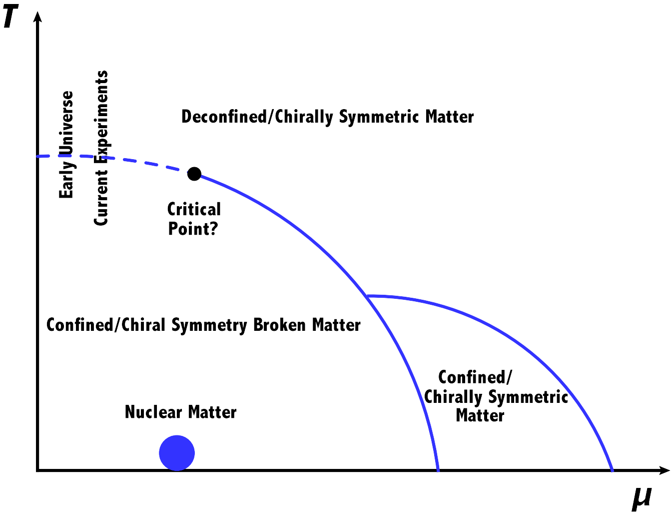
# Hadronic Matter at the Edge

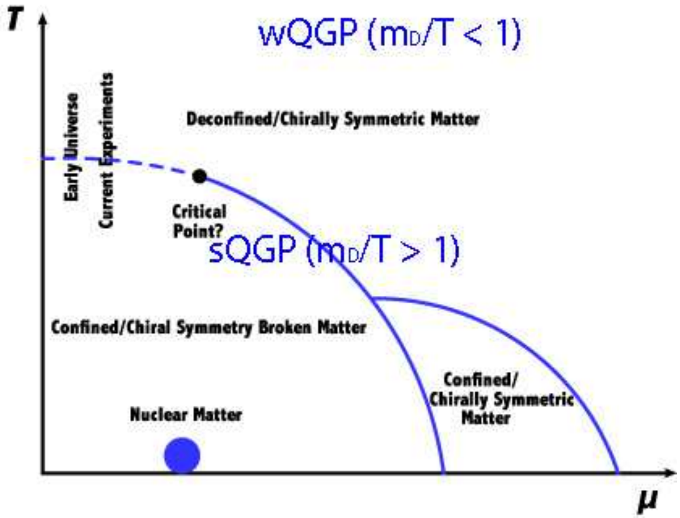
Alejandro Ayala (ICN-UNAM)

XV Mexican Workshop on Particles and Fields
Mazatlan, 2015

# QCD Phase Diagram



# QCD Phase Diagram



# **Edges** on the phase diagram

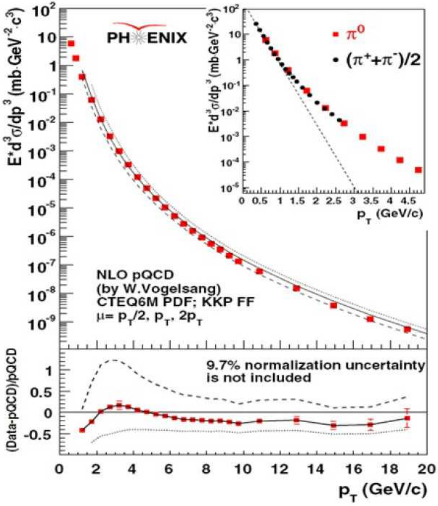
Can we locate the boundaries?

- Soft/hard boundary (transition between weak and strong coupling regime)
- Microscopic/macroscopic boundary (transition between large and small mean free path)
- **Critical End Point**

## Soft/hard boundary

- How small should $p_T$ be before non-perturbative effects dominate?
- What are the conditions to describe colliding hadrons in terms of **perturbative** quarks and gluons?
- What are the conditions to describe colliding hadrons in terms of **non-perturbative** constituent quarks or strings?

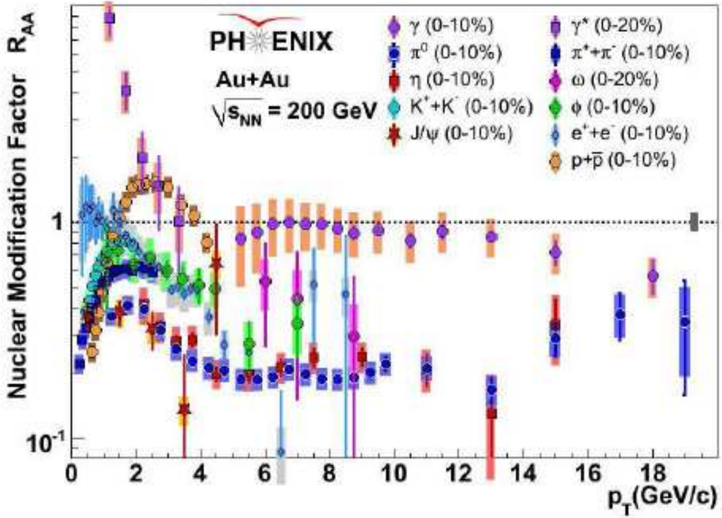
# pQCD does a good job in p+p for $p_T \geq 2$ GeV



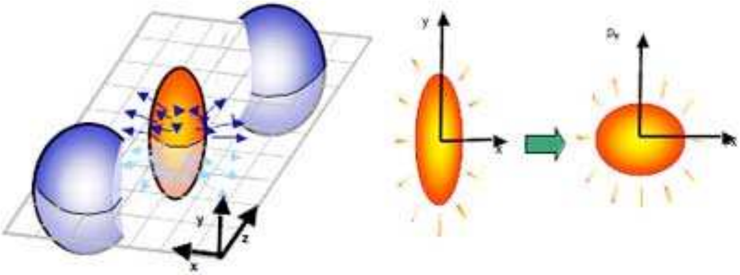
# Microscopic/macroscopic boundary

- The microscopic scale is the **mean free path**. On general grounds one can employ **macroscopic theories** when the mean free path is small compared to the system's size.
- A+A, p+A p+p collisions with a large spread in multiplicity show **collective behavior ($R_{AA}$ suppression, flow)**
- Important to study these systems as a function of multiplicity to look for a change of regime

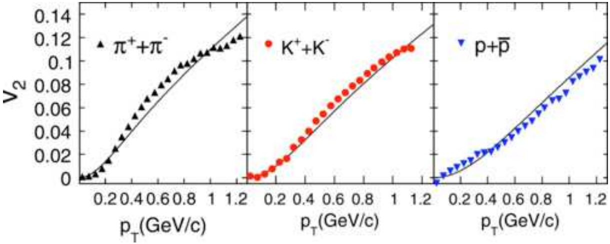
# Collective behavior in AA



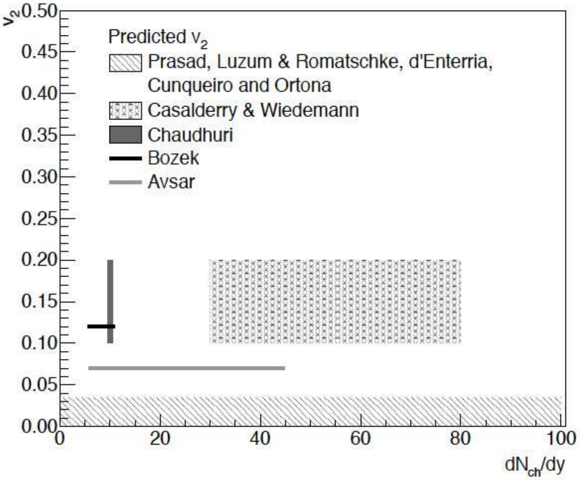
# Collective behavior in AA



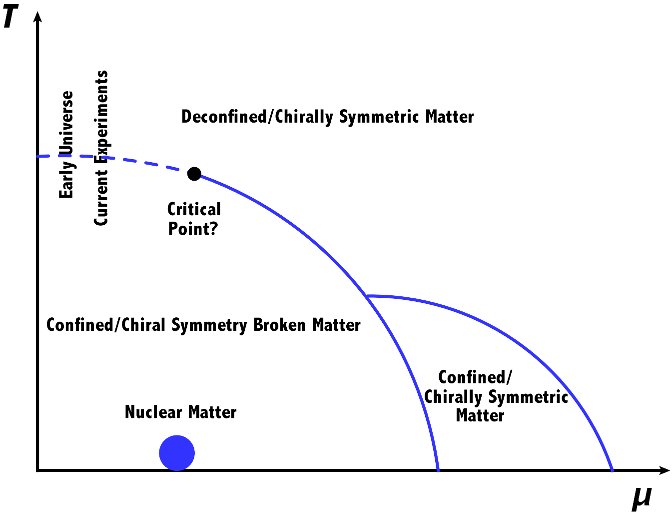
# Collective behavior in AA



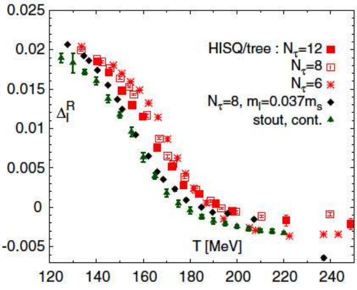
# Collective behavior in pp for high multiplicity events

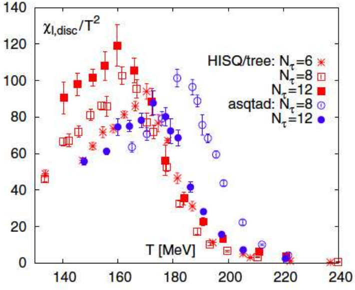


# Coming back to the QCD phase diagram



# Theoretical tools: light quark condensate $\langle\bar{\psi}\psi\rangle$ from lattice QCD ($\mu = 0$)

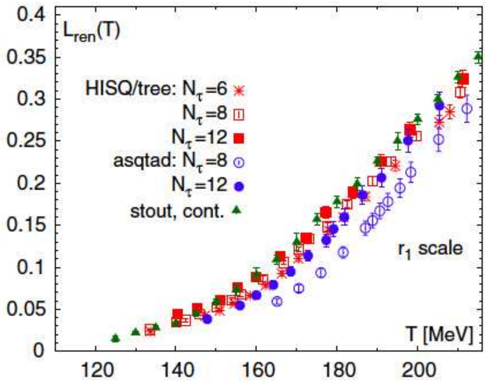



A. Bazavov *et al.*, Phys. Rev. D **85**, 054503 (2012).

# Theoretical Tools: Polyakov Loop form lattice QCD

$$\langle \mathrm{Tr}\ L \rangle \propto e^{-\Delta F_q / T}$$




A. Bazavov *et al.*, Phys. Rev. D **85**, 054503 (2012).

# Critical temperatures from lattice QCD ($\mu = 0$)

- $T_c$ from the susceptibility's peak for 2+1 flavors using different kinds of fermion representations.
- Values show some discrepancies:
- The MILC collaboration obtains $T_c = 169(12)(4)$ MeV.
- The RBC-Bielefeld collaboration reports $T_c = 192(7)(4)$ MeV.
- The Wuppertal-Budapest collaboration has consistently obtained smaller values, the last being $T_c = 147(2)(3)$ MeV.
- The HotQCD collaboration has reported $T_c = 154(9)$ MeV.
- Differences may be attributed to different lattice spacings.

# For $\mu \neq 0$ matters get complicated

- Lattice QCD is affected by the **sign problem**
- The calculation of the partition function produces a fermion determinant.

$$\mathrm{Det}M = \mathrm{Det}(\not{D} + m + \mu\gamma_0)$$

- Consider a complex value for $\mu$. Take the determinant on both sides of the identity

$$\gamma_5(\not{D} + m + \mu\gamma_0)\gamma_5 = (\not{D} + m - \mu^*\gamma_0)^\dagger,$$

we obtain

$$\mathrm{Det}(\not{D} + m + \mu\gamma_0) = [\mathrm{Det}(\not{D} + m - \mu^*\gamma_0)]^*\,,$$

This shows that **the determinant is not real unless** $\mu = 0$ **or purely imaginary**.

## The sign problem

- For real $\mu$ **it is not possible to carry out the direct sampling on a finite density ensemble by Monte Carlo methods**
- It'd seem that the problem is not so bad since we could naively write

$$\mathrm{Det}M = |\mathrm{Det}M| e^{i\theta}$$

- To compute the thermal average of an observable $O$ we write

$$\langle O \rangle = \frac{\int DUe^{-S_{YM}} \mathrm{Det}M \; O}{\int DUe^{-S_{YM}} \mathrm{Det}M} = \frac{\int DUe^{-S_{YM}} |\mathrm{Det}M| e^{i\theta} \; O}{\int DUe^{-S_{YM}} |\mathrm{Det}M| e^{i\theta}},$$

- $S_{YM}$ is the Yang-Mills action.

# The sign problem

- Note that written in this way, the simulations can be made in terms of the *phase quenched theory* where the measure involves $|\text{Det}M|$ and the thermal average can be written as

$$\langle O \rangle = \frac{\langle O e^{i\theta} \rangle_{\text{pq}}}{\langle e^{i\theta} \rangle_{\text{pq}}}.$$

- The average phase factor (also called the average sign) in the*phase quenched theory* can be written as

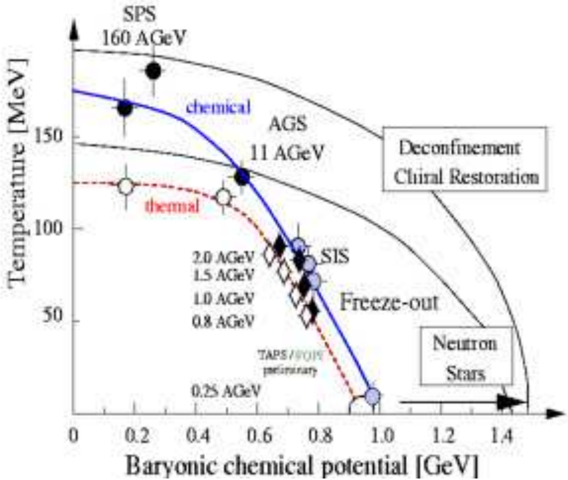
$$\langle e^{i\theta} \rangle_{\text{pq}} = e^{-V(f - f_{pq})/T},$$

  where $f$ y $f_{pq}$ are the free energy densities of the full and the phase quenched theories, respectively and $V$ is the 3-dimensional volume.
- If $f - f_{pq} \neq 0$, the average phase factor decreaces exponentially when $V$ grows (thermodynamical limit) and/or when $T$ goes to zero.
- Under these circumstances the signal/noise ratio worsens. This is known as the *severe sign problem*.

# Alternatives for $\mu \neq 0$

- In lattice QCD it is possible to make a Taylor expansion for small $\mu$.
- The expansion coefficients can be expressed as the expectation values of traces of polynomial matrices taken on the ensemble with $\mu = 0$.
- Although care has to be taken with the growing of the statistical error, this strategy gives rise to an important result: The **curvature** $\kappa$ **of the transition curve** para $\mu = 0$.
- Values for $\kappa$=0.01–0.04 have been reported.
- These values are considerably smaller than those of the **chemical freeze-out** curve.

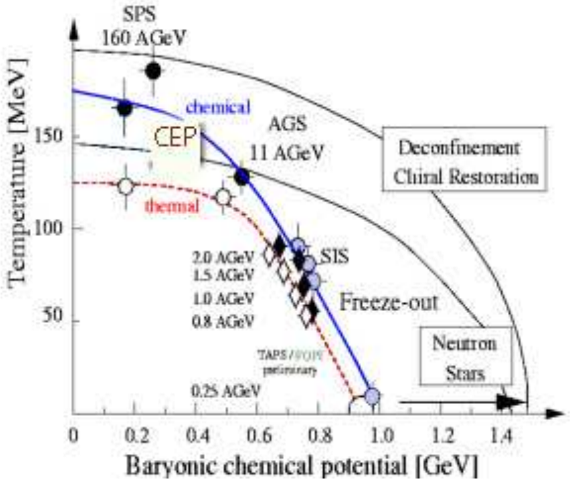
# Chemical freeze-out



# CEP's Location

- Mathematical extensions of Lattice QCD:
  $(\mu^{\text{CEP}}/T_c, T^{\text{CEP}}/T_c) \sim (1.0\text{–}1.4\ ,\ 0.9\text{–}9.5)$

## Chemical freeze-out and CEP location

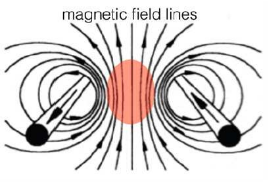


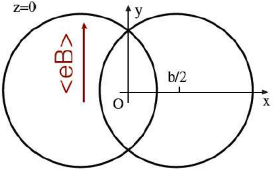
Q: Can we get any help from an external probe?

A: Try using a magnetic field

# Magnetic fields in peripheral Heavy-Ion Collisions

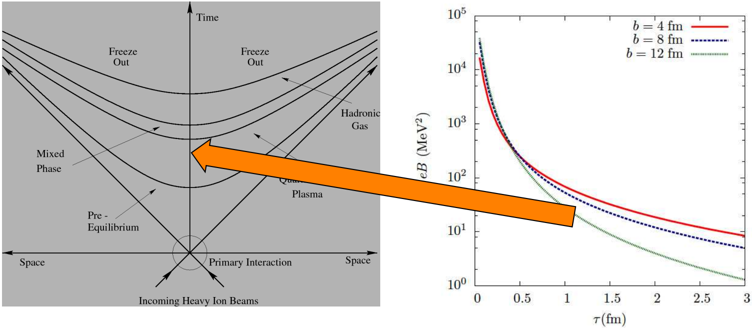
- Generated in the interaction region by the (charged) colliding nuclei



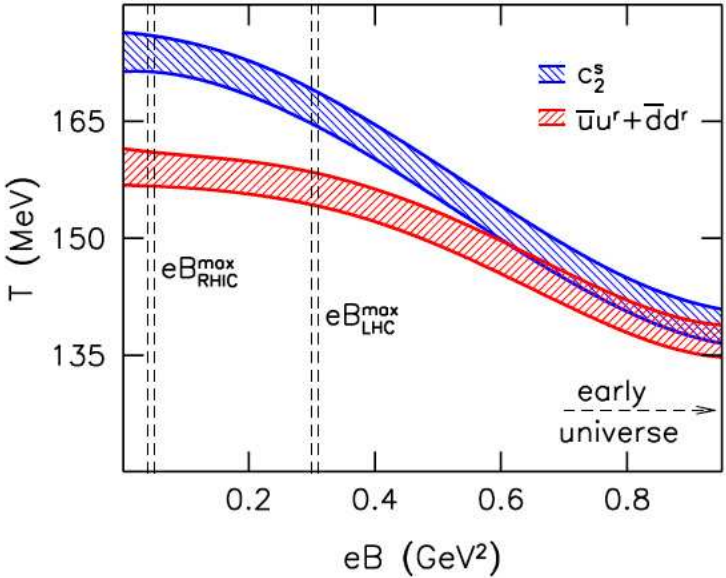
# Time evolution of magnetic fields in Heavy-Ion Collisions

- Field intensity is a rapidly decreasing function of time

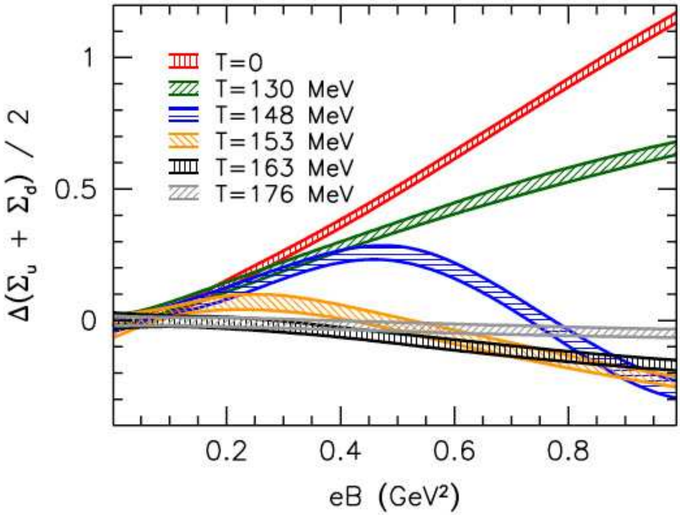



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

# Lattice results for $T_c$ [G. S. Bali et al., JHEP 02 (2012) 044] Inverse magnetic catalysis



Lattice results for the condensate **[G. S. Bali et al., Phys. Rev. D 86, 071502 (2012)]** **Inverse magnetic catalysis**



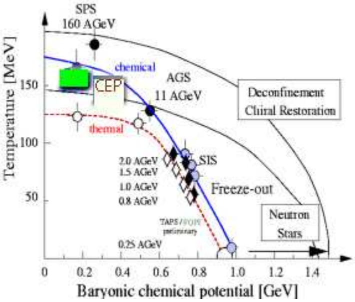
# Inverse magnetic catalysis is obtained in some models

- ✓ Deconfinement transition for large $N_c$ in the bag model:
  [E. Fraga, J. Noronha, L. Palhares, Phys. Rev. D **87**, 114014 (2013)]

- ✓ Coupling constant decreases with magnetic field intensity in effective QCD models:
  R. L. S. Farias, K. P. Gomes, G. Krein and M. B. Pinto, arXiv:1404.3931 [hep-ph];
  M. Ferreira, P. Costa, O. Lourenço, T. Frederico, C. Providência, arXiv:1404.5577 [hep-ph];
  A. A., M. Loewe, A. Mizher, R. Zamora, Phys. Rev. D **90**, 036001 (2014); A. A., M. Loewe, R. Zamora, Phys. Rev. D **91**, 016002.

- ✓ Paramagnetic phase (quarks and gluons) preferred over diamagnetic phase (pions):
  N. O. Agasian, S. M. Federov, Phys. Lett. **B** 663, 445 (2008)

## Higher $T_c$, chemical freeze-out curve closer to transition curve. **Visible effects**

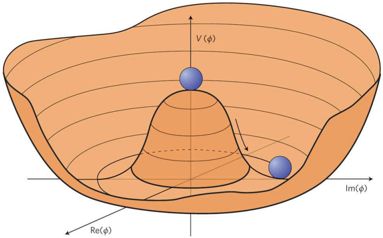



- If the pseudo critical line for $B \neq 0$ **happens for higher temperatures and lower densities,** this can be closer to the chemical freeze-out curve.
- Distance between CEP and freeze-out curve decreases.
- **Signals of criticality can be revealed.**

# Model QCD: Linear sigma model

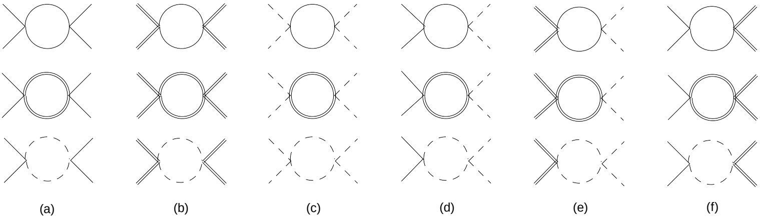
- Effective QCD models (linear sigma model with quarks)

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \\
&+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi,
\end{aligned}
$$

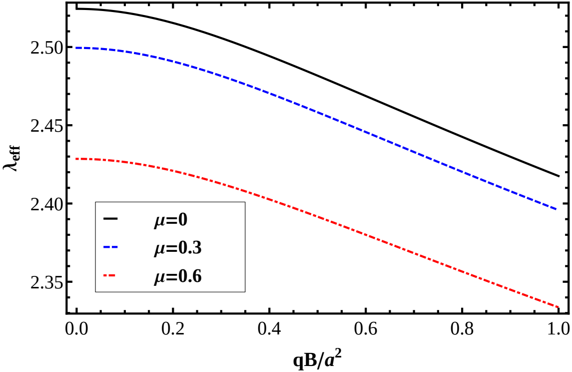



$$
\begin{aligned}
\sigma &\to \sigma + v, \\
m_\sigma^2 &= \frac{3}{4}\lambda v^2 - a^2, \\
m_\pi^2 &= \frac{1}{4}\lambda v^2 - a^2 \\
m_f &= gv \\
v_0 &= \frac{2a}{\sqrt{\lambda}}
\end{aligned}
$$

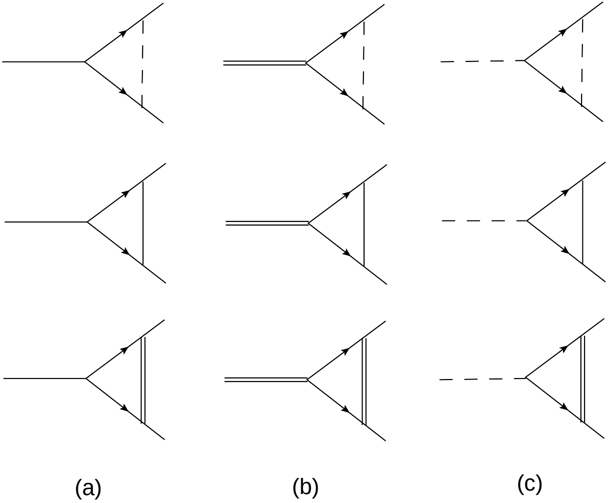
# Effective thermomagnetic scalar coupling $\lambda$ as a function of magnetic field strength



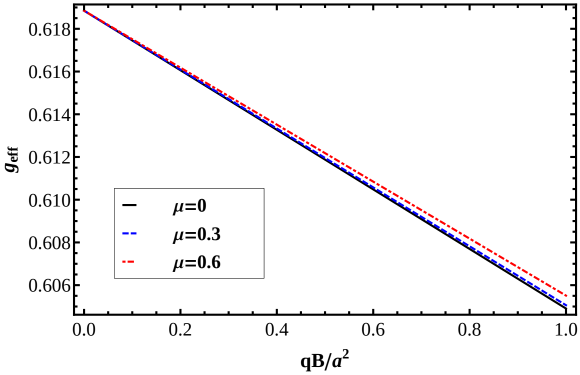
# Effective thermomagnetic scalar coupling $\lambda$ as a function of magnetic field strength



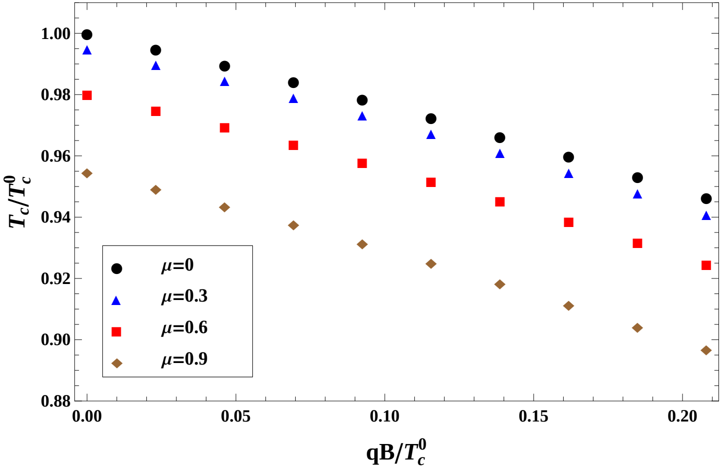
# Effective thermomagnetic fermion-scalar coupling $g$ as a function of magnetic field strength



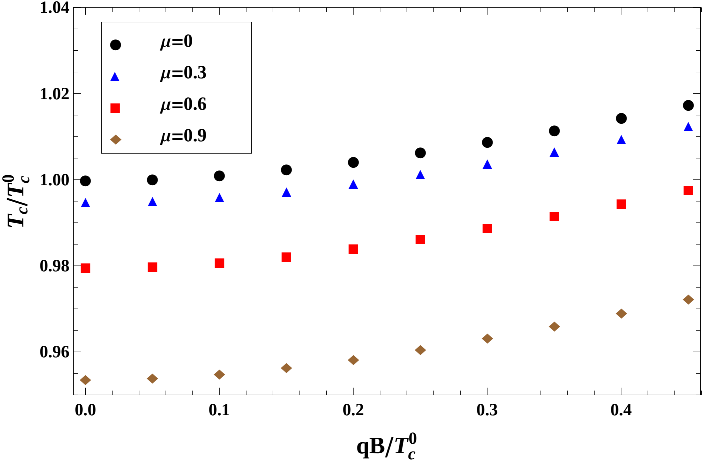
# Effective thermomagnetic fermion-scalar coupling $g$ as a function of magnetic field strength



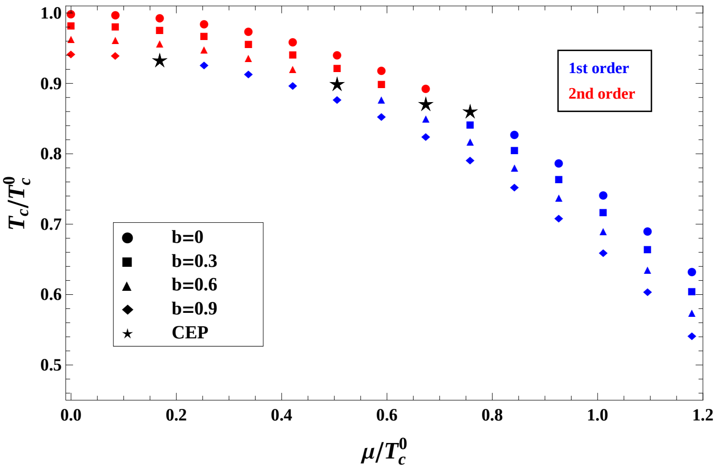
# Inverse magnetic catalysis: Critical temperature decreases with field strength



# Inverse magnetic catalysis: **Without** $B$**-dependence of couplings**, critical temperature **increases** with field strength

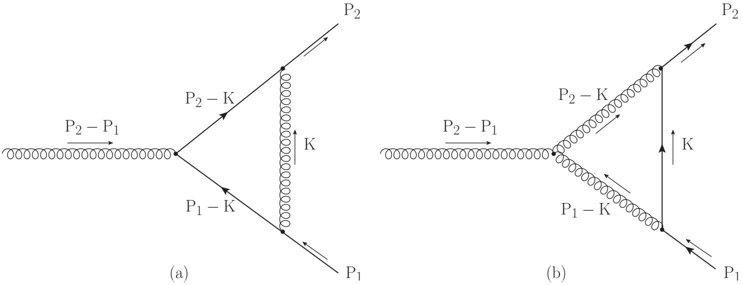


# Magnetized phase diagram




A. A., C. Dominguez, L. A. Hernández, M. Loewe, R. Zamora, arXiv:1509.03345 [hep-ph] (accepted for publication in PRD)

# QCD case: Quark-gluon vertex with a magnetic field




$$S(K) = \frac{m - \not{K}}{K^2 + m^2} - i\gamma_1\gamma_2 \frac{m - \not{K}_\parallel}{(K^2 + m^2)^2}(qB)$$

A. A., M. Loewe, J. Cobos-Martínez, M. E. Tejeda-Yeomans, R. Zamora, Phys. Rev. D **91**, 016007 (2015)

# QCD case: Quark-gluon vertex with a magnetic field at **high** temperature

$$
\begin{aligned}
\delta\Gamma_\mu^{(a)} &= -ig^2(C_F - C_A/2)(qB)T\sum_n\int\frac{d^3k}{(2\pi)^3} \\
&\times \gamma_\nu\left[\gamma_1\gamma_2\not{K}_\parallel\gamma_\mu\not{K}\widetilde{\Delta}(P_2-K)\right. \\
&+ \left.\not{K}\gamma_\mu\gamma_1\gamma_2\not{K}_\parallel\widetilde{\Delta}(P_1-K)\right]\gamma_\nu \\
&\times \Delta(K)\widetilde{\Delta}(P_2-K)\widetilde{\Delta}(P_1-K)
\end{aligned}
$$

$$
\begin{aligned}
\delta\Gamma_\mu^{(b)} &= -2ig^2\frac{C_A}{2}(qB)T\sum_n\int\frac{d^3k}{(2\pi)^3} \\
&\times \left[-\not{K}\gamma_1\gamma_2\not{K}_\parallel\gamma_\mu + 2\gamma_\nu\gamma_1\gamma_2\not{K}_\parallel\gamma_\nu K_\mu\right. \\
&- \left.\gamma_\mu\gamma_1\gamma_2\not{K}_\parallel\not{K}\right] \\
&\times \widetilde{\Delta}(K)^2\Delta(P_1-K)\Delta(P_2-K).
\end{aligned}
$$

# Effective thermomagnetic QCD coupling as a function of magnetic field strength at **high** temperature

$$\delta\vec{\Gamma}_{\parallel}(p_0) = \left(\frac{2}{3p_0^2}\right) 4g^2 C_F M^2(T, m, qB)\vec{\gamma}_{\parallel}\Sigma_3$$

$$M^2(T, m, qB) = \frac{qB}{16\pi^2}\left[\ln(2) - \frac{\pi}{2}\frac{T}{m}\right].$$

$$g_{\text{eff}}^{\text{therm}} = g\left[1 - \frac{m_f^2}{T^2} + \left(\frac{8}{3T^2}\right) g^2 C_F M^2(T, m_f, qB)\right],$$

## QCD case: Quark-gluon vertex with a magnetic field at **zero** temperature

$$\begin{aligned}
\delta\Gamma^{\mu}_{(a)} &= ig^3(qB)\left(C_F-\frac{C_A}{2}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\\
&\times \left\{\gamma^{\nu}\frac{(\not{p}_2-\not{k})}{(p_2-k)^2}\gamma^{\mu}\frac{\gamma_1\gamma_2\left[\gamma\cdot(p_1-k)\right]_{\parallel}}{(p_1-k)^4}\gamma_{\nu}\right.\\
&+\left.\gamma^{\nu}\frac{\gamma_1\gamma_2\left[\gamma\cdot(p_2-k)\right]_{\parallel}}{(p_2-k)^4}\gamma^{\mu}\frac{(\not{p}_1-\not{k})}{(p_1-k)^2}\gamma_{\nu}\right\},
\end{aligned}$$

$$\begin{aligned}
\delta\Gamma^{\mu}_{(b)} &= -2ig^3(qB)\frac{C_A}{2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^4}\left[g^{\mu\nu}(2p_2-p_1-k)^{\rho}\right.\\
&+ \left.g^{\nu\rho}(2k-p_2-p_1)^{\mu}+g^{\rho\mu}(2p_1-k-p_2)^{\nu}\right]\\
&\times \gamma_{\rho}\frac{\gamma_1\gamma_2(\gamma\cdot k)_{\parallel}}{(p_2-k)^2(p_1-k)^2}\gamma_{\nu},
\end{aligned}$$

# Effective magnetic QCD coupling as a function of magnetic field strength at **zero** temperature

$$
\begin{aligned}
g_{\rm eff}^{\rm vac} &= g - \left[ g^2 \frac{1}{3\pi^2} \frac{q\vec{\Sigma}\cdot\vec{B}}{Q^2} \right] \\
&\times \left\{ \left( C_F - \frac{C_A}{2} \right) [1 + \ln(4)] + \frac{C_A}{5} [-1 + \ln(4)] \right\} \\
&= g - \left[ g^2 \frac{1}{3\pi^2} \frac{q\vec{\Sigma}\cdot\vec{B}}{Q^2} \right] \\
&\times \left\{ [1 + \ln(4)]\, C_F - \frac{[7 + 3\ln(4)]}{10} C_A \right\} . \\
C_F &= \frac{N^2 - 1}{2N} \qquad C_A = N
\end{aligned}
$$

For $N = 3$, $g_{\rm eff}^{\rm vac}$ **grows** whereas $g_{\rm eff}^{\rm therm}$ **decreases** with $B$.

# Conclusions

- Efforts to find CEP location **key** to understand transition between **soft/hard** and **microscopic/macroscopic** regimes in QCD
- Use of effective models **important tool** to gain insight
- Magnetic fields can serve as an external probe to explore CEP location
- Important example: Change of behavior of QCD coupling with magnetic field strength from low to high temperatures allows to understand inverse magnetic catalysis