Conditions for the emergence of gauge bosons from spontaneous Lorentz symmetry breaking

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Outline

- Introduction and motivations
- Review of the Yang Mills (YM) theory
- The non abelian Nambu model (NANM)
- Conditions for the equivalence between the NANM and the YM theory
- Summary

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Introduction and motivations

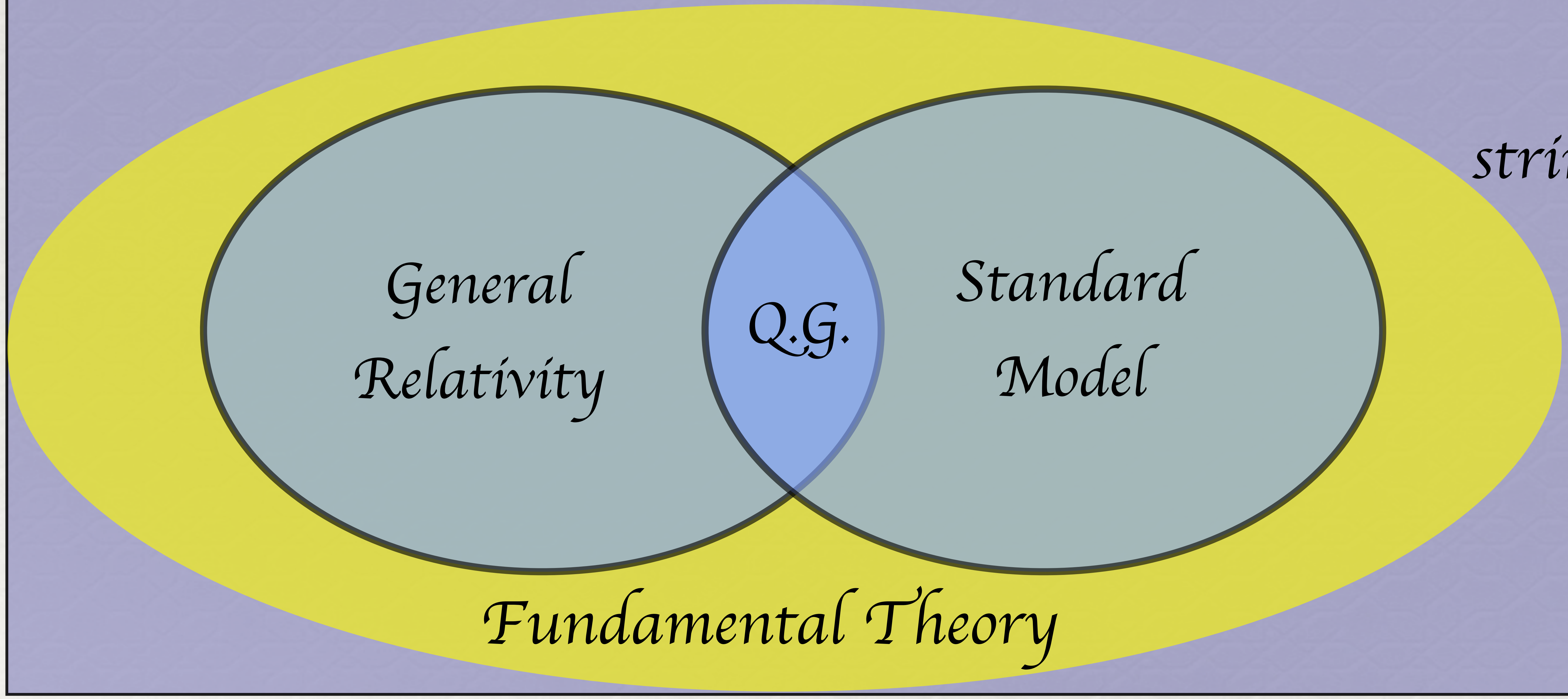
- Standard Model
- General Relativity

Lorentz invariant

No violation has been discovered to date

Motivation 1: Such an important principle should be tested

Motivation 2: There could be Lorentz violation coming from a fundamental theory



Fundamental Theory:

strings?, noncommutative spacetime?, loop quantum gravity?,...

Physics to the Plack's scale

Lorentz violation

Two possible ways to implement the breaking of Lorentz invariance:

- Explicit

  Tensor Fields act as fixed background in any observer frame

  Incompatibility with the Bianchi identities

- Spontaneous symmetry breaking

  Tensor fields acquire non zero expectation values $\langle 0|A_\mu|0\rangle \neq 0$

  Tensor Fields transform under coordinate transformations

Yang Mills

The Lagrangian density is given by

$$\mathcal{L} = Tr\left[-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{J}^{\mu}\mathbf{A}_{\mu}\right], \qquad \mathbf{M} = M^a t^a, \qquad [t^a, t^b] = C^{abc} t^c.$$

The canonical momenta are

$$\Pi_0^a = 0, \qquad \Pi_i^a = F_{i0}^a \equiv -E_i^a,$$

which satisfy the non-zero Poisson brackets

$$\{A_0^a(\mathbf{x}, t), \Pi_0^b(\mathbf{y}, t)\} = \delta^{ab}\delta(\mathbf{x}-\mathbf{y}), \qquad \{A_i^a(\mathbf{x}, t), \Pi^{bj}(\mathbf{y}, t)\} = \delta_i^j \delta^{ab}\delta(\mathbf{x}-\mathbf{y}).$$

Yang Mills

The Hamiltonian density is given by

$$\mathcal{H}_c = \Pi_a^i \dot{A}_i^a - \mathcal{L} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - A_0^a (D_i E_i - J^0)^a - J_i^a A_i^a.$$

We employ the Dirac's method to construct the canonical theory due to the fact that primary constraints, $\Sigma^a = \Pi_0^a \simeq 0$, *are present. The evolution condition of the primary constraints,*

$$\dot{\Sigma}^a(\mathbf{x}) = \{\Sigma^a(\mathbf{x}), \int d^3y\, \mathcal{H}_E(\mathbf{y})\} = \{\Sigma^a(\mathbf{x}), \int d^3y\, \mathcal{H}_c(\mathbf{y}) + \lambda^a \Sigma_a\} \simeq 0,$$

leads to the Gauss laws: $\Omega^a = (D_i E_i - J^0)^a \simeq 0.$

Yang Mills

The time evolution for the Gauss laws gives

$$\dot{\Omega}^a = -C^{abc} A_0^b \Omega^c - D_\mu J^{a\mu},$$

which is zero, modulo the constraints and using current conservation. The theory only contains the first class constraints

$$\Sigma^a = \Pi_0^a \simeq 0, \qquad \Omega^a = (D_i E_i - J^0)^a \simeq 0.$$

Normally one fixes

$$\Pi_0^a \simeq 0, \qquad A_0^a \simeq \Theta^a,$$

with Θ^a being arbitrary functions to be consistently determined after the remaining first class constraints Ω^a are fixed.

Yang Mills

The final Hamiltonian density is

$$\mathcal{H}_E = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \Theta^a (D_i E_i - J^0)_a + J_i^a A_a^i.$$

Once Π_0^a and A_0^a are fixed strongly, the Dirac brackets among the remaining variables are

$$\{A_i^a(\mathbf{x},t), A_j^b(\mathbf{y},t)\}^* = 0, \qquad \{E^{ai}(\mathbf{x},t), E^{bj}(\mathbf{y},t)\}^* = 0, \quad \{A_i^a(\mathbf{x},t), E^{bj}(\mathbf{y},t)\}^* = -\delta_i^j \delta^{ab} \delta(\mathbf{x}-\mathbf{y}).$$

The final count of degrees of freedom (DOF) per point in coordinate space yields

$$\#d.o.f. = \frac{1}{2}(2 \times 4N - 2 \times 2N) = 2N.$$

$$\{A(\mathbf{x}), B(\mathbf{y})\}^* = \{A(\mathbf{x}), B(\mathbf{y})\}_P - \int d^3u\, d^3v \{A(\mathbf{x}), \chi_i(\mathbf{u})\}_P (Q^{-1})^{ij} \{\chi_j(\mathbf{v}), B(\mathbf{y})\}_P\,.$$

The non abelian Nambu Model

The Lagrangian density is given by

$$\mathcal{L} = Tr\left[-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \mathbf{J}^{\mu}\mathbf{A}_{\mu}\right], \qquad \mathbf{M} = M^a t^a, \qquad [t^a, t^b] = C^{abc}t^c.$$

with the constraint

$$A^a_\mu A^{a\mu} = n^2 M^2. \qquad \langle A_\mu \rangle = n_\mu M$$

- There is not gauge symmetry.
- The number of degrees of freedom are 3N vs. 2N of the Yang Mills theory.
- The equations of motion do not correspond to standard Yang Mills theory.
- The conservation of the current does not follow as a consistency condition from the equations of motion in the NANM, as it happens in the YM case.

We employ the parameterization

$$A_0^a = B^a \left(1 + \frac{N}{4B^2}\right), \qquad A_3^a = B^a \left(1 - \frac{N}{4B^2}\right), \qquad N = \left(A_{\bar{i}}^b A_{\bar{i}}^b + n^2 M^2\right), \quad 4B^2 \pm N \neq 0, \ \bar{i} = 1, 2.$$

which it is written in terms of the 3N independent d.o.f. $B^a, A_{\bar{i}}^b$ .In order to unify the notation when going to the Hamiltonian formulation we introduce the 3N d.o.f.$\Phi_A^a, \ A = 1, 2, 3,$

$$\Phi_1^a = A_1^a, \quad \Phi_2^a = A_2^a \ , \quad \Phi_3^a = B^a,$$

in such a way that the coordinate transformation$A_i^a = A_i^a(\Phi_A^b)$ is invertible. In fact, the inverses are

$$\Phi_1^a = A_1^a, \quad \Phi_2^a = A_2^a, \qquad \Phi_3^a = \frac{A_3^a}{2\sqrt{A_3^b A_3^b}} \left(\sqrt{A_3^b A_3^b} + \sqrt{A_i^b A_i^b + n^2 M^2}\right).$$

The relevant property of the transformation is that

$$\dot{A}_i^a = \frac{\partial A_i^a}{\partial \Phi_B^b}\dot{\Phi}_B^b, \quad \rightarrow \quad \frac{\partial \dot{A}_i^a}{\partial \dot{\Phi}_B^b} = \frac{\partial A_i^a}{\partial \Phi_B^b}; \qquad \dot{\Phi}_A^a = \frac{\partial \Phi_A^a}{\partial A_i^b}\dot{A}_i^b.$$

Next we proceed to calculate the Hamiltonian density of the NANM in terms of the canonically conjugated variables $\Phi_A^b,\ \Pi_A^b$ *. We rewrite*

$$\mathcal{L}_{\rm NANM}(\Phi,\dot{\Phi}) = \frac{1}{2}E_i^a E_i^a - \frac{1}{2}B_i^a B_i^a - J^{a\mu}A_\mu^a,$$

$$E_i^a = \dot{A}_i^a - D_i A_0^a,$$
$$B_i^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a.$$

with $E_i^a = E_i^a(\Phi,\dot{\Phi}), \quad B_i^a = B_i^a(\Phi)$ *. The canonically conjugated momenta are calculated as*

$$\Pi_A^a = \frac{\partial \mathcal{L}_{\rm NANM}(\Phi,\dot{\Phi})}{\partial \dot{\Phi}_A^a} = E_i^b\frac{\partial \dot{A}_i^b}{\partial \dot{\Phi}_A^a} = E_i^b\frac{\partial A_i^b}{\partial \Phi_A^a}; \qquad E_i^b(\Phi,\Pi) = \frac{\partial \Phi_A^a}{\partial A_i^b}\Pi_A^a.$$

The NANM Hamiltonian density is

$$\mathcal{H}_{\mathrm{NANM}} = \Pi^a_A \dot{\Phi}^a_A - \left(\frac{1}{2}E^a_i E^a_i - \frac{1}{2}B^a_i B^a_i - J^{a\mu} A^a_\mu\right),$$

which we rewrite in successive steps

$$\mathcal{H}_{\mathrm{NANM}} = \Pi^a_A \frac{\partial \Phi^a_A}{\partial A^b_i} \dot{A}^b_i - \left(\frac{1}{2}E^a_i E^a_i - \frac{1}{2}B^a_i B^a_i - J^{a\mu} A^a_\mu\right),$$

$$\mathcal{H}_{\mathrm{NANM}} = E^b_i \dot{A}^b_i - \left(\frac{1}{2}E^a_i E^a_i - \frac{1}{2}B^a_i B^a_i - J^{a\mu} A^a_\mu\right),$$

$$\mathcal{H}_{\mathrm{NANM}}(\Phi, \Pi) = \frac{1}{2}E^a_i E^a_i + \frac{1}{2}B^a_i B^a_i - A^b_0 \left(D_i E^b_i - J^{b0}\right) + J^{ai} A^a_i,$$

with the canonical algebra

$$\{\Phi^a_A(\mathbf{x}), \Phi^b_B(\mathbf{y})\} = 0, \quad \{\Pi^a_A(\mathbf{x}), \Pi^b_B(\mathbf{y})\} = 0, \quad \{\Phi^a_A(\mathbf{x}), \Pi^b_B(\mathbf{y})\} = \delta^{ab}\delta_{AB}\delta^3(\mathbf{x} - \mathbf{y}).$$

Since the transformations $(\Phi, \Pi) \to (A, E)$ are generated by the change of variables in coordinate space, we know from classical mechanics that the full transformation in phase space is a canonical transformation. In this way we automatically recover the PB algebra

$$\{A_i^a(\mathbf{x}), A_j^b(\mathbf{y})\} = 0, \quad \{E^{ai}(\mathbf{x}), E^{bj}(\mathbf{y})\} = 0, \quad \{A_i^a(\mathbf{x}), E^{bj}(\mathbf{y})\} = -\delta^{ab}\delta_i^j\delta^3(\mathbf{x}-\mathbf{y}).$$

- The A_0^b fields into the Hamiltonian density are not arbitrary functions, as it happens in the YM case.
- The Gauss laws $(D_iE_i - J^0)^b$ are missing in the NANM.

The time evolution of the functions $\Omega^b = \left(D_i E_i - J^0\right)^b$, according to the NANM dynamics, gives

$$\dot{\Omega}^a = -gC^{abc}A_0^b\Omega^c - D_\mu J^{\mu a} + D_3\left(\frac{A_0^1}{A_3^1}\Omega^a\right) - D_3\left(\frac{A_3^a}{A_3^c A_3^c}A_0^b\Omega^b\right) + D_i\left(\frac{A_i^a}{N}A_0^b\Omega^b\right).$$

(i) Imposing current conservation at some initial time t=0.

(ii) Demanding also the Gauss laws to hold at t=0, we obtain $\partial_0\Omega^a = 0$ (a=1,2,...,N) as well at t=0.

In this way we can recover the Yang-Mills theory by imposing the Gauss laws as Hamiltonian constraints, with arbitrary functions N^a adding $-N^a\Omega^a$ to $\mathcal{H}_E$ and redefining $A_0^a + N^a = \Theta^a$. This leads to

$$\mathcal{H}_E = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \Theta^a\Omega^a + J_i^a A^{ia}.$$

Summary

- The non abelian Nambu model (NANM) is motived by Spontaneous Lorentz Symmetry Breaking (SLSB).
- Using a nonperturbative Hamiltonian analysis, we prove that the Yang-Mills theory is equivalent to the corresponding NANM, after both current conservation and the Gauss laws are imposed as initial conditions for the latter.
- The gauge fields or photons (abelian case) can be interpreted as Goldstone bosons arising from an SLSB.

Summary

- There are no physical effects from Lorentz violation in the non abelian Nambu model when the previous initial conditions are imposed.
- The gauge symmetry emerges from a dynamical setting imposing suitable initial condition.
- Generalizations of this idea can be considered.