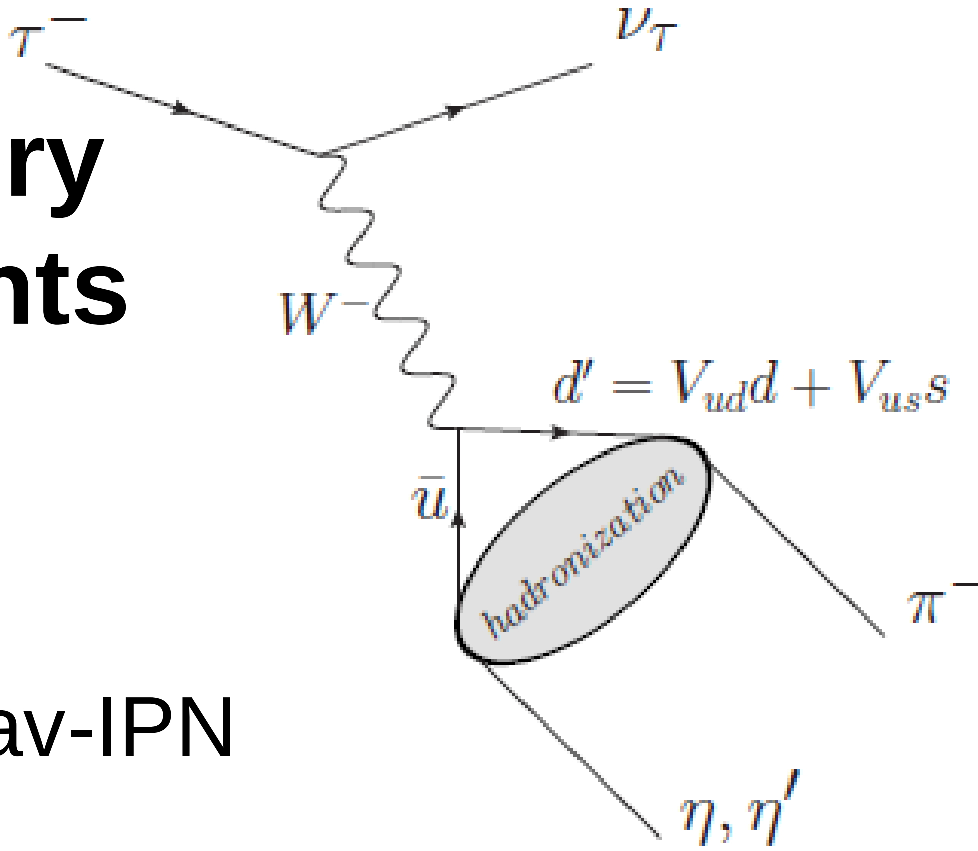
XV MEXICAN WORKSHOP ON PARTICLES AND FIELDS

Mazatlán (Sinaloa) 2-6 November 2015

Towards the (Mexican) discovery of 2nd class currents @ Belle-II



Pablo Roig

Dpto. de Física Cinvestav-IPN

México DF

Collaboration with R. Escribano and S. González-Solís
(IFAE & UAB, Barcelona), to appear soon

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$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

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Spectra and branching ratio predictions

CONCLUSIONS

MOTIVATION

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

Non-strange V-A currents can be split into

1st class currents: $J^{PG} = 0^{++}, 0^{--}, 1^{+-}, 1^{-+}$

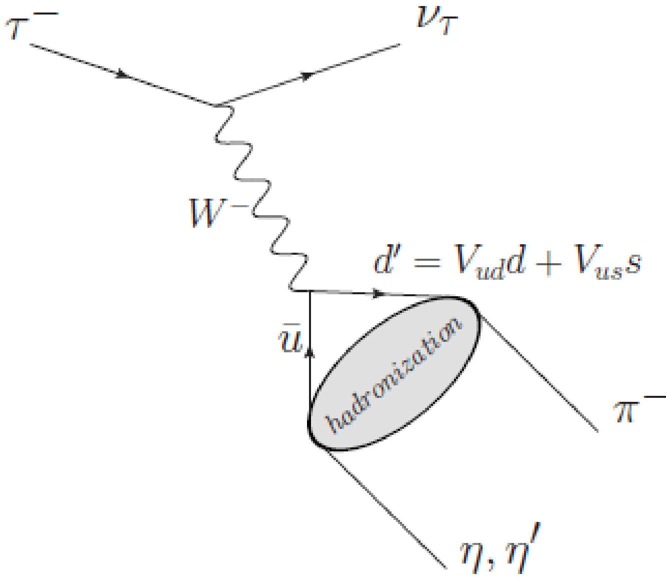
SCC 2nd class current: $J^{PG} = 0^{+-}, 0^{-+}, 1^{++}, 1^{--}$

$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle$$

$$G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \neq G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

G-Parity violation

Irrespective of the underlying resonance mechanism



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G-Parity violation

Irrespective of the underlying resonance mechanism

Note: There are/have been several attempts to discover SCC in nuclear processes, but they mostly rely on CVC (and SCC should be effects of the order of isospin breaking corrections to CVC) and have large uncertainties.

RevModPhys.78.991

It is an isospin violating process $(\underline{m_u \neq m_d}, e \neq 0)$

SCC

MOTIVATION

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle \qquad G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \neq G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

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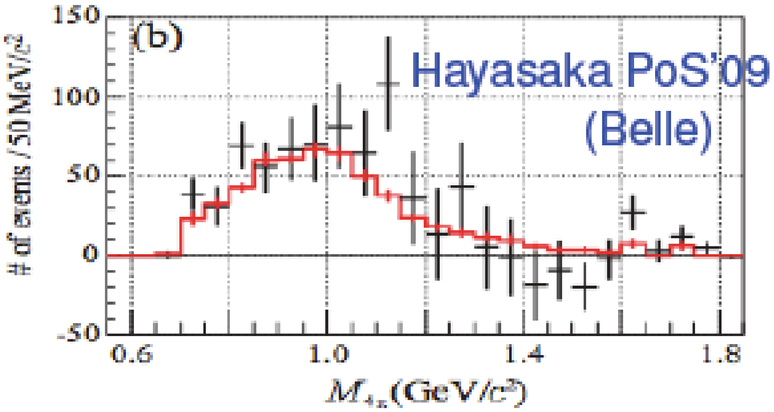
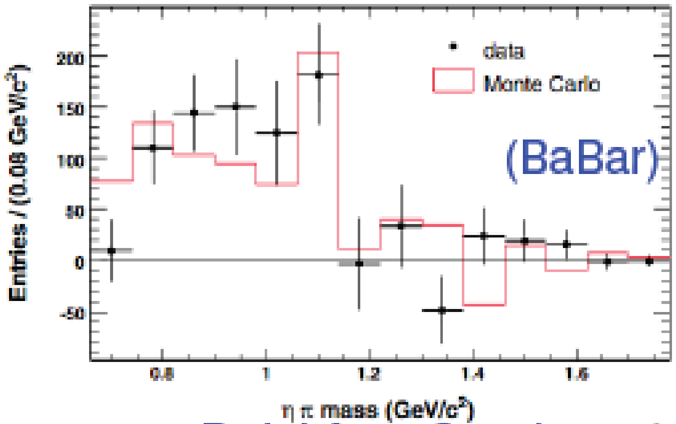


Hayasaka PoS'09 (Belle)



(BaBar)

P. del Amo Sanchez *et.al*
(PRD 83 032002 '11)

$BR_{exp}^{Belle} < 7.3 \cdot 10^{-5} \quad 90\% CL$

$BR_{exp}^{BaBar} < 9.9 \cdot 10^{-5} \; 95\% CL$

These decay modes should have already been discovered if it was not for the strong bkg

MOTIVATION

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle$ $\quad G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \neq G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$

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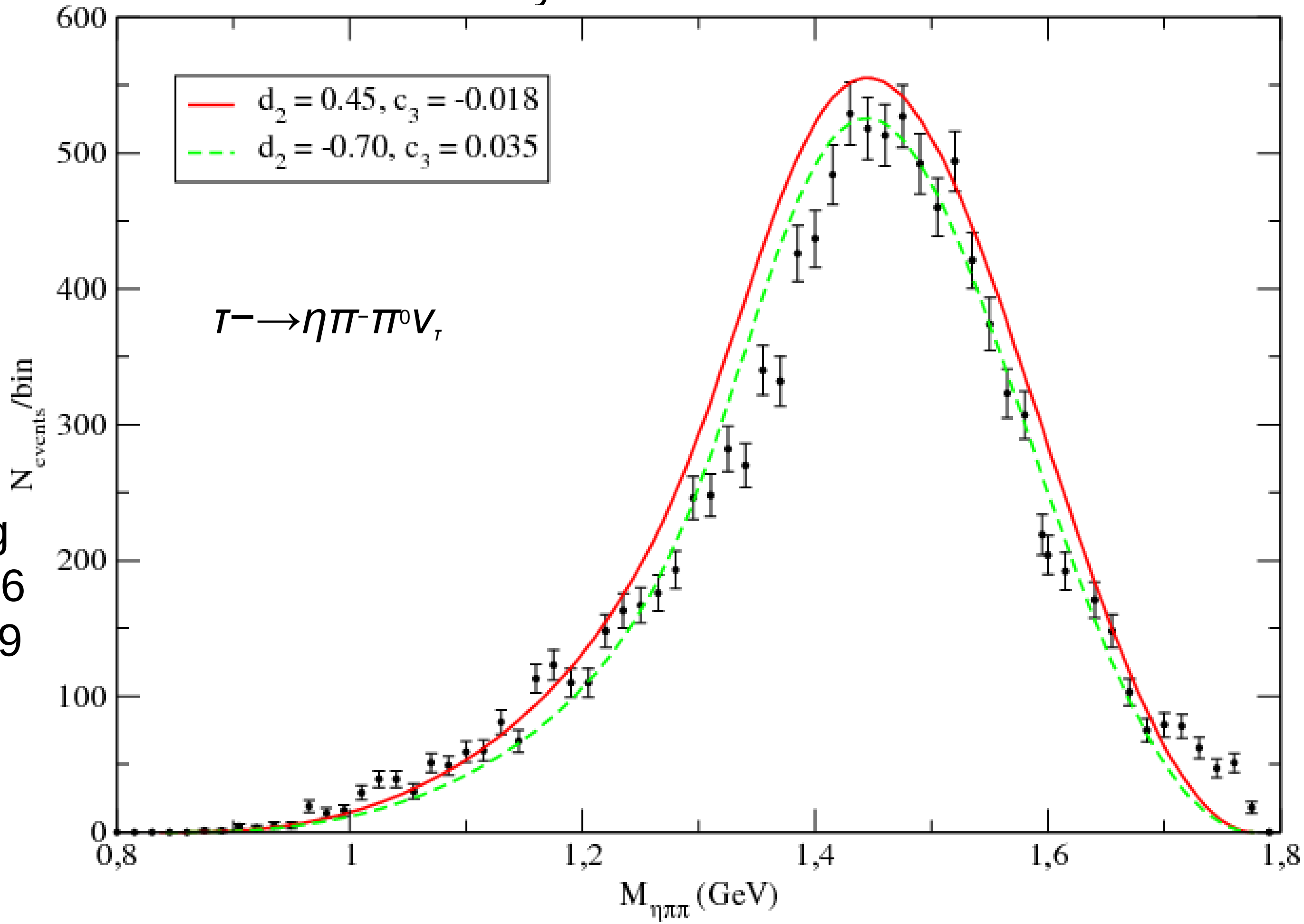

Dumm & Roig
Phys.Rev. D86
(2012) 076009

SCC

MOTIVATION

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Irrespective of the underlying resonance mechanism G-Parity violation

It is an isospin violating process $(\underline{m_u \neq m_d}, e \neq 0)$

$$\epsilon = \frac{\sqrt{3}(m_d - m_u)}{4(m_s - (m_u + m_d)/2)}$$

The considered processes could provide complementary information to η→3π

SCC

MOTIVATION

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

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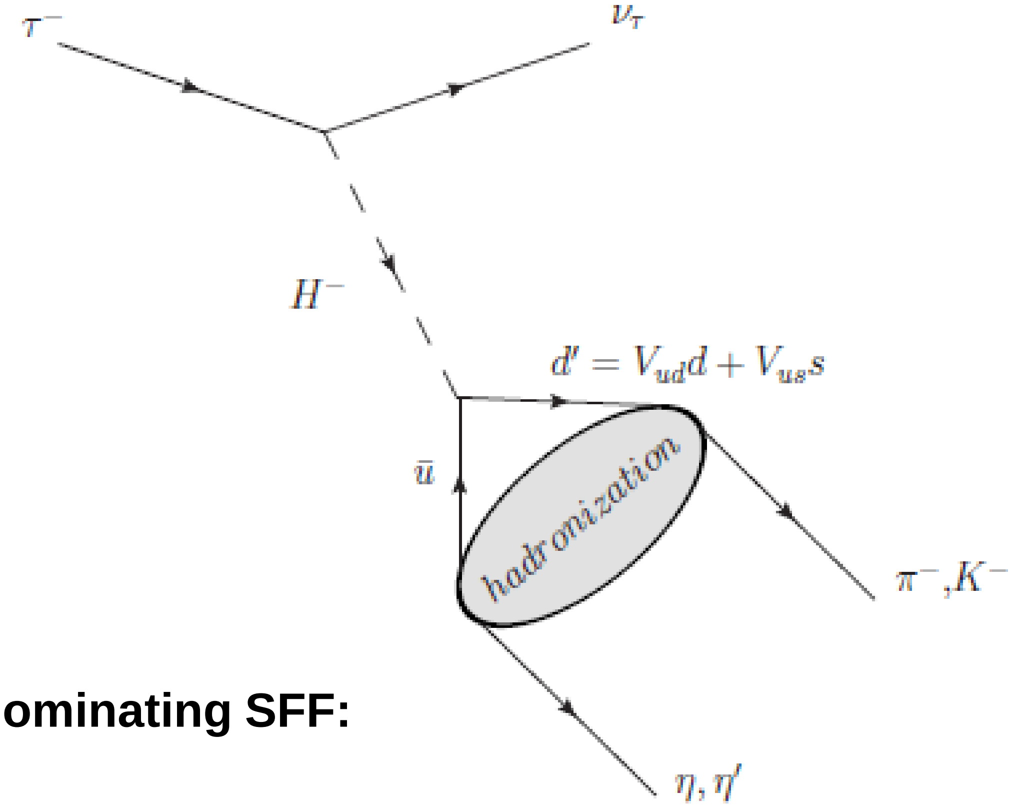
The considered processes could provide complementary information to $\eta \to 3\pi$

The corresponding suppression of the SM contribution can make NP visible

MOTIVATION

$\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

Possible new physics contributions: Charged Higgs



Enhancement of the dominating SFF:

$$f_0^{\eta\pi}(s) = f_0^{\eta\pi}(s)|_{SM} \left(1 - \frac{\zeta_\tau^* (\zeta_u m_u - \zeta_d m_d)}{m_u - m_d} \times \frac{s}{m^2_{H^+}} \right)$$

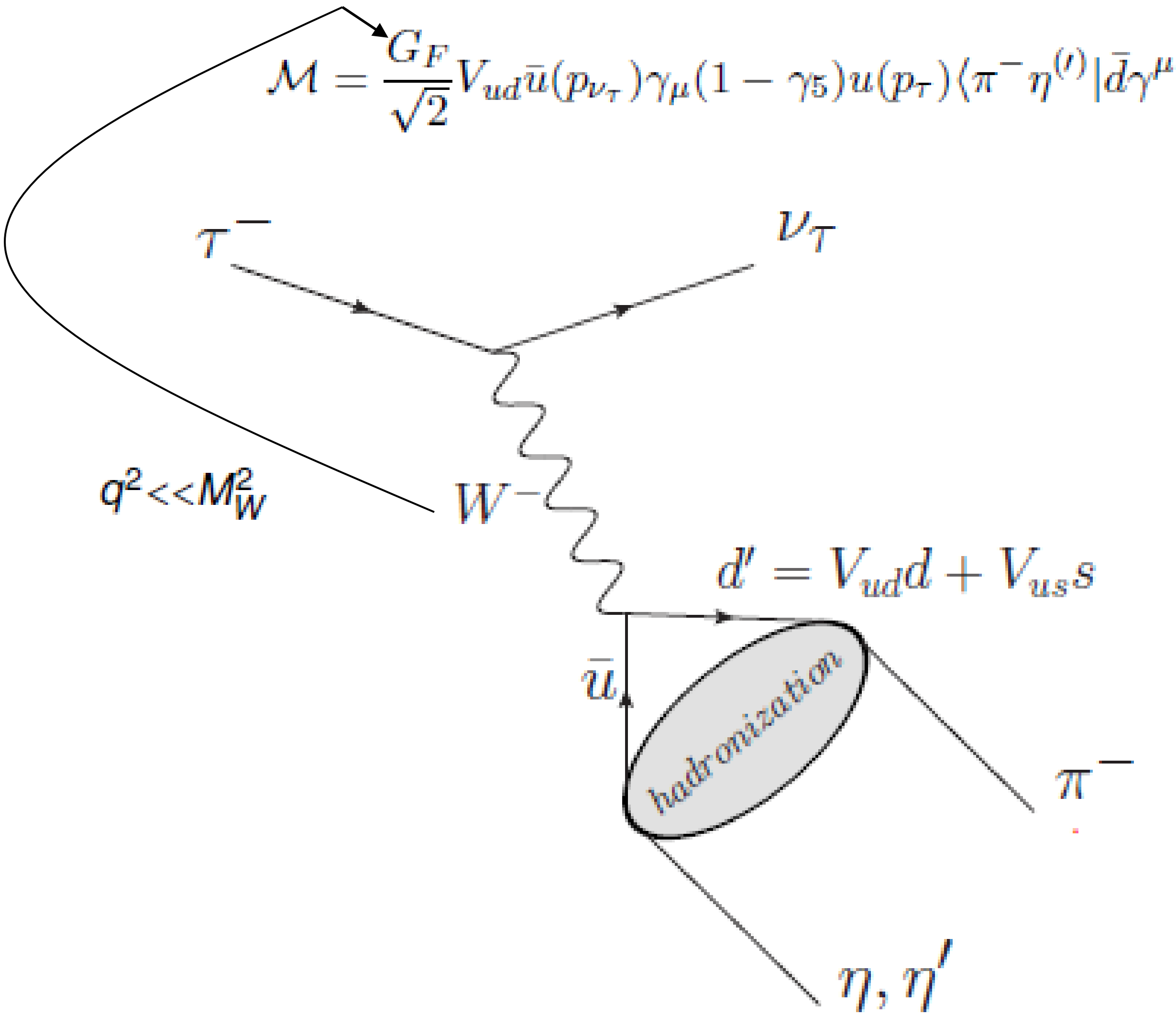
From B→τν_τ

$$|\zeta_l \zeta_d / m^2_{H^+}| < 0.1 \text{ GeV}^{-2}$$

Improvable if we know the SFF with ≤ 20% accuracy

Hadronic matrix element and decay width $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ud} \bar{u}(p_{\nu_\tau}) \gamma_\mu (1-\gamma_5) u(p_\tau) \langle \pi^- \eta^{(\prime)} | \bar{d} \gamma^\mu u | 0 \rangle$$



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Following Gasser & Leutwyler:

$$\langle \pi^- \eta^{(\prime)} | \bar{d} \gamma^\mu u | 0 \rangle = c^V_{\pi^- \eta^{(\prime)}} \left[(p_{\eta^{(\prime)}} - p_{\pi^-})^\mu F_+^{\pi^- \eta^{(\prime)}}(s) - (p_{\eta^{(\prime)}} + p_{\pi^-})^\mu F_-^{\pi^- \eta^{(\prime)}}(s) \right]$$

$$c^V_{\pi^- \eta^{(\prime)}} = \sqrt{2}, \; s = q^2 = (p_{\eta^{(\prime)}} + p_{\pi^-})^2$$

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Another FF (F_0) is employed instead of F_- in order to have both FFs in correspondence with S- (F_0) and P-wave (F_+).

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∂_μ

$$\langle \pi^- \eta^{(\prime)} | \partial_\mu (\bar{d} \gamma^\mu u) | 0 \rangle = i \underbrace{(m_d - m_u)}_{\mathcal{O}(\varepsilon_{\pi\eta^{(\prime)}}) \Rightarrow \text{suppresion}} \langle \pi^- \eta^{(\prime)} | \bar{d} u | 0 \rangle \equiv \underbrace{i \Delta^{QCD}_{K^0 K^+}}_{\text{suppression}} c^S_{\pi^- \eta^{(\prime)}} F_0^{\pi^- \eta^{(\prime)}}(s)$$

$$c^S_{\pi^- \eta} = \sqrt{\frac{2}{3}}, \quad c^S_{\pi^- \eta'} = \frac{2}{\sqrt{3}}, \quad \Delta_{PQ} = m_P^2 - m_Q^2$$

Hadronic matrix element and decay width $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

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∂_μ

$$i q_\mu \langle \pi^- \eta^{(\prime)} | \bar{d} \gamma^\mu u | 0 \rangle = i c^V_{\pi^-\eta^{(\prime)}} \left[(m^2_{\eta^{(\prime)}} - m^2_{\pi^-}) F_+^{\pi^-\eta^{(\prime)}}(s) - s F_-^{\pi^-\eta^{(\prime)}}(s) \right]$$

Hadronic matrix element and decay width $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$

$$\left\{ \begin{array}{c} \langle\pi^-\eta^{(\prime)}|\partial_\mu(\bar{d}\gamma^\mu u)|0\rangle = i(m_d-m_u)\langle\pi^-\eta^{(\prime)}|\bar{d}u|0\rangle \equiv i\Delta^{QCD}_{K^0K^+}c^S_{\pi^-\eta^{(\prime)}}F_0^{\pi^-\eta^{(\prime)}}(s) \\ iq_\mu\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = ic^V_{\pi^-\eta^{(\prime)}}\left[(m^2_{\eta^{(\prime)}}-m^2_{\pi^-})F_+^{\pi^-\eta^{(\prime)}}(s) - sF_-^{\pi^-\eta^{(\prime)}}(s)\right] \end{array} \right\}$$

$$\Downarrow$$

$$F_-^{\pi\eta^{(\prime)}}(s) = -\frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}\left[\frac{c^S_{\pi\eta^{(\prime)}}}{c^V_{\pi\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}}F_0^{\pi\eta^{(\prime)}}(s) + F_+^{\pi^-\eta^{(\prime)}}(s)\right]$$

$$\Downarrow$$

$$\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = \left[(p_{\eta^{(\prime)}}-p_\pi)^\mu + \frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}q^\mu\right]c^V_{\pi\eta^{(\prime)}}F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0K^+}}{s}q^\mu c^S_{\pi^-\eta^{(\prime)}}F_0^{\pi^-\eta^{(\prime)}}(s)$$

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$$\Downarrow$$

$$F_-^{\pi\eta^{(\prime)}}(s) = -\frac{\Delta_{\pi^-\eta^{(\prime)}}}{s} \left[\frac{c^S_{\pi\eta^{(\prime)}}}{c^V_{\pi\eta^{(\prime)}}} \frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}} F_0^{\pi\eta^{(\prime)}}(s) + F_+^{\pi^-\eta^{(\prime)}}(s) \right]$$

$$\Downarrow$$

$$\langle \pi^-\eta^{(\prime)} | \bar{d}\gamma^\mu u | 0 \rangle = \left[(p_{\eta^{(\prime)}} - p_\pi)^\mu + \frac{\Delta_{\pi^-\eta^{(\prime)}}}{s} q^\mu \right] c^V_{\pi\eta^{(\prime)}} F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0K^+}}{s} q^\mu c^S_{\pi^-\eta^{(\prime)}} F_0^{\pi^-\eta^{(\prime)}}(s)$$

The finiteness of the matrix element at the origin imposes

$$F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{c^S_{\pi^-\eta^{(\prime)}}}{c^V_{\pi^-\eta^{(\prime)}}} \frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}} F_0^{\pi^-\eta^{(\prime)}}(0) \,.$$

Hadronic matrix element and decay width $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

$$\frac{d\Gamma\left(\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} S_{EW} |V_{ud} \overbrace{F_+^{\pi^-\eta^{(\prime)}}(0)}^{\mathcal{O}(\varepsilon_{\pi\eta}) \Rightarrow \text{suppression}}|^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q^3_{\pi^-\eta^{(\prime)}}(s) |\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s} q_{\pi^-\eta^{(\prime)}}(s) |\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2 \right\}$$

$$q_{PQ}(s) = \frac{\sqrt{s^2 - 2s\Sigma_{PQ} + \Delta_{PQ}^2}}{2\sqrt{s}}, \quad \Sigma_{PQ} = m_P^2 + m_Q^2$$

Hadronic matrix element and decay width $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$

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$$q_{PQ}(s) = \frac{\sqrt{s^2 - 2s\Sigma_{PQ} + \Delta^2_{PQ}}}{2\sqrt{s}}, \quad \Sigma_{PQ} = m_P^2 + m_Q^2$$

All dynamics is encoded in the normalized FFs

$$\widetilde{F}_{+,0}^{\pi^-\eta^{(\prime)}}(s) = \frac{F_{+,0}^{\pi^-\eta^{(\prime)}}(s)}{F_{+,0}^{\pi^-\eta^{(\prime)}}(0)}$$

$S_{EW} = 1.0201$ (Erler) $\quad V_{ud} = 0.97425(8)(10)(18)$

Hadronic matrix element and decay width $\tau^-\to\pi^-\eta^{(\prime)}\nu_\tau$

$$\frac{d\Gamma\left(\tau^-\to\pi^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}}=\frac{G_F^2M_\tau^3}{24\pi^3 s}S_{EW}|V_{ud}\overbrace{\boxed{F_+^{\pi^-\eta^{(\prime)}}(0)|}}^{\mathcal{O}(\varepsilon_{\pi\eta})\Rightarrow\text{suppression}}{}^2\left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1+\frac{2s}{M_\tau^2}\right)q^3_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2+\frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s}q_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2\right\}$$

$$q_{PQ}(s)=\frac{\sqrt{s^2-2s\Sigma_{PQ}+\Delta^2_{PQ}}}{2\sqrt{s}}\,,\quad \Sigma_{PQ}=m_P^2+m_Q^2$$

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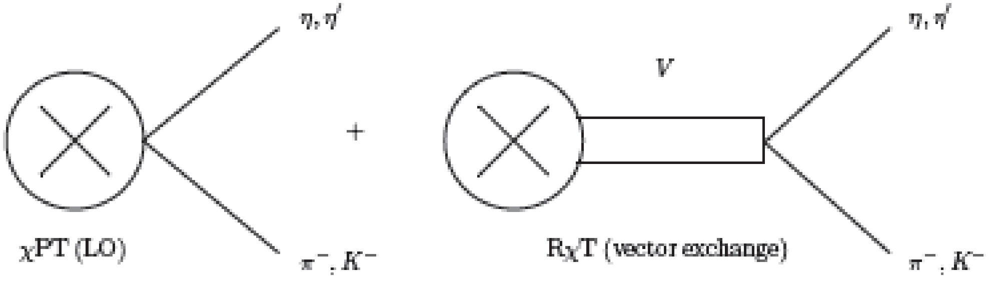
$S_{EW}\;=\;1.0201$ (Erler) $\qquad V_{ud}=0.97425(8)(10)(18)$

Isospin-violating quantity $\mathcal{O}(m_d-m_u)$

explaining the overall suppression of these decays

Relation between ππ,πη,πη’ VFFs

χPT in the large-N_C limit: simultaneous expansion in p^2, m^2 and $1/N_C$

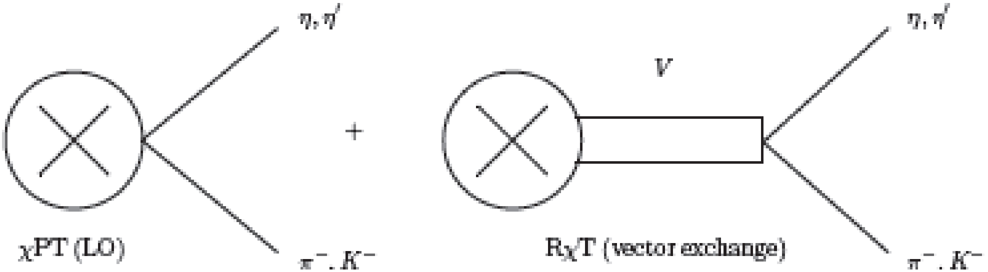


Relation between ππ,πη,πη’ VFFs

χPT in the large-N_C limit: simultaneous expansion in p^2, m^2 and $1/N_C$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}$$

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon_{\pi\eta} & \varepsilon_{\pi\eta'} \\ \sin\left(\theta_{\eta\eta'}\right)\varepsilon_{\pi\eta'} - \cos\left(\theta_{\eta\eta'}\right)\varepsilon_{\pi\eta} & \cos\left(\theta_{\eta\eta'}\right) & -\sin\left(\theta_{\eta\eta'}\right) \\ -\sin\left(\theta_{\eta\eta'}\right)\varepsilon_{\pi\eta} - \cos\left(\theta_{\eta\eta'}\right)\varepsilon_{\pi\eta'} & \sin\left(\theta_{\eta\eta'}\right) & \cos\left(\theta_{\eta\eta'}\right) \end{pmatrix} \begin{pmatrix} \pi^3 \\ \eta_8 \\ \eta_1 \end{pmatrix}$$



Relation between ππ,πη,πη' VFFs

χPT in the large-N_C limit: simultaneous expansion in p^2, m^2 and $1/N_C$

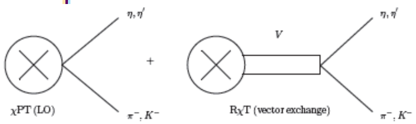
$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}$$

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon_{\pi\eta} & \varepsilon_{\pi\eta'} \\ \sin(\theta_{\eta\eta'})\,\varepsilon_{\pi\eta'} - \cos(\theta_{\eta\eta'})\,\varepsilon_{\pi\eta} & \cos(\theta_{\eta\eta'}) & -\sin(\theta_{\eta\eta'}) \\ -\sin(\theta_{\eta\eta'})\,\varepsilon_{\pi\eta} - \cos(\theta_{\eta\eta'})\,\varepsilon_{\pi\eta'} & \sin(\theta_{\eta\eta'}) & \cos(\theta_{\eta\eta'}) \end{pmatrix} \begin{pmatrix} \pi^3 \\ \eta_8 \\ \eta_1 \end{pmatrix}$$

$$F_+^{\pi^-\eta}(s) = (\varepsilon_{\pi\eta}\cos\theta - \varepsilon_{\pi\eta'}\sin\theta)\left[1 + \sum_V \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s}\right]$$

$\theta_{\eta\eta'} \sim \theta_P = (-13.3 \pm 1.0)^\circ$

(KLOE Coll. PLB 648 '07 267)



$$F_+^{\pi^-\eta'}(s) = (\varepsilon_{\pi\eta'}\cos\theta + \varepsilon_{\pi\eta}\sin\theta)\left[1 + \sum_V \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s}\right]$$

$\varepsilon_{\pi\eta} \sim 0.018(2)$

$\varepsilon_{\pi\eta'} \sim 0.005(1)$

(Kroll Mod.Phys.Lett. A20 (2005))

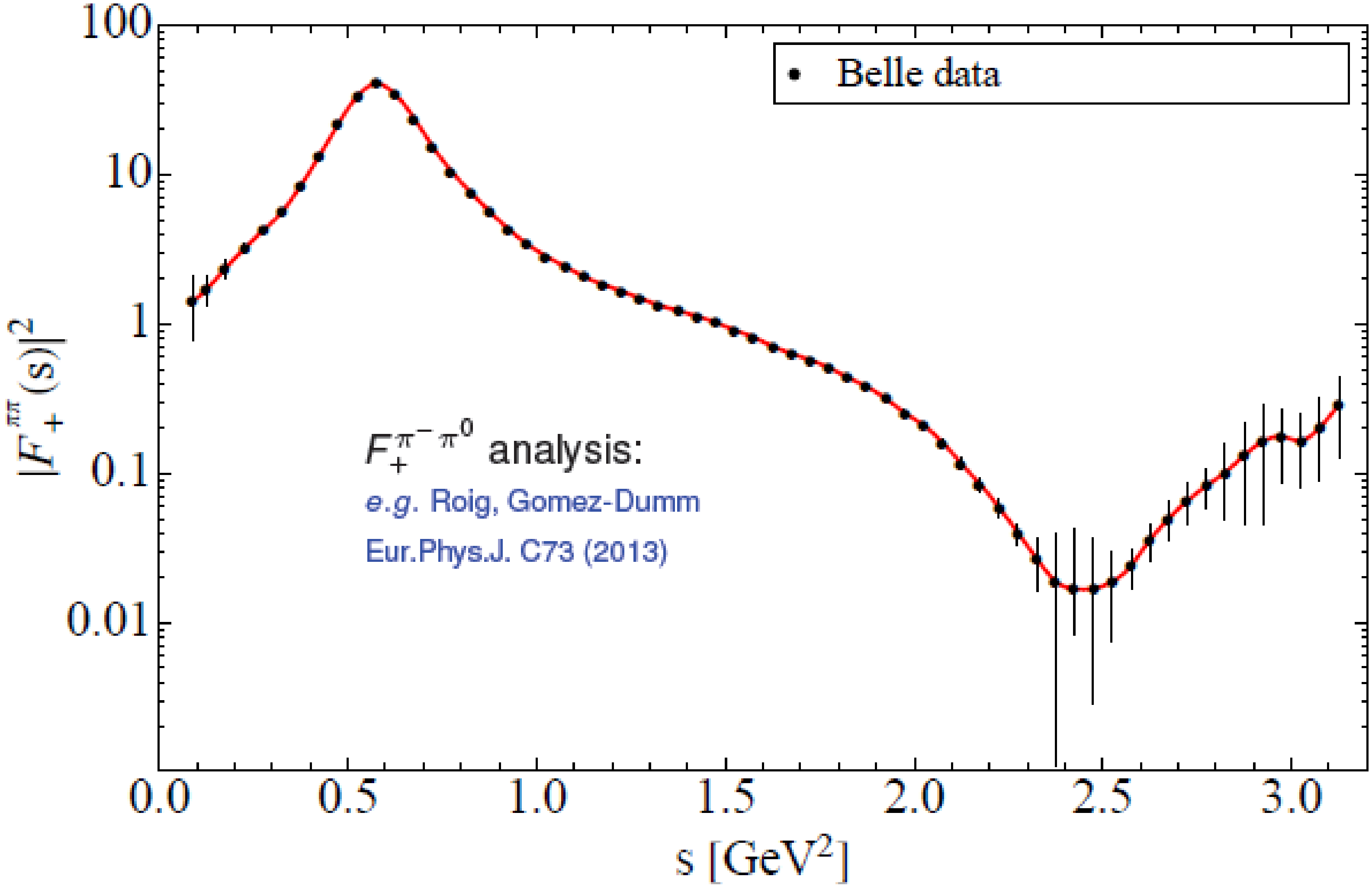
$\mathcal{O}(\varepsilon_{\pi\eta}) \Rightarrow$ suppresion

$F_+^{\pi^-\pi^0}(s)$

$$F_+^{\pi^-\eta}(0) = \varepsilon_{\pi\eta}\cos\theta - \varepsilon_{\pi\eta'}\sin\theta\,, \quad F_+^{\pi^-\eta'}(0) = \varepsilon_{\pi\eta'}\cos\theta + \varepsilon_{\pi\eta}\sin\theta$$

Data-driven πη & πη' VFFs

$$\widetilde{F}_+^{\pi^-\eta}(s) = \widetilde{F}_+^{\pi^-\eta'}(s) = \widetilde{F}_+^{\pi^-\pi^0}(s)$$



$F_+^{\pi^-\pi^0}$ analysis:

e.g. Roig, Gomez-Dumm

Eur.Phys.J. C73 (2013)

Relation between KK,$\pi\eta$,$\pi\eta'$ SFFs

(Escribano, González-Solís and Roig JHEP 1310 (2013) 039)
(Escribano, González-Solís, Jamin and Roig JHEP 1409 (2014) 042)

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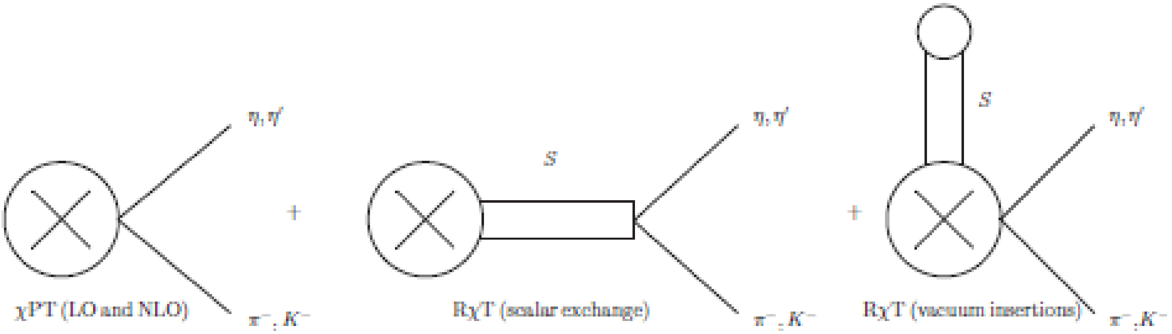
Although theory says BWs should not be applied, sometimes they are an easy solution fo the experimentalists.

That is why we decided to start our analyses with them (again).

Relation between KK,πη,πη' SFFs

$$(m_d - m_u)\langle \pi^-\eta^{(\prime)}|\bar{d}u|0\rangle = \Delta^{QCD}_{K^0K^+}\, C^S_{\pi^-\eta^{(\prime)}}\, F_0^{\pi^-\eta^{(\prime)}}(s) =$$

Breit-Wigner



Imposing $F_0^{\pi^-\eta^{(\prime)}}(s)$ to vanish for $s \to \infty$ we arrive at

$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \frac{M_S^2+\Delta_{\pi^-\eta^{(\prime)}}}{M_S^2-s-iM_S\Gamma_S(s)}, \qquad \Gamma_S(s) = \Gamma_{a_0}(M_{a_0}^2)\left(\frac{s}{M_{a_0}^2}\right)^{3/2}\frac{h(s)}{h(M_{a_0}^2)}$$

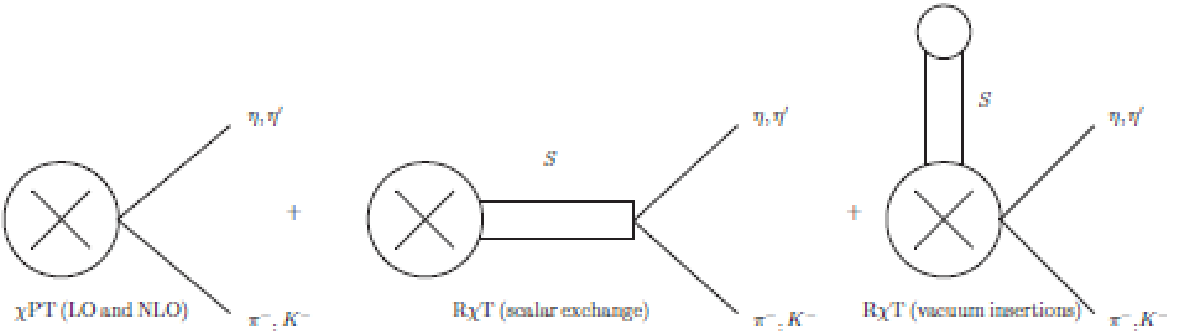
$$h(s) = \sigma_{KK}(s) + \frac{2}{3}\sigma_{\pi\eta}(s)\left(c_0^{\pi^-\eta}\right)^2\left(1+\frac{\Delta_{\pi\eta}}{s}\right)^2 + \frac{4}{3}\sigma_{\pi\eta'}(s)\left(c_0^{\pi^-\eta'}\right)^2\left(1+\frac{\Delta_{\pi\eta'}}{s}\right)^2$$

$$c_0^{\pi^-\eta} = \cos\theta_{\eta\eta'} - \sqrt{2}\sin\theta_{\eta\eta'}, \qquad c_0^{\pi^-\eta'} = \cos\theta_{\eta\eta'} + \frac{1}{\sqrt{2}}\sin\theta_{\eta\eta'}$$

Relation between KK,πη,πη’ SFFs

$$(m_d - m_u)\langle \pi^- \eta^{(\prime)} | \bar{d} u | 0 \rangle = \Delta^{QCD}_{K^0 K^+} C^S_{\pi^- \eta^{(\prime)}} F_0^{\pi^- \eta^{(\prime)}}(s) =$$

Breit-Wigner



Imposing $F_0^{\pi^- \eta^{(\prime)}}(s)$ to vanish for $s \to \infty$ we arrive at

(Brodsky-Lepage)

$$F_0^{\pi^- \eta^{(\prime)}}(s) = c_0^{\pi^- \eta^{(\prime)}} \frac{M_S^2 + \Delta_{\pi^- \eta^{(\prime)}}}{M_S^2 - s - i M_S \Gamma_S(s)},$$

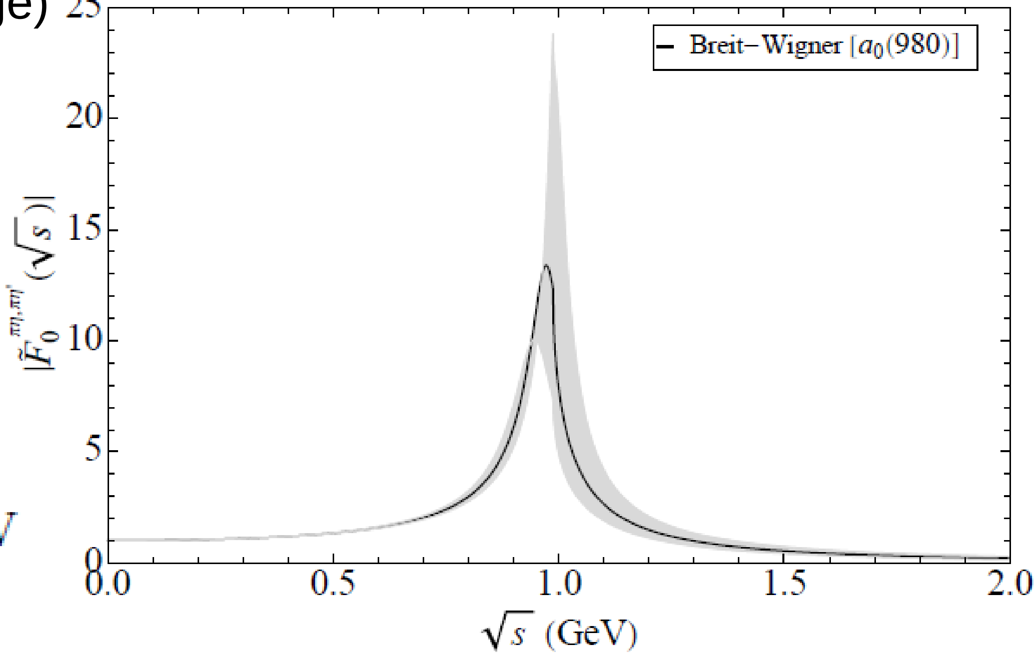
$$F_0^{\pi\eta}(0) = 0.92 \text{ and } F_0^{\pi\eta'}(0) = 0.05$$

Real part of the loop neglected

Violation of analyticity

$$M_S = 980(20) \text{ MeV and } \Gamma = 75(25) \text{ MeV}$$

$$\sqrt{s_p} = (940 - i77/2) \text{ MeV}$$



Relation between KK,πη,πη' SFFs

Breit-Wigner We have also considered a two-resonance BW SFF

$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \left\{ \frac{\left(M_S^2 - s\right)\left(M_{S'}^2 - s\right) + \frac{4}{F_\pi^2}\left(s + M_\pi^2 - M_{\eta^{(\prime)}}^2\right)\left[c_m^2\left(M_{S'}^2 - s\right) + c_{m'}^2\left(M_S^2 - s\right)\right]}{\left(M_S^2 - s - iM_S\Gamma_S(s)\right)\left(M_{S'}^2 - s - iM_{S'}\Gamma_{S'}(s)\right)} \right\}$$

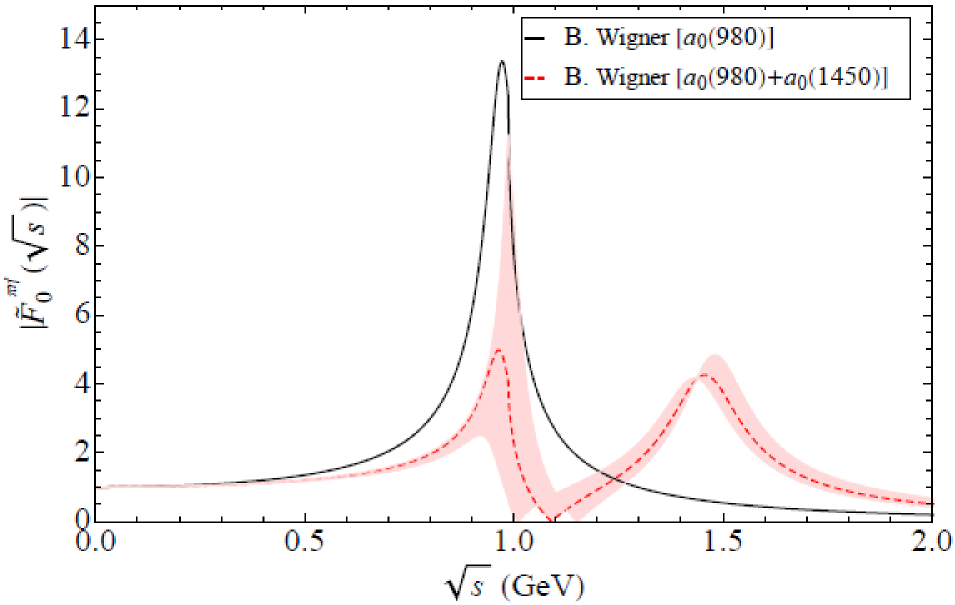
$M_{S'} = 1474(19)$ MeV and $\Gamma = 265(13)$ MeV

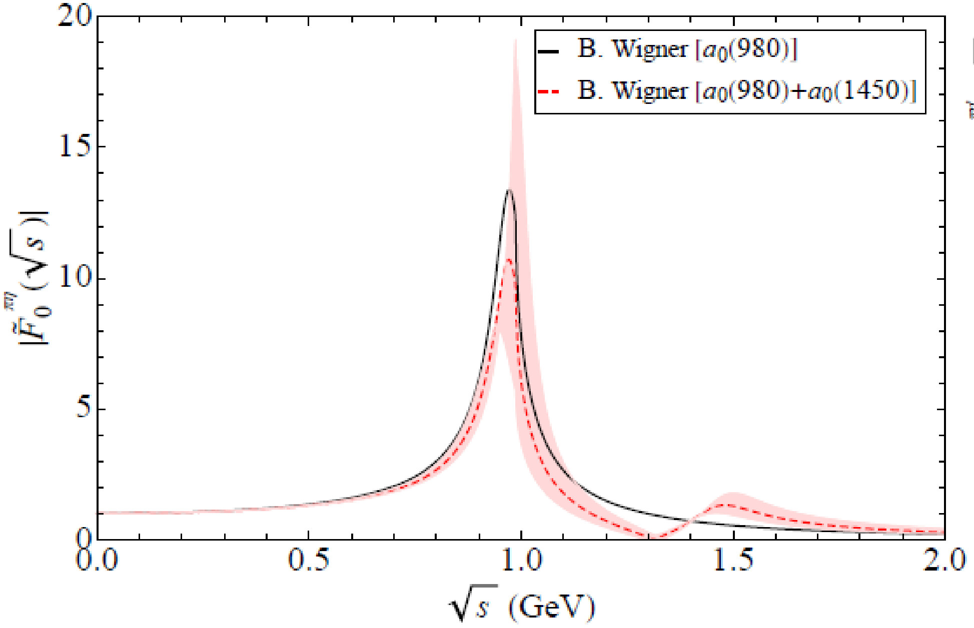
$\downarrow$

$\sqrt{s_p} = (1457 - i161/2)$ MeV

$c_m^2 + c_m'^2 = F^2/4$

$c_m = 41.9$ MeV



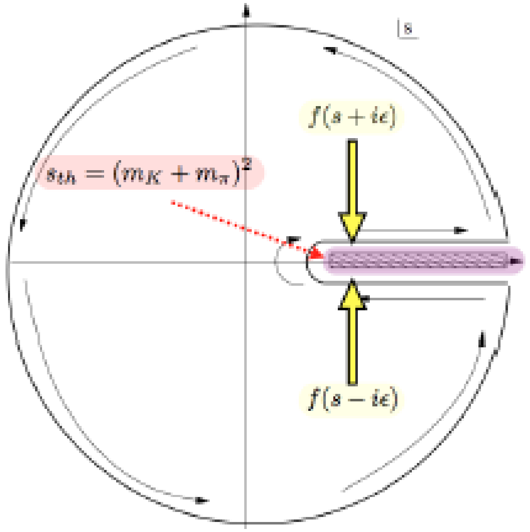


Now both normalized FFs differ!

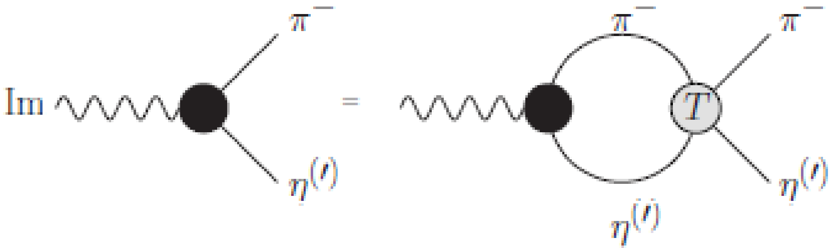
Relation between KK,πη,πη' SFFs

Elastic final state interactions

Analyticity and elastic unitarity ensured through a dispersion relation



$$F_0^{\pi^-\eta^{(\prime)}}(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty}ds'\frac{\mathrm{Im}F_0^{\pi^-\eta^{(\prime)}}(s')}{s'-s-i\varepsilon}$$



$$\mathrm{Im}F_0^{\pi^-\eta^{(\prime)}}(s)=\sigma_{\pi\eta^{(\prime)}}(s)F_0^{\pi^-\eta^{(\prime)}}T^*(s)$$
$$=F_0^{\pi^-\eta^{(\prime)}}\sin\delta_{1,0}^{\pi\eta^{(\prime)}}(s)e^{-i\delta_{1,0}^{\pi\eta^{(\prime)}}(s)}$$

§ (once subtracted) Omnès solution (Omnès '58)

$$F_0^{\pi^-\eta^{(\prime)}}(s)=P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty}ds'\frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right]=P(s)\Omega(s)$$

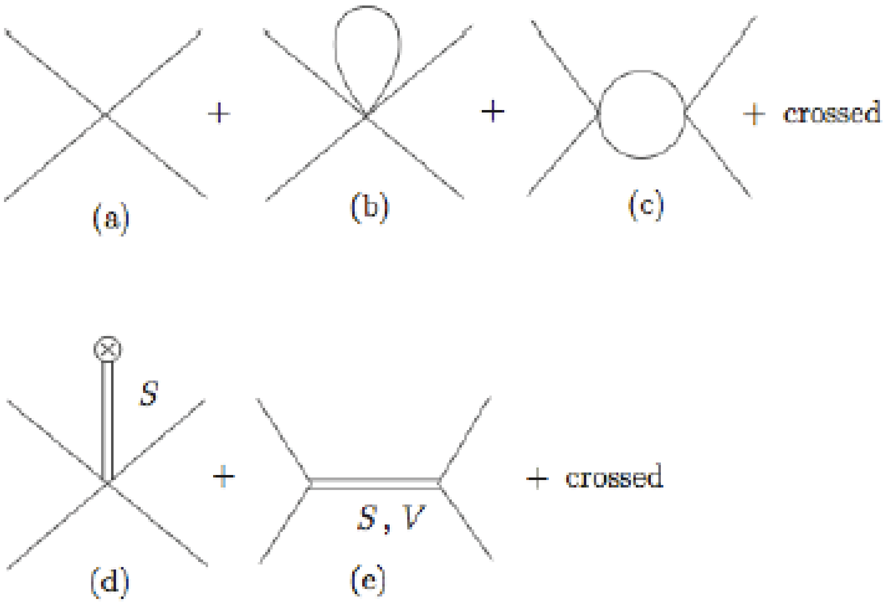
Relation between KK,πη,πη' SFFs

Elastic final state interactions

Elastic unitarity: Form factor phase= $\delta_{\pi^-\eta^{(\prime)}}$ $2 \to 2$ elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan\frac{\mathrm{Im}\,t_{1,0}(s)}{\mathrm{Re}\,t_{1,0}(s)}$$

$t_{1,0}$: unitarized S-waves of the $U(3) \times U(3)$ amplitudes in χPT at one-loop including resonances (Guo-Oller: Phys.Rev. D84 (2011) 034005)



$\widetilde{c}_d = c_d/\sqrt{3}, \quad \widetilde{c}_m = c_m/\sqrt{3}$

$c_d = 17.4$ MeV , $\quad c_m = 28.1$ MeV

$M_{a_0,S_8} = 1390$ MeV , $\quad M_{S_1} = 1020$ MeV

$a_{SL}^{10,\pi\eta} = 2, \quad a_{SL}^{10,\pi\eta'} = -1.14$

$\Lambda_2 = -0.22$

Errors known, correlations unknown!

Relation between KK,πη,πη’ SFFs

Elastic final state interactions

Elastic unitarity: Form factor phase= $\delta_{1,0}^{\pi^-\eta^{(\prime)}}$ 2 → 2 elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan\frac{\mathrm{Im}\, t_{1,0}(s)}{\mathrm{Re}\, t_{1,0}(s)}, \quad t_{1,0}(s) = \frac{N_{1,0}(s)}{D_{1,0}(s)}$$

$$D(s) = D(s_0) + \frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}D(s')}{(s'-s_0)(s'-s-i\varepsilon)}, \quad N(s) = \frac{s-s_0}{\pi}\int_{-\infty}^{s_L} ds' \frac{\mathrm{Im}N(s')}{(s'-s_0)(s'-s-i\varepsilon)}$$

Simplified perturbative solution

$$t_{1,0}(s) = \frac{N_{1,0}(s)}{1+g(s)N_{1,0}(s)}, \quad N_{1,0}(s) = T_{1,0}^{\mathcal{O}(p^2)+res+loop} - g(s)\left(T_{1,0}^{\mathcal{O}(p^2)}\right)^2$$

$g(s)$: meson one-loop scalar functions

Relation between KK,πη,πη’ SFFs

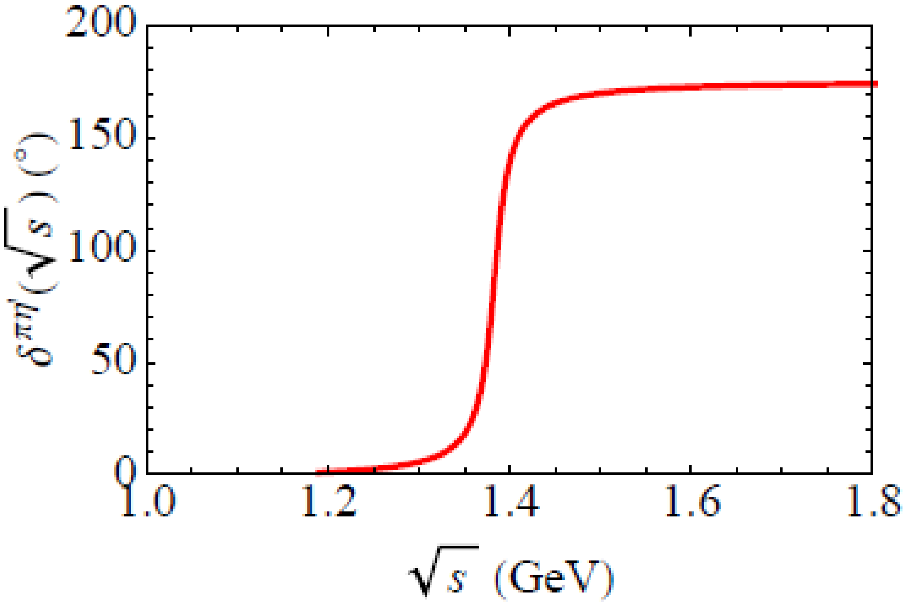
Elastic final state interactions

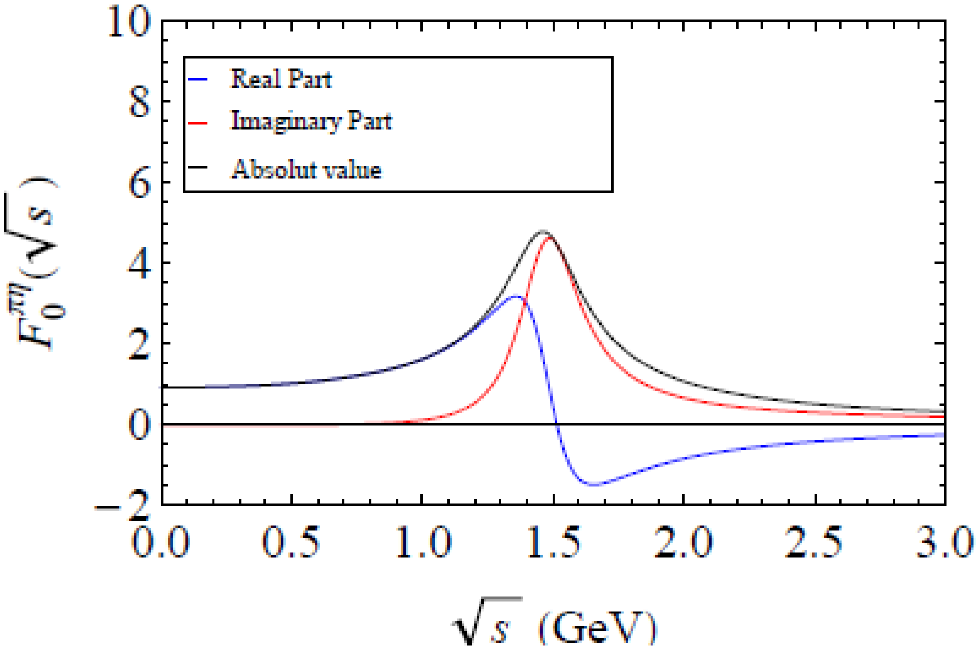
Assuming $F_0^{\pi^-\eta^{(\prime)}}(s)$ to behave as s^{-1} : $F_0^{\pi\eta^{(\prime)}}(s) = P(s)\Omega(s)$,

(Brodsky-Lepage)

$$\Omega(s) \sim s^{-\ell}, \quad \ell = \frac{1}{\pi}(\delta(\infty) - \delta(s_{sth})); \quad \delta(\infty) = n\pi \Rightarrow P(s) \text{ constant } (n = 1)$$

our choice: $$P(s) = F_0^{Breit-Wigner}(0)$$





Relation between KK,πη,πη’ SFFs

SFF: Closed expression

Once subtracted dispersion relation

$$F(s+i\varepsilon)=F(s_0)+\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty}ds'\,\frac{\sigma(s')t^*_{IJ}(s')F(s')}{(s'-s_0)(s'-s-i\varepsilon)}=F(s_0)+\widetilde{F}(s+i\varepsilon)$$

$$\begin{aligned}\widetilde{F}(s+i\varepsilon)-\widetilde{F}(s-i\varepsilon)&=2i\sigma(s)t^*(s+i\varepsilon)F(s+i\varepsilon)\\&=2i\sigma(s)t^*(s+i\varepsilon)[F(s_0)+\widetilde{F}(s+i\varepsilon)]\end{aligned}$$

$$\left.\begin{array}{l}t=N/D\\ \mathrm{Im}\,t^{-1}=-\sigma(s)\\ \mathrm{Im}\,D(s)=-N\sigma(s)\end{array}\right\}\qquad \begin{aligned}&\widetilde{F}(s+i\varepsilon)D(s+i\varepsilon)-\widetilde{F}(s-i\varepsilon)D(s-i\varepsilon)\\&=-2i\,\mathrm{Im}\,D(s)F(s_0),\end{aligned}$$

$$\begin{aligned}\widetilde{F}(s+i\varepsilon)&=\frac{1}{D(s+i\varepsilon)}\frac{-(s-s_0)}{\pi}\int_{s_{th}}^{\infty}ds'\,\frac{\mathrm{Im}\,D(s')F(s_0)}{(s'-s_0)(s'-s)}\\&=-D(s+i\varepsilon)^{-1}\left[D(s+i\varepsilon)-D(s_0)\right]F(s_0)\end{aligned}$$

Relation between KK,πη,πη’ SFFs

SFF: Closed expression

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i=1} \frac{s - s_z^i}{s - s_p^i} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of $D(s) = 1 - g(s)N(s)$

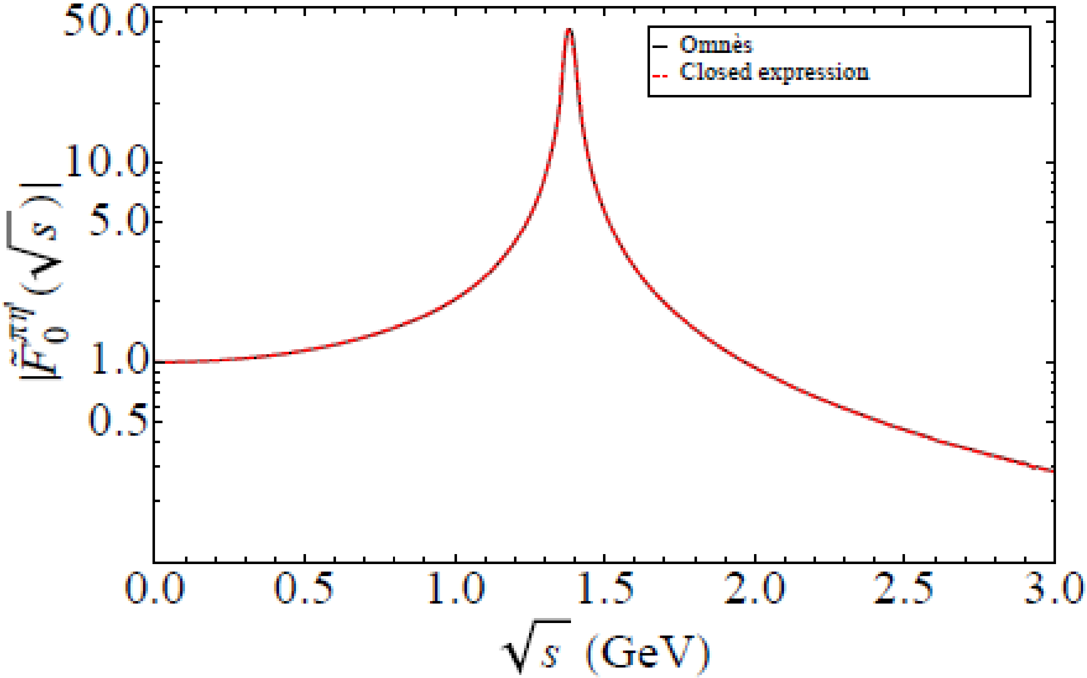
Iwamura, Kurihara, Takahashi ’77
Kamal ’79, Kamal, Cooper ’80
Jamin, Oller, Pich ’01

$s_0 = 0$

$F_0(s_0) = F_0^{BW}(0)$

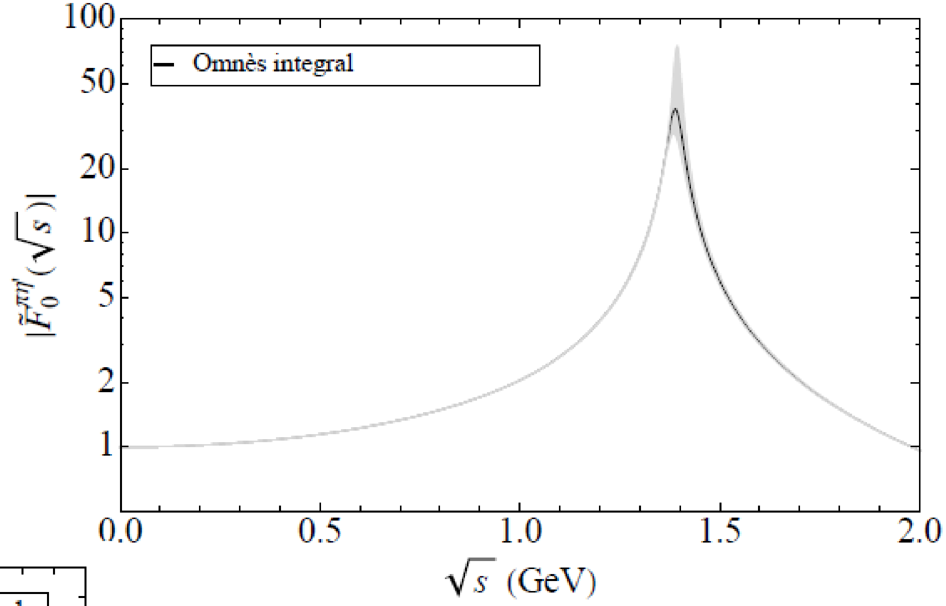
$s_p = 1.390$ GeV

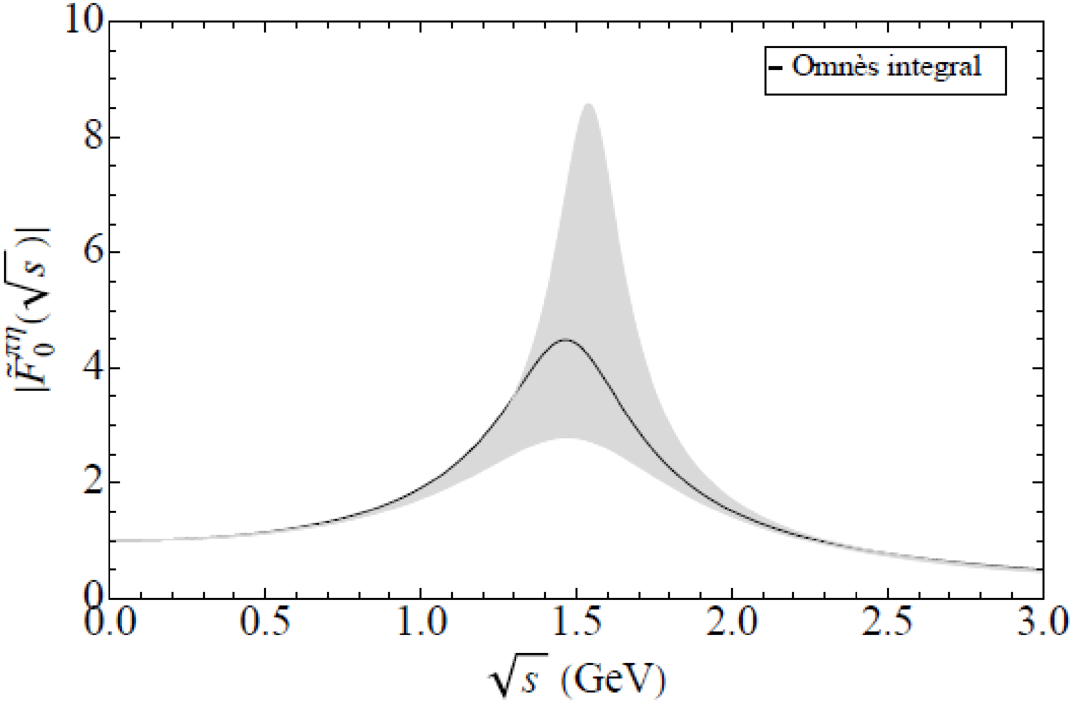
$N(s) = T_{1,0}^{\mathcal{O}(p^2)+res+\cancel{loop}}$



Relation between KK,πη,πη’ SFFs

SFF: Closed expression

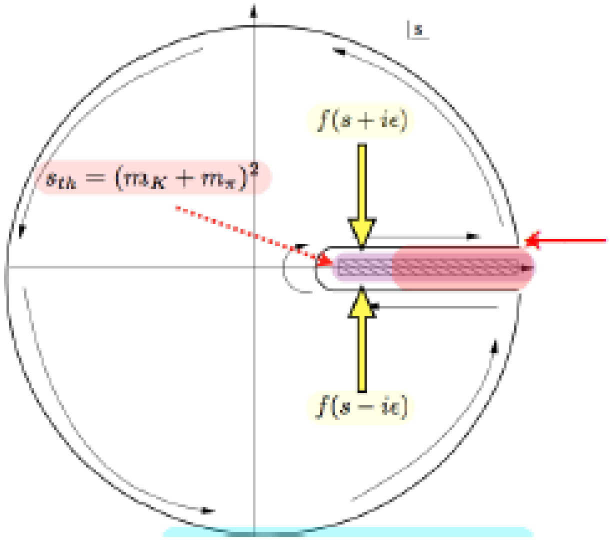




Very different normalized FFs!

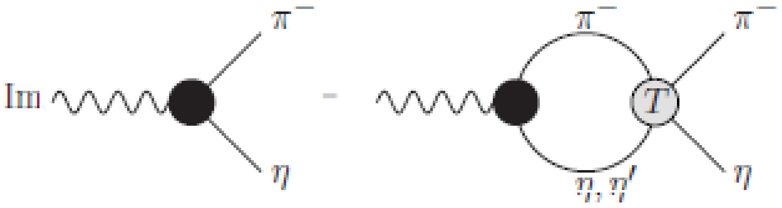
Relation between KK,πη,πη’ SFFs

SFF: Coupled channels case



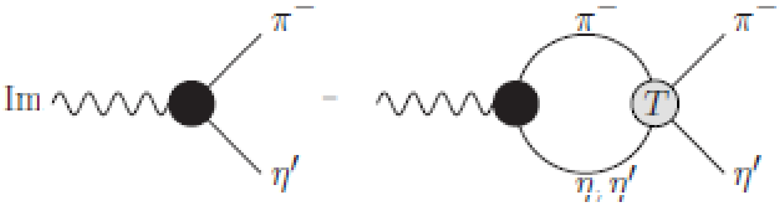
$$F_0^i(s) = \frac{1}{\pi}\sum_{j=1}^{2}\int_{s_j}^{\infty} ds' \frac{\Sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^*}{(s'-s-i\varepsilon)}$$

Other cuts $(K\bar{K}, \pi\eta'...)$



$$F_0^{\pi\eta}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta}(s')}{s'-s-i\varepsilon}$$

Two coupled channels



‘traditional’ iterative solution

$$F_0^{\pi\eta'}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta'}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta'}(s')}{s'-s-i\varepsilon}$$

Relation between KK,πη,πη’ SFFs

SFF: Coupled channels case

Two coupled channels

Closed-form solution

$$g_{1,0}(s) = \begin{pmatrix} g_{\pi^-\eta} & 0 \\ 0 & g_{\pi^-\eta'} \end{pmatrix}, \qquad N_{1,0}(s) = \begin{pmatrix} N_{\pi^-\eta\to\pi^-\eta} & N_{\pi^-\eta\to\pi^-\eta'} \\ N_{\pi^-\eta'\to\pi^-\eta} & N_{\pi^-\eta'\to\pi^-\eta'} \end{pmatrix}$$

$$N_{ij}(s) = T_{ij}^{\mathcal{O}(p^4)}(s) - g_i(s)\left(T_{ij}^{\mathcal{O}(p^2)}(s)\right)^2 \ (i,j=1,2)$$

$$\begin{pmatrix} F_0^{\pi^-\eta}(s) \\ F_0^{\pi^-\eta'}(s) \end{pmatrix} = \frac{1}{Det[D_{IJ}(s)} \begin{pmatrix} 1+g_{\pi^-\eta'}(s)N_{\pi^-\eta'\to\pi^-\eta'}(s) & -g_{\pi^-\eta}(s)N_{\pi^-\eta\to\pi^-\eta'}(s) \\ -g_{\pi^-\eta'}(s)N_{\pi^-\eta'\to\pi^-\eta}(s) & 1+g_{\pi^-\eta}(s)N_{\pi^-\eta\to\pi^-\eta}(s) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} F_0^{\pi^-\eta}(0) \\ F_0^{\pi^-\eta'}(0) \end{pmatrix}$$

A closed-form solution is much less time-consuming than the ‘traditional’ iterative method, which is great for fits and MC generators (TAUOLA)

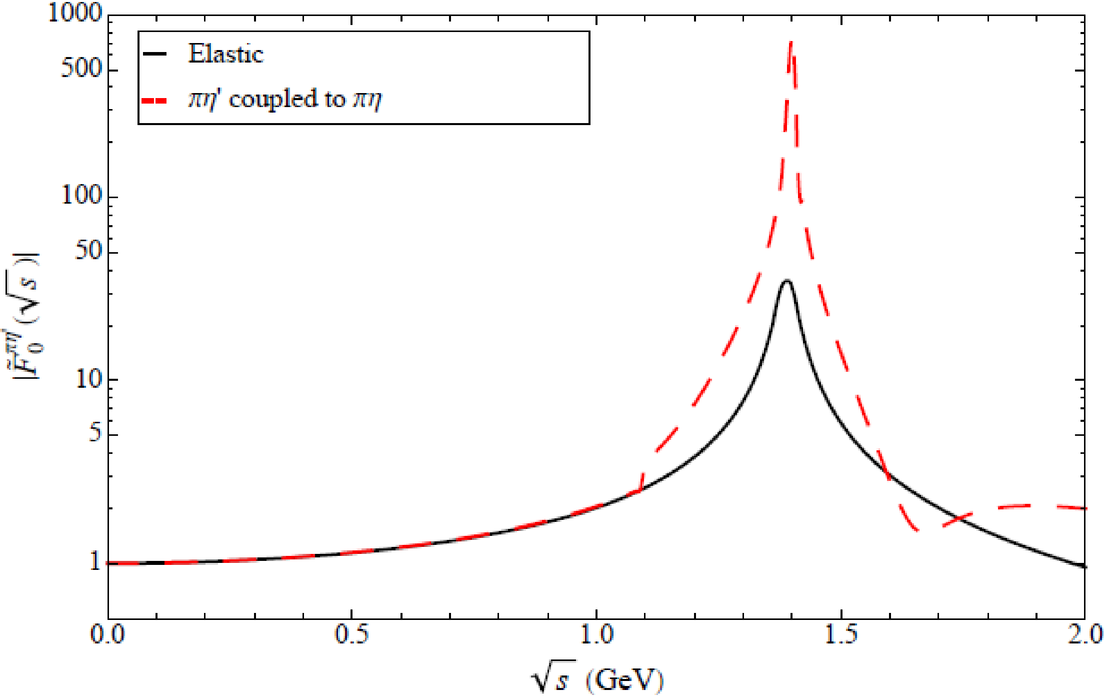
O. Shekhovtsova , T. Przedzinski, P. Roig & Z. Was. **Phys.Rev. D86 (2012) 113008**

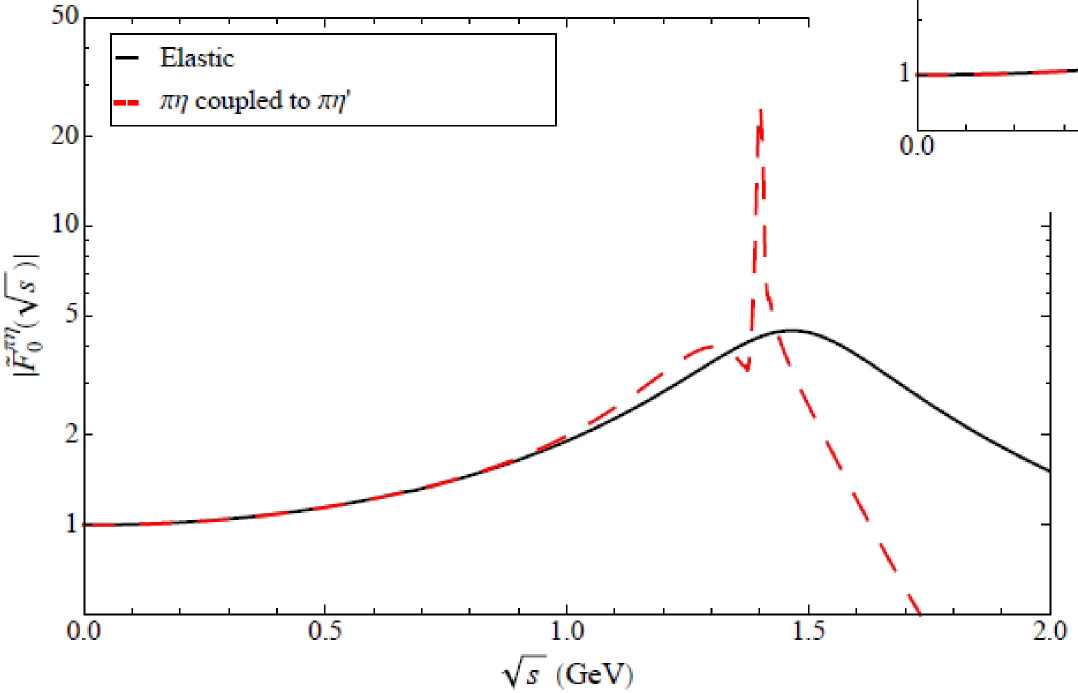
Relation between KK,πη,πη’ SFFs

SFF: Coupled channels case

Crucial coupled channels corrections!

Two coupled channels





Very different normalized FFs!

Relation between KK,πη,πη' SFFs

SFF: Coupled channels case

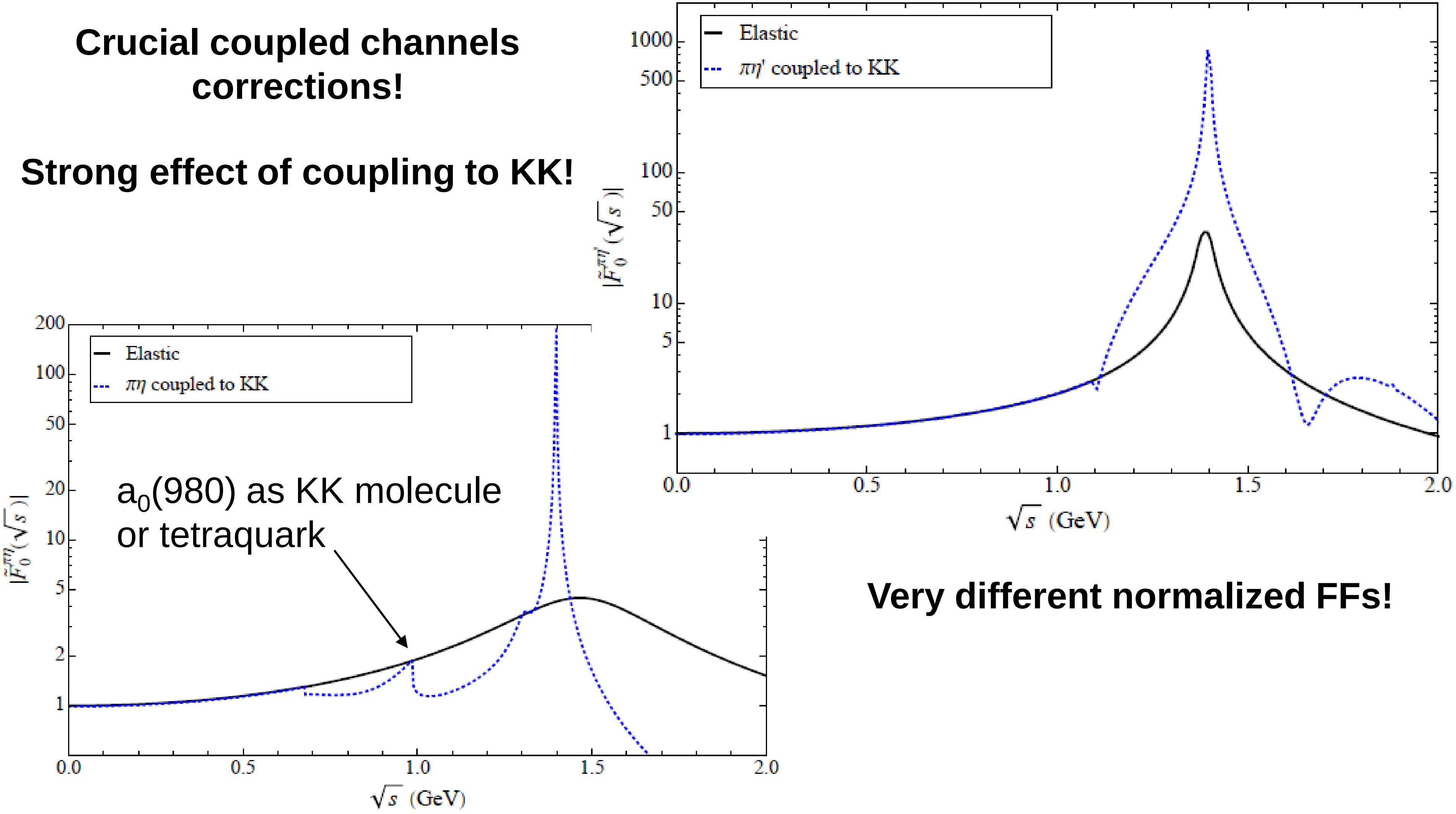
Two coupled channels

Closed-form solution

We can also consider coupling πη & πη' to KK

Crucial coupled channels corrections!

Strong effect of coupling to KK!



$a_0(980)$ as KK molecule or tetraquark

Very different normalized FFs!

Relation between KK,πη,πη' SFFs

SFF: Coupled channels case

Three coupled channels

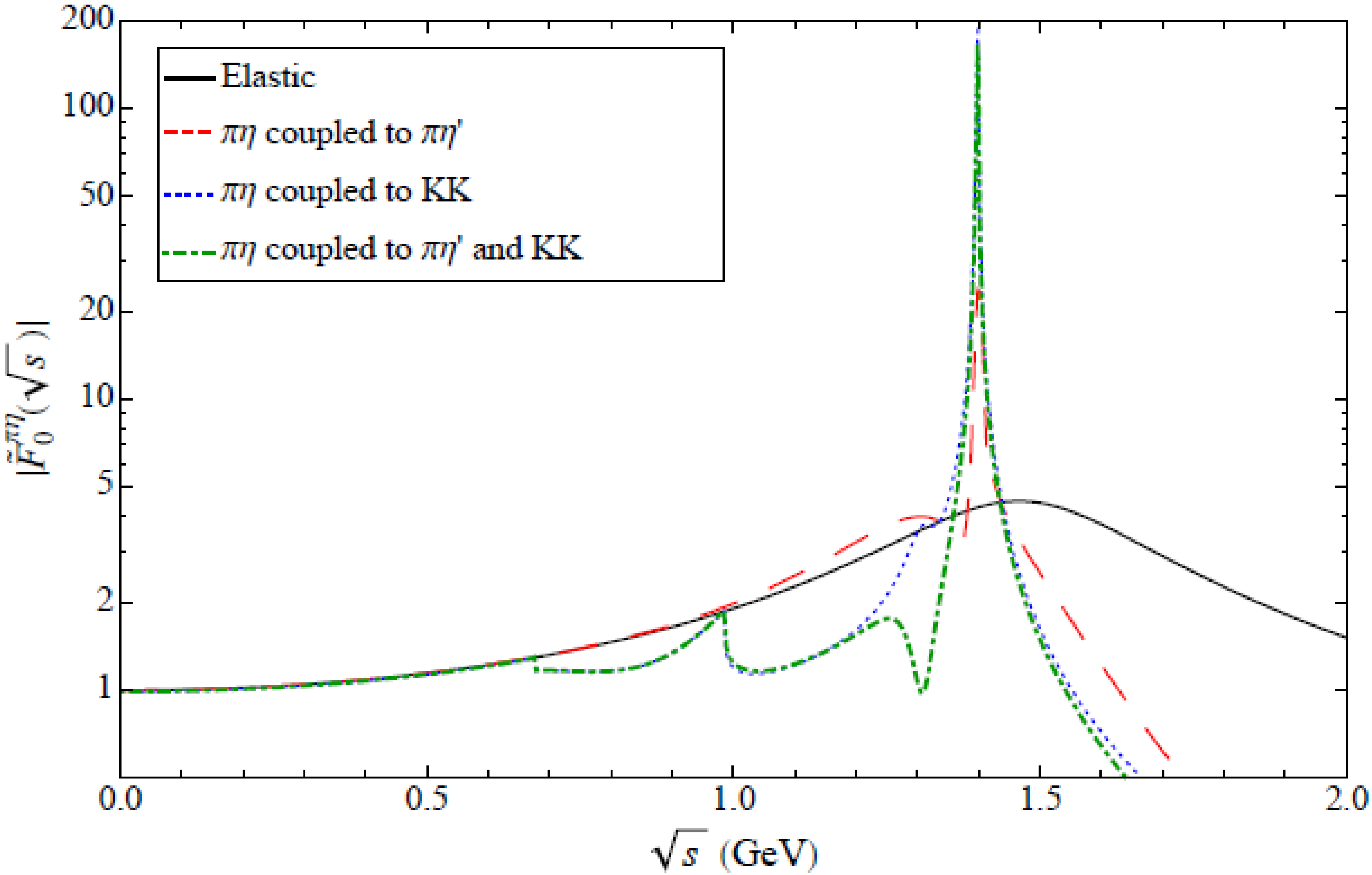
$$g_{1,0}(s) = \begin{pmatrix} g_{\pi^-\eta} & 0 & 0 \\ 0 & g_{KK} & 0 \\ 0 & 0 & g_{\pi^-\eta'} \end{pmatrix} \quad N_{1,0}(s) = \begin{pmatrix} N_{\pi^-\eta\to\pi^-\eta} & N_{\pi^-\eta\to\pi^-\eta'} & N_{\pi^-\eta\to K^-K^0} \\ N_{\pi^-\eta'\to\pi^-\eta} & N_{\pi^-\eta'\to\pi^-\eta'} & N_{\pi^-\eta'\to K^-K^0} \\ N_{K^-K^0\to\pi^-\eta} & N_{K^-K^0\to\pi^-\eta'} & N_{K^-K^0\to K^-K^0} \end{pmatrix}$$

Straightforward generalization of 2-coupled channels case (closed-form solution)

Relation between KK,πη,πη’ SFFs

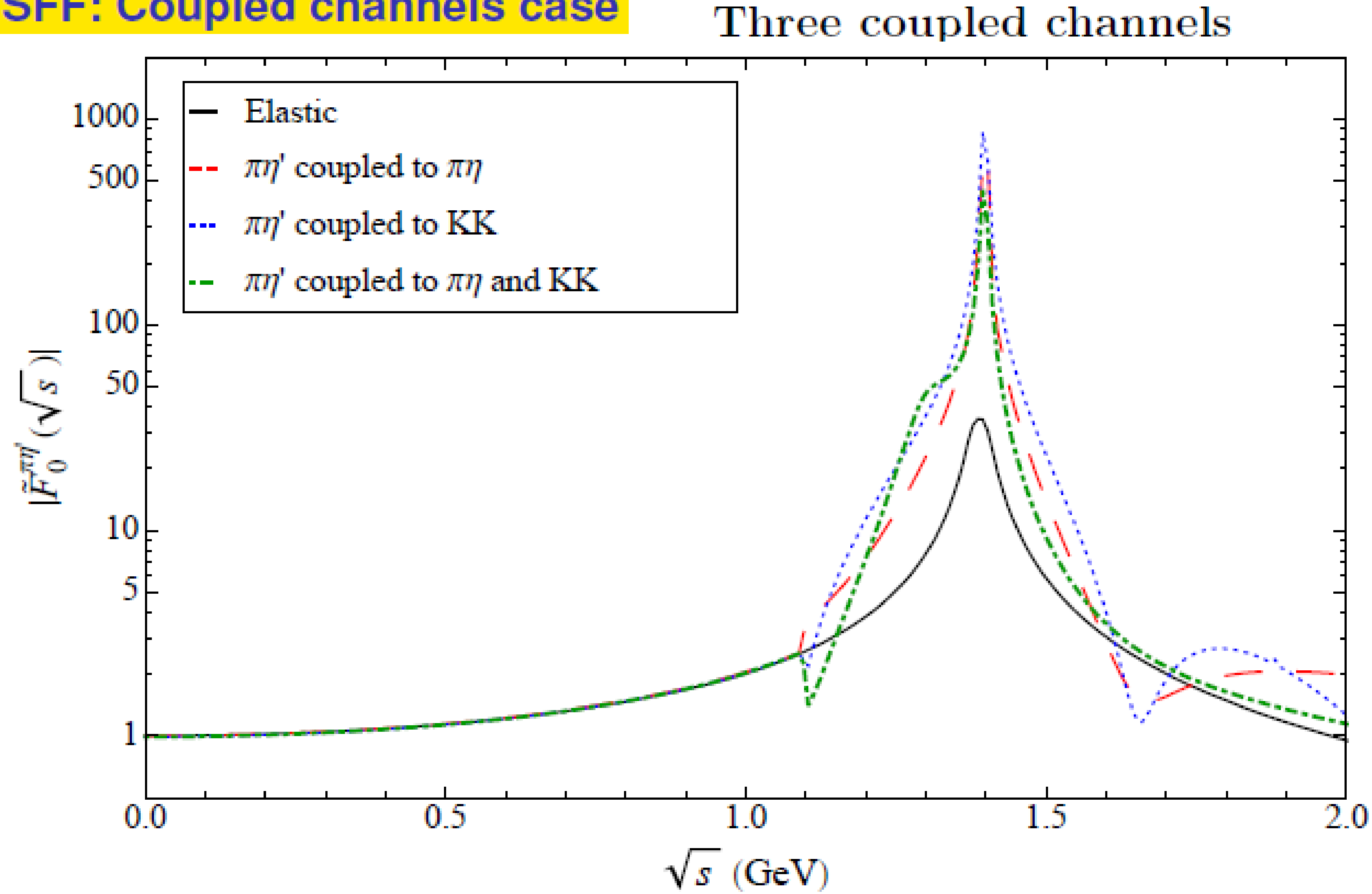
SFF: Coupled channels case

Three coupled channels



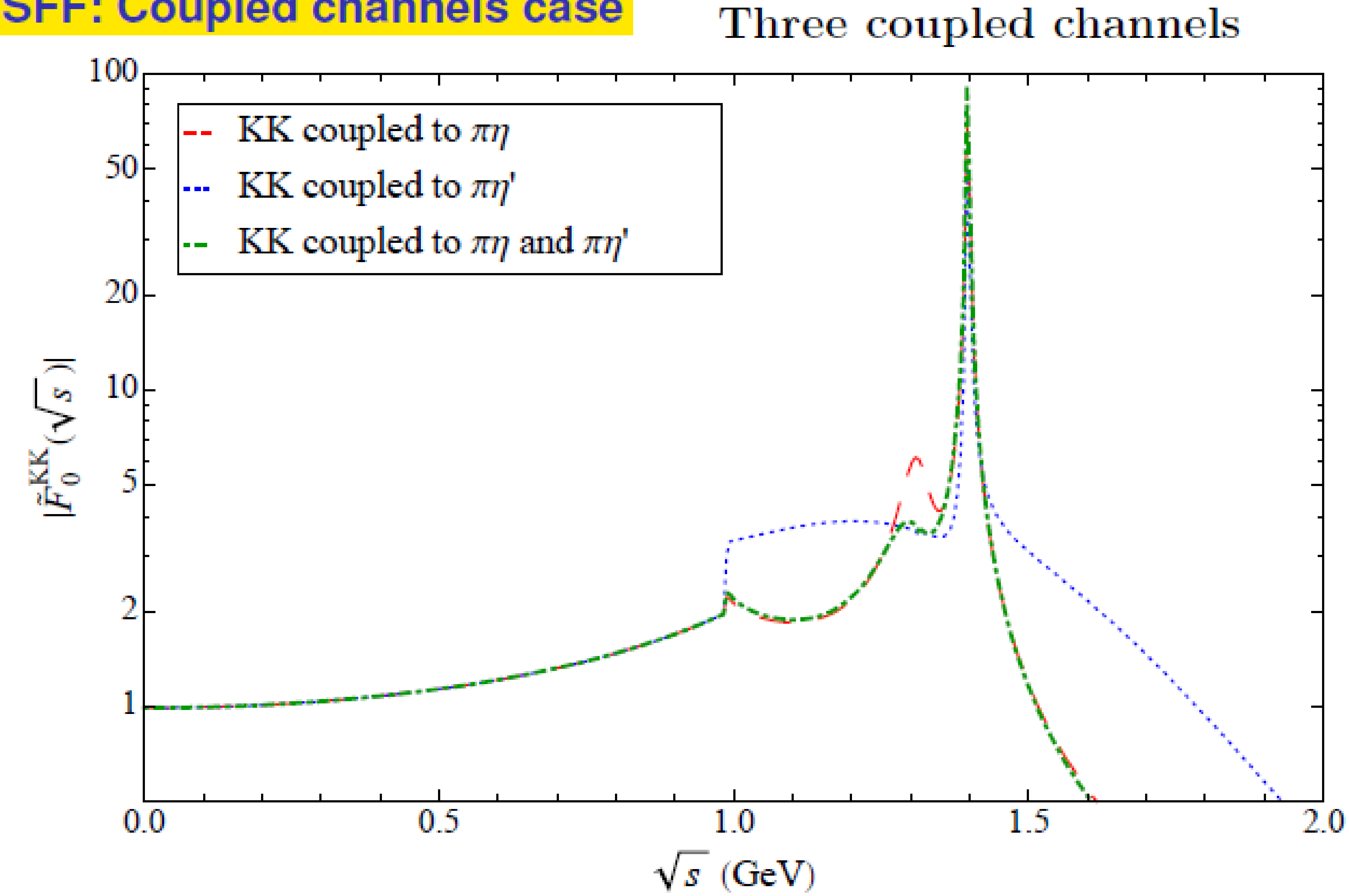
Relation between KK,πη,πη’ SFFs

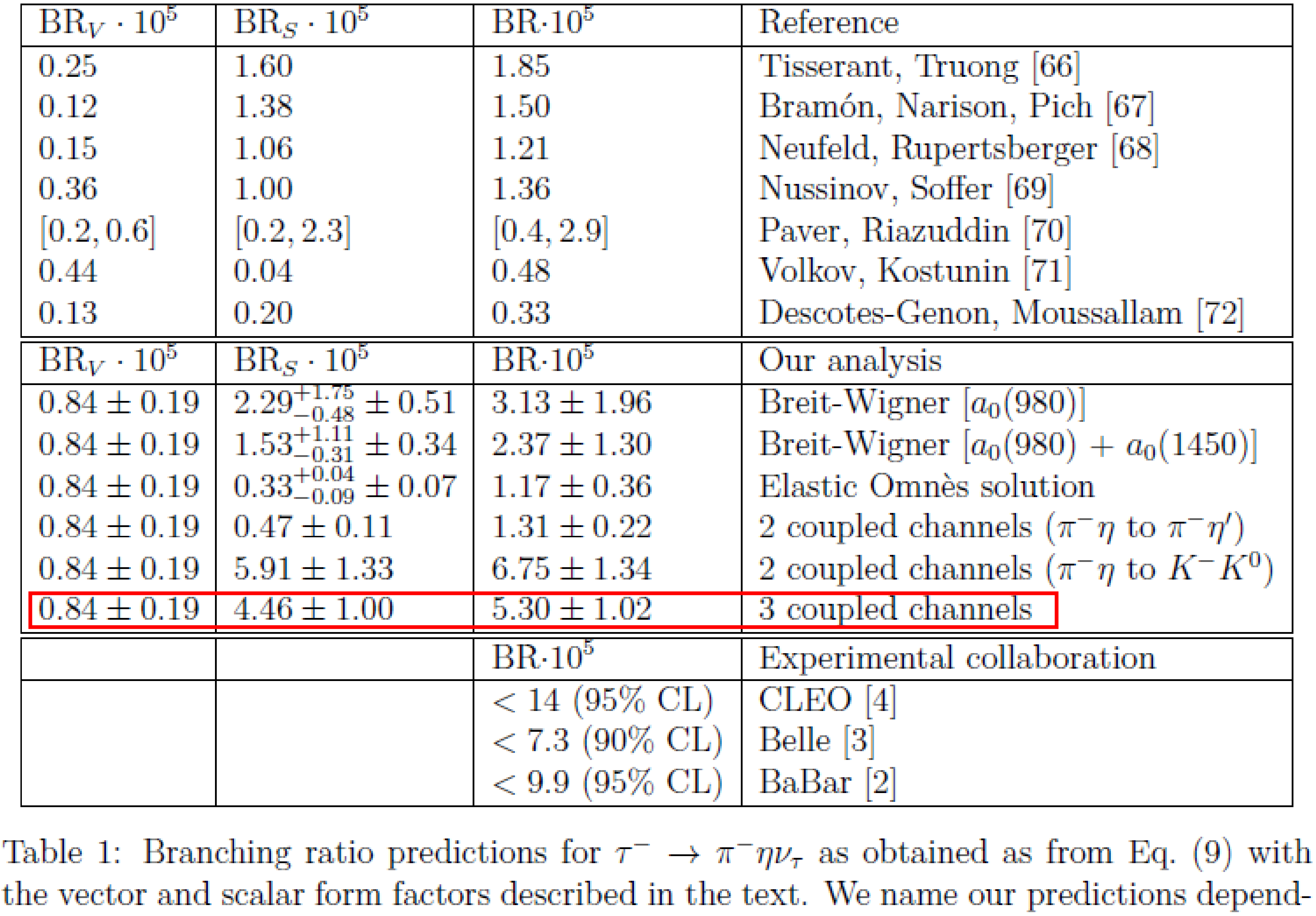
SFF: Coupled channels case



Relation between KK,πη,πη' SFFs

SFF: Coupled channels case





$BR_V \cdot 10^5$	$BR_S \cdot 10^5$	$BR \cdot 10^5$	Reference
0.25	1.60	1.85	Tisserant, Truong [66]
0.12	1.38	1.50	Bramón, Narison, Pich [67]
0.15	1.06	1.21	Neufeld, Rupertsberger [68]
0.36	1.00	1.36	Nussinov, Soffer [69]
$[0.2, 0.6]$	$[0.2, 2.3]$	$[0.4, 2.9]$	Paver, Riazuddin [70]
0.44	0.04	0.48	Volkov, Kostunin [71]
0.13	0.20	0.33	Descotes-Genon, Moussallam [72]

$BR_V \cdot 10^5$	$BR_S \cdot 10^5$	$BR \cdot 10^5$	Our analysis
0.84 ± 0.19	$2.29^{+1.75}_{-0.48} \pm 0.51$	3.13 ± 1.96	Breit-Wigner $[a_0(980)]$
0.84 ± 0.19	$1.53^{+1.11}_{-0.31} \pm 0.34$	2.37 ± 1.30	Breit-Wigner $[a_0(980) + a_0(1450)]$
0.84 ± 0.19	$0.33^{+0.04}_{-0.09} \pm 0.07$	1.17 ± 0.36	Elastic Omnès solution
0.84 ± 0.19	0.47 ± 0.11	1.31 ± 0.22	2 coupled channels ($\pi^-\eta$ to $\pi^-\eta'$)
0.84 ± 0.19	5.91 ± 1.33	6.75 ± 1.34	2 coupled channels ($\pi^-\eta$ to K^-K^0)
0.84 ± 0.19	4.46 ± 1.00	5.30 ± 1.02	3 coupled channels

		$BR \cdot 10^5$	Experimental collaboration
		< 14 (95% CL)	CLEO [4]
		< 7.3 (90% CL)	Belle [3]
		< 9.9 (95% CL)	BaBar [2]

Table 1: Branching ratio predictions for $\tau^- \to \pi^-\eta\nu_\tau$ as obtained as from Eq. (9) with the vector and scalar form factors described in the text. We name our predictions depend-

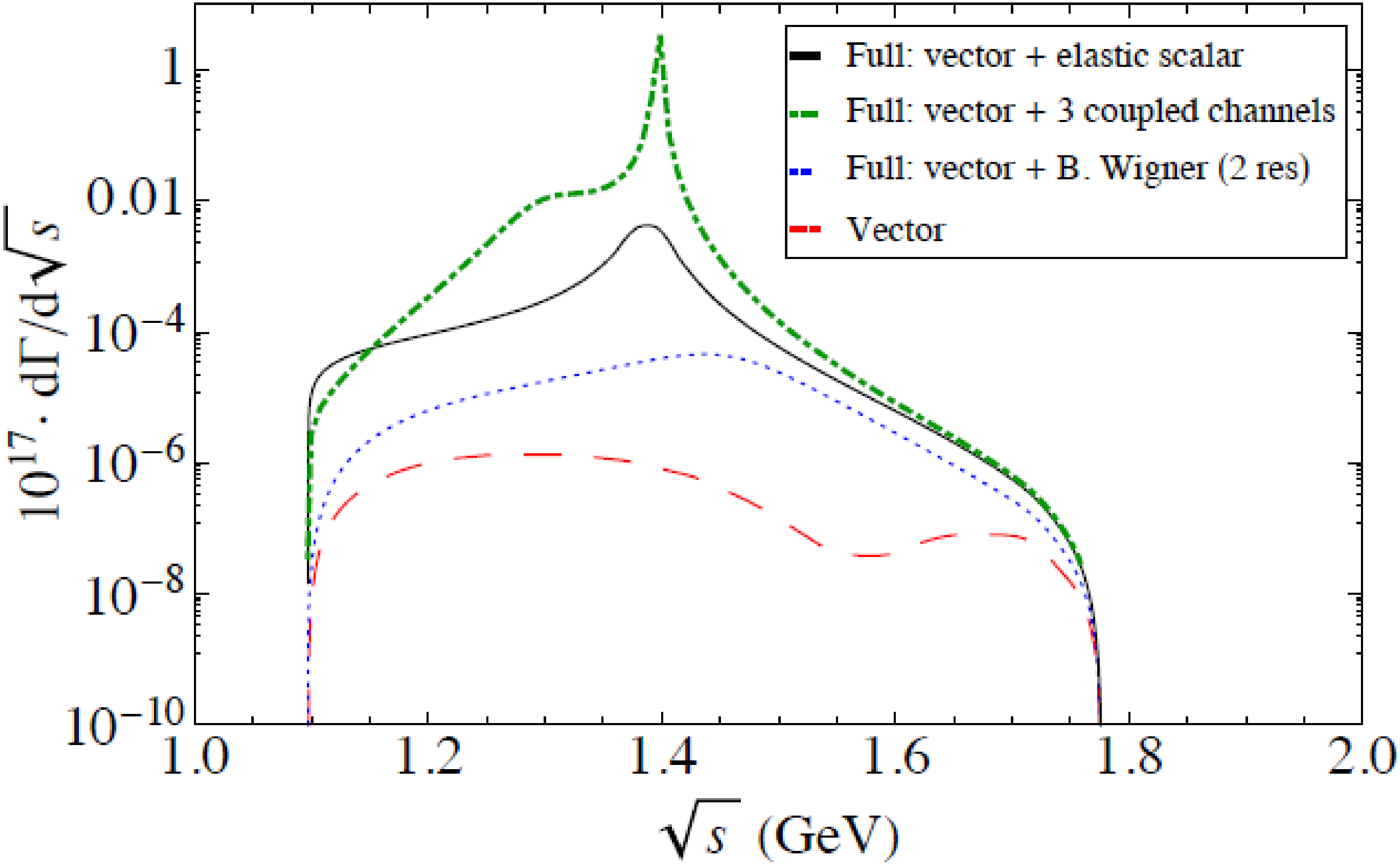


Figure 10: Decay distribution for $\tau^- \to \pi^- \eta' \nu_\tau$ as obtained as from Eq. (9)

Errors due to π-η-η' mixing

$\varepsilon_{\pi\eta}$ ~ 0.018(2)
$\varepsilon_{\pi\eta'}$ ~ 0.005(1)
(Kroll Mod.Phys.Lett. A20 (2005))

$$\varepsilon_{\pi\eta} = 0.0134$$
$$\varepsilon_{\pi\eta'} = (3 \pm 1) \cdot 10^{-3}$$

$$\varepsilon_{\pi\eta} = 0.0155$$
$$\varepsilon_{\pi\eta'} = 0.00679$$

Source	table 1		*set 1*		*set 2*	
	$\mathrm{BR}_S \cdot 10^5$	$\mathrm{BR}\cdot 10^5$	$\mathrm{BR}_S \cdot 10^5$	$\mathrm{BR}\cdot 10^5$	$\mathrm{BR}_S \cdot 10^5$	$\mathrm{BR}\cdot 10^5$
B. W. (1 res)	$2.29^{+1.75}_{-0.48} \pm 0.51$	3.13 ± 1.96	$1.42^{+1.08}_{-0.30} \pm 0.04$	1.94 ± 1.13	$2.08^{+1.59}_{-0.43}$	2.84 ± 1.65
B. W. (2 res)	$1.53^{+1.11}_{-0.31} \pm 0.34$	2.37 ± 1.30	$0.94^{+0.69}_{-0.19} \pm 0.03$	1.46 ± 0.74	$1.39^{+1.01}_{-0.28}$	2.15 ± 1.05
Omnès	$0.33^{+0.04}_{-0.09} \pm 0.07$	1.17 ± 0.36	$0.21^{+0.02}_{-0.06} \pm 0.06$	0.73 ± 0.19	$0.31^{+0.04}_{-0.09}$	1.06 ± 0.10
$\pi^-\eta$ to $\pi^-\eta'$	0.47 ± 0.11	1.31 ± 0.22	0.29(1)	0.81(17)	0.43	1.19
$\pi^-\eta$ to K^-K^0	5.91 ± 1.33	6.75 ± 1.34	3.66(12)	4.18(21)	5.38	6.14
3 c.c.	4.46 ± 1.00	5.30 ± 1.02	2.76(9)	3.28(19)	4.06	4.82

Table 2: Branching ratio predictions for $\tau^- \to \pi^-\eta\nu_\tau$ obtained by considering $\varepsilon_{\pi\eta} = 0.0134$ and $\varepsilon_{\pi\eta'} = (3 \pm 1) \cdot 10^{-3}$ for the *set 1* and $\varepsilon_{\pi\eta} = 0.0155$ and $\varepsilon_{\pi\eta'} = 0.00679$ for the *set 2*, as explained in the text, compared with results given in Table 1. The source of the uncertainty in *set 1* arise from the errors on $\varepsilon_{\pi\eta}$ and $\varepsilon_{\pi\eta'}$ (first and second error on the vector and scalar form factor respectively) and from the (uncorrelated) errors on the input values (second error on the scalar form factor). In *set 2* the error is only due to the latter.

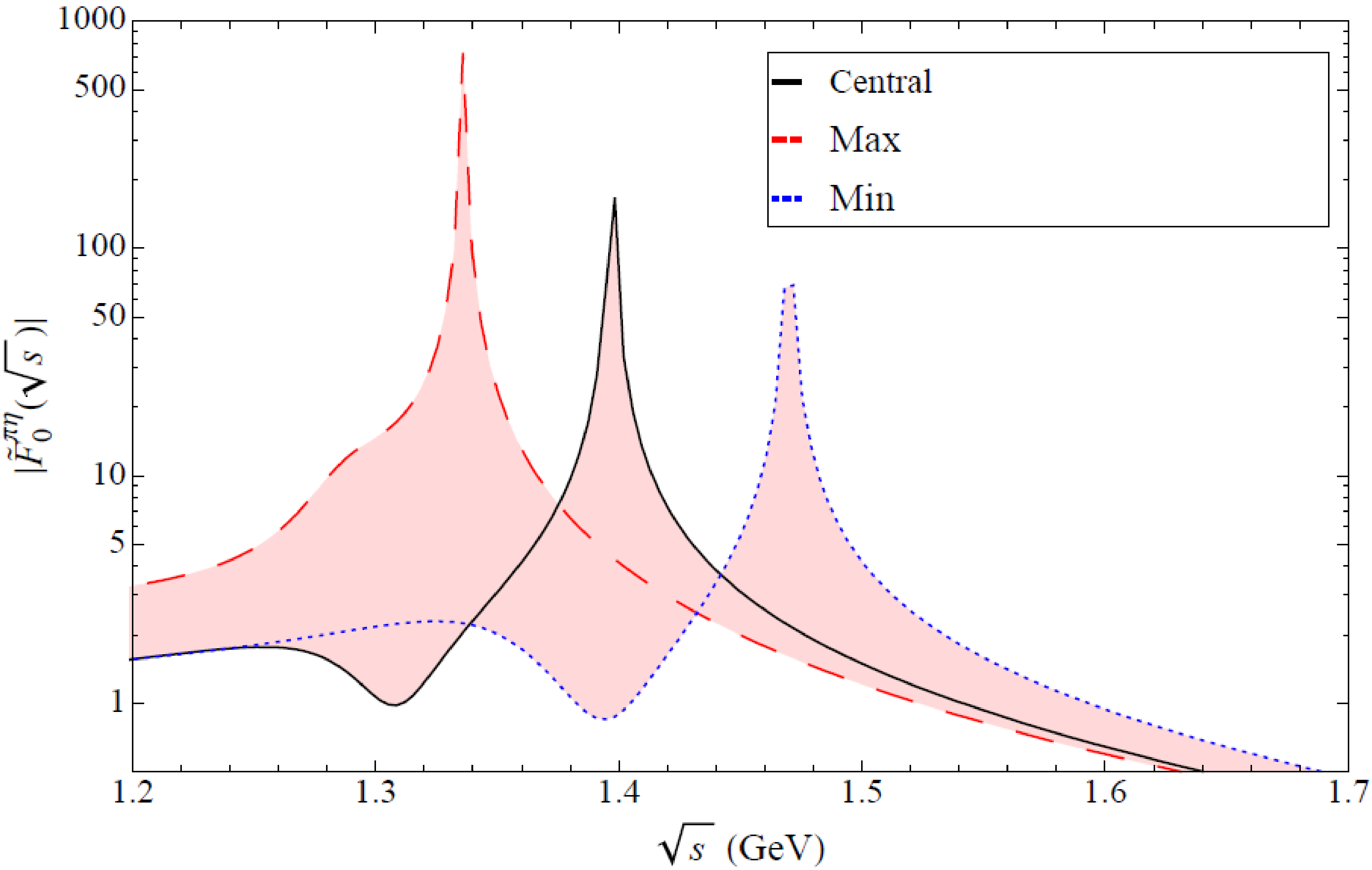
Our best theoretical estimate,

$$F_+^{\pi^-\eta}(0) = \varepsilon_{\pi\eta}\cos\theta - \varepsilon_{\pi\eta'}\sin\theta$$
$$F_+^{\pi\ \eta'}(0) = \varepsilon_{\pi\eta'}\cos\theta + \varepsilon_{\pi\eta}\sin\theta$$

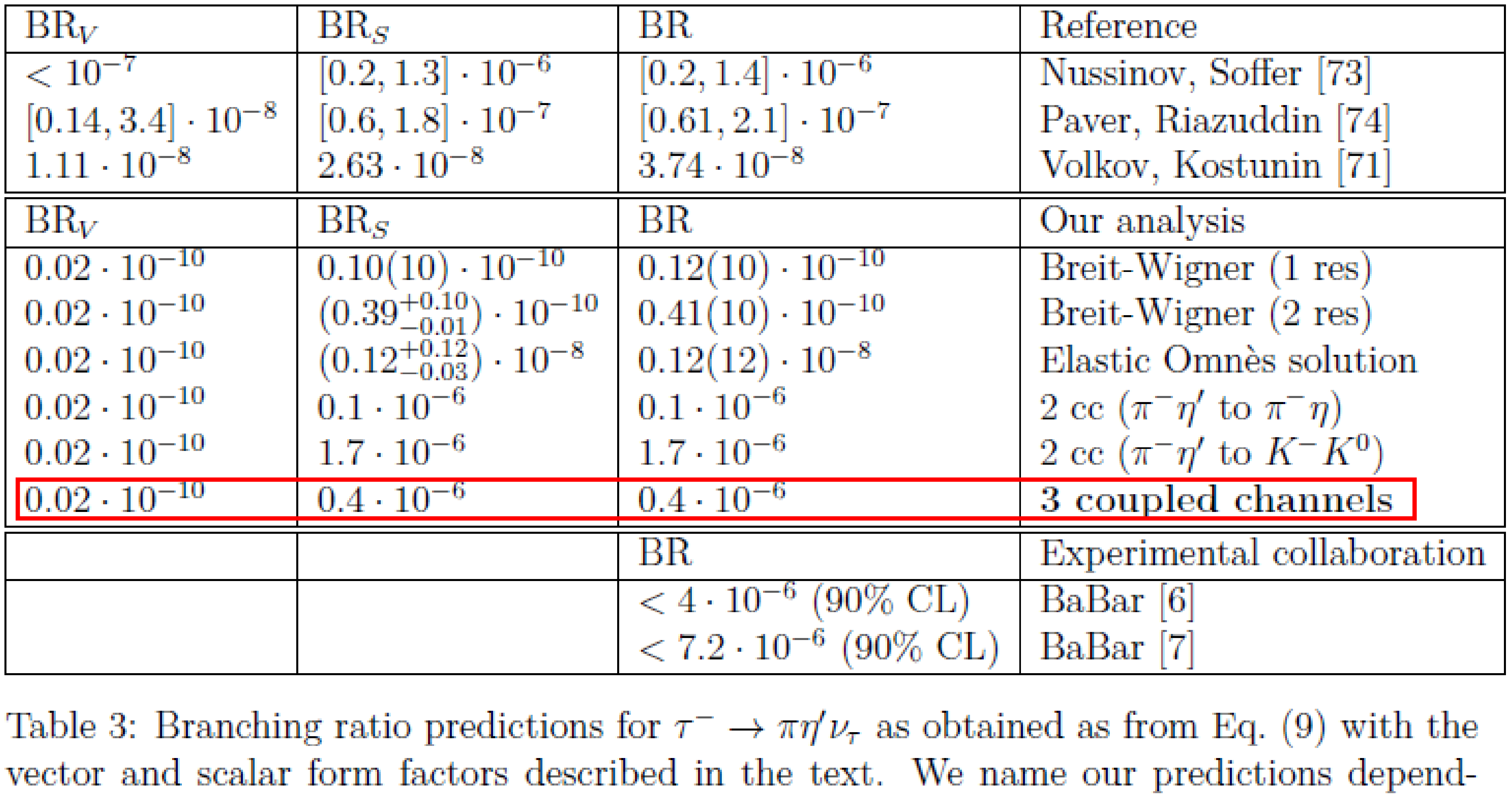
The finiteness of the matrix element at the origin imposes

$$F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{c^S_{\pi^-\eta^{(\prime)}}}{c^V_{\pi^-\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}}F_0^{\pi^-\eta^{(\prime)}}(0)\,.$$

Meaningless uncorrelated error analysis



Predictions for $\pi\eta'$



BR_V	BR_S	BR	Reference
$< 10^{-7}$	$[0.2, 1.3] \cdot 10^{-6}$	$[0.2, 1.4] \cdot 10^{-6}$	Nussinov, Soffer [73]
$[0.14, 3.4] \cdot 10^{-8}$	$[0.6, 1.8] \cdot 10^{-7}$	$[0.61, 2.1] \cdot 10^{-7}$	Paver, Riazuddin [74]
$1.11 \cdot 10^{-8}$	$2.63 \cdot 10^{-8}$	$3.74 \cdot 10^{-8}$	Volkov, Kostunin [71]
BR_V	BR_S	BR	Our analysis
$0.02 \cdot 10^{-10}$	$0.10(10) \cdot 10^{-10}$	$0.12(10) \cdot 10^{-10}$	Breit-Wigner (1 res)
$0.02 \cdot 10^{-10}$	$(0.39^{+0.10}_{-0.01}) \cdot 10^{-10}$	$0.41(10) \cdot 10^{-10}$	Breit-Wigner (2 res)
$0.02 \cdot 10^{-10}$	$(0.12^{+0.12}_{-0.03}) \cdot 10^{-8}$	$0.12(12) \cdot 10^{-8}$	Elastic Omnès solution
$0.02 \cdot 10^{-10}$	$0.1 \cdot 10^{-6}$	$0.1 \cdot 10^{-6}$	2 cc ($\pi^-\eta'$ to $\pi^-\eta$)
$0.02 \cdot 10^{-10}$	$1.7 \cdot 10^{-6}$	$1.7 \cdot 10^{-6}$	2 cc ($\pi^-\eta'$ to K^-K^0)
$0.02 \cdot 10^{-10}$	$0.4 \cdot 10^{-6}$	$0.4 \cdot 10^{-6}$	**3 coupled channels**
		BR	Experimental collaboration
		$< 4 \cdot 10^{-6}$ (90% CL)	BaBar [6]
		$< 7.2 \cdot 10^{-6}$ (90% CL)	BaBar [7]

Table 3: Branching ratio predictions for $\tau^- \to \pi\eta'\nu_\tau$ as obtained as from Eq. (9) with the vector and scalar form factors described in the text. We name our predictions depend-

Science-fiction measurement?

$\eta_{\ell 3}^{(\prime)}$ $\quad\quad$ $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$

$m_\ell^2 < s < (m_{\eta^{(\prime)}} - m_\pi)^2$ $\quad\quad$ $(m_{\eta^{(\prime)}} + m_\pi)^2 < s < m_\tau^2$

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{G_F^2 |V_{ud} F_+(0)|^2 (c_V^{\pi\eta^{(\prime)}})^2 (s - m_l^2)^2}{24\pi^3 M_\eta^3 s}$$
$$\left\{ (2s + m_\ell^2) q_{\pi\eta^{(\prime)}}^3 \widetilde{F}_+(s)^2 + \frac{3}{4s} \Delta_{\pi\eta^{(\prime)}}^2 m_\ell^2 q_{\pi\eta^{(\prime)}} \widetilde{F}_0(s)^2 \right\}$$

Same FF's but they are real in η_{l3} while complex in τ decays

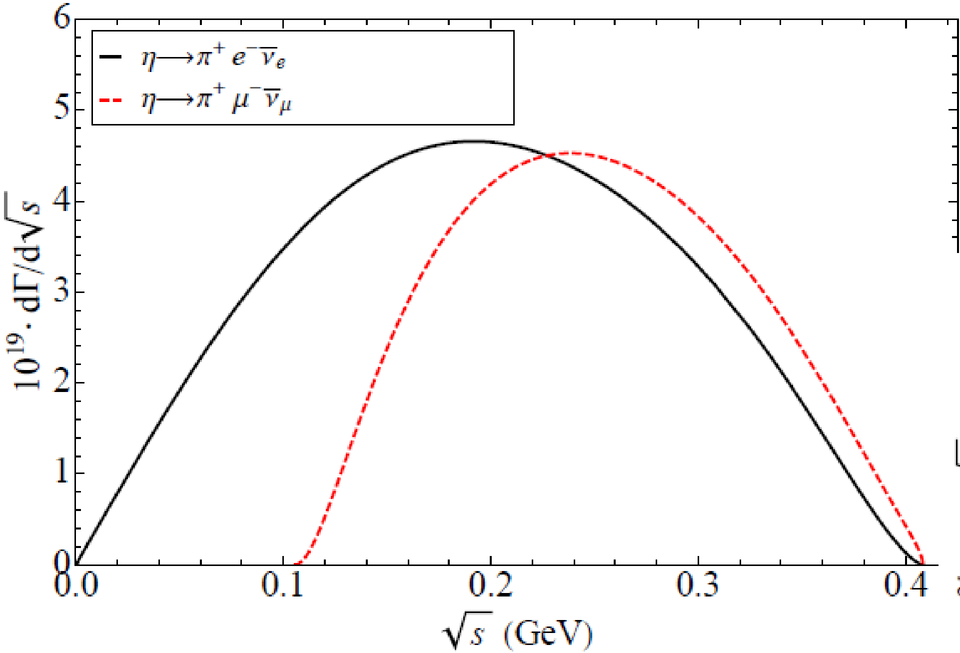
In η decays SFF is suppressed by m_l^2, while it dominates the corresponding τ decays

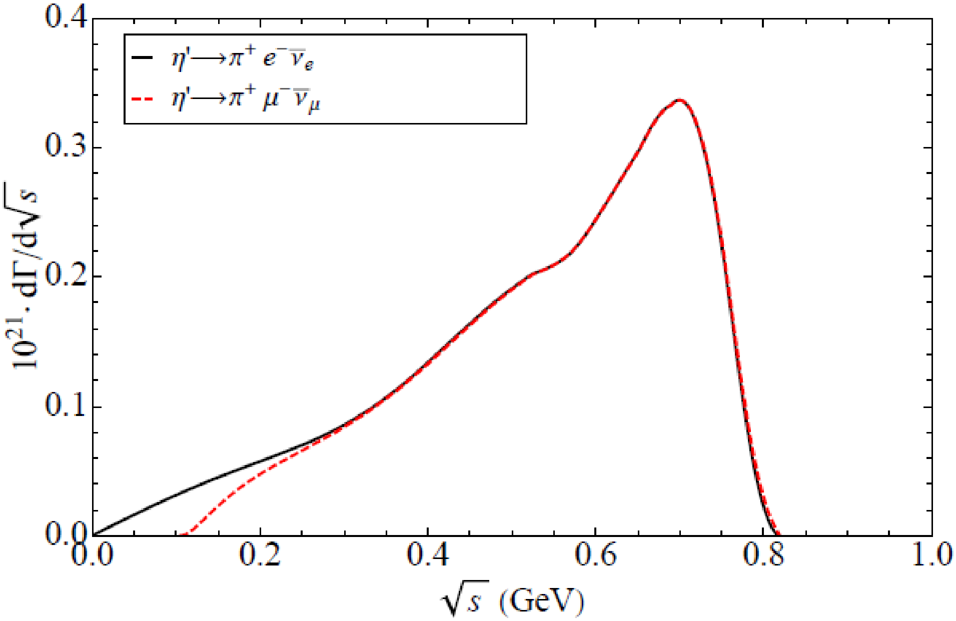
$$\mathcal{B}(\eta \to \pi^+ e^- \bar{\nu}_e + c.c.) < 1.7 \cdot 10^{-4} \text{ and } \mathcal{B}(\eta' \to \pi^+ e^- \bar{\nu}_e + c.c.) < 2.2 \cdot 10^{-4}$$

BESIII @ 90% CL

Science-fiction measurement?

Decay	Descotes-Genon, Moussallam [72]	Our estimate
$\eta \to \pi^+ e^- \nu_e + c.c.$	$\sim 1.40 \cdot 10^{-13}$	$1.8 \cdot 10^{-13}$
$\eta \to \pi^+ \mu^- \nu_\mu + c.c.$	$\sim 1.02 \cdot 10^{-13}$	$1.4 \cdot 10^{-13}$
$\eta' \to \pi^+ e^- \nu_\mu + c.c.$		$1.1 \cdot 10^{-18}$
$\eta' \to \pi^+ \mu^- \nu_\mu + c.c.$		$1.1 \cdot 10^{-18}$





CONCLUSIONS

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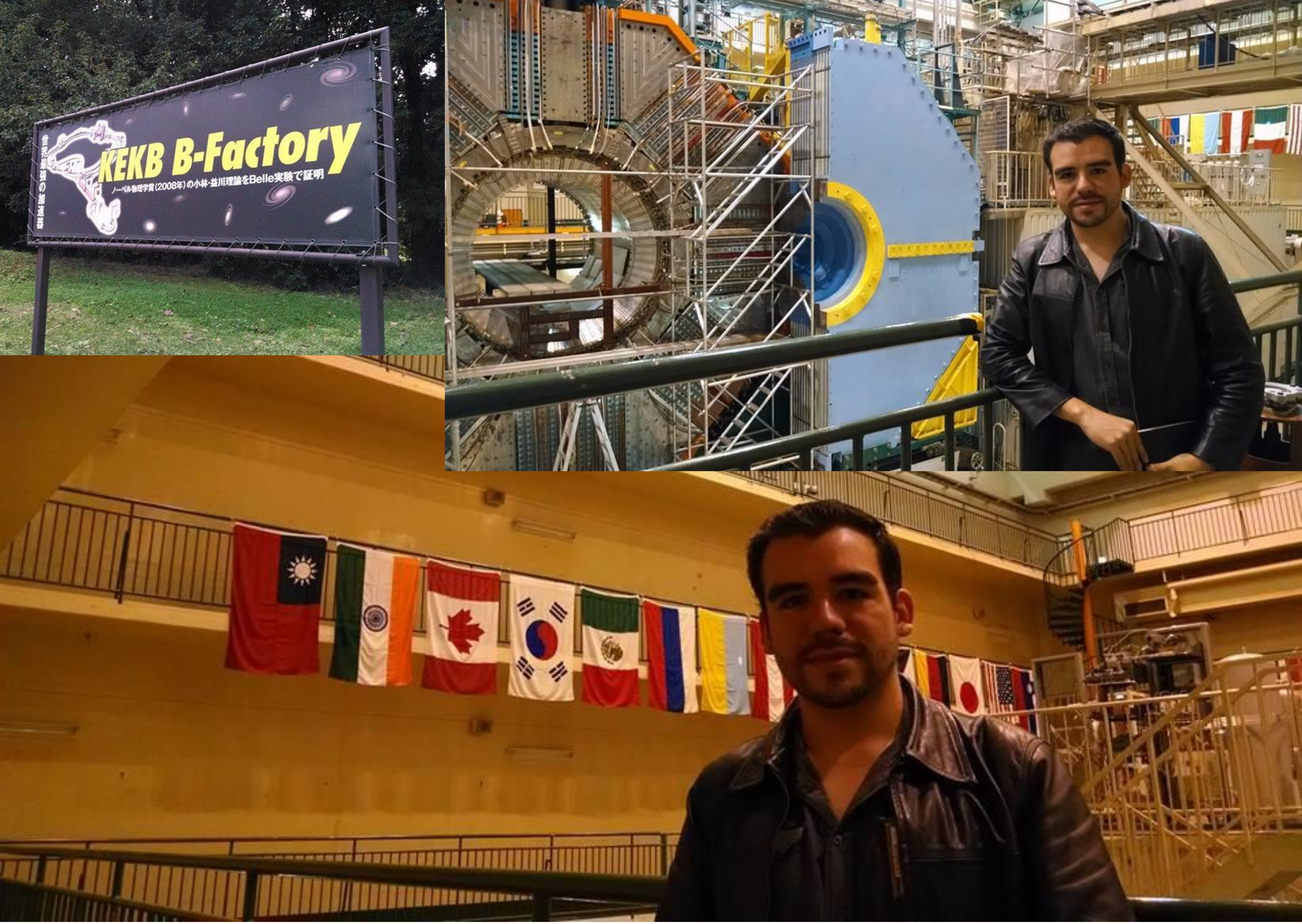
- The search for these decays will be driven by the Mexican node of the Belle-II Collaboration (Michel Hernández, Eduard & Gabriel’s student).


世界最強の加速器
KEKB B-Factory
ノーベル物理学賞（2008年）の小林・益川理論をBelle実験で証明

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- México is also contributing coding the relevant SM FFs in TAUOLA and predicting the most important SM bkg ($\tau\to\pi\eta^{(')}\gamma\nu_\tau$ Adolfo Guevara, Gabriel & PR).

ANUNCIO

Se oferta un posdoc de dos años (1+1) asociado al proyecto de Ciencia Básica del Dr. Gabriel López Castro (Cinvestav, México DF) en el área de "Física de sabor: fenomenología" para comenzar en Abril de 2016. La convocatoria se realizará próximamente a través de las listas de correo de la DPyC y RED-FAE.