell_{13}, \ \theta^\ell_{23}, \delta^\ell_{cp}$

Standard Model parameters



$g,\ g',\ \alpha_s$

$\lambda,\ v,$

$m_u,\ m_c,\ m_t,$
$m_d,\ m_s,\ m_b,$

$m_e,\ m_\mu,\ m_\tau,$
$m_{\nu 1},\ m_{\nu 2},\ m_{\nu 3},$

$\theta^q_{12},\ \theta^q_{13},\ \theta^q_{23}, \delta^q_{cp}$

$\theta^\ell_{12},\ \theta^\ell_{13},\ \theta^\ell_{23}, \delta^\ell_{cp}$

Mixing parametrizations

$V = L_a L_b^\dagger \quad a = u, e \quad b = d, \nu$ (Mass basis)

$VV^\dagger = V^\dagger V = 1 \qquad \Rightarrow U(n) \to n^2$

$-(2n-1)$ (Independent field rephasings)

Num. of mixing parameters: $(n-1)^2$

$n = 3 \quad \Rightarrow \quad 4\ (3+1)$

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PDG, Chau, Maiani, Kobayashi-Maskawa, Wolfenstein, Fritzsch-Xing, Valle-Rodejohann

Mass ratios

$$\tan^2\theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),
Mohapatra (1977), Weinberg (1977),
Fritzsch (1977), Ramond (1993), Xing (1996),
Rasin (1997), Chkareuli (1998), Mondragón (1998),
Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

$$m_1^a,\ m_2^a,\ m_3^a,\ m_1^b,\ m_2^b,\ m_3^b,$$

$$\Rightarrow 2(n-1) \qquad \Rightarrow n \leq 3$$

$$V = V\left(\frac{m_1^a}{m_2^a},\ \frac{m_2^a}{m_3^a},\ \frac{m_1^b}{m_2^b},\ \frac{m_2^b}{m_3^b}\right)$$

Complex phases I

$$V = V(\frac{m_i}{m_j}, \delta_1, \delta_2, \cdots, \delta_k)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b})$$

$$\Rightarrow L_f = L_f(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f})$$

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

Complex phases I

$$V = V(\frac{m_i}{m_j}, \delta_1, \delta_2, \cdots, \delta_k)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b})$$

$$\Rightarrow L_f = L_f(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f})$$

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

No exact solution.

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$\Downarrow$$

$$m_1 \ll m_2 \ll m_3$$

$$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$$

$$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$$

$$\Delta m^2_{31(32)} : \Delta m^2_{21} = 1 : 10^{-2}$$

Schmidt-Mirsky approximation theorem

(Schmidt, Mirsky, Eckart, Young)

$$\text{rank}[A] = n \qquad \sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$$

$$s_k = \{\sigma_k, \sigma_{k-1}, \ldots, \sigma_1\} \ll \sigma_{k+1}$$

$$||A - B||_X \geq ||A - A(s_k = 0)||_X$$

$$\text{rank}[B] = n - k \qquad ||A||_F = \sqrt{Tr(AA^\dagger)}$$

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$$

$$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$$

$$m_{1,2} = 0$$

$+$

rank 1 $\quad m_1 \ll m_2 \ll m_3$

rank 2

$$\Delta m^2_{31(32)} : \Delta m^2_{21} = 1 : 10^{-2}$$

$$m_1 = 0$$

Schmidt-Mirsky approximation theorem (Schmidt, Mirsky, Eckart, Young)

$$\text{rank}[A] = n \qquad \sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$$

$$s_k = \{\sigma_k, \sigma_{k-1}, \ldots, \sigma_1\} \ll \sigma_{k+1}$$

$$||A - B||_X \geq ||A - A(s_k = 0)||_X$$

$$\text{rank}[B] = n - k \qquad ||A||_F = \sqrt{Tr(AA^\dagger)}$$

Minimal flavor violation (MFV)

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i\gamma^\mu \partial_\mu \psi$$

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i\gamma^\mu D_\mu \psi$$

$$\mathcal{L}_\psi = \sum_f \bar{\psi}_f (i\gamma^\mu D^f_\mu - \mathcal{M}_f)\psi_f$$

$U(48)$ → $U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu$ → $U(1)_B \times U(1)_L$

Rank 0 $U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu$ $m_1, m_2, m_3 = 0$

↓

Rank 1 $U(2)^Q \times U(2)^u \times U(2)^d \times U(2)^\ell \times U(2)^e \times U(2)^\nu$ $m_1, m_2 = 0$

↓

Rank 2 $U(1)^Q \times U(1)^u \times U(1)^d \times U(1)^\ell \times U(1)^e \times U(1)^\nu$ $m_1 = 0$

↓

Rank 3 $U(1)_B \times U(1)_L$

(Barbieri, Hall, Romanino, Dvali, Straub, Blankenburg, Grinstein)

Electroweak basis*

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\uparrow$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\uparrow$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

↑

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\uparrow$

$$M_{f,r=2} M_{f,r=2}^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \theta_{23}^2 & \theta_{23} \\ 0 & \theta_{23} & 1+\theta_{23}^2 \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

$\uparrow$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

?

$$\tan^2\theta_{23} = \frac{m_2}{m_3}$$

$$M_{f,r=2} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_{23} \\ 0 & m_{23} & m_{33} \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

$$|M_{f,r=1}| = m_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Complex phases II

$$V = L_a L_b^\dagger \rightarrow V_{23} = L_{23}^a L_{23}^b{}^\dagger$$

$$V_{ij} = \sqrt{\frac{\hat{m}_{ij}^a + \hat{m}_{ij}^b - 2\hat{m}_{ij}^a \hat{m}_{ij}^b \cos(\delta_{ij}^a - \delta_{ij}^b)}{(1+\hat{m}_{ij}^a)(1+\hat{m}_{ij}^b)}}$$

- Minimal mixing

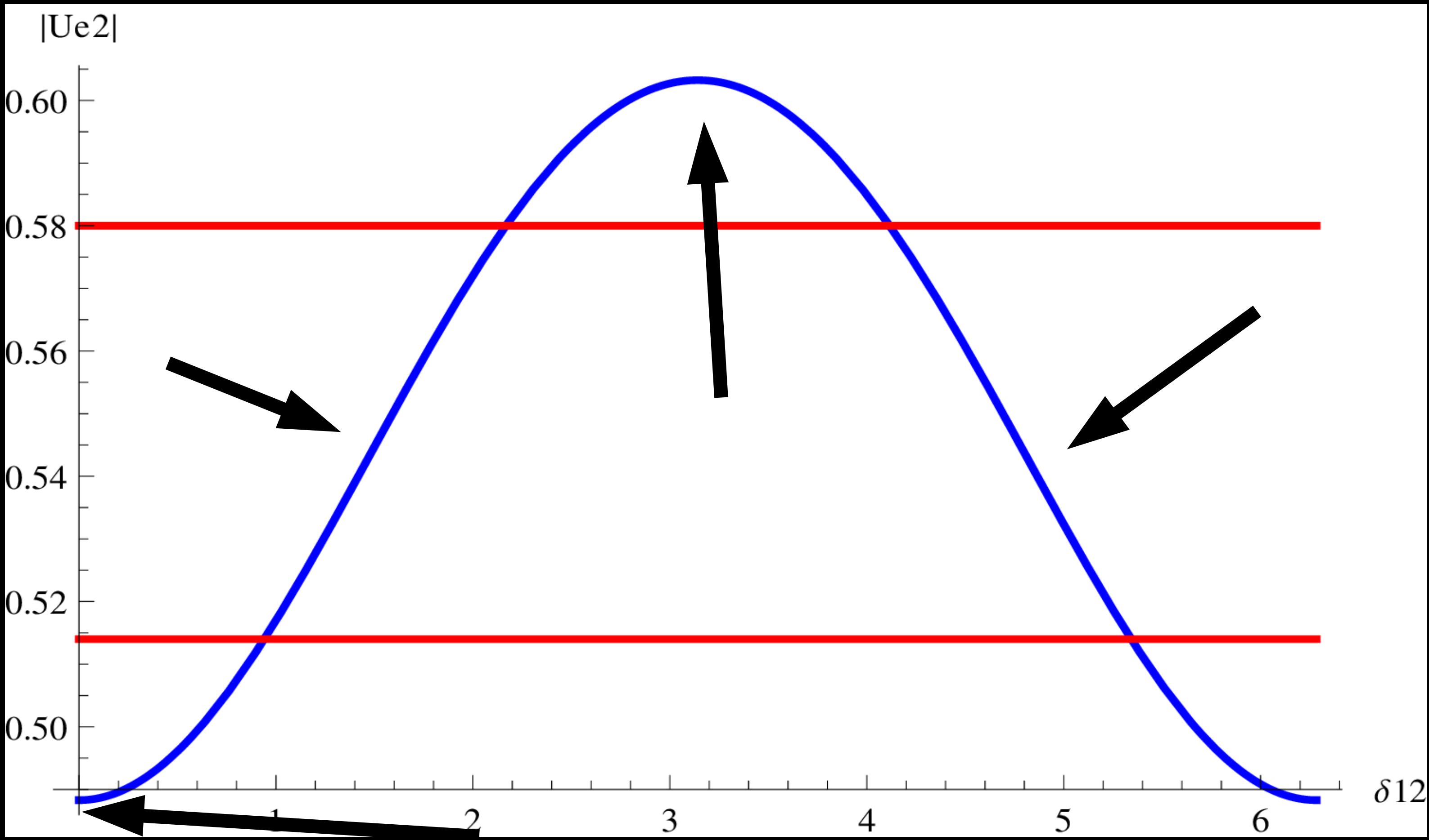
$$\Delta\delta_{ij} = 0$$

- Maximal mixing

$$\Delta\delta_{ij} = \pi$$

- CP Violation

$$\Delta\delta_{ij} = (3)\frac{\pi}{2}$$



Ansatz

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^f = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{23}^{(1)}(\frac{m_1}{m_3}) L_{23}^{(0)}(\frac{m_2}{m_3})$$

$$L_f = L_{12}(\theta_{12}^f, \pi/2) L_{13}(\theta_{13}^f, 0) L_{23}(\theta_{23}^f, 0)$$

CKM (other authors)

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$V_{CKM}^{\text{th}} = \begin{pmatrix} 0.974 & 0.225 & 0.032 \\ 0.225 & 0.971 & 0.076 \\ 0.033 & 0.076 & 0.997 \end{pmatrix}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

CKM (ours)

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^{f} = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{23}^{(1)}(\frac{m_1}{m_3}) L_{23}^{(0)}(\frac{m_2}{m_3})$$

$$|V_{\mathrm{CKM}}^{\mathrm{th}}| = \begin{pmatrix} 0.974^{+0.004}_{-0.003} & 0.225^{+0.016}_{-0.011} & 0.0031^{+0.0018}_{-0.0015} \\ 0.225^{+0.016}_{-0.011} & 0.974^{+0.004}_{-0.003} & 0.039^{+0.005}_{-0.004} \\ 0.0087^{+0.0010}_{-0.0008} & 0.038^{+0.004}_{-0.004} & 0.9992^{+0.0002}_{-0.0001} \end{pmatrix}$$

$$J_q^{\mathrm{th}} = (2.6^{+1.3}_{-1.0}) \times 10^{-5}$$

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

$$J_q = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$$

PMNS (Neutrino masses)

$$m_{\nu 2} = \sqrt{\Delta m_{21}^2/(1-\hat{m}_{\nu 12}^2)},$$
$$m_{\nu 1} = \sqrt{m_{\nu 2}^2 - \Delta m_{21}^2},$$
$$m_{\nu 3} = \sqrt{\Delta m_{31}^2 - \Delta m_{21}^2 + m_{\nu 2}^2}.$$

$$|V_{12}^{f=q,\ell}| \approx \sqrt{\frac{\hat{m}_{12}^a + \hat{m}_{12}^b}{(1+\hat{m}_{12}^a)(1+\hat{m}_{12}^b)}}$$

$$\frac{m_e}{m_\mu},\ |U_{e2}|,\ \Delta m_{21}^2,\ \Delta m_{31}^2$$

$$\text{NH:}\quad \Delta m_{31}^2 = +2.457 \pm 0.002 \times 10^{-3}\,\text{eV}^2,$$
$$\text{IH:}\quad \Delta m_{32}^2 = -2.448 \pm 0.047 \times 10^{-3}\,\text{eV}^2,$$
$$\Delta m_{21}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\text{eV}^2,$$

NuFit14

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\text{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\text{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\text{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

PMNS (our predictions)

$$|U_{PMNS}| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

$$J_\ell = -0.033 \pm 0.010$$

NuFit14

$$|U^{\rm th}_{\rm PMNS}| = \begin{pmatrix} 0.83^{+0.04}_{-0.05} & 0.54^{+0.06}_{-0.09} & 0.14 \pm 0.03 \\ 0.38^{+0.04}_{-0.06} & 0.57^{+0.03}_{-0.04} & 0.73 \pm 0.02 \\ 0.41^{+0.04}_{-0.06} & 0.61^{+0.03}_{-0.04} & 0.67 \pm 0.02 \end{pmatrix},$$

$$J_\ell = -0.031^{+0.006}_{-0.007}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\mathrm{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\mathrm{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\mathrm{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

PMNS (without fine tuning)

See Prof. Valle's talk

$$\sin^2\theta_{12}^{\ell} = 0.323 \pm 0.016, \qquad \sin^2\theta_{23}^{\ell} = 0.567^{+0.032}_{-0.128}, \qquad \sin^2\theta_{13}^{\ell} = 0.0234 \pm 0.020,$$

Forero et al

$$\frac{\delta_{\mathrm{CP}}}{\pi} = 1.34^{+0.64}_{-0.38}$$

$$\sin^2\theta_{12}^{\ell,\mathrm{th}} = 0.30^{+0.07}_{-0.09}, \qquad \sin^2\theta_{23}^{\ell,\mathrm{th}} = 0.54 \pm 0.03, \qquad \sin^2\theta_{13}^{\ell,\mathrm{th}} = 0.020^{+0.009}_{-0.007},$$

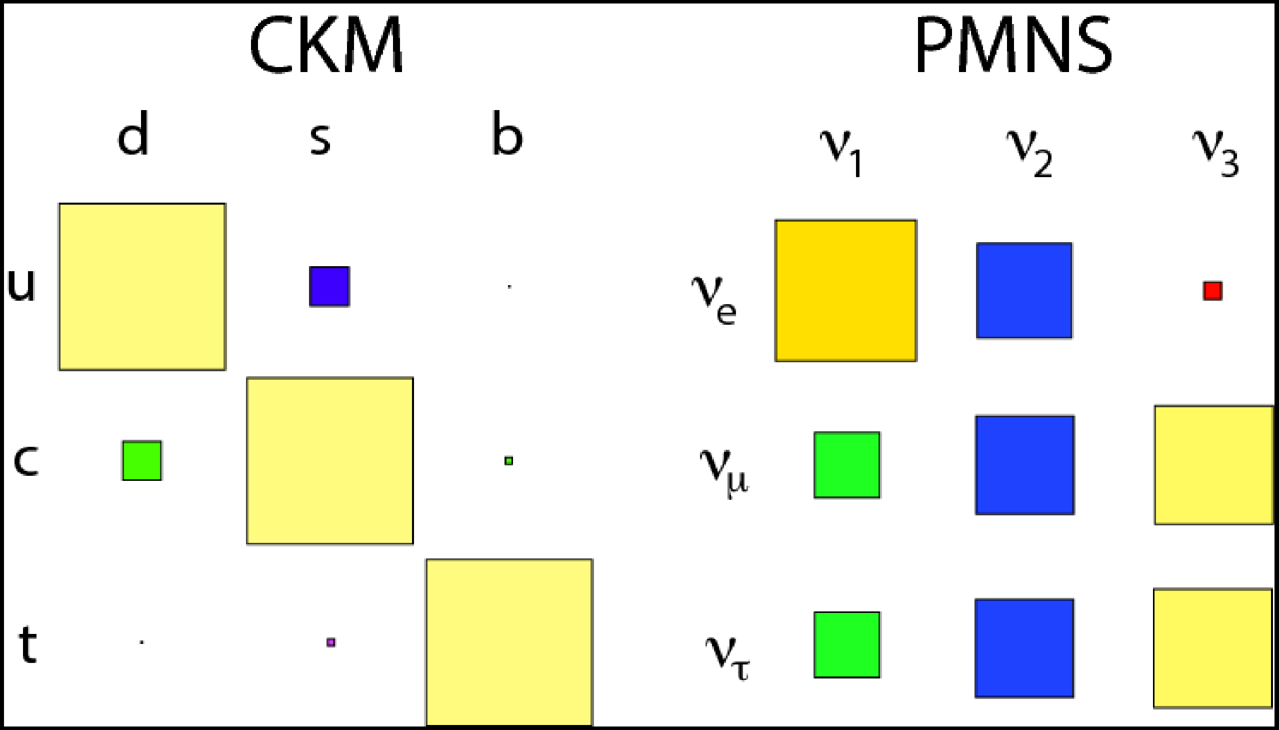
$$\frac{\delta_{\mathrm{CP}}^{\mathrm{th}}}{\pi} = 1.36^{+0.05}_{-0.16}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\mathrm{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\mathrm{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\mathrm{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

Some insight into the *flavor puzzle*



- Strong hierarchical masses
- Minimal mixing in the 1-3 and 2-3 sectors
- CP Violation in the 1-2 sector

- Weak hierarchy in neutrino masses
- Minimal mixing in the 1-3 sector
- Maximal mixing in the 2-3 sector
- CP Violation in the 1-2 sector

The Gatto-Sartori-Tonin condition

Two inquiries

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$m_3 \gg m_2 \gg m_1$$

- Hierarchical contributions*
- Ordered Yukawas

What principle lies behind such sequential Yukawas?

*(Froggatt-Nielsen, Arkani Hamed, Ibarra-Solaguren, Altmannshofer, Knapen-Robinson)

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$ $\quad$ $|m_{22}| \neq |m_{22}|(\delta_{ij})$

$$|\mathbf{m}| = \begin{pmatrix} 0 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

ArXiv: 1509.08877
UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$ $|m_{22}| \neq |m_{22}|(\delta_{ij})$

$$|\mathbf{m}| = \begin{pmatrix} 0 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$M = L^\dagger \Sigma R$

$\theta_L = \theta_R$

$\delta_L = \delta_R + \pi$

ArXiv: 1509.08877
UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$$\tan^2 \theta_{ij} = \frac{m_i}{m_j}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$ $\quad$ $|m_{22}| \neq |m_{22}|(\delta_{ij})$

Antisymmetric

$\delta = 0,\ \pi$

$$M = \begin{pmatrix} 0 & \sqrt{m_1 m_2} e^{-i\delta} \\ -\sqrt{m_1 m_2} e^{i\delta} & m_2 - m_1 \end{pmatrix}$$

Symmetric

$\delta = \frac{\pi}{2},\ \frac{3\pi}{2}$

ArXiv: 1509.08877 UJSS

The Gatto-Sartori-Tonin condition:

"Yukawa couplings should be added as a sum of matrices each obeying distinct symmetry properties under permutations of the corresponding massless fields."

$$S_{nL} \otimes S_{nR} \to S_{(n-1)L} \otimes S_{(n-1)R} \to \cdots \to S_{2L} \otimes S_{2R} \to S_{2A}$$

$$\mathcal{Y}^{1\leftrightarrow 2\leftrightarrow \cdots \ (n-1)\leftrightarrow n} + \mathcal{Y}^{1\leftrightarrow 2\leftrightarrow \cdots (n-2)\leftrightarrow (n-1)} + \cdots + \mathcal{Y}^{1\leftrightarrow 2} + \mathcal{Y}^{2A}$$

ArXiv: 1509.08877, UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann $$\mathcal{Y} = y \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} \beta & \alpha \\ -\alpha & -\beta \end{pmatrix}$$

$\psi_1, \ \psi_2$

$S_{2L} \times S_{2R}$

S_{2A}

ArXiv: 1509.08877, UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann

$$\mathcal{Y} = 2y \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \alpha + \beta \\ \beta - \alpha & 0 \end{pmatrix}$$

$\psi_1^m, \; \psi_2^m$

$S_{2L} \times S_{2R}$

$$O_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

S_{2A}

$m_2 \neq 0, \quad m_1 = 0$

$m_1 \neq 0$

ArXiv: 1509.08877, UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann $$\mathcal{Y} = x_1 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & x_2 e^{-i\delta} \\ x_2 e^{i\delta} & 0 \end{pmatrix}$$

$$x_1 = m_2 - m_1$$

$$x_2 = \sqrt{m_1 m_2}$$

$$\delta = 0,\ \pi$$

Symmetric

$$\tan^2 \theta_{ij} = \frac{m_i}{m_j}$$

$$\delta = \frac{\pi}{2},\ \frac{3\pi}{2}$$

Antisymmetric

ArXiv: 1509.08877, UJSS

Three family case

Harari 78, Kaus, Lavoura, Fritzsch, Tanimoto, Meshkov, Babu, Mohapatra, Mondragón, Rodríguez Jauregui, González Canales, Barranco

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$\mathcal{Y} = y \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \longrightarrow S_{3L} \times S_{3R} \quad m_3 \neq 0,\ m_2, m_1 = 0$$

$$+ \begin{pmatrix} \beta & \beta & \alpha_1 \\ \beta & \beta & \alpha_1 \\ \alpha_2 & \alpha_2 & \gamma \end{pmatrix} \longrightarrow S_{2L} \times S_{2R} \quad m_3, m_2 \neq 0,\ m_1 = 0$$

$$+ \begin{pmatrix} \tau & i\mu & \nu_1 \\ -i\mu & -\tau & -\nu 1 \\ \nu_2 & -\nu_2 & 0 \end{pmatrix} \longrightarrow S_{2A} \quad m_3, m_2, m_1 \neq 0$$

ArXiv: 1509.08877, UJSS

Three family case

Harari 78, Kaus, Lavoura, Fritzsch, Tanimoto, Meshkov, Babu, Mohapatra, Mondragón, Rodríguez Jauregui, González Canales, Barranco

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$\mathcal{Y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} \longrightarrow U(2)^3 \quad m_3 \neq 0,\ m_2, m_1 = 0$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} \longrightarrow U(1)^3 \quad m_3, m_2 \neq 0,\ m_1 = 0$$

$$+ \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix} \longrightarrow U(1)_{B(L)} \quad m_3, m_2, m_1 \neq 0$$

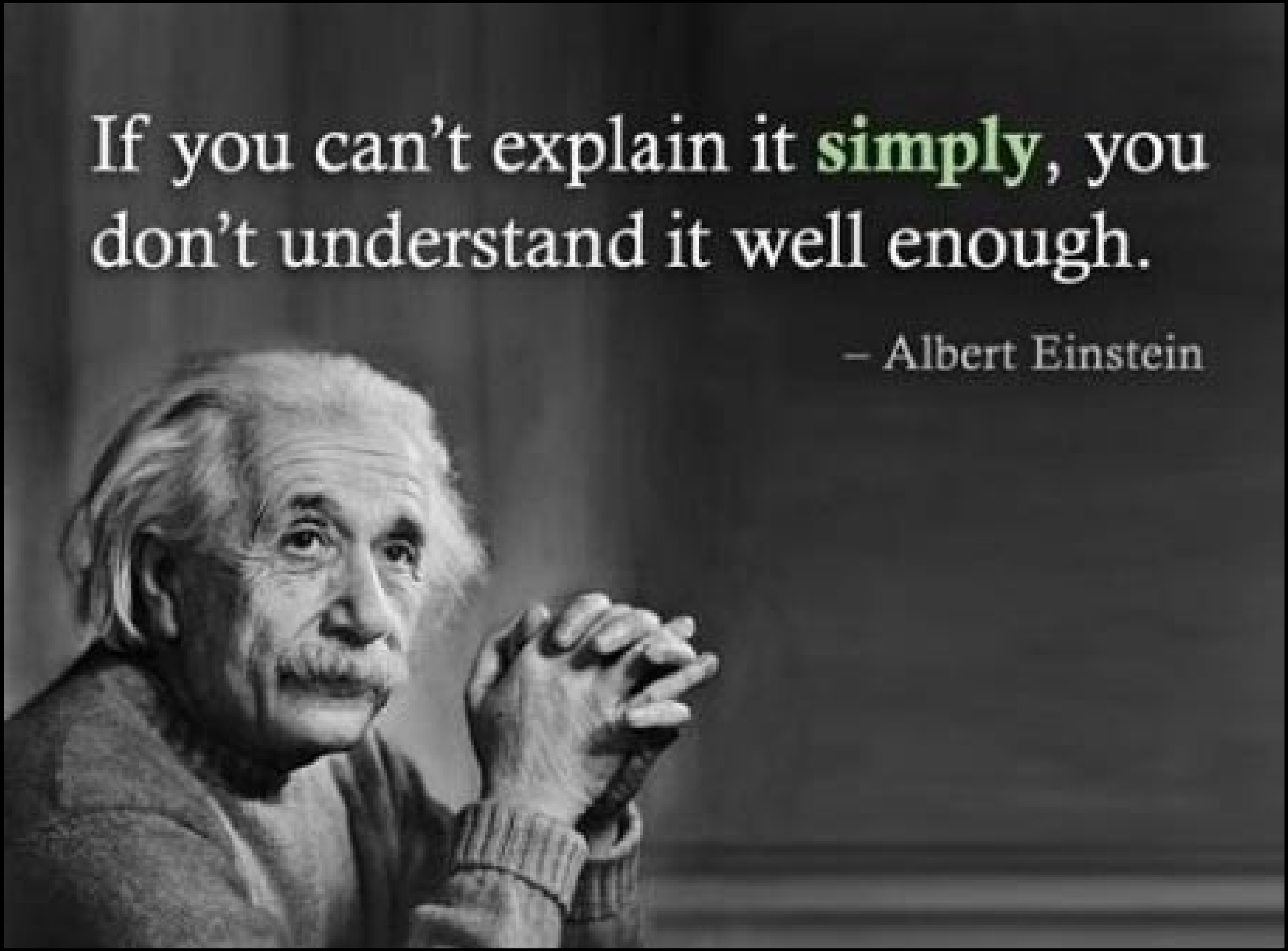
ArXiv: 1509.08877, UJSS

Conclusions

Conclusions

- The hierarchy in the masses provides the *simplest* way to study fermion mixing
- We have built a new mixing parametrization using four mass ratios
- The flavor puzzle is understood as a direct consequence of the fermion masses
- For the parametrization it was necessary to use the Schmidt-Mirsky approximation theorem
- Application of this theorem was equivalent to ask Minimal Flavor Violation
- We found an excellent agreement in the quark mixing sector (CKM)
- Application to the lepton sector provided the absolute value of neutrino masses (which give an excellent agreement to the PMNS matrix) and pointed to which 2-3 octant
- The study of the Gatto-Sartori-Tonin condition provided a way to understand the sequential Yukawa terms noticed in the study of the four mass ratios parametrization

Thanks for your attention!



Back up slides

	δ_{12}	$\delta_{13}^{(0)}$	$\delta_{13}^{(1)}$	$\delta_{13}^{(2)}$	$\delta_{23}^{(0)}$	$\delta_{23}^{(1)}$	$\delta_{23}^{(2)}$
CKM	$\frac{\pi}{2}$	0	π	π	0	π	π
PMNS	$\frac{\pi}{2}$	0	π	π	π	π	0

$$L_{ij}(\theta_{ij}, \delta_{ij}) = \begin{pmatrix} c\theta_{ij} & s\theta_{ij}e^{-i\delta_{ij}} \\ -s\theta_{ij}e^{i\delta_{ij}} & s\theta_{ij} \end{pmatrix}$$

$$L_{23}^{f} = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2})L_{23}^{(1)}(\frac{m_1}{m_3})L_{23}^{(0)}(\frac{m_2}{m_3})$$

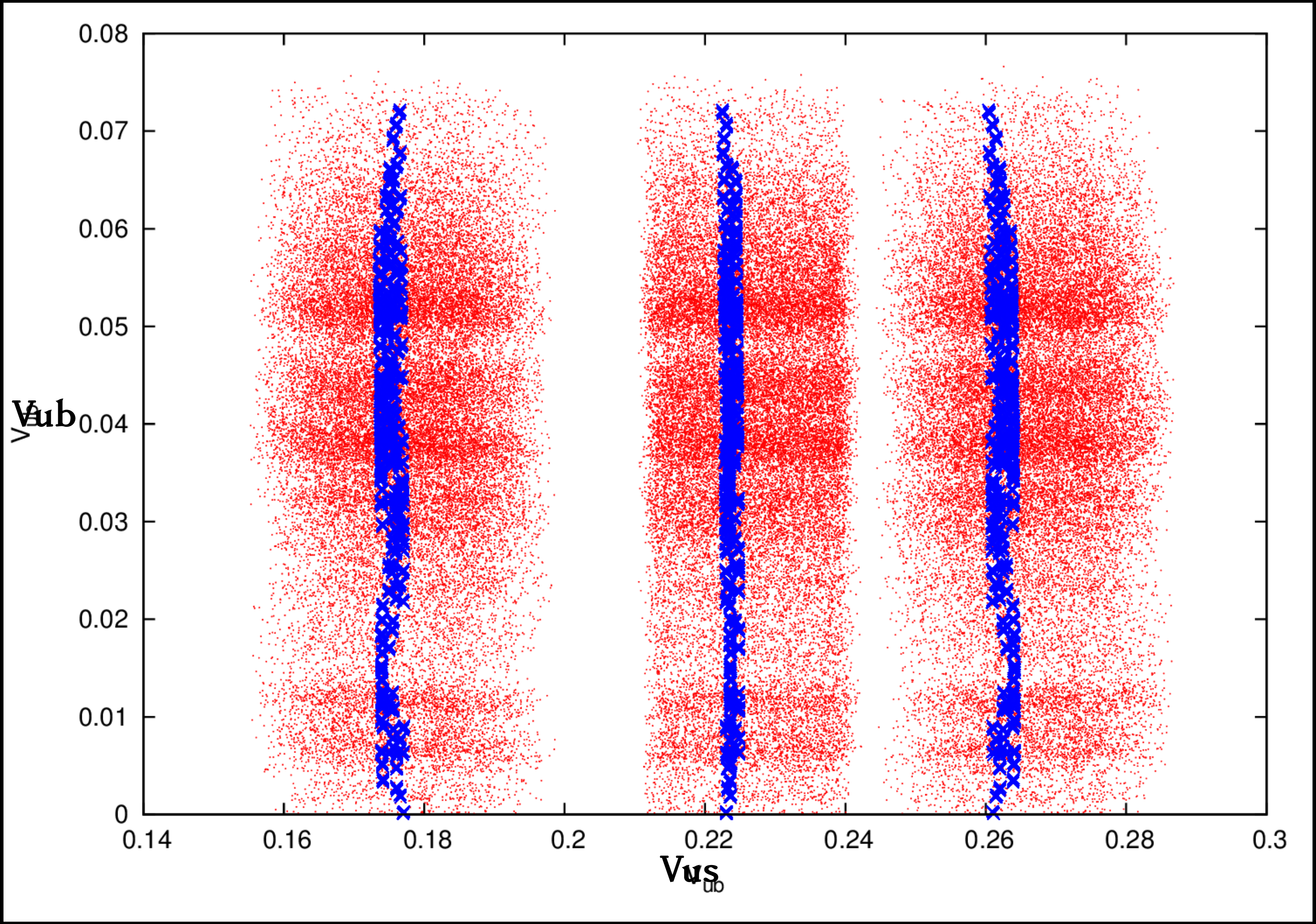
$$L_{13}^{f} = L_{13}^{(2)}(\frac{m_1 m_2}{m_3^2})L_{13}^{(1)}(\frac{m_2^2}{m_3^2})L_{13}^{(0)}(\frac{m_1}{m_3})$$

$$L_{12}^{f} = L_{12}^{(0)}(\frac{m_1}{m_2})$$

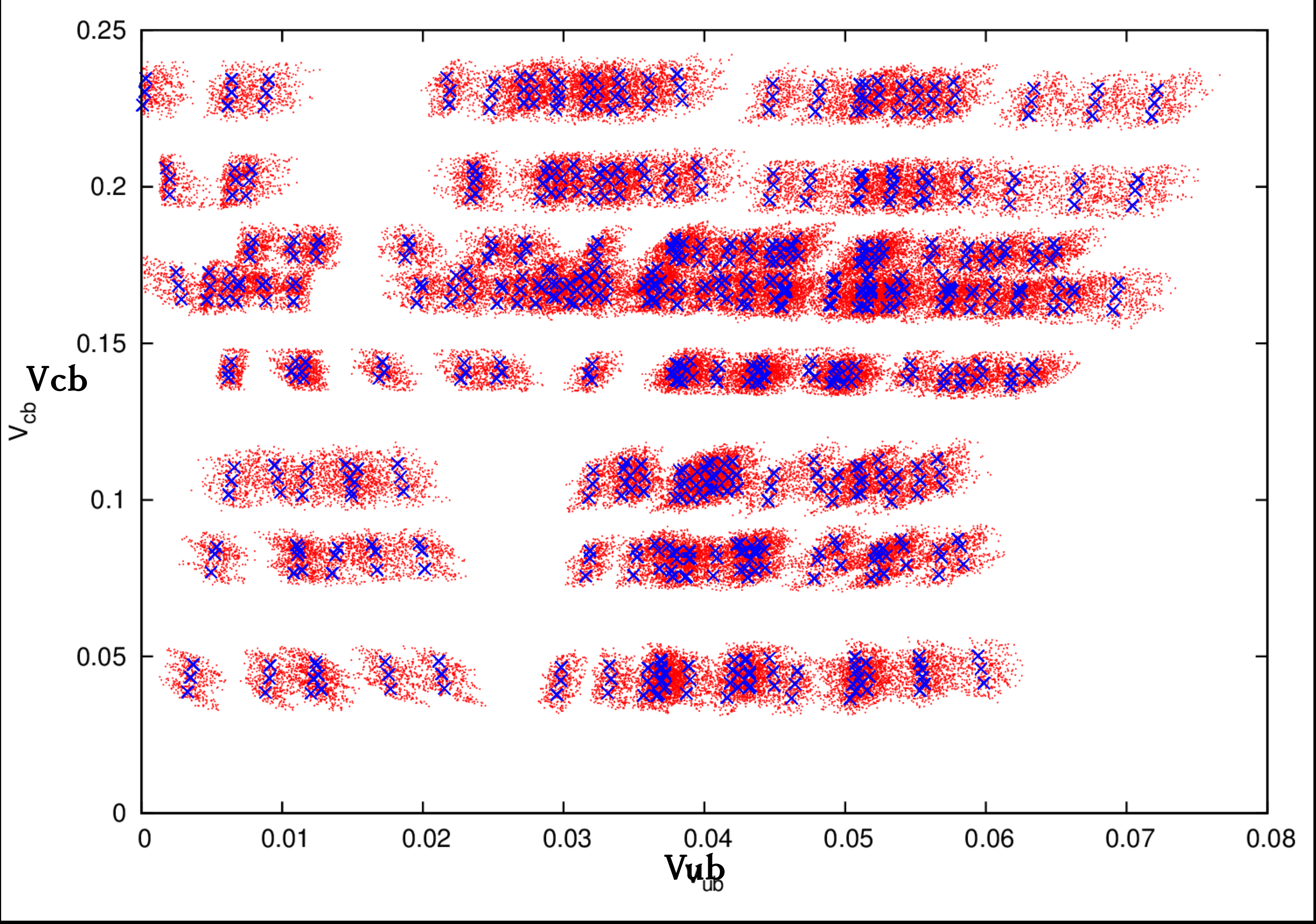
$$x_f^r \equiv \frac{\sqrt{(r-1)m_{f,2}^2 + m_{f,3}^2}}{\parallel \mathcal{M}_f \parallel_{\mathbf{F}}} = \sqrt{\frac{(r-1)m_{f,2}^2 + m_{f,3}^2}{m_{f,1}^2 + m_{f,2}^2 + m_{f,3}^2}},$$

x_f^r	u	d	e	ν
$r = 1$	0.999993	0.999816	0.998274	0.978894
$r = 2$	0.999999	0.999999	0.999999	0.996773

$$\parallel \mathcal{M}_f \parallel_{\mathbf{F}} = \sqrt{\sum_{i=1,2,3} m_{f,i}^2}.$$



0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.14
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
Vub
Vus



0.25
0.2
0.15
0.1
0.05
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Vcb
Vub