

CP breaking in $S(3)$ flavoured Higgs model

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Abstract

We analyze the Higgs sector of the minimal $S(3)$ -invariant extension of the Standard Model including CP violation arising from the spontaneous breaking of the electroweak symmetry. This extended Higgs sector includes three $SU(2)$ doublets Higgs fields with complex vev's provide an interesting scenario to analyze the Higgs masses spectrum, trilinear self-couplings and CP violation. We present how the spontaneous electroweak symmetry breaking coming from three $S(3)$ Higgs fields gives an interesting scenario with nine physical Higgs and three Goldstone bosons when spontaneous CP violation arises from the Higgs field $S(3)$ singlet H_S . Furthermore, numerical analysis of the Higgs masses and trilinear self-couplings is presented, particularly we find a physical solution for the scenario in which spontaneous CPB is provided by the single field H_S . The scalar Higgs H_4^0 is identified whose mass is 125 GeV and $\lambda_{H_4^0 H_4^0 H_4^0} \sim \lambda_{h^0 h^0 h^0}^{SM}$ in good agreement with SM.

Introduction

In the SM, only one $SU(2)_L$ doublet Higgs field is included, which, upon acquiring a vacuum expectation value, breaks the $SU(2)_L \times U(1)_Y$ symmetry.

In the SM each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory.

An extended Higgs sector opened up the window for CP violation scenarios coming from the Higgs sector, we look for the conditions under which CP violation arises from spontaneous gauge symmetry breaking $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$.

In this direction interesting work has been done with the addition of discrete symmetries to the SM.

It is noticeable that many interesting features of masses and mixing of the SM can be understood using a minimal discrete group, namely the permutational group $S(3)$.

[Derman(1979), Derman and Tsao(1979), Pakvasa and Sugawara(1978)]

[Pakvasa and Sugawara(1979), Mondragon and Rodriguez-Jauregui(1998)]

[Mondragon and Rodriguez-Jauregui(1999)]

[Mondragon and Rodriguez-Jauregui(2000), Harrison and Scott(2003)]

[Kubo et al.(2004)Kubo, Okada, and Sakamaki]

[Caravaglios and Morisi(2005), Araki et al.(2006)Araki, Kubo, and Paschos]

[Kubo et al.(2005)]

[Koide(2006), Grimus and Lavoura(2005), Teshima(2006)]

[Kimura(2005), Koide(2007), Mohapatra et al.(2006)Mohapatra, Nasri, and Yu]

[Kaneko et al.(2007)Kaneko, Sawanaka, Shingai, Tanimoto, and Yoshioka]

[Beltran et al.(2009)Beltran, Mondragon, and Rodriguez-Jauregui, Morisi and Peinado(2010)].

Introduction

To give mass to all fermions and, at the same time, preserve the $S(3)$ flavour symmetry of the theory, an extended flavoured Higgs sector is required with three Higgs $SU(2)$ doublets, one in a singlet and the other two in a doublet irreducible representation of $S(3)$

[Kubo et al.(2003)Kubo, Mondragon, Mondragon, and Rodriguez-Jauregui]

[Felix et al.(2007)Felix, Mondragon, Mondragon, and Peinado]

[Mondragon et al.(2007)Mondragon, Mondragon, and Peinado]

Introduction

We carry out a detailed analysis of a minimal $S(3)$ -invariant extension of the Standard Model, with an extended $S(3)$ -Higgs sector.

We describe the different CPB scenarios of the model and give expressions for the corresponding Higgs boson mass matrix.

A CP breaking minimum should be deepest than a normal stationary point (N) and a charge breaking (CB) stationary point.

A precise measurement of the trilinear Higgs self-coupling will also make it possible to test this extended $S(3)$ -Standard Model which has a different trilinear Higgs couplings as compared to the Standard Model.

We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space.

Higgs Boson in the Standard Model

Higgs Boson in the Standard Model

In the Standard Model, one $SU(2)$ doublet Higgs Field is included for the symmetry breaking of the $SU(2) \times U(1)$ gauge groups.

$$V(\Phi) = -\mu^2 |\Phi|^2 + \lambda |\Phi|^4 \quad \text{where} \quad \Phi(X) = \begin{pmatrix} \phi(X)^+ \\ \phi(X)^0 \end{pmatrix}$$

The parameter λ must be positive to produce a stable vacuum.

The parameter μ can be either sign.

In fact if the sign of the quadratic term is negative namely $\mu^2 > 0$, at the origin the potential has a maximum, hence, the stable vacuum state corresponds to a non-zero value of the Φ field.

- ▶ The states satisfying $|\phi(X)^+|^2 + |\phi(X)^0|^2 = \mu^2/2\lambda = v^2/2$ are degenerate minima of the potential.
- ▶ We can choose the vacuum expectation value in the $\langle \phi^0 \rangle = v/\sqrt{2}$ direction.
- ▶ There is one important prediction of this model, one scalar particle appears in the physical spectrum which is called the Higgs boson.
- ▶ The mass of the Higgs boson is given by $m_h = \sqrt{2\lambda}v$, the W and Z masses are $m_W = (g/2)v$, $m_Z = \left(\sqrt{g^2 + g'^2}/2\right)v$.
- ▶ Through the Yukawa couplings, the Higgs gives mass to the quarks and leptons $m_f = Y_f v/\sqrt{2}$.

- ▶ Prior to the introduction of the Higgs boson, and mass terms the Lagrangian of the Standard Model is chiral and invariant with respect to any permutations of the left and right quark and lepton fields $\leftrightarrow S(3)$ flavour symmetry.
- ▶ If we assume that the $S(3)$ permutational symmetry is not broken, the Higgs in the S. M. is an $S(3)$ singlet and only one fermion can acquire mass.
- ▶ Although the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, we believe that this is not the final form of the theory but rather an effective description of a more fundamental theory.

The $S(3)$ flavour symmetry

The ingredients of the extension of the Standard Model are the following:

- ▶ To extend the flavour and family **concepts** to the Higgs sector
- ▶ To associate each family to an irreducible representation of the flavour group
- ▶ To construct a Lagrangian invariant under the action of the $SU(3)_c \times SU(2) \times U(1) \times S(3)^f$ group

$S(3)$ irreducible representations

The group $S(3)$ has two one dimensional irreducible representations (singlets) and a two dimensional irreducible representation (doublet)

- ▶ One dimensional representations: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- ▶ Bi - dimensional: $\mathbf{2}$ doublet

Direct product of an $S(3)$ irreducible representations

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \quad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s,$$

$$\mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}$$

Direct product of two $S(3)$ doublets

$$\mathbf{p}_D \otimes \mathbf{q}_D = r_s \oplus r_A \oplus \mathbf{r}_D$$

$$\mathbf{p}_D = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \quad \text{and} \quad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

it has two singlets, r_s and r_A , just one doublet r_D^T

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2} \quad \text{which is invariant,}$$

$$r_A = p_{D1}q_{D2} - p_{D2}q_{D1} \quad \text{it is not invariant}$$

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

The Higgs sector is modified

$$\Phi \rightarrow H = (\Phi_a, \Phi_b, \Phi_c)^T$$

H is a reducible representation to $\mathbf{1}_s \oplus \mathbf{2}$ of $S(3)$

$$H_s = \frac{1}{\sqrt{3}}(\Phi_a + \Phi_b + \Phi_c), \quad \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_a - \Phi_b) \\ \frac{1}{\sqrt{6}}(\Phi_a + \Phi_b - 2\Phi_c) \end{pmatrix}$$

The quark, lepton and Higgs fields are given by

$$Q^T = (u_L, d_L), u_R, d_R, \quad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \quad H$$

All the fields have three species (flavours) and belong to a representation reducible to $\mathbf{1} \oplus \mathbf{2}$ de $S(3)$

The $S(3)$ extended Higgs doublet model

The Lagrangian \mathcal{L}_Φ of the Higgs sector is given by

$$\mathcal{L}_\Phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S),$$

where D_μ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S(3)$.

The $S(3)$ extended Higgs doublet model

The analysis of the stability properties of the potential V is of great relevance to study the phenomenological implications of this model.

The $S(3)$ extended Higgs doublet model

There are many different ways of writing the Higgs potential for this model, but for the purpose of this work the best basis is

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix},$$

$$H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

The $S(3)$ extended Higgs doublet model

The numbering of the real scalar ϕ fields is chosen for convenience in writing the mass matrices for the scalar particles and the subscript S is the flavour index for the Higgs field singlet.

The $S(3)$ extended Higgs doublet model

$$\begin{aligned}
 V = & \mu_1^2 (x_1 + x_2) + \mu_0^2 x_3 + ax_3^2 + b(x_1 + x_2)x_3 + c(x_1 + x_2)^2 \\
 & - 4dx_7^2 + 2e[(x_1 - x_2)x_6 + 2x_4x_5] + f(x_5^2 + x_6^2 + x_8^2 + x_9^2) \\
 & + g[(x_1 + x_2)^2 + 4x_4^2] + 2h(x_5^2 + x_6^2 - x_8^2 - x_9^2).
 \end{aligned}$$

where

- ▶ the $\mu_{0,1}^2$ parameters have dimensions of mass squared,
- ▶ the a, \dots, h parameters are dimensionless.

The $S(3)$ extended Higgs doublet model

The invariants x_i , the potential V depends on the fields ϕ_i through x_i , considering our assignment as

$$\begin{aligned}
 x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R} \left(H_1^\dagger H_2 \right), & x_7 &= \mathcal{I} \left(H_1^\dagger H_2 \right), \\
 x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R} \left(H_1^\dagger H_S \right), & x_8 &= \mathcal{I} \left(H_1^\dagger H_S \right), \\
 x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R} \left(H_2^\dagger H_S \right), & x_9 &= \mathcal{I} \left(H_2^\dagger H_S \right).
 \end{aligned} \tag{1}$$

The $S(3)$ extended Higgs doublet model

If the Higgs potential $S(3)$ invariant is bounded from below, being a quartic polynomial function it will certainly have a global minimum somewhere. We have three types of minima: the “Normal” one, where electroweak symmetry breaking occurs, away from the origin, for

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9,$$

the CP violating minimum with VEVs which do have a complex relative phase.

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{10} = \gamma_1, \phi_{11} = \gamma_2, \phi_{12} = \gamma_3, \phi_i = 0, \quad i \neq 7, 8, 9, 10, 11,$$

and the CB minimum.

After diagonalizing the mass matrices the masses of the physical scalars and pseudoscalars are obtained. In our analysis we are not taking into account the parameter space with negative eigenvalues solutions for the squared masses of the physical Higgs fields.

Of the original twelve scalar degrees of freedom,

- ▶ three Goldstone bosons (G^\pm and G) are absorbed by W^\pm and Z .

The remaining nine physical Higgs particles are

- ▶ three CP -even scalar (h and H_1, H_2 , with $m_h \geq m_{H_1} \geq m_{H_2}$),
- ▶ two CP -odd scalar (A_1 , and A_2 , with $m_{A_1} \leq m_{A_2}$), and
- ▶ two charged Higgs pair ($H_{1,2}^\pm$, mass degenerate).

To generate the correct W^\pm and Z^0 masses, with the assignments $v_1^2 + v_2^2 + v_3^2 + \gamma_1^2 + \gamma_2^2 + \gamma_3^2 = v^2$ has to hold, where $v = 246$ GeV.

As a result, we have six CPB possibilities or scenarios:

- 1 $\gamma_1 \neq 0$, and $\gamma_2 = \gamma_3 = 0$,
- 2 $\gamma_2 \neq 0$, and $\gamma_1 = \gamma_3 = 0$,
- 3 $\gamma_3 \neq 0$, and $\gamma_1 = \gamma_2 = 0$,
- 4 $\gamma_1 \neq 0$, $\gamma_2 \neq 0$, and $\gamma_3 = 0$,
- 5 $\gamma_1 \neq 0$, $\gamma_3 \neq 0$, and $\gamma_2 = 0$, and
- 6 $\gamma_2 \neq 0$, $\gamma_3 \neq 0$, and $\gamma_1 = 0$.

The squared-mass parameters μ_0^2 and μ_1^2 can be eliminated by minimizing the scalar potential,

$$\frac{\partial V}{\partial v_i} = 0 \quad \Longleftrightarrow \quad \frac{\partial V}{\partial x_j} \frac{\partial x_j}{\partial v_i} = 0, \quad (2)$$

where $i = 1, 2, 3, 10, 11, 12$, and $j = \overline{1}, \overline{9}$, and we get

$$\mu_1^2 = -(v_3)^2(b + f + 2h) - 8(v_2)^2(c + g) - 6ev_2v_3, \quad (3)$$

$$\mu_0^2 = -2a(v_3)^2 - 4(v_2)^2(b + f + 2h) - \frac{8e(v_2)^3}{v_3}. \quad (4)$$

From eq.(2) and using the resulting minimization conditions to eliminate μ_0^2 and μ_1^2 , one obtains the elements

$$(\mathcal{M}_H^2)_{ij} = \frac{1}{2} \left[\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right]_{\text{CPBmin}} ,$$

of the tree-level mass squared matrix with $i, j = 1, 2, \dots, 12$. We have

$$\mathcal{M}_H^2 = \text{diag} (\mathbf{M}_{C,\gamma}^2, \mathbf{M}_{N,\gamma}^2) ,$$

with $\mathbf{M}_{C,\gamma}^2$ corresponding to the mass matrix of electrically charged Higgs bosons and $\mathbf{M}_{N,\gamma}^2$ to the neutral Higgs boson matrix, which are the 6×6 symmetric and Hermitian sub-matrices

For each of the corresponding scenarios we have a matrix for charged and neutral Higgs bosons, that we specify with the gamma index, $\gamma = \gamma_1, \gamma_2, \gamma_3$, as the corresponding scenario, where

$$\mathbf{M}_{C,\gamma}^2 = \begin{pmatrix} \mathbf{M}_{C11}^2(\gamma) & \mathbf{M}_{C12}^2(\gamma) \\ \mathbf{M}_{C21}^2(\gamma) & \mathbf{M}_{C22}^2(\gamma) \end{pmatrix}, \quad (5)$$

The neutral Higgs mass matrix is given by

$$\mathbf{M}_{N,\gamma}^2 = \begin{pmatrix} \mathbf{M}_{N11}^2(\gamma) & \mathbf{M}_{N12}^2(\gamma) \\ \mathbf{M}_{N21}^2(\gamma) & \mathbf{M}_{N22}^2(\gamma) \end{pmatrix}, \quad (6)$$

For this model, with CP violation arising from the Higgs $S(3)$ doublet sector, among the nine physical Higgs fields, we obtain four charged bosons which are mass degenerate two by two, and five non degenerated bosons in the neutral sector

Defining the Scalar physical mass eigenstates $m_{H_1^0}^2$, $m_{H_2^0}^2$, $m_{H_3^0}^2$, $m_{H_4^0}^2$ and $m_{H_5^0}^2$, the masses are found from the diagonalization process

$$\mathbf{M}_{N,\gamma}^2{}_{diag} = R^T \mathbf{M}_{N,\gamma}^2 R = \text{diag}(m_{H_1^0}^2, m_{H_2^0}^2, m_{H_3^0}^2, m_{H_4^0}^2, m_{H_5^0}^2,),$$

we perform the minimum conditions and the parameter space analysis for scenario 3.

The scenario 3 corresponds to $\gamma_3 \neq 0$ and $\gamma_1 = \gamma_2 = 0$, The fermions in the $S(3)$ SM acquire mass through the Yukawa interactions, in particular for real Yukawa couplings the corresponding Yukawa Lagrangian is given in Ref. [Kubo et al.(2003) ?]. The fermionic mass matrix \mathbf{M}_f including spontaneous CP violation ($\gamma_3 \neq 0$ and $\gamma_1 = \gamma_2 = 0$) as

$$\mathbf{M}_f = \begin{pmatrix} m_1^{CP} + m_6 & m_2 & m_5 \\ m_2 & m_1^{CP} - m_6 & m_8 \\ m_4 & m_7 & m_3^{CP} \end{pmatrix}, \quad (7)$$

where

$$m_1^{CP} = m_1 - Y_1^f(i\gamma_3), \quad (8)$$

$$m_3^{CP} = m_3 - Y_3^f(i\gamma_3). \quad (9)$$

m_i ($i = 1, 2, \dots, 8$) are the expressions in the case of CP conserving [Kubo et al.(2003)K]. Then, the fermionic mass matrices are complex with a complex contribution γ_3 arising from the Higgs sector. Thus, the EWSB mechanism provides a source for CP violation in the fermionic sector contributing to CP violation in the quark and lepton mixing matrices.

The mass sub-matrices for charged Higgs bosons in eq.(5) can be analytically diagonalized. The eigenvalues are two massless bosons and four charged

$$\left\{ 0, -\frac{(\gamma_3^2 + 4v_2^2 + v_3^2)(ev_2 + fv_3)}{2v_3}, -\frac{ev_2(\gamma_3^2 + 9v_3^2) + fv_3(\gamma_3^2 + v_3^2) + 16gv_3v_2^2}{2v_3} \right\}. \quad (10)$$

We also compute the diagonalization for the neutral matrix (6), the eigenvalues are: five neutral bosons and one Goldstone boson, in accordance with the SM and current experimental data.

We end up with five neutral, four charged, and three massless bosons

The trilinear self-couplings of the neutral Higgs bosons are defined as

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}, \quad (11)$$

which are most easily obtained from the corresponding derivatives of V with respect to the fields $\{\eta_i\}$ with $i = 1, 2, 3$.

[Osland et al.(2008)Osland, Pandita, and Selbuz]

[Carena et al.(2003)Carena, Ellis, Mrenna, Pilaftsis, and Wagner]

[Carena et al.(2000)Carena, Ellis, Pilaftsis, and Wagner]

We can then write the trilinear couplings in terms of the derivatives of the potential V with respect to η_i and the elements of the rotation matrix R as

$$\lambda_{ijk} = N \sum_{lmn} R_{il} R_{jm} R_{kn} \frac{\partial^3 V}{\partial \eta_l \partial \eta_m \partial \eta_n}, \quad (12)$$

where the indices l, m, n refer to the weak field basis, and $l \leq m \leq n = 1, 2, 3$, N is a factor of $n!$ for n identical fields.

In the general case, we find it convenient to study the dimensionless ratios of the couplings

$$\tilde{\lambda}_{ijk} = \frac{\lambda_{H_i^0 H_j^0 H_k^0}}{\lambda_{HHH}^{SM}},$$

where,

$$\lambda_{HHH}^{SM} = \frac{3M_H^2}{v},$$

with $M_H = 125$ GeV and $v = 246$ GeV.

The allowed values of the Higgs boson masses:



$$M_{H_1^0} = 0 - 300 \text{ GeV},$$



$$M_{H_2^0} = 0 - 500 \text{ GeV},$$



$$M_{H_3^0} = 240 - 600 \text{ GeV},$$



$$M_{H_4^0} = 0 - 150 \text{ GeV},$$

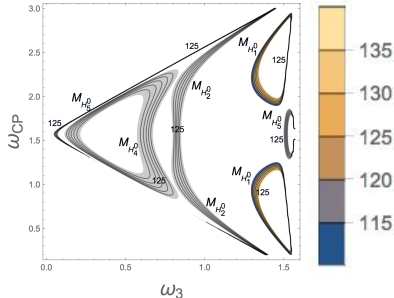


$$M_{H_5^0} = 0 - 960 \text{ GeV}.$$

The figures are plotted for parameters approach with

- ▶ $a = b = c = -d = -e \rightarrow 1$
- ▶ $f \leftrightarrow 3$
- ▶ $g \leftrightarrow 2$
- ▶ $h = \frac{e(v_2^2 - v_1^2)}{4v_2v_1}$
- ▶ $v_1 = \sqrt{3}v_2$
- ▶ $v = 246 = \sqrt{4v_2^2 + v_3^2 + \gamma_3^2} \text{ GeV}$

It is convenient to define $v_i = v \cos w_i$ $i = 1, 2, 3$ and $\gamma_3 = v \cos w_{CP}$

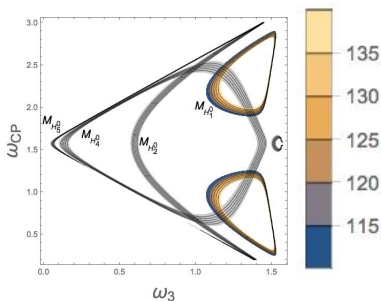


(a)

Figure: Higgs masses $M_{H_i^0}$ ($i = 1, \dots, 5$) as a function of the angles ω_3 , ω_{CP} , where

$0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$, with parameters: (a)

$a = b = c = -d = -e \rightarrow 1$, $f \rightarrow 3$, $g \rightarrow 2$,



(a)

Figure: Higgs masses $M_{H_i^0}$ ($i = 1, \dots, 5$) as a function of the angles ω_3, ω_{CP} , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$, with parameters: (b) $a = b = c = -d = -e = f = g \rightarrow 1$. The mass range plotted runs from 110-140 GeV.

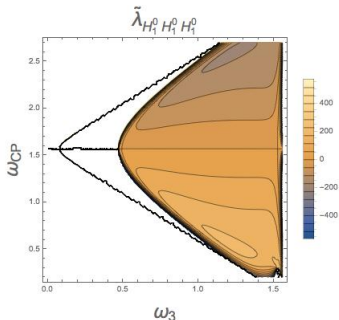


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e \rightarrow 1, f \rightarrow 3, g \rightarrow 2$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

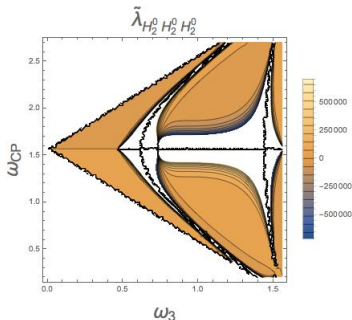


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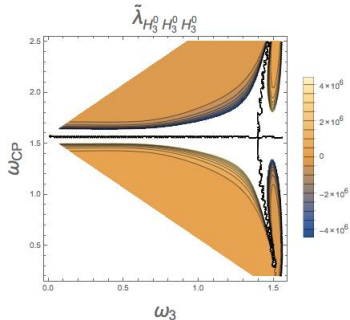


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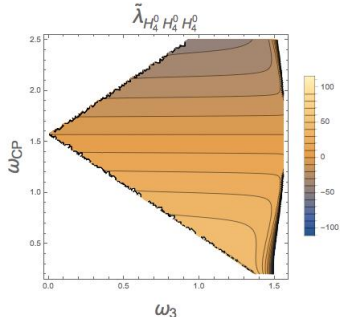


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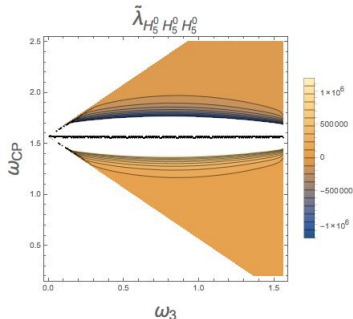
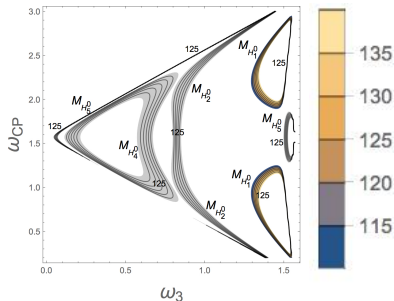


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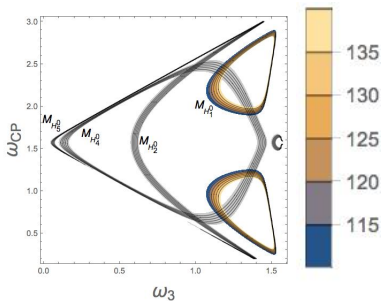


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$a = b = c = -d = -e \rightarrow 1$, $f \rightarrow 3$, $g \rightarrow 2$,



(a)

Figure: Higgs masses $M_{H_i^0}$ ($i = 1, \dots, 5$) as a function of the angles ω_3, ω_{CP} , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$, with parameters: (b) $a = b = c = -d = -e = f = g \rightarrow 1$. The mass range plotted runs from 110-140 GeV.

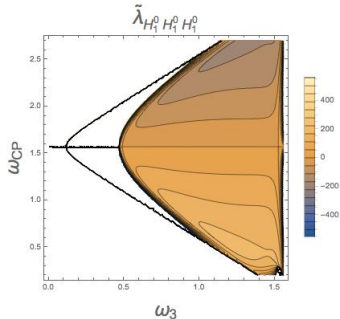


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

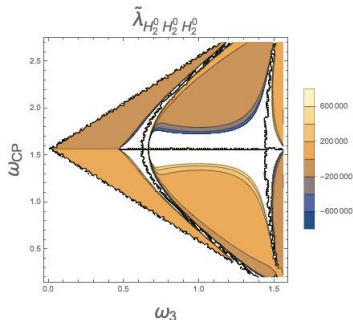


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

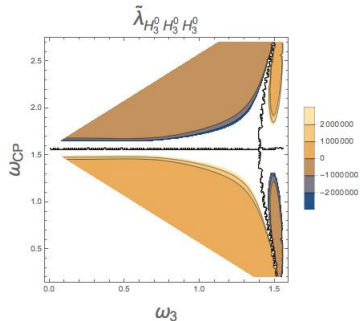


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

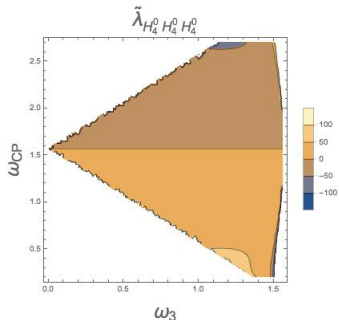


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

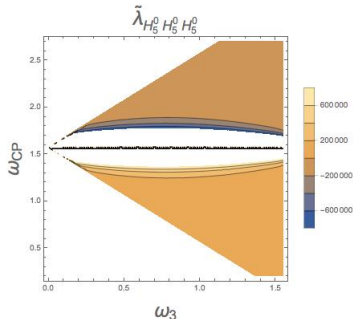


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, \dots, 5$) for

$a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $0.0 \leq \omega_3 \leq 1.56$ and $0.2 \leq \omega_{CP} \leq 2.7$.

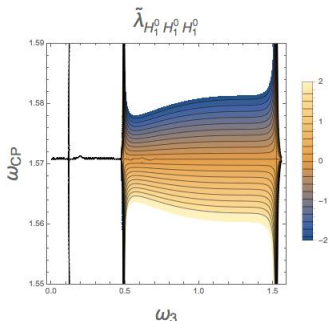


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, 2, 4$) for $a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $-2.0 \leq \tilde{\lambda}_{H_i^0 H_i^0 H_i^0} \leq 2.0$.

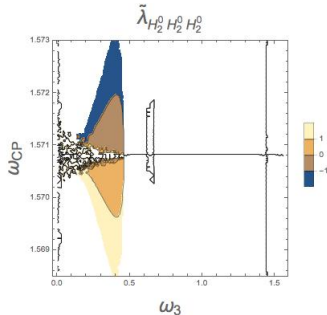


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, 2, 4$) for $a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $-2.0 \leq \tilde{\lambda}_{H_i^0 H_i^0 H_i^0} \leq 2.0$.

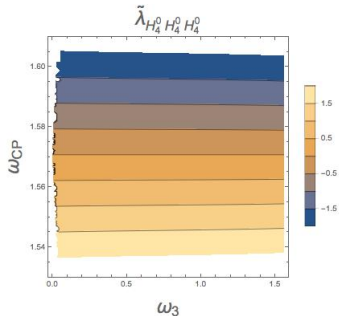


Figure: The Higgs trilinear self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ ($i = 1, 2, 4$) for $a = b = c = -d = -e = f = g \rightarrow 1$, in the space (ω_3, ω_{CP}) , where $-2.0 \leq \tilde{\lambda}_{H_i^0 H_i^0 H_i^0} \leq 2.0$.

Summary and Conclusions

In the $S(3)$ SM we introduced three Higgs $SU(2)$ doublets eq. (18) with twelve real fields.

We found a parameter space region where the minimum of the potential defines a CPB ground state.

One see that all the masses of the Higgs bosons are decoupled as was shown in Figure 9 for a mass range from 110 to 140 GeV. The parameter matrix B and the

Higgs mass eigenvalues are positive defined if we simultaneously demand that a Higgs mass is of the order of 125 GeV and $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$ of order one. We have

found that one Higgs is excluded for this parameter values $a \rightarrow 1, b \rightarrow 1, c \rightarrow 1, d \rightarrow -1, e \rightarrow -1, f \rightarrow 3, g \rightarrow 2$.

A different set of parameters is obtained if $f = g = 1$, then we can find for $(\omega_3, \omega_{CP}) = (1.4, 1.33)$ a set of five Higgs masses $M_{H_1^0} = 80$ GeV, $M_{H_2^0} = 383$ GeV, $M_{H_3^0} = 479$ GeV, $M_{H_4^0} = 122$ GeV, and $M_{H_5^0} = 243$ GeV.

For this allowed values the trilinear self-coupling of H_4^0 is in the range $2 \leq \tilde{\lambda}_{H_4^0 H_4^0 H_4^0} \leq 50$. A better fit can be found for the range of parameters $1.4 \leq$








$\omega_3 \leq 1.5$ and $1.54 \leq \omega_{CP} \leq 1.56$, in this case $0.5 \leq \tilde{\lambda}_{H_4^0 H_4^0 H_4^0} \leq 1.5$ and $M_{H_1^0} \leq 60$ GeV, $350 \leq M_{H_2^0} \leq 400$ GeV, $400 \leq M_{H_3^0} \leq 600$ GeV, $100 \leq M_{H_4^0} \leq 140$ GeV, and $120 \leq M_{H_5^0} \leq 240$ GeV.

We have shown the Higgs masses and trilinear couplings for two allowed parameter sets, and shown that the Higgs mass of H_4^0 is sensitive to the potential parameters f, g .

In this case, the trilinear self-couplings analysis confirms our hypothesis: one can have CP violation resulting from the neutral Higgs sector with a trilinear self coupling in accordance with the Standard Model one. Also, we have found a

CPB ground state in the scenario where the CPB comes from the $S(3)$ Higgs singlet. Furthermore, we also computed the Higgs masses and Higgs trilinear

self-couplings $\tilde{\lambda}_{H_i^0 H_i^0 H_i^0}$, $i = 1, \dots, 5$, in terms of (ω_3, ω_{CP}) . Particularly, in scenario 3 which CP violation comes from the $S(3)$ singlet H_S , one observed H_4^0 as possible candidate like the SM Higgs.

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