# Amplitude Analysis in Hadron Spectroscopy

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# Joint Physics Analysis Center

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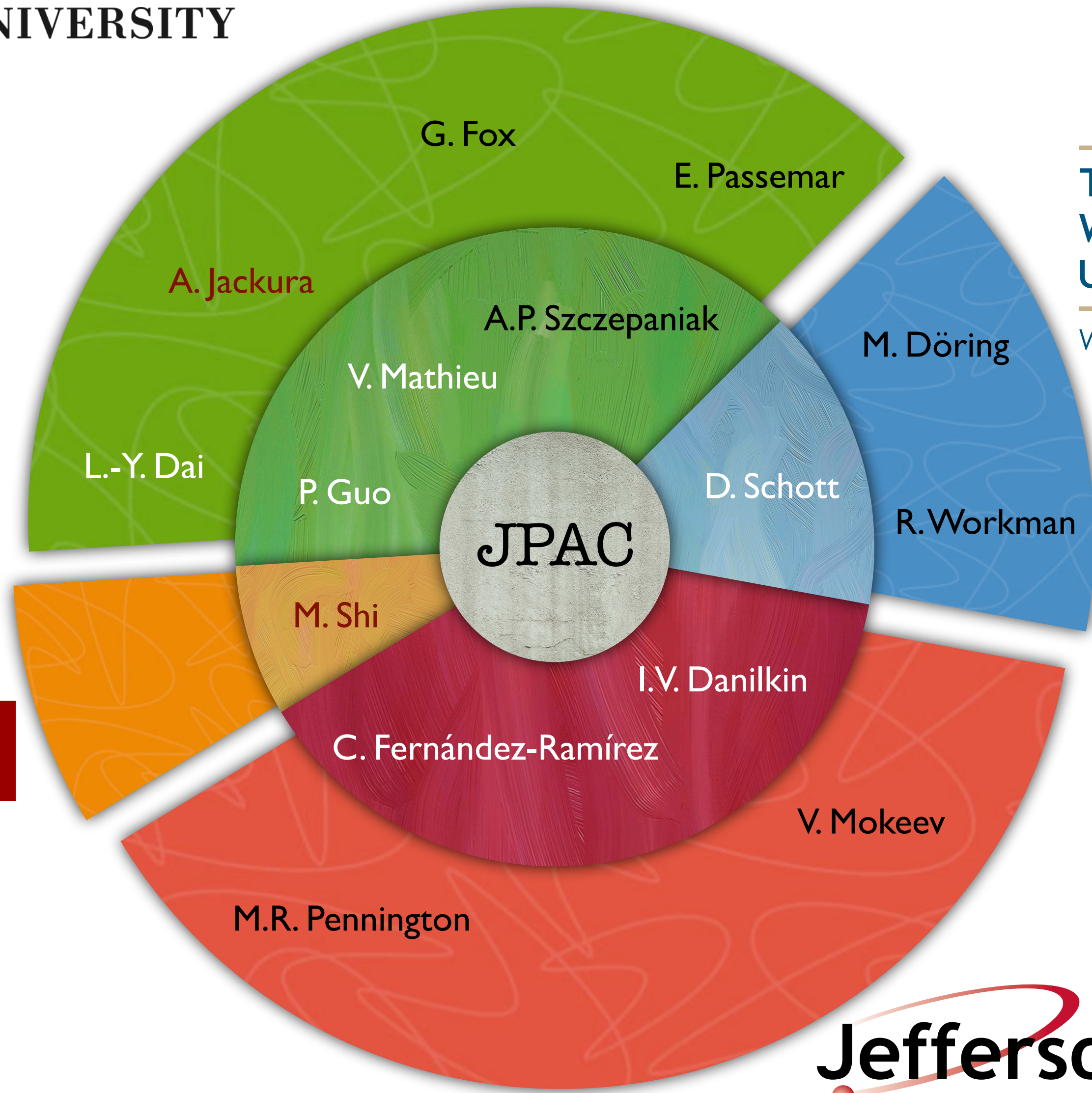


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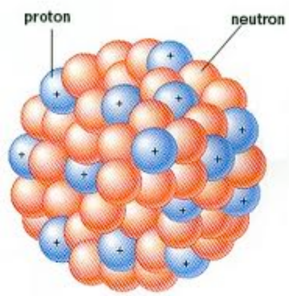
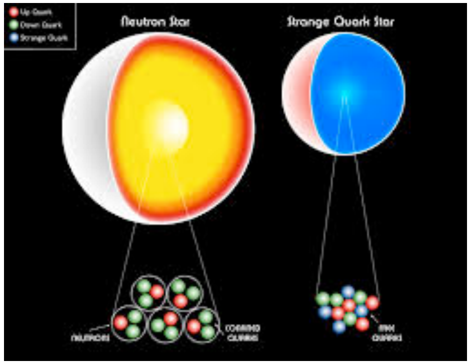
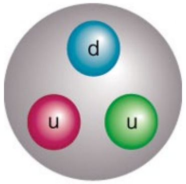
# Why is QCD special?

✓ A single theory is responsible for phenomena at distance scales of the order of $10^{-15}$m as well as of the order $10^{4}$m.

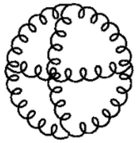


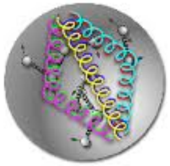



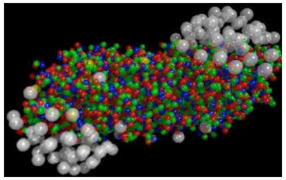

✓ It builds from objects (quarks and gluons) that do not exist in a common sense. >90% mass comes from interactions!



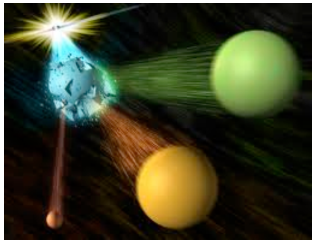

✓ Predicts existence of exotic matter, e.g. made from radiation (glueballs,hybrids) or novel plasmas.





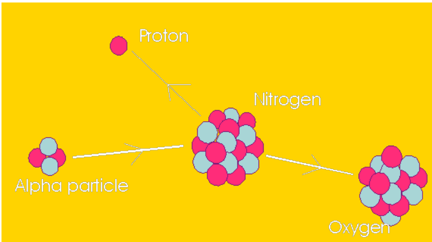


✓ A possible template for physics beyond the Standard Model



✓ It is challenging!

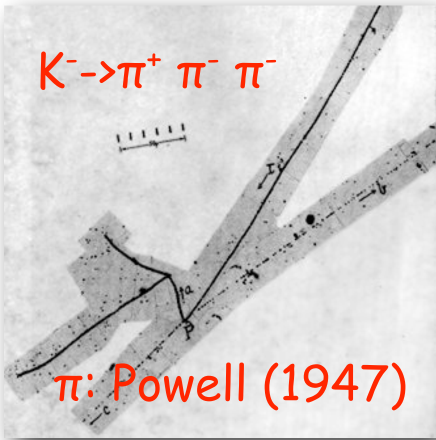
# This talk is about hadrons

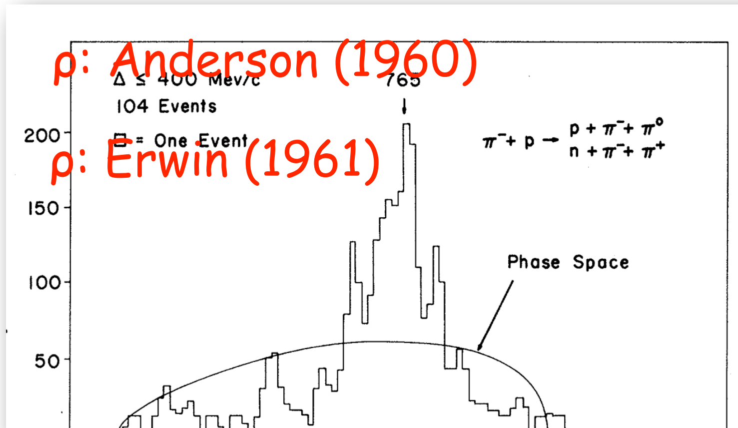


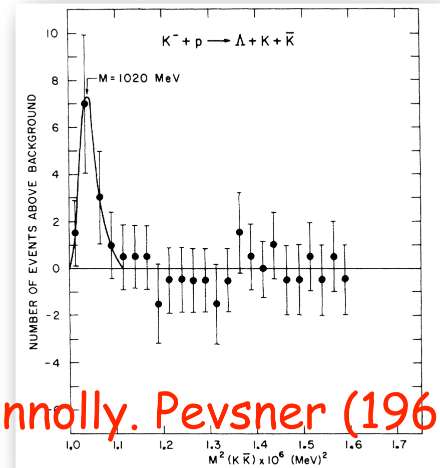

1909/1911 Rutherford/Geiger/Marsden discover the nucleus

1919 Rutherford discovers the proton

1932 Chadwick discovers the neutron



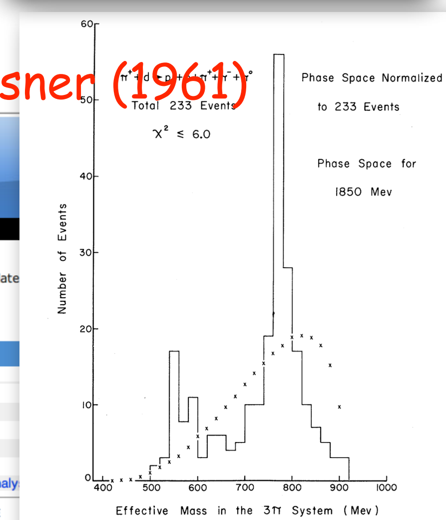
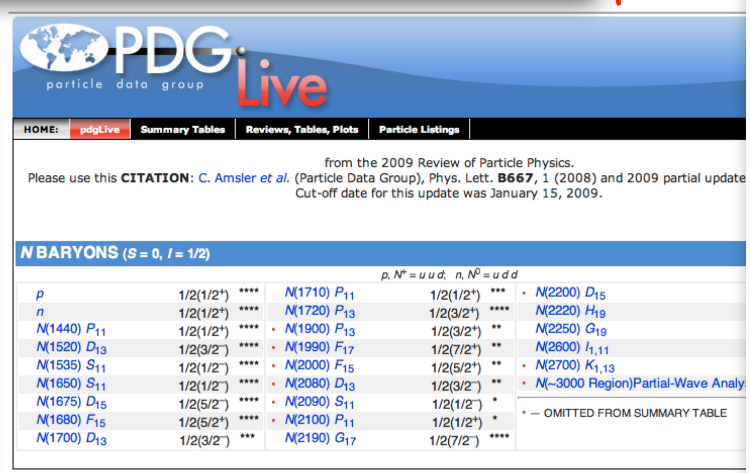




FIG. 2. Histogram of the effective mass of the three-pion system for 233 events.



PDG particle data group Live

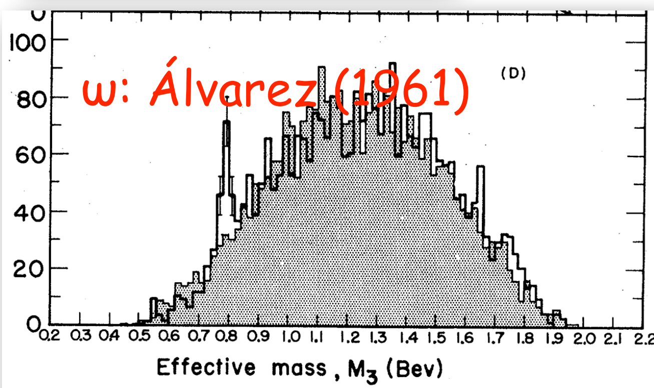
HOME: pdgLive | Summary Tables | Reviews, Tables, Plots | Particle Listings

from the 2009 Review of Particle Physics.
Please use this **CITATION**: C. Amsler *et al.* (Particle Data Group), Phys. Lett. **B667**, 1 (2008) and 2009 partial update
Cut-off date for this update was January 15, 2009.

**N BARYONS** ($S = 0$, $I = 1/2$)

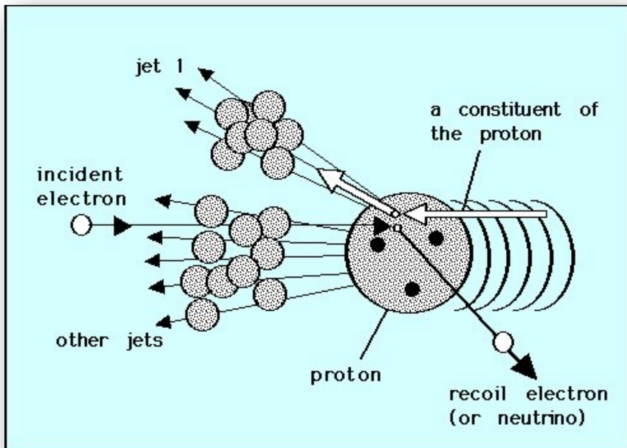
$p$, $N^+ = uud$; $n$, $N^0 = udd$

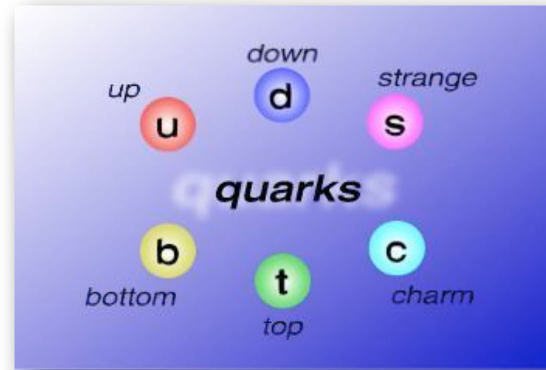
| | | | | | | |
|---|---|---|---|---|---|---|
| $p$ | 1/2(1/2$^+$) | **** | $N(1710)$ $P_{11}$ | 1/2(1/2$^+$) | *** | • $N(2200)$ $D_{15}$ |
| $n$ | 1/2(1/2$^+$) | **** | $N(1720)$ $P_{13}$ | 1/2(3/2$^+$) | **** | • $N(2220)$ $H_{19}$ |
| $N(1440)$ $P_{11}$ | 1/2(1/2$^+$) | **** | • $N(1900)$ $P_{13}$ | 1/2(3/2$^+$) | ** | $N(2250)$ $G_{19}$ |
| $N(1520)$ $D_{13}$ | 1/2(3/2$^-$) | **** | • $N(1990)$ $F_{17}$ | 1/2(7/2$^+$) | ** | $N(2600)$ $I_{1,11}$ |
| $N(1535)$ $S_{11}$ | 1/2(1/2$^-$) | **** | • $N(2000)$ $F_{15}$ | 1/2(5/2$^+$) | ** | • $N(2700)$ $K_{1,13}$ |
| $N(1650)$ $S_{11}$ | 1/2(1/2$^-$) | **** | • $N(2080)$ $D_{13}$ | 1/2(3/2$^-$) | ** | • $N$(~3000 Region)Partial-Wave Analy |
| $N(1675)$ $D_{15}$ | 1/2(5/2$^-$) | **** | • $N(2090)$ $S_{11}$ | 1/2(1/2$^-$) | * | |
| $N(1680)$ $F_{15}$ | 1/2(5/2$^+$) | **** | • $N(2100)$ $P_{11}$ | 1/2(1/2$^+$) | * | • — OMITTED FROM SUMMARY TABLE |
| $N(1700)$ $D_{13}$ | 1/2(3/2$^-$) | *** | $N(2190)$ $G_{17}$ | 1/2(7/2$^-$) | **** | |



# “Bare (free)” particles of QCD: quarks and gluons

e.g. as seen in high energy collisions



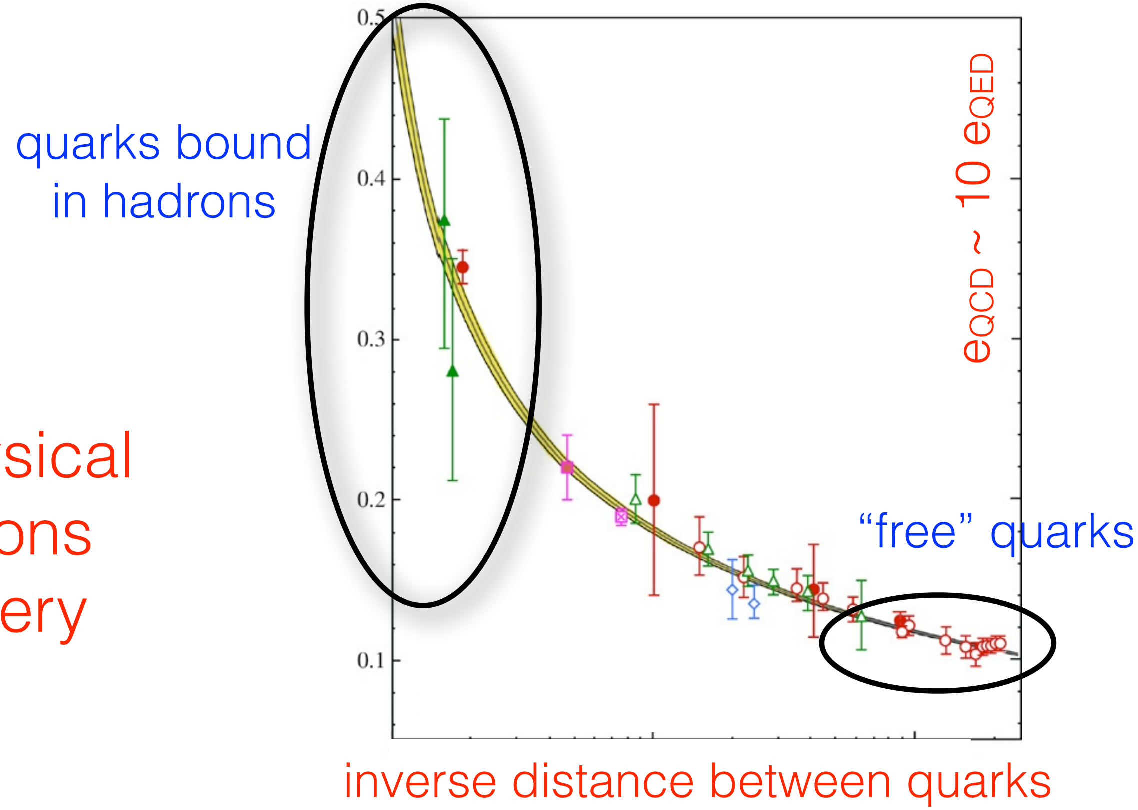




analogous to 8 (color) x 6 (flavor) copies of QED:
quark → electron
gluon → photon
(but non-abelian)

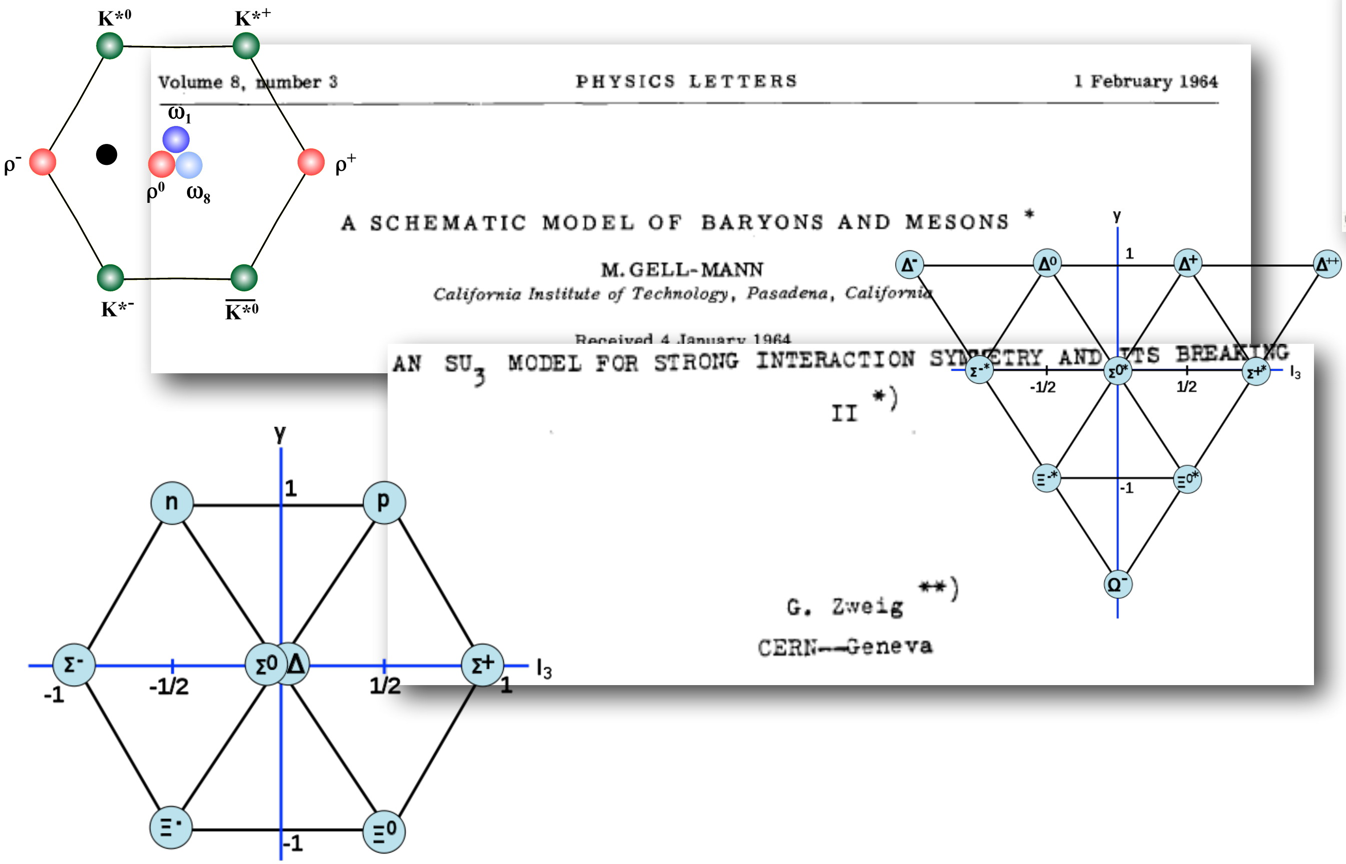


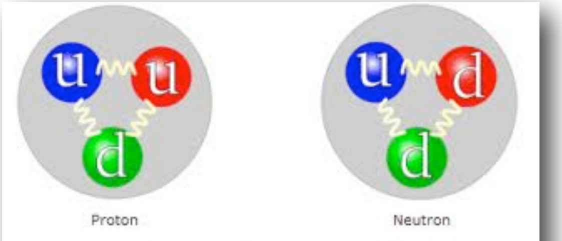
x8 combinations

the nature of physical quarks and gluons remains a mystery

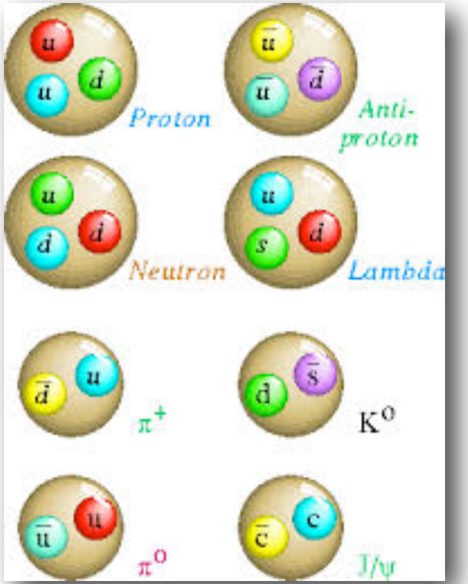


# Quark models



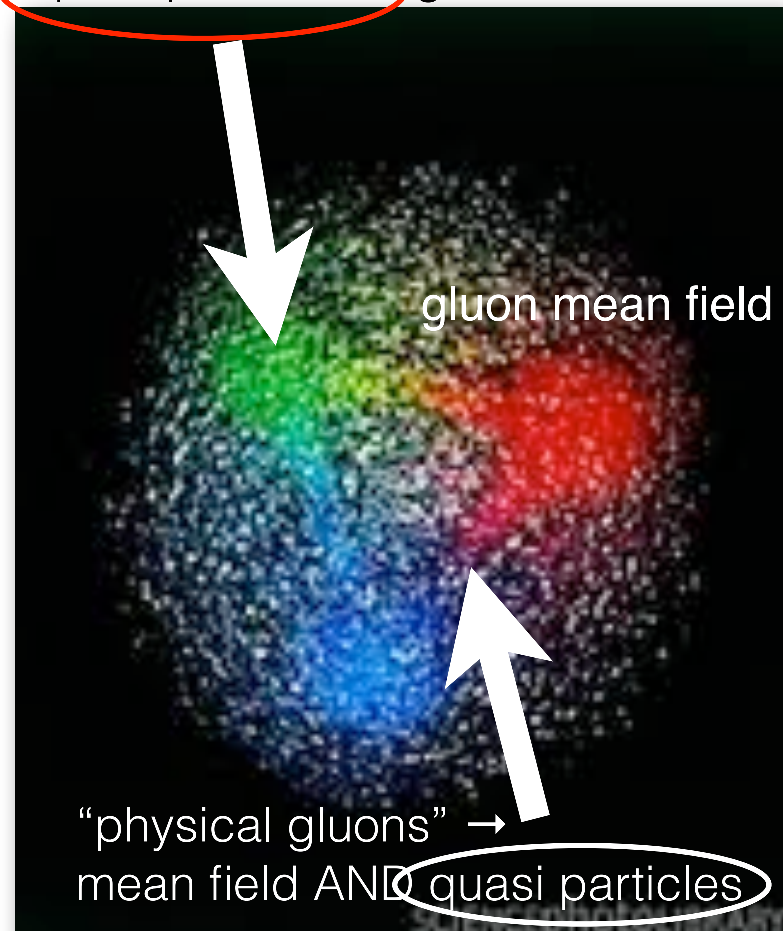
Quark composition of a proton and a neutron (diagrams from *Wikipedia*)




physical quarks appear to move in a kind of "mean, gluonic field"

# Plausible model?

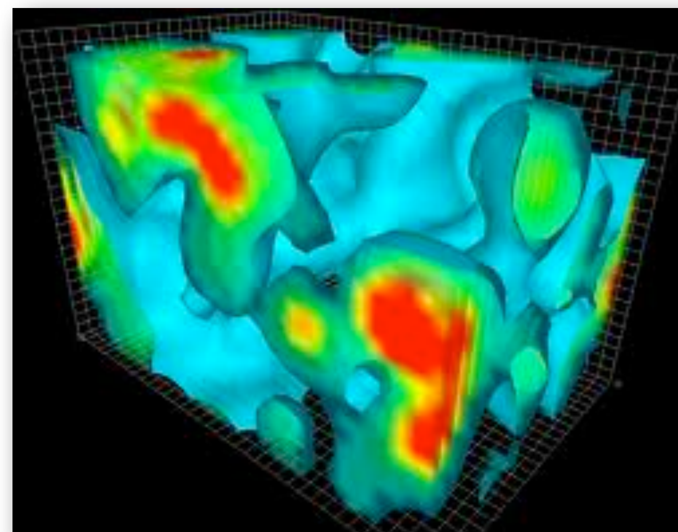
“physical quarks” → quasi particles in gluon mean field



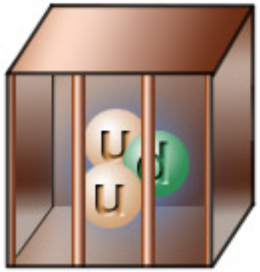

$H_{QCD} = H_{c.h.o.} + \text{non-linear}$

finite energy, localized solutions: solitons (monopoles, vortices , ...)

The QCD vacuum is not empty. Rather it contains quantum fluctuations in the gluon field at all scales. (Image: University of Adelaide)



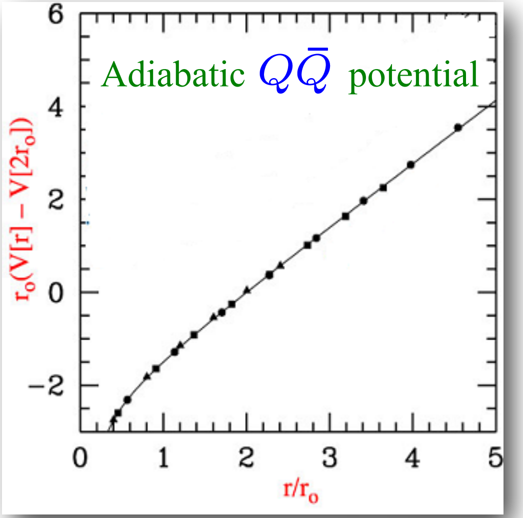
# Confinement in QCD



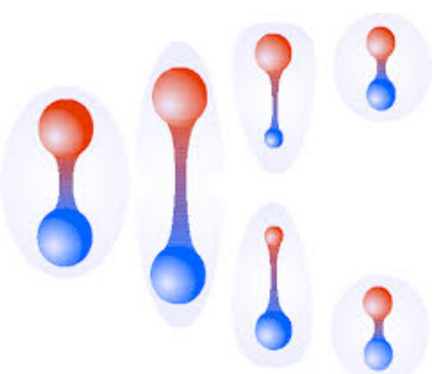

Absence of isolated quarks

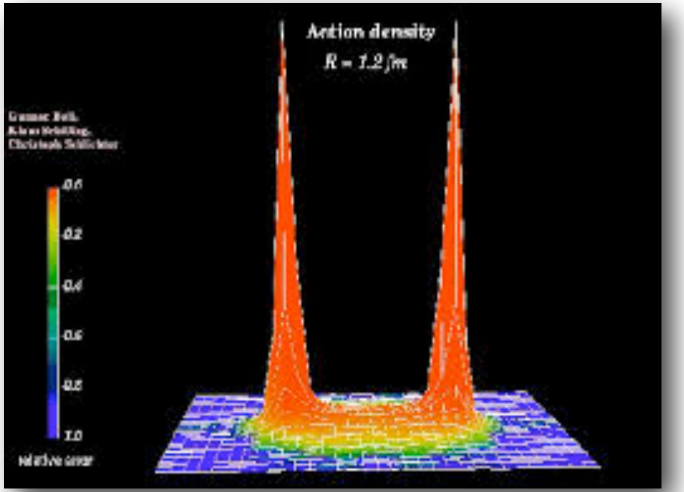
Properties of confinement:

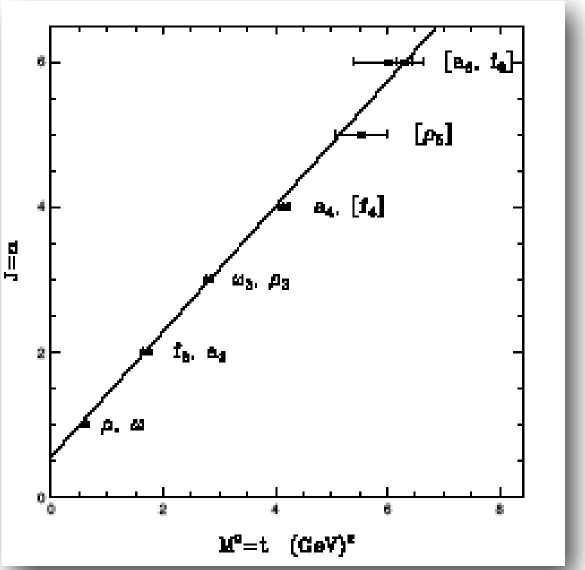
- Linearly rising potential
- Regge trajectories
- String behavior




$r_0 = 0.5$ fm





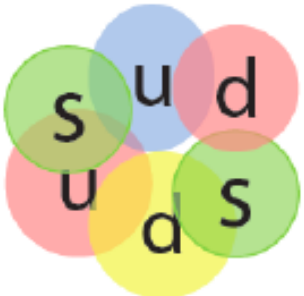




Spectroscopy of Hadrons can teach us about "workings" of QCD

1. Hadrons with gluon excitations (confinement)

2. Hadron molecules (residual forces)

# QCD: There are many other possible color singlets.

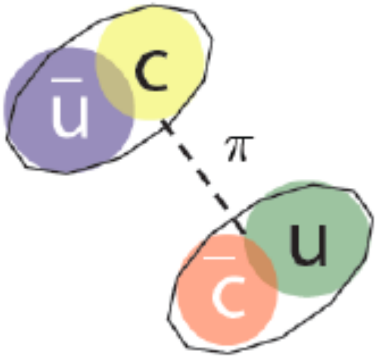



dibaryon

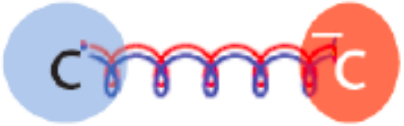

pentaquark


glueball


diquark + di-antiquark



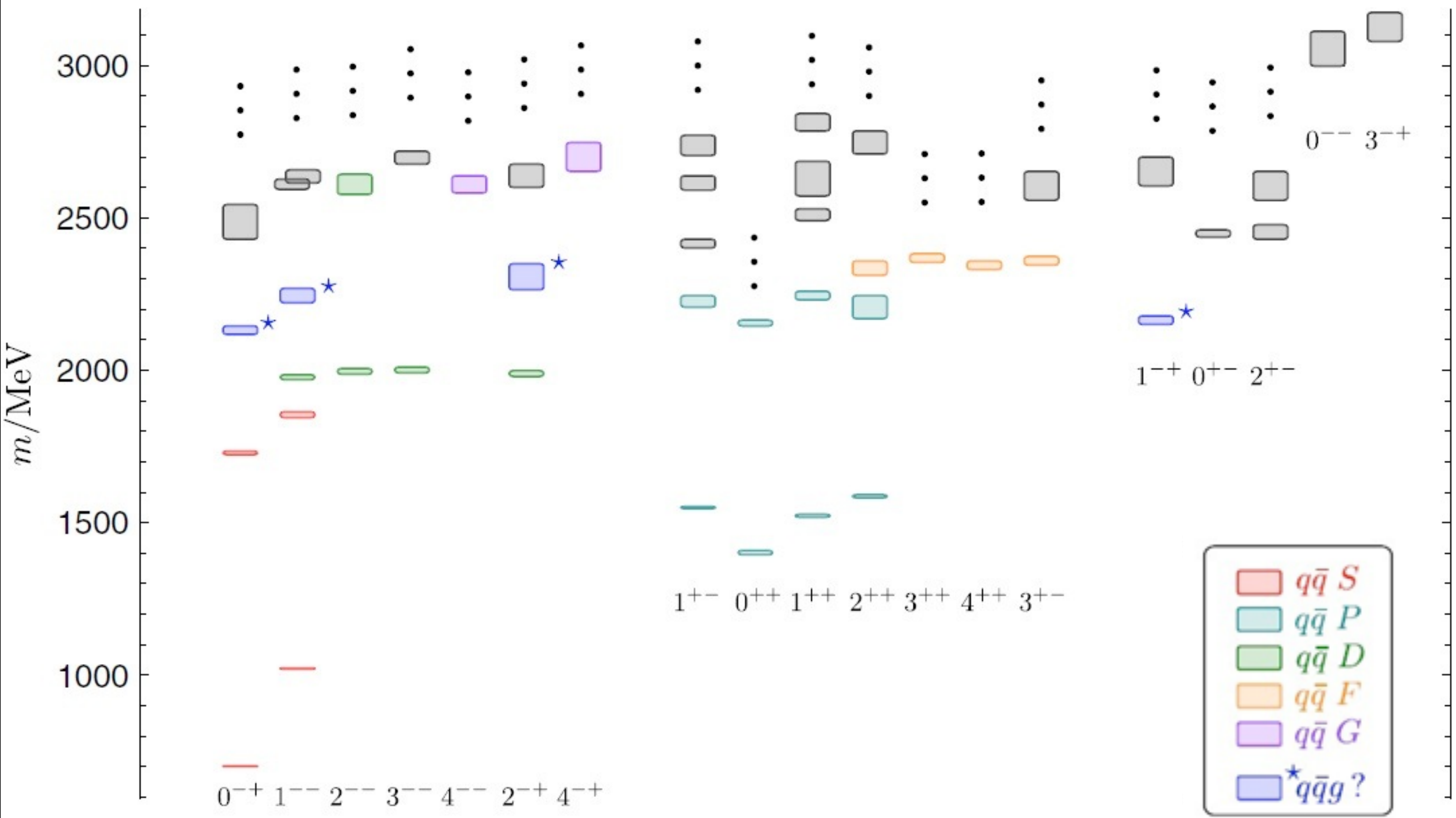

dimeson molecule




$q\,\bar{q}\,g$ hybrid

# Lattice meson spectrum

[Dudek 2011]



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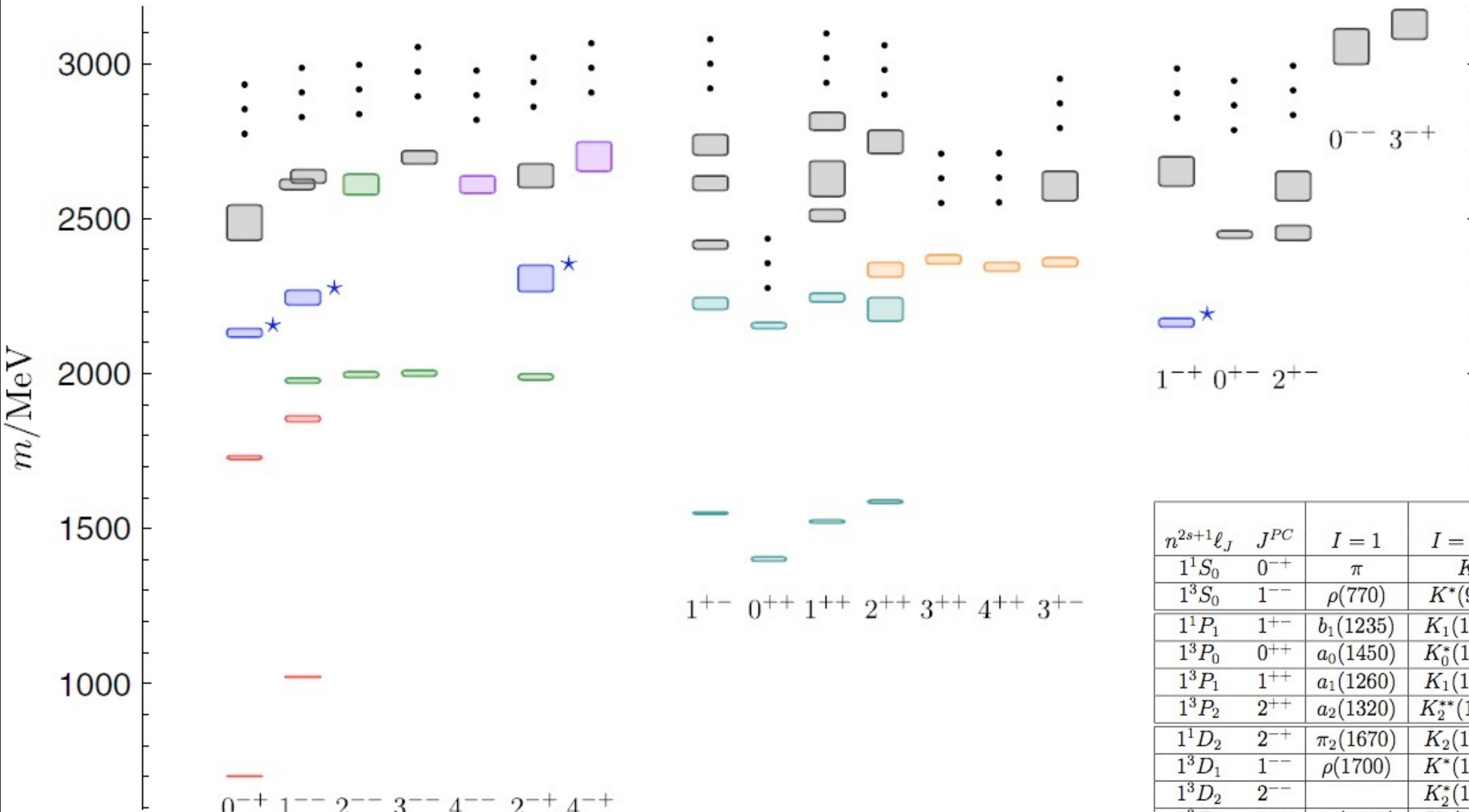
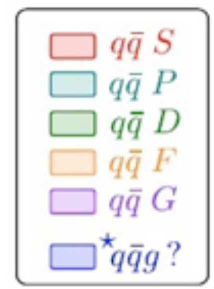
[Dudek 2011]

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| $1^3S_0$ | $1^{--}$ | $\rho(770)$ | $K^*(982)$ | $\omega(782)$ | $\phi(1020)$ | $R1$ |
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| $1^3P_0$ | $0^{++}$ | $a_0(1450)$ | $K_0^*(1430)$ | $f_0(1370)$ | $f_0(1710)$ | $R4$ |
| $1^3P_1$ | $1^{++}$ | $a_1(1260)$ | $K_1(1270)$ | $f_1(1285)$ | $f_1(1420)$ | $R3$ |
| $1^3P_2$ | $2^{++}$ | $a_2(1320)$ | $K_2^{**}(1430)$ | $f_2(1270)$ | $f_2'(1525)$ | $R1$ |
| $1^1D_2$ | $2^{-+}$ | $\pi_2(1670)$ | $K_2(1770)$ | $\eta_2(1645)$ | $\eta_2(1870)$ | $R2$ |
| $1^3D_1$ | $1^{--}$ | $\rho(1700)$ | $K^*(1680)$ | $\omega(1650)$ | | $R4$ |
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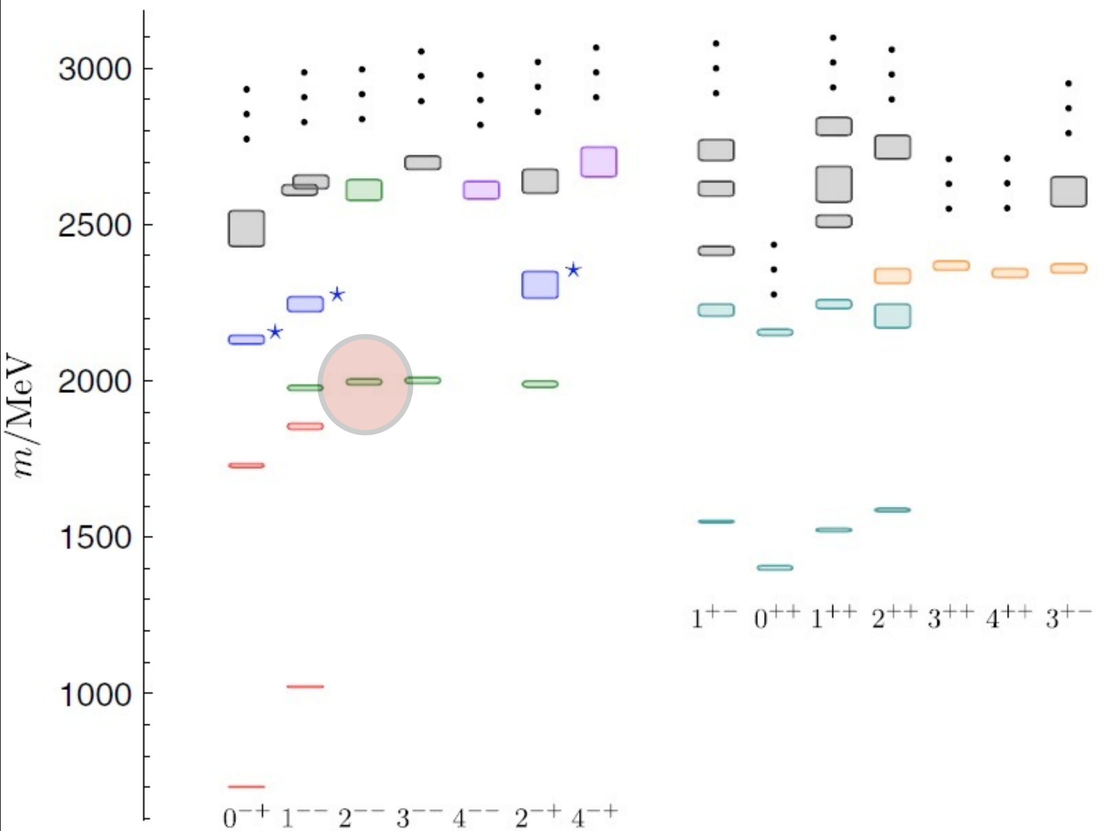
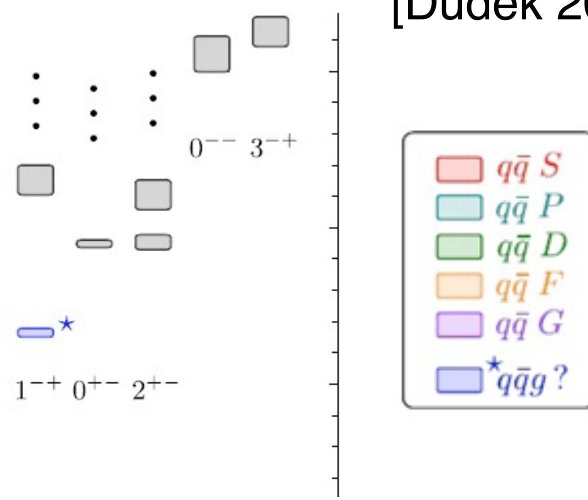
# Lattice meson spectrum

[Dudek 2011]

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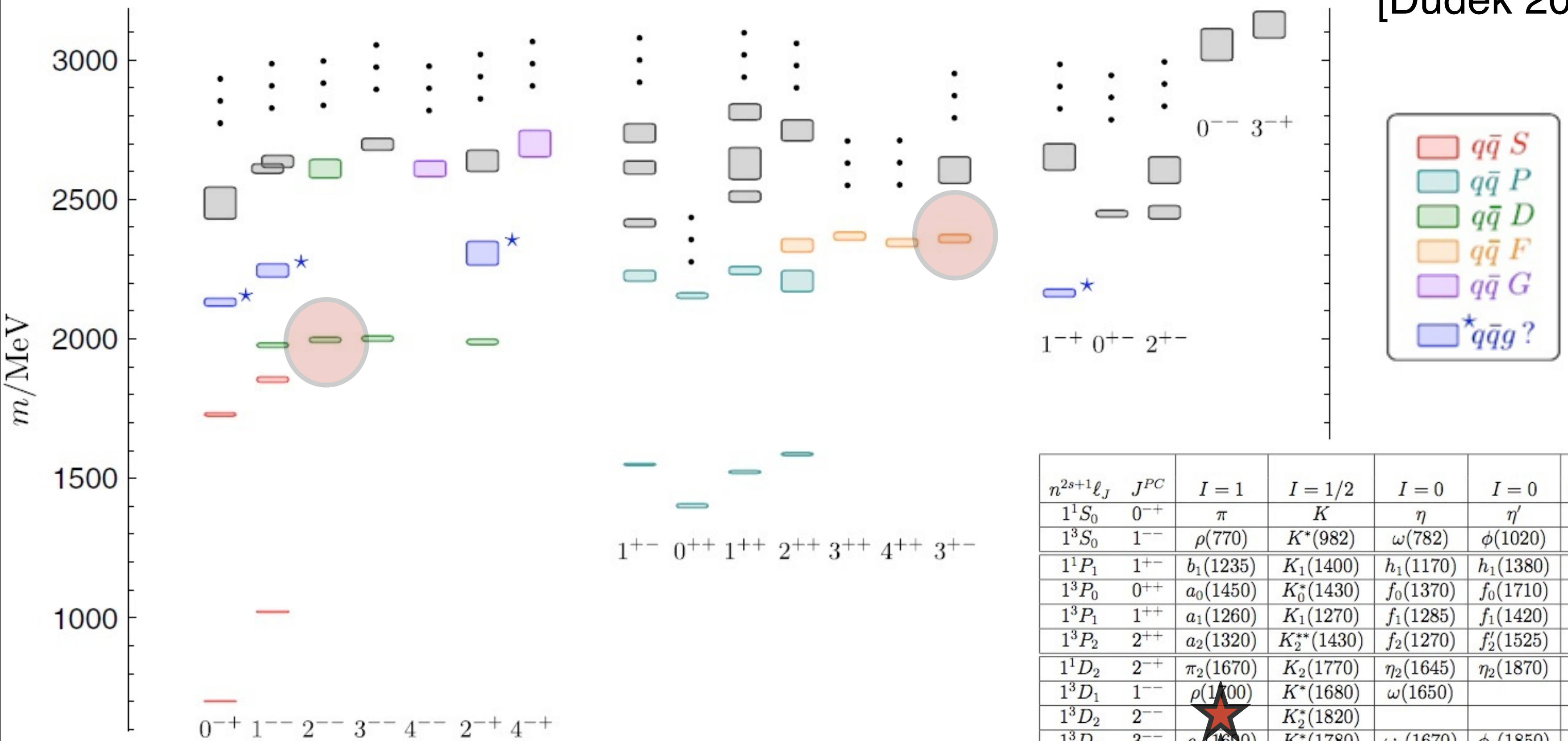
[Dudek 2011]

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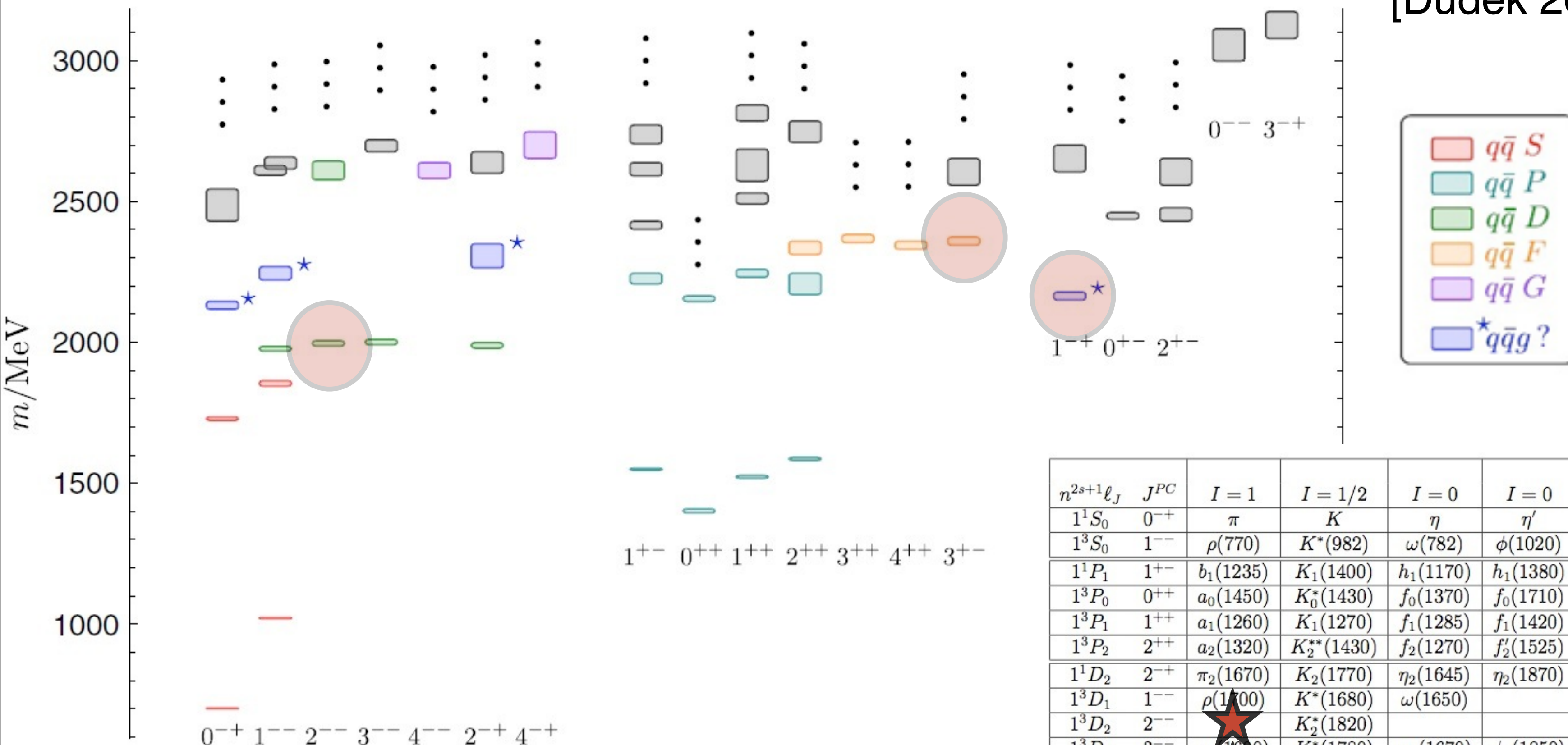


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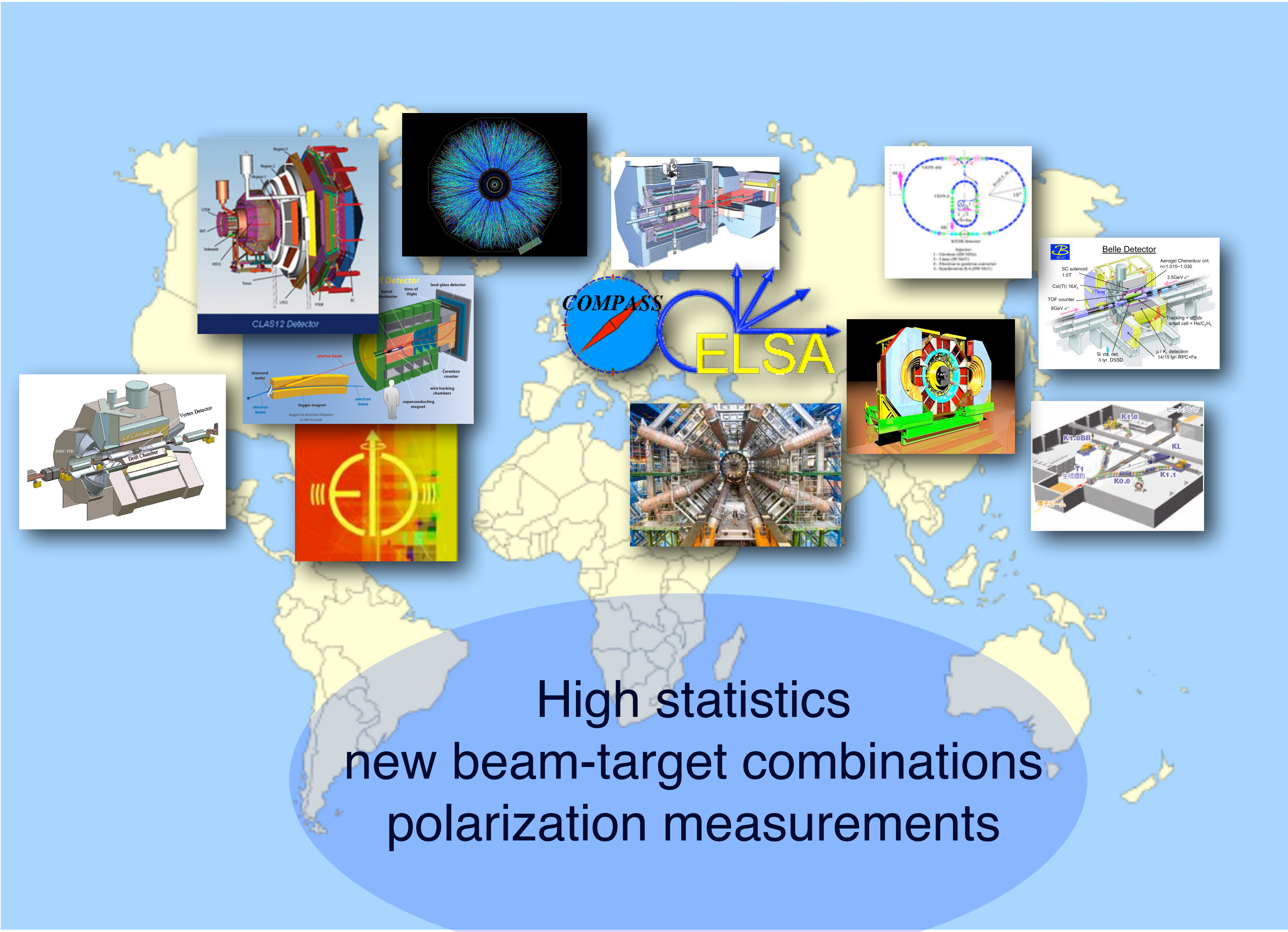
| $1^3D_1$ | $1^{--}$ | $\rho(1700)$ | $K^*(1680)$ | $\omega(1650)$ | | $R4$ |
| $1^3D_2$ | $2^{--}$ | | $K_2^*(1820)$ | | | $R3$ |
| $1^3D_3$ | $3^{--}$ | $\rho_3(1690)$ | $K_3^*(1780)$ | $\omega_3(1670)$ | $\phi_3(1850)$ | $R1$ |
| $1^1F_3$ | $3^{+-}$ | | | | | $R2$ |
| $1^3F_2$ | $2^{++}$ | | $K_2^*(1980)$ | $f_2(1910)$ | $f_2(2010)$ | $R4$ |
| $1^3F_3$ | $3^{++}$ | | $K_3(2320)$ | | | $R3$ |
| $1^3F_4$ | $4^{++}$ | $a_4(2040)$ | $K_4^{**}(2045)$ | | $f_4(2050)$ | $R1$ |

- Strong, theoretical evidence (lattice) for gluon field excitations in hadron spectrum
- Phenomenologically, gluons behave as axial vector, quasiparticles $J^{PC}=1^{+-}$
- Lowest multiplet of “hybrid mesons” has $J^{PC} = 0^{-+}$, $1^{-+}$, $2^{-+}$, $1^{--}$ states
- What about other non-quark model possibilities?
- Can these be detected and distinguished?

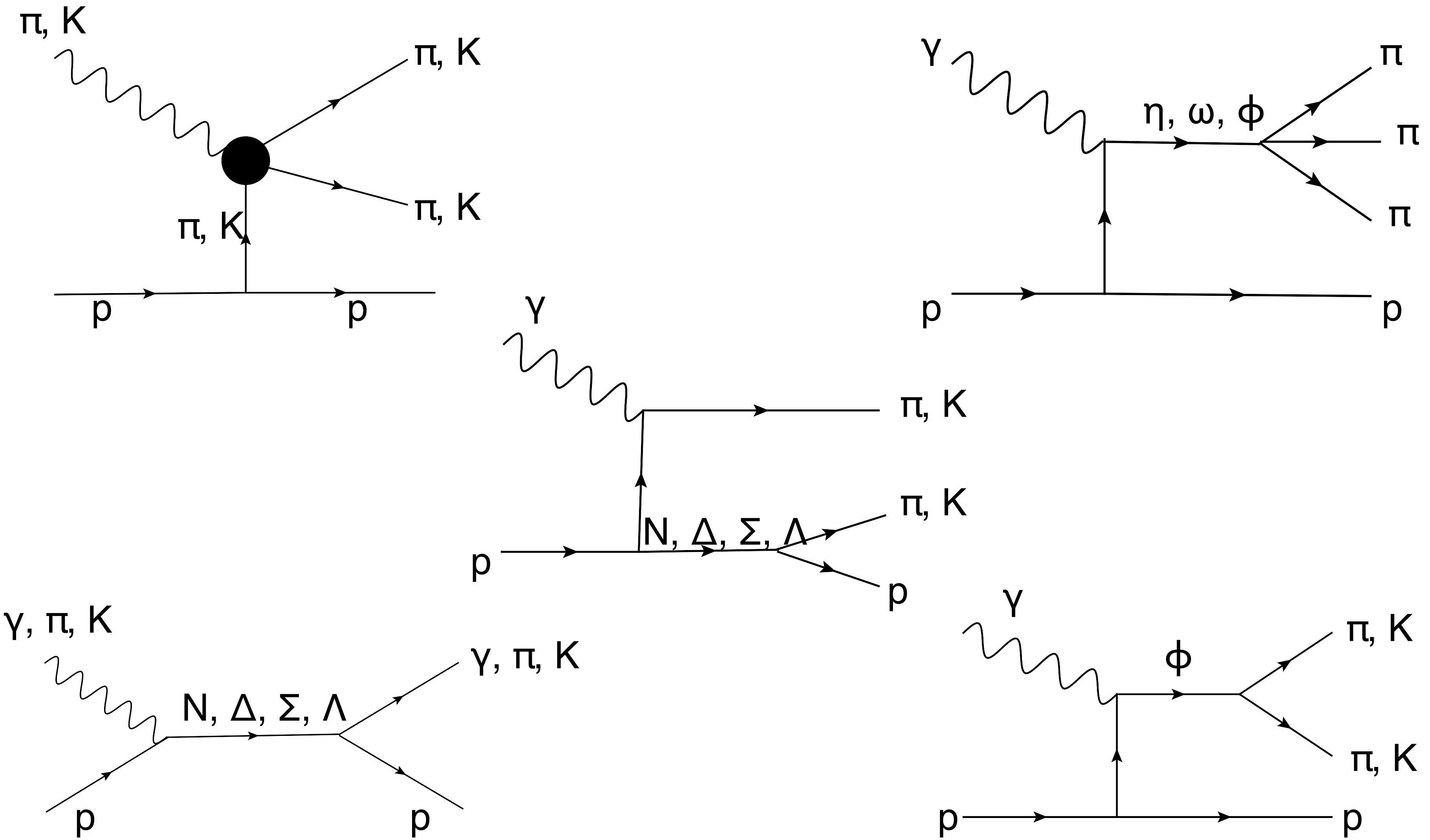
# New opportunities




High statistics
new beam-target combinations
polarization measurements

# What kind of experiments?

2→ 2, 3 processes

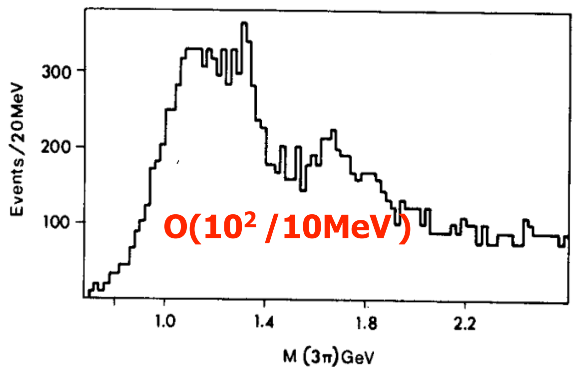


# Evolution in statistics

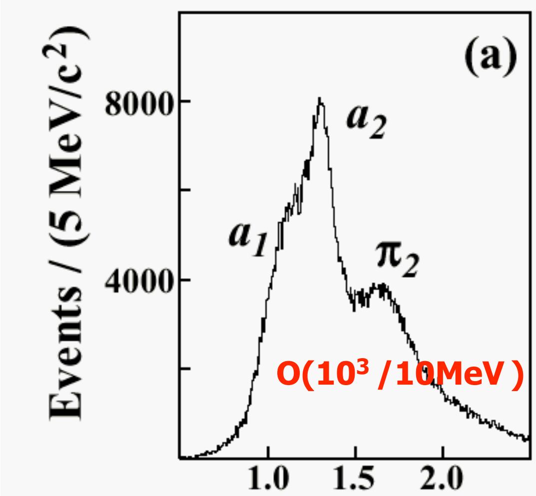
$\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

CERN ca. 1970

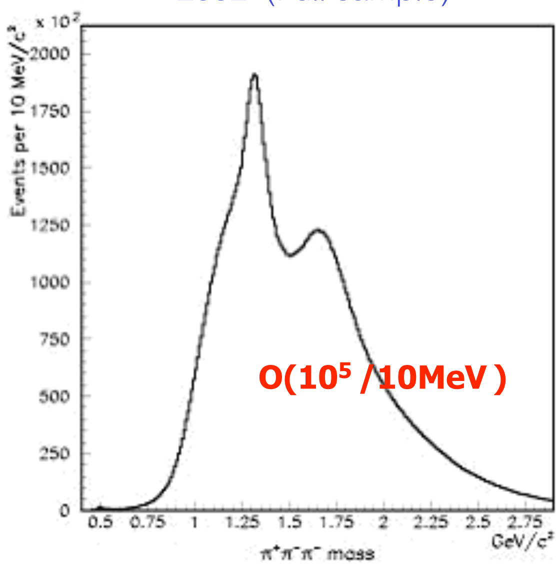
- t



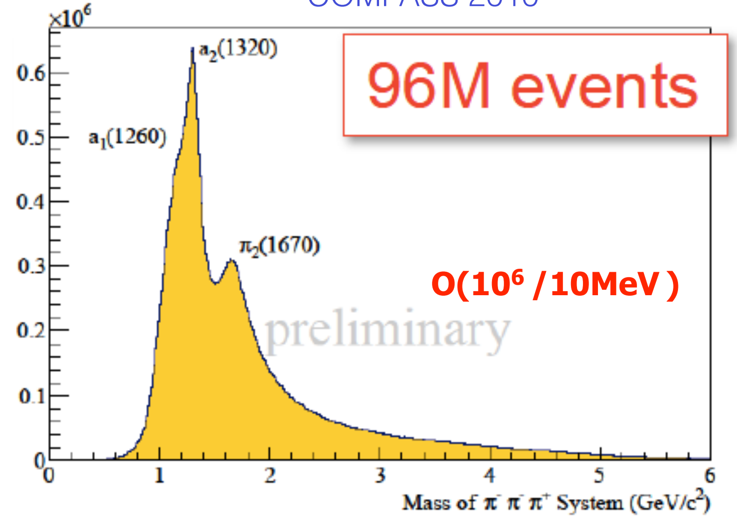

BNL (E852) ca 1995




E852 (Full sample)




COMPASS 2010



# Resonance paradigm: Δ(1232)

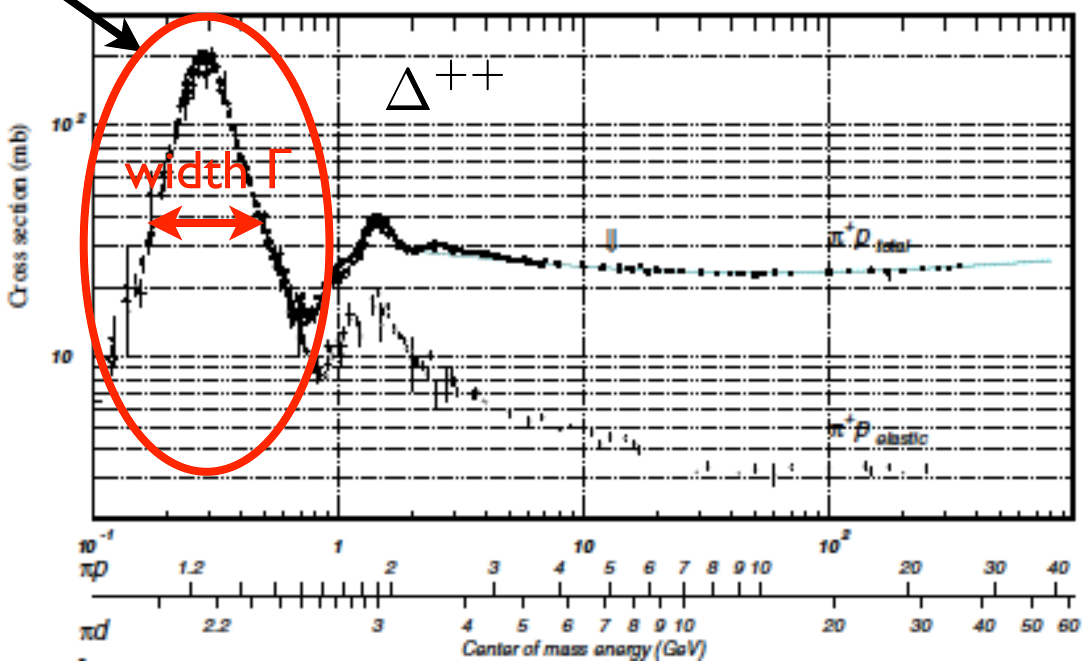
peak in intensity
(cross section)

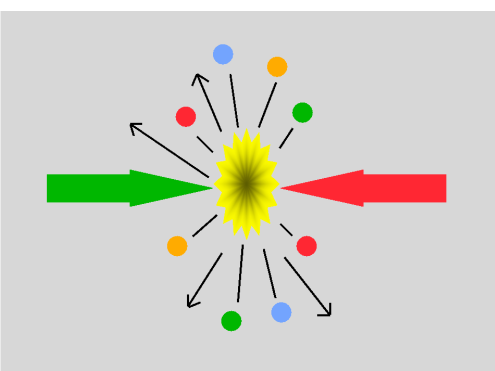
In 1952, Fermi and collaborators measured the cross section $\pi^+ p \to \pi^+ p$ and found it steeply raising.

mass ~ 30% above proton

lifetime ~ 4.5 x $10^{-24}$ s

width ~ lifetime$^{-1}$ = 150 MeV



# How do we catch resonances?

- Is every bump a resonance?
- Is every resonance a bump?
  - The answer to both questions is NO
- Examples
  - $\sigma$ meson in $\pi\pi$ scattering has a broad structure
  - Threshold effects can appear as resonances

# Actually, what is a resonance?

- A resonance is a pole of the scattering amplitude in the unphysical Riemann sheets

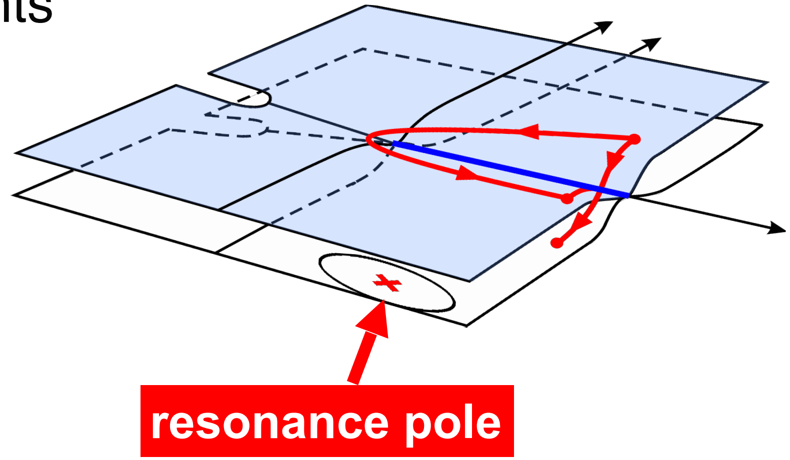
- 12 GeV upgrade at JLab: *CLAS, GlueX, etc.*
  - Aim: ➾ Complete understanding of the *hadron spectrum*

    ➾ discover *new resonances* e.g, gluonic excitations (states where glue builds their $J^{PC}$)

  - Tools: *Amplitude analysis of data*

    To find new resonances not bump-hunting, but search for poles

    ➡ must build in S-Matrix constraints
    + state-of-the-art knowledge
    of reaction dynamics




**Dispersive analytic continuation**

- 12 GeV upgrade at JLab: *CLAS, GlueX, etc.*
  - Aim: ➩ Complete understanding of the *hadron spectrum*

    ➩ discover *new resonances* e.g, gluonic excitations (states where glue builds their $J^{PC}$)

  - Tools: *Amplitude analysis of data*

    To find new resonances not bump-hunting, but search for poles

    ⇨ must build in S-Matrix constraints

    \+ state-of-the-art knowledge of reaction dynamics

*Hadron spectrum, exotics*

ChPT + Analyticity + Unitarity
Dispersion Relations
Regge Theory, Models

Experimental Data
*CLAS, GlueX, JEF, COMPASS, BESS, LHCb, PANDA,…*

FFs, resonance parameters: $M_R$, $\Gamma_P$, couplings

# Scattering theory

- *S* matrix

$$S = I + 2i\rho T$$

- *S* is unitary (probability conservation)

$$S^\dagger S = I$$
$$\mathrm{Disc}\, T = \mathrm{Im}\, T = T^\dagger \rho T$$

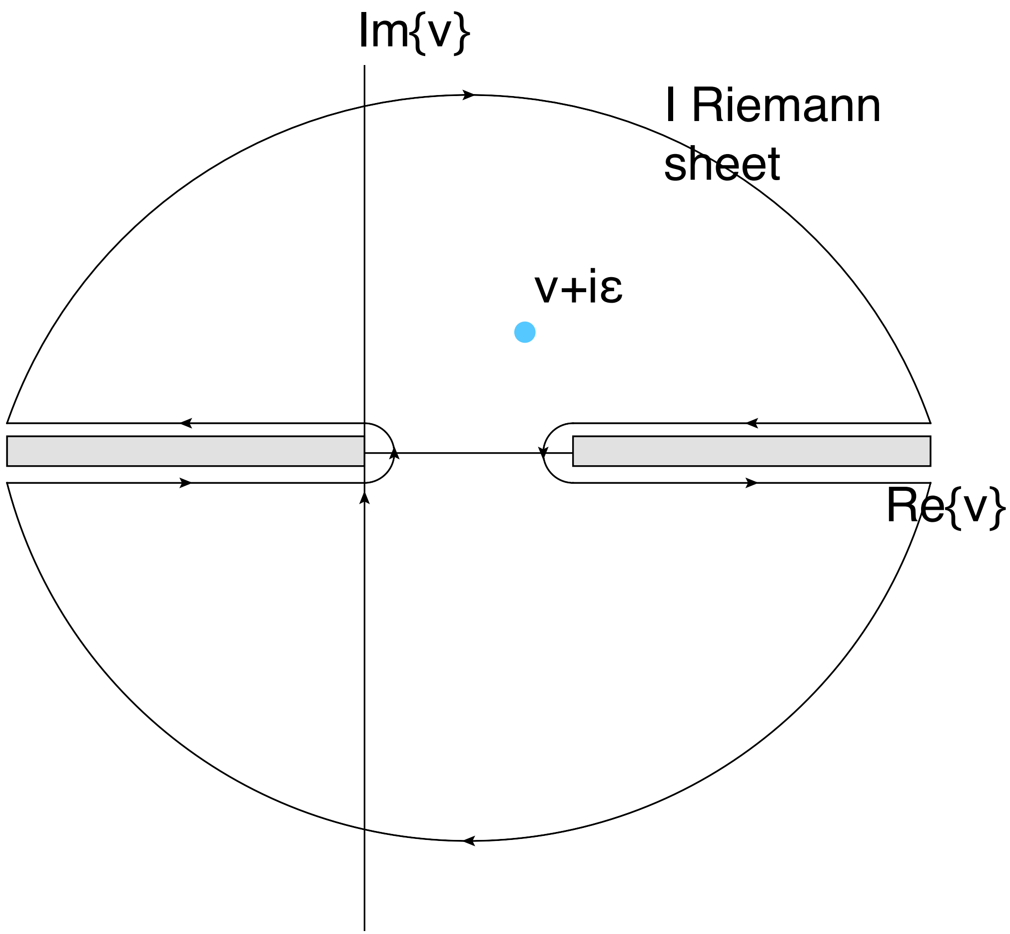
- *T* is an analytical function (causality)
- *T* has branch cuts at thresholds (several Riemann sheets)
- *T* has no poles in the first Riemann sheet

# Dispersion theory

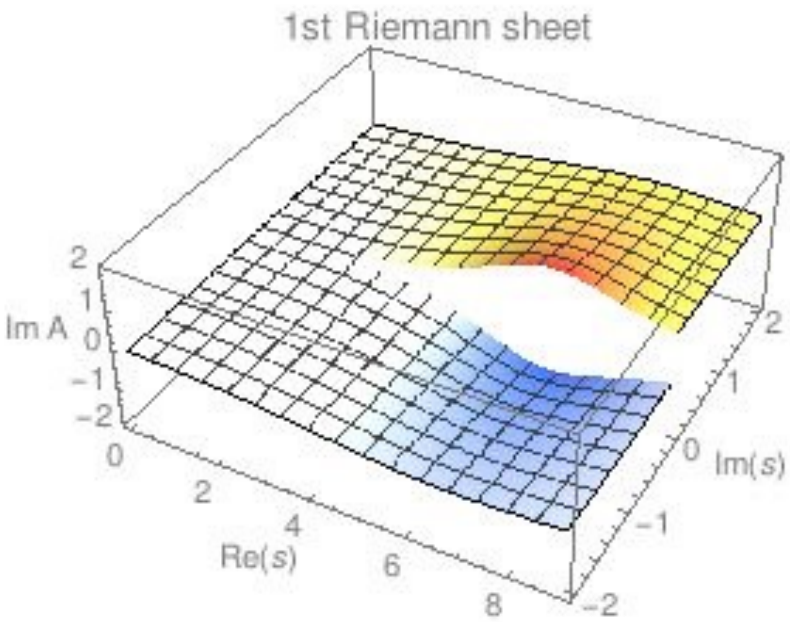
- Cauchy Theorem

$$t(\nu + i\epsilon) = \frac{1}{2\pi i}\int_C \frac{t(\nu')}{\nu' - \nu - i\epsilon} d\nu'$$

- Amplitude has no complex poles in the 1st Riemann sheet and has two branch cuts
- We can reconstruct the full amplitude in the whole complex plane

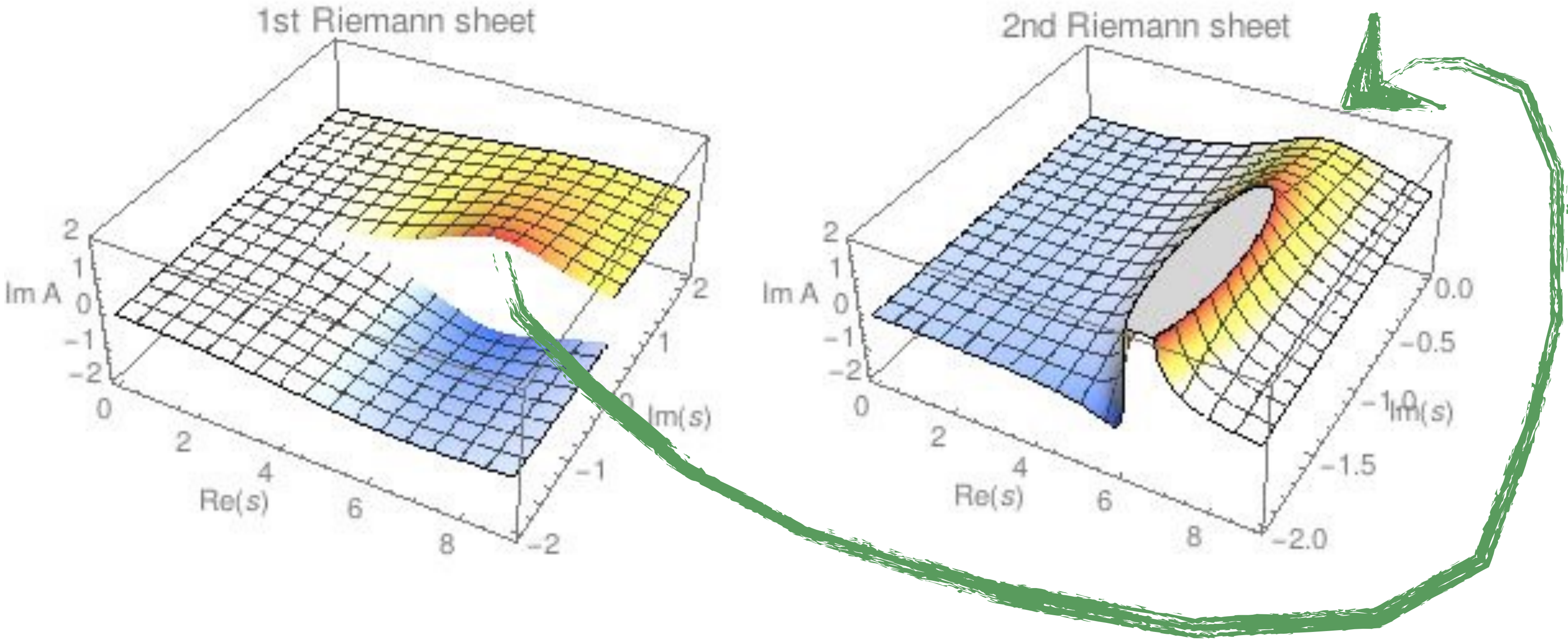


# Example: Breit-Wigner amplitude




$$A_I(s) = \frac{M}{M^2 - s - iMq_I(s)}$$

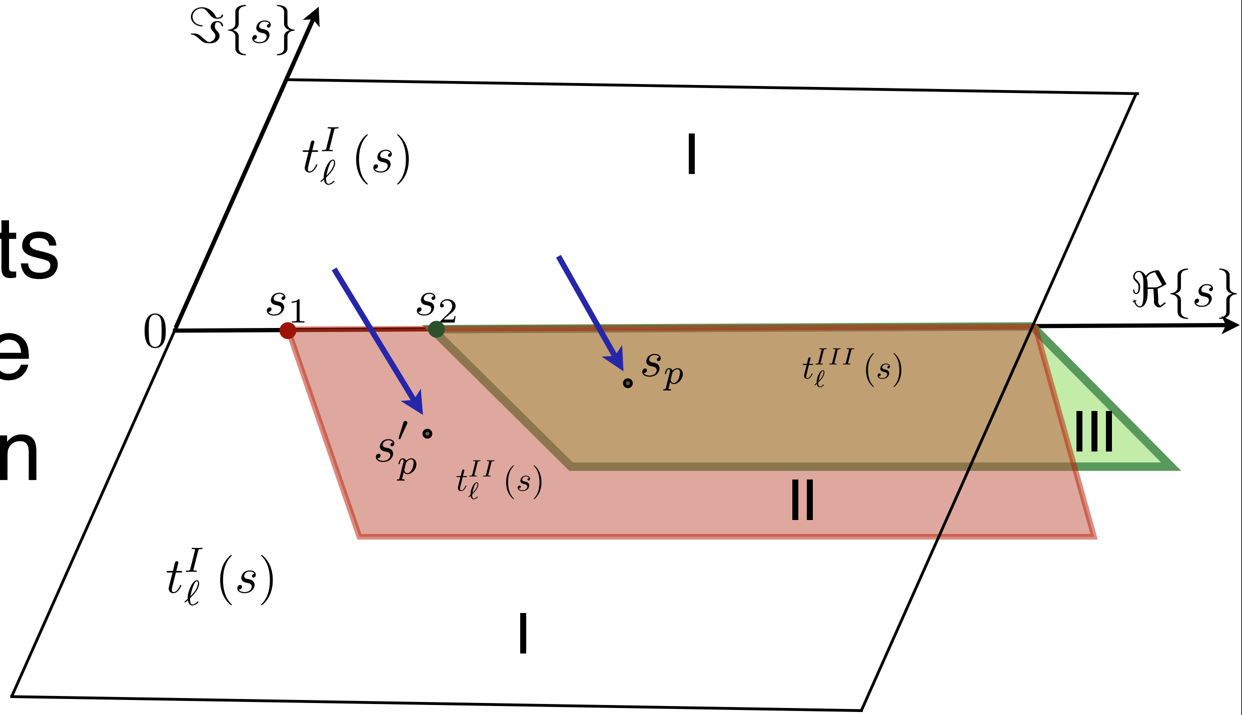
# Example: Breit-Wigner amplitude




$$A_I(s) = \frac{M}{M^2 - s - iMq_I(s)}$$

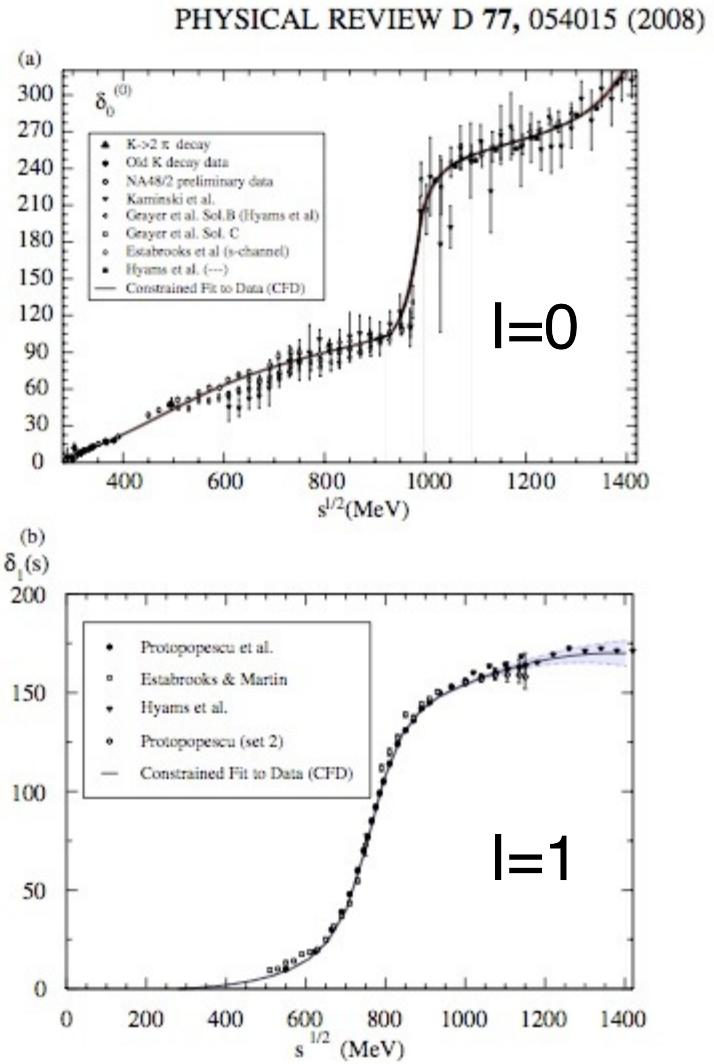
# Riemann sheets

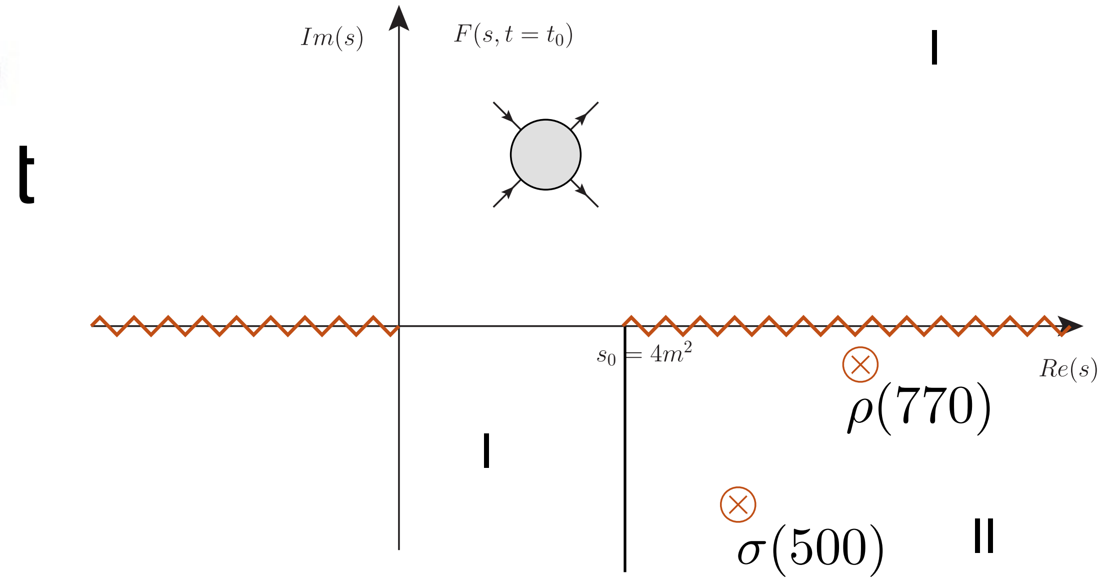
- The amount of Riemann sheets depends on the number of open channels: $2^N$



# Amplitude analysis: ππ scattering

Peláez et al.



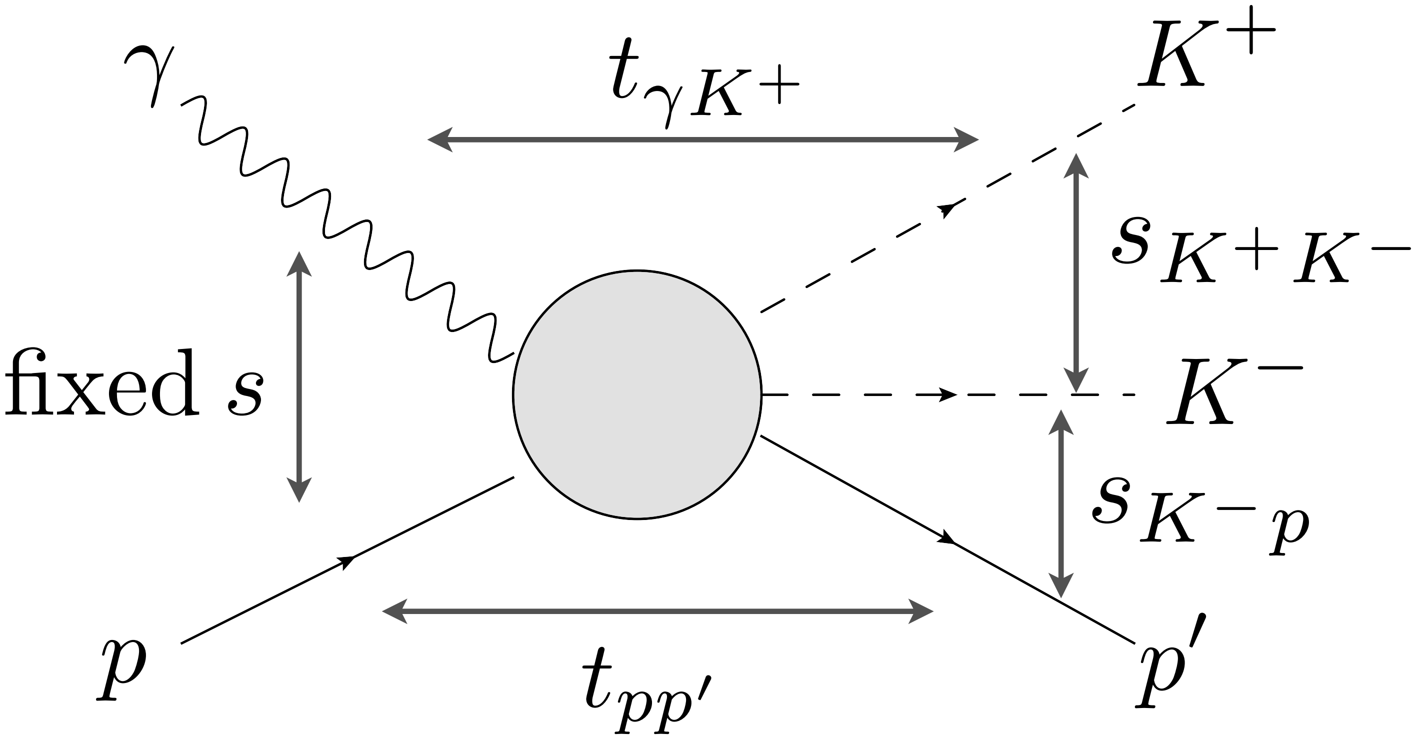




1. Parametrize the data
2. Check constrains (unitarity)
3. Continuation to extract the pole
4. Interpretation (hybrid, glueball, tetraquark, ...)

# $\gamma p \rightarrow K^+K^-p$

Amplitude depends on 5 Mandelstam variables:

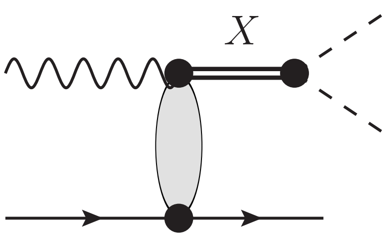
- One is fixed: *s*
- Two are measured: $s_{K+K-}$ and $s_{K-p}$
- Two are integrated out: $t_{\gamma K+}$ and $t_{pp'}$



# Interesting channels

GlueX (Hall D@JLab) invariant mass range 2-3 GeV

Glueballs?

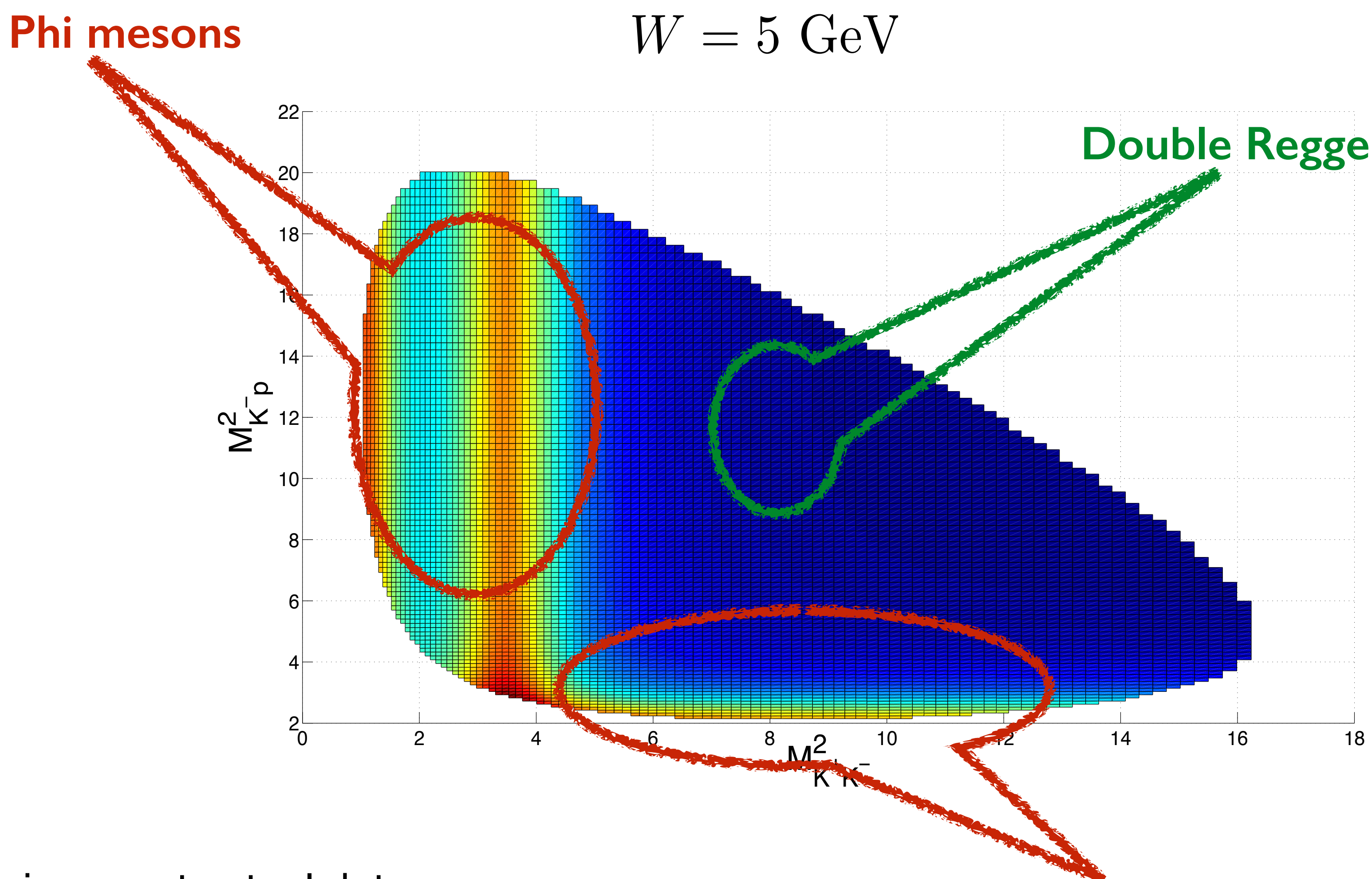



$$\pi\pi, K\bar{K} \quad X = \rho, \rho_3, \phi, \phi_3, f_0, f_2, f_2', a_0, a_2$$

$$\pi\eta, \pi\eta' \quad X = a_0, a_2, \pi_1$$

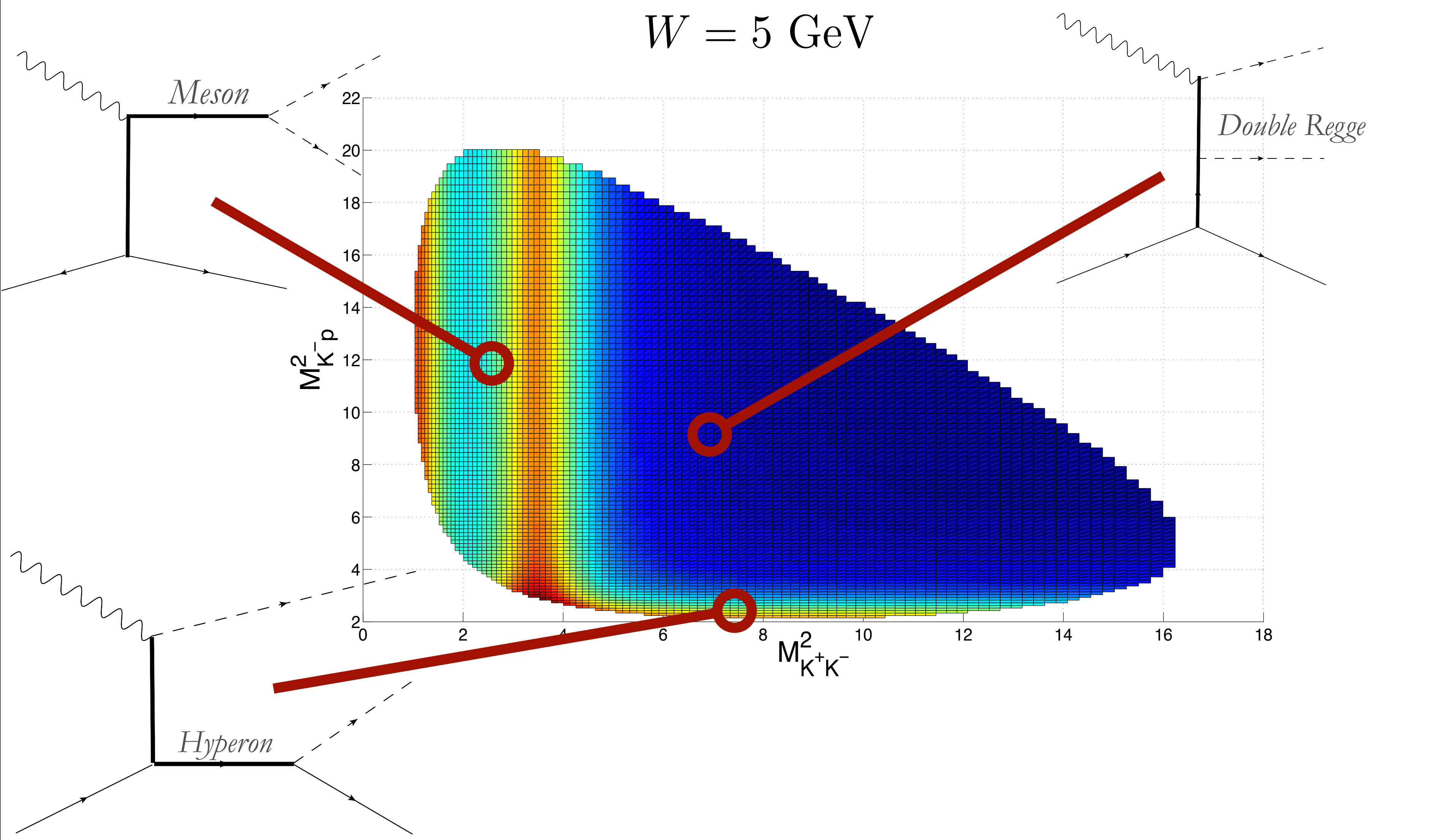
Hybrid?

# $K^+K^-$ Dalitz plot




Disclaimer: not actual data
(actual data from g12 CLAS@JLab under analysis)

# K+K- Dalitz plot



# Things we expect/hope to study

- Exotics
  - new physics that will help us understand the role of the gluon and confinement
- Strangeonia $(s\bar{s})$
  - this spectrum is not well studied and looks pretty empty. Also information on gluons and virtual quarks
- Hyperons
  - Strange content of hadrons, impact in hypernuclei physic and strangeness in neutron stars

# Models

- Dual model (B5)
  - Generalization of the Veneziano model
  - In the double-Regge limit:

Shi et al., PRD91, 034007 (2015)

- Deck model
  - More elaborated
  - Resonance region: K-matrix, coupled channels
  - High energy: Regge
  - Connection through dispersion theory

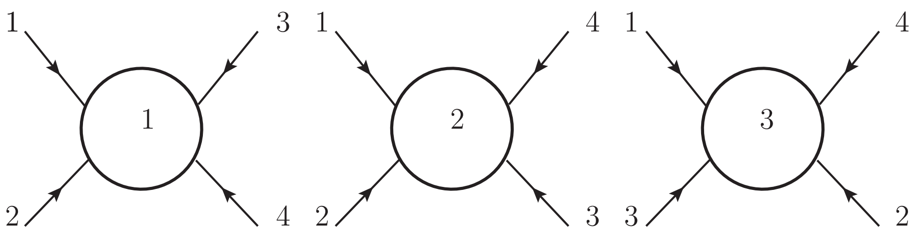
# Veneziano model (B4)

$$B_4(s,t) = \frac{\Gamma(-\alpha(s)\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} = \sum_{n=0}^{\infty} \frac{\beta_n(t)}{n - \alpha(s)} = \sum_{n=0}^{\infty} \frac{\beta_n(s)}{n - \alpha(t)}$$

$$\lim_{s\to\infty} B_4(s,t) = [-\alpha(s)]^{\alpha(t)}\,\Gamma(-\alpha(t))$$

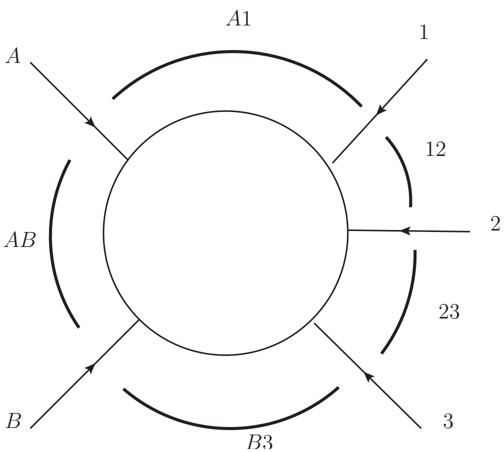
$$B_4(s,t) = \int_0^1 dx\, x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1}$$

- Advantages
  - Analyticity
  - Incorporates Regge-resonance duality
  - Right behavior at high energy
  - Generalization to N legs
- Caveats:
  - Unitarity is violated
  - Resonance details get lost
  - Only for high energy
  - Spinless particles (spin factor)



# B5 for $\gamma p \to K^+ K^- p$

$$B_5(s_{AB}, s_{A1}, s_{12}, s_{23}, s_{B3}) = \int_0^1 dt \int_0^1 dx\, x^{-\alpha_{12}-1} t^{-\alpha_{23}-1} (1-t)^{-\alpha_{A1}-1} (1-x)^{-\alpha_{B3}-1} (1-xt)^{-\alpha_{AB}+\alpha_{12}+\alpha_{23}}$$



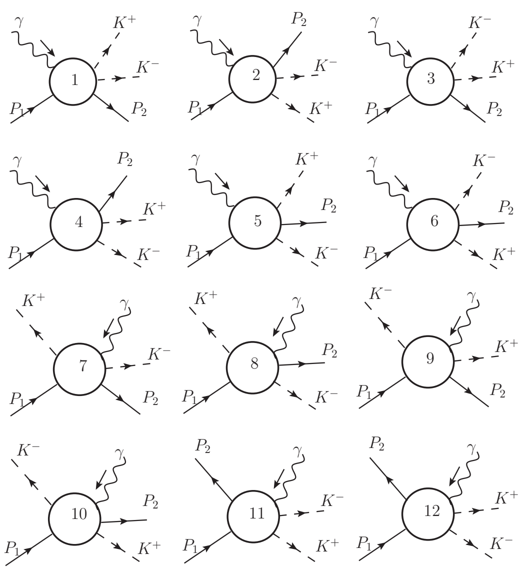

- A lot more complicated to compute
- We consider the double-Regge limit

$$s_{AB}, s_{12}, s_{23} \to \infty; \quad \frac{s_{12} s_{23}}{s_{AB}} = \text{fixed}; \quad \frac{t_{A1}}{s_{AB}}, \frac{t_{B3}}{s_{AB}} \to 0$$

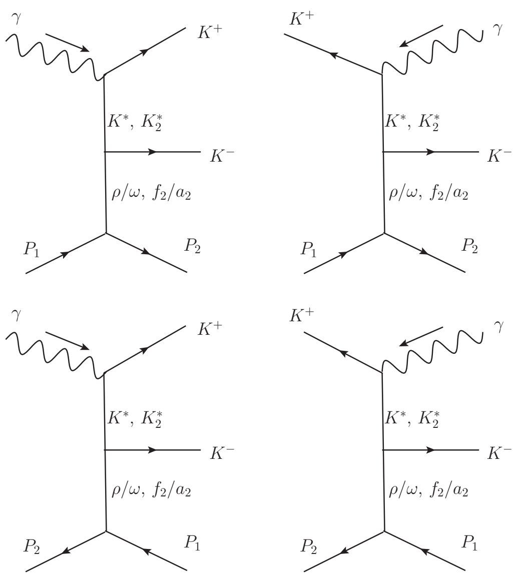
$$A + B \to \bar{1} + \bar{2} + \bar{3}$$

$$\gamma + p \to K^+ + K^- + p$$

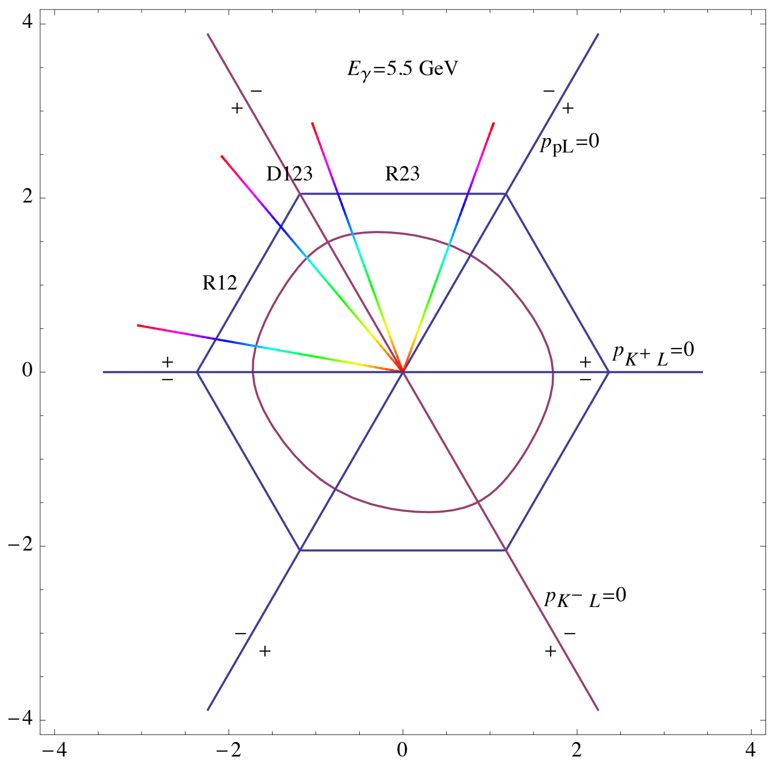
Shi et al., PRD91, 034007 (2015)




$\Rightarrow$



# Double-Regge Limit: Van Hove Plots



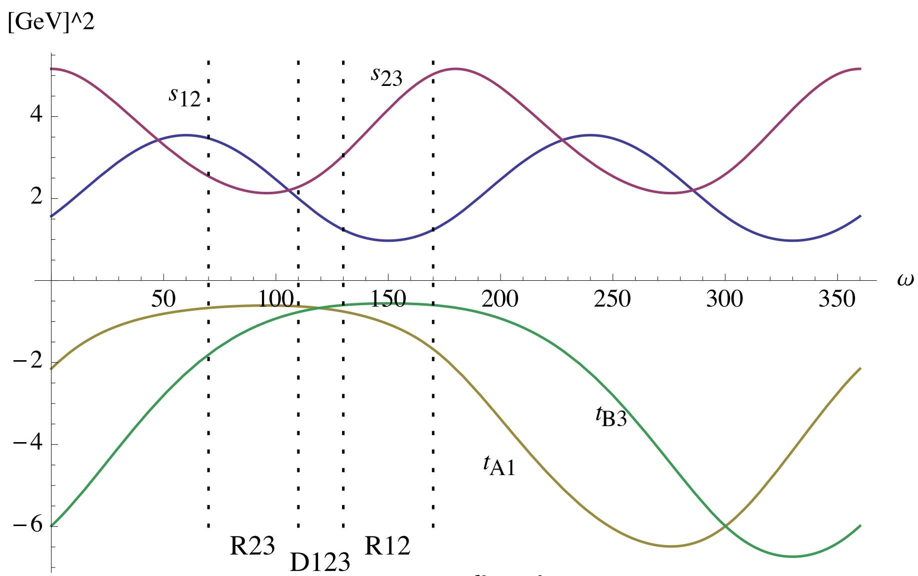

2-dimensional plot of the longitudinal momenta

$$q = \sqrt{p_{K+L}^2 + p_{K-L}^2 + p_{pL}^2}$$

$$p_{K+L} = \sqrt{2/3}q \sin\omega$$

$$p_{K-L} = \sqrt{2/3}q \sin(2\pi/3 + \omega)$$

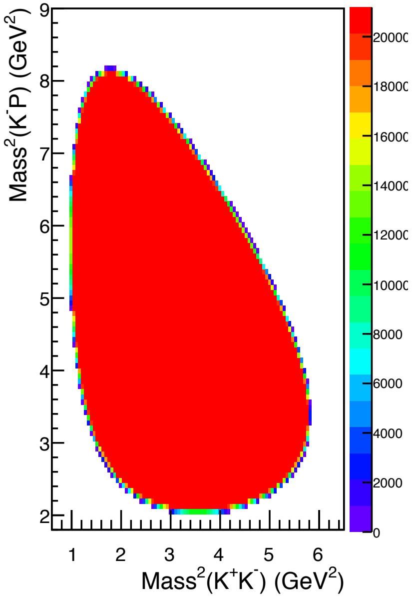
$$p_{pL} = \sqrt{2/3}q \sin(4\pi/3 + \omega)$$

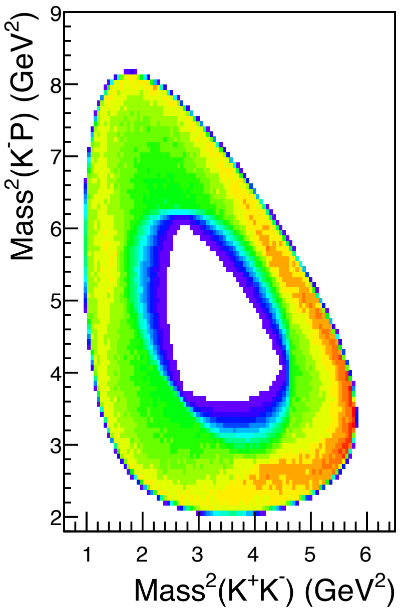


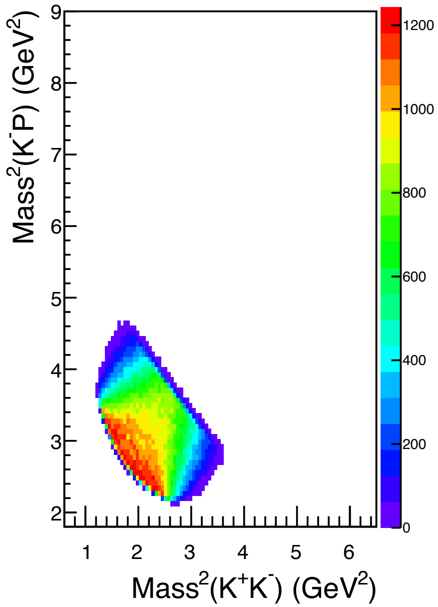

In the DRL we want *t* to be small and *s* large

fixed transverse momentum

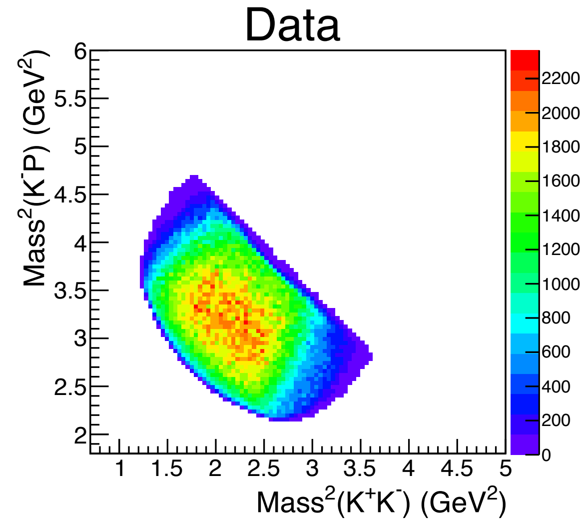
# Double-Regge Limit: Synthetic Data



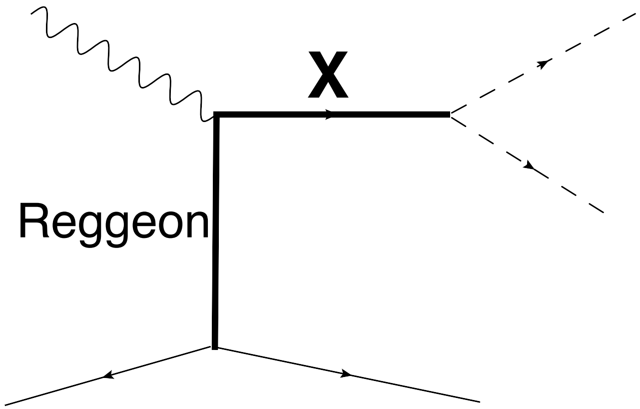



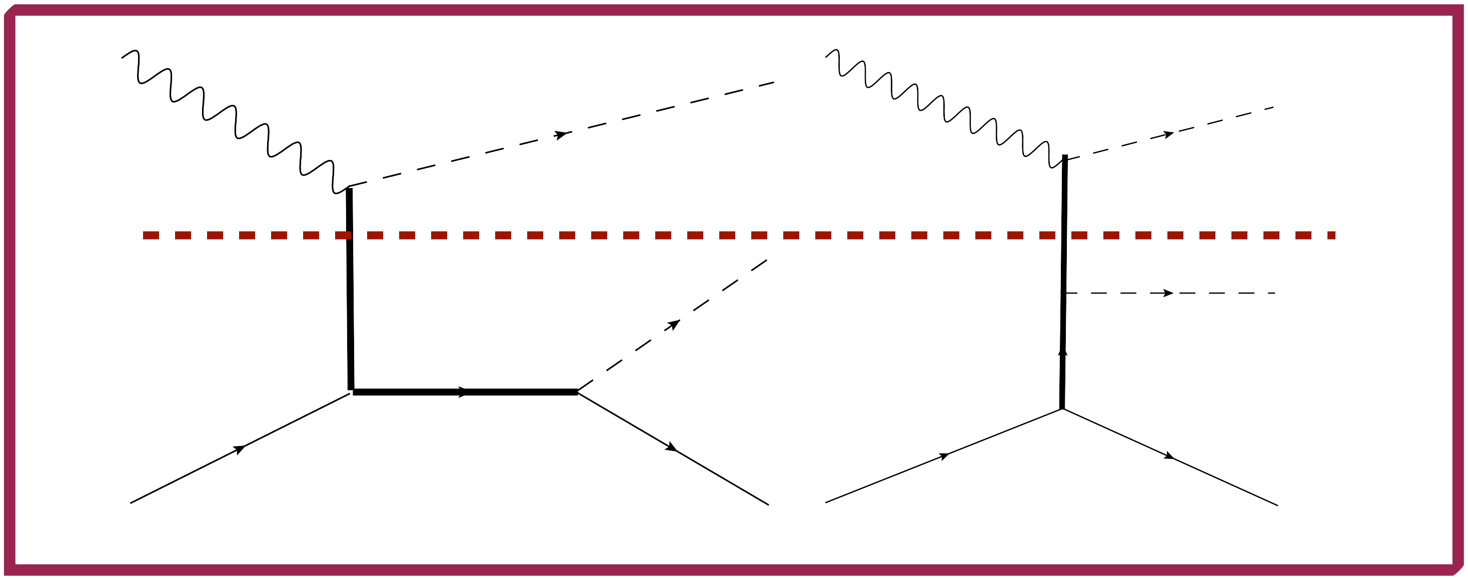




1. We generate an uniform distribution (phase space)
2. We cut the large transverse momenta
3. We perform the Van Hove selection
4. We incorporate spin

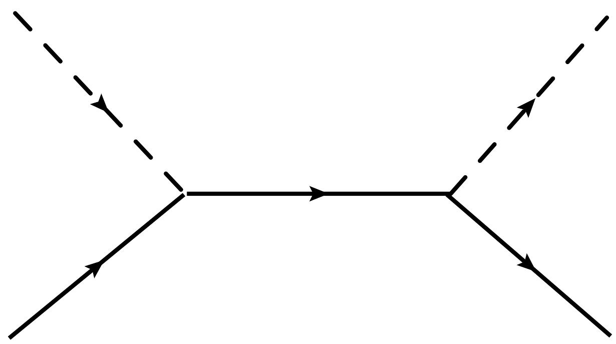


# Deck Model: one step at a time

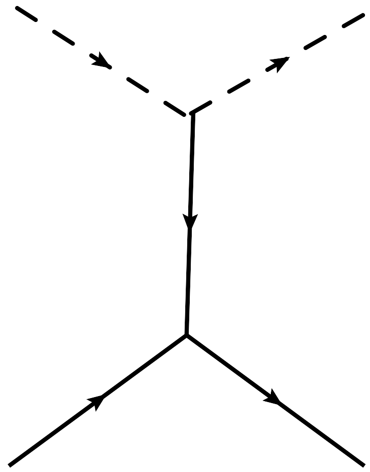


# KN scattering



Resonance region



High energy region

# KN in resonance region: Hyperons

- Partial-wave analysis

$$t^I(s,\cos\theta) = \sum_\ell f_\ell^I(s) P_\ell(\cos\theta)$$

- Coupled channels

$$\bar{K}N \to \bar{K}N, \pi\Sigma, \pi\Lambda$$

- Unitarity
- Analyticity
- Right threshold behavior (angular momentum barrier)
- Resonances are incorporated “by-hand” through Breit-Wigner amplitudes

# Single channel

$$S = I + 2i\,\rho(s)T(s)$$

$$\rho(s) = q\,\frac{q^{2\ell}}{1+q^{2\ell}}$$

$$T(s) = \frac{K(s)}{1 - i\rho_a(s)K(s)}$$

$$K(s) = \frac{M\Gamma}{M^2 - s}$$

$$i\rho_a(s) = \frac{s - s_{th}}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s-s'}\frac{\rho(s')}{s'-s_{th}}$$

# Coupled channels: 2 amplitudes

$$(K)_{kj} = x_k^1\, x_j^1\, K_1 + x_k^2(s)\, x_j^2(s)\, K_2$$

$$(T)_{kj} = c_{11}\, x_k^1\, x_j^1 + c_{12}\, x_k^1\, x_j^2 + c_{21}\, x_k^2\, x_j^a(s) + c_{22}\, x_k^2\, x_j^2$$

$$c_{11} = \mathcal{T}_1(s)/\mathcal{C}(s)$$

$$c_{22} = \mathcal{T}_2(s)/\mathcal{C}(s)$$

$$c_{12} = c_{21} = i\left[\sum_{k=1}^{n_C} x_k^1\, x_k^2 [\rho_a(s)]_k\right](s)\mathcal{T}_1(s)\mathcal{T}_2(s)/\mathcal{C}(s)$$

$$\mathcal{T}_1(s) = \frac{M_1}{M_1^2 - s - i\sum_{k=1}^{n_C}[\rho_a(s)]_k M_1 [x_k^1]^2}$$

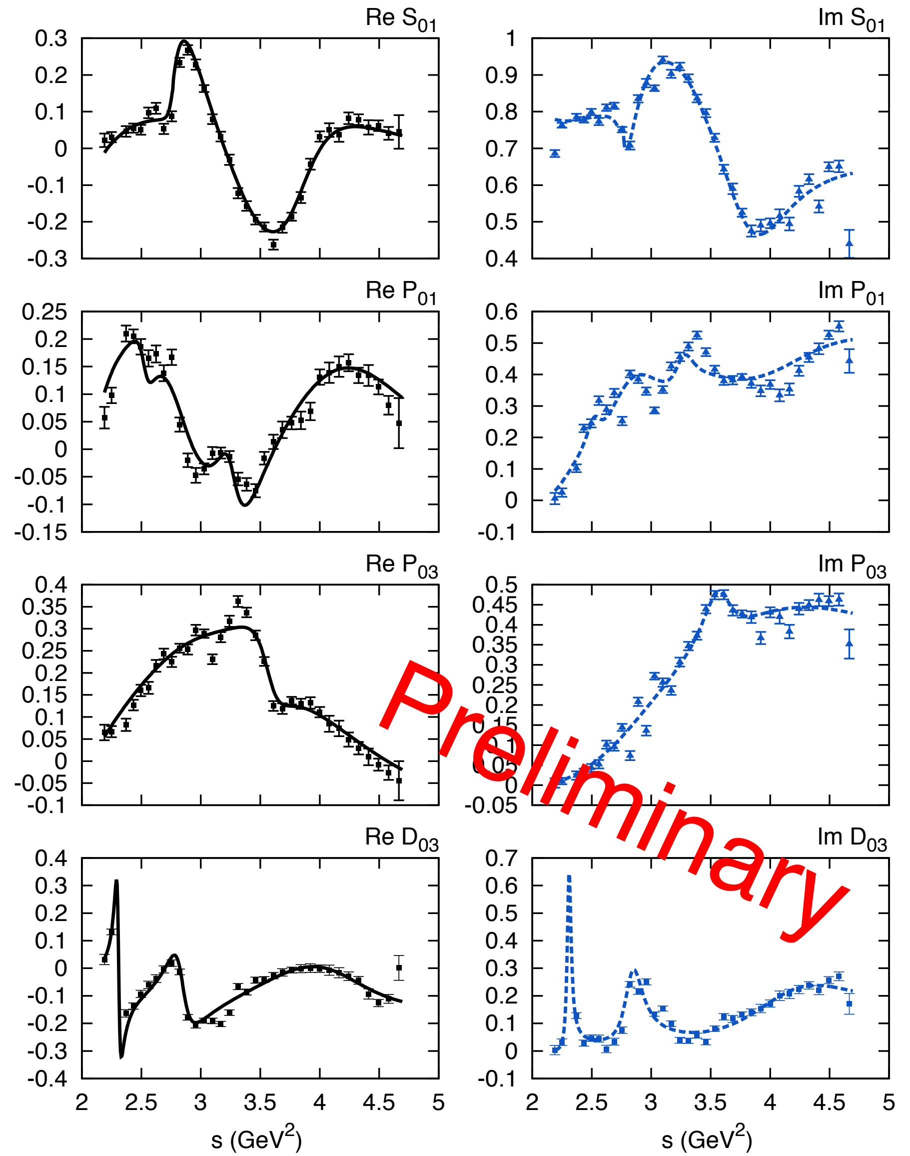
$$\mathcal{C}(s) = 1 + \left[\sum_{k=1}^{n_C} x_k^1\, x_k^2 [\rho_a(s)]_k\right]^2 \mathcal{T}_1(s)\mathcal{T}_2(s)$$

# KN in resonance region

Fit single-energy partial waves from Kent State University analysis of:

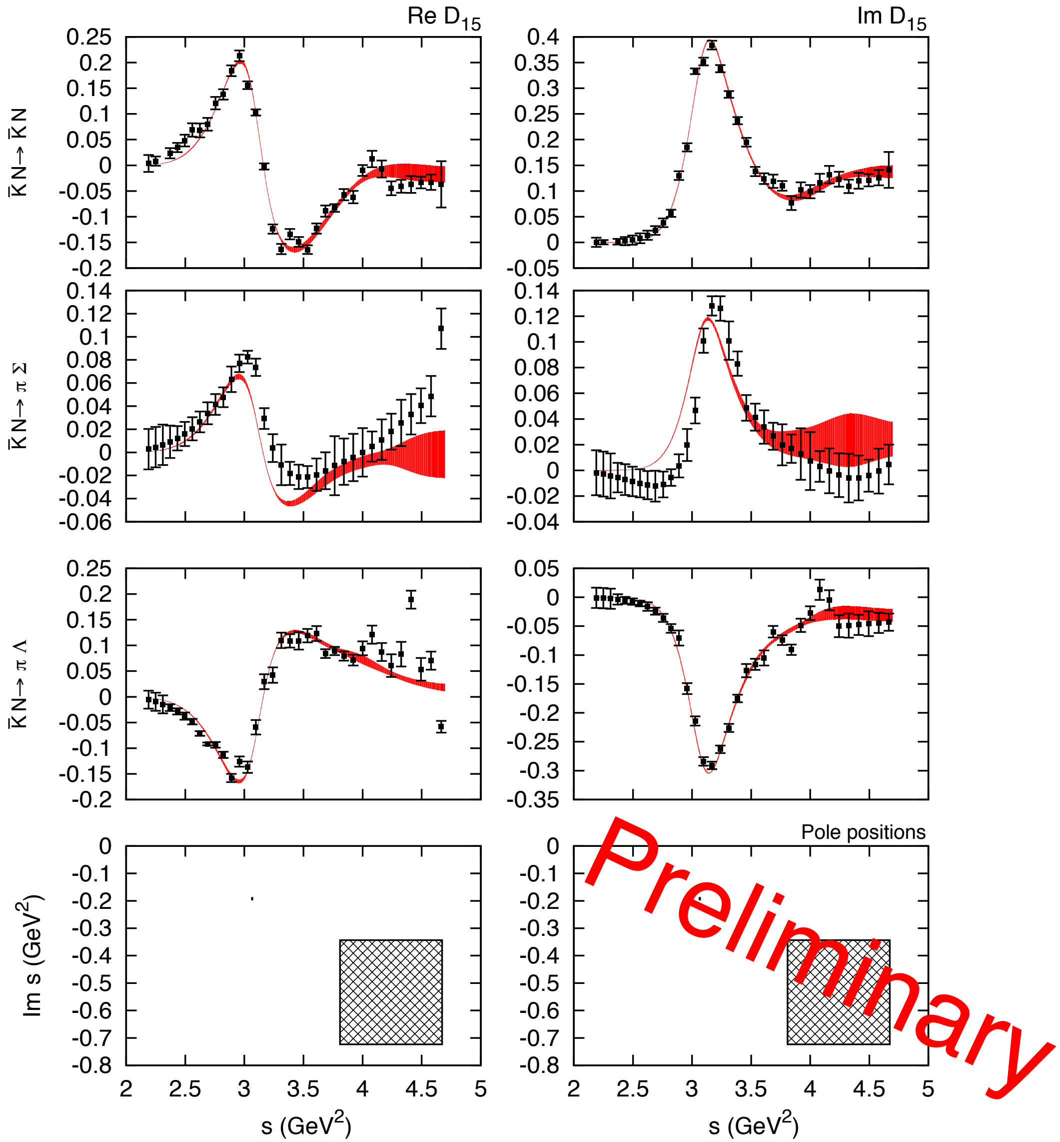
- ~8000 exp. data for $\bar{K}N \to \bar{K}N$
- ~4500 exp. data for $\bar{K}N \to \pi\Lambda$
- ~5000 exp. data for $\bar{K}N \to \pi\Sigma$

Model can be readjusted once we extend to the two kaon photoproduction process, getting more insight on hyperons

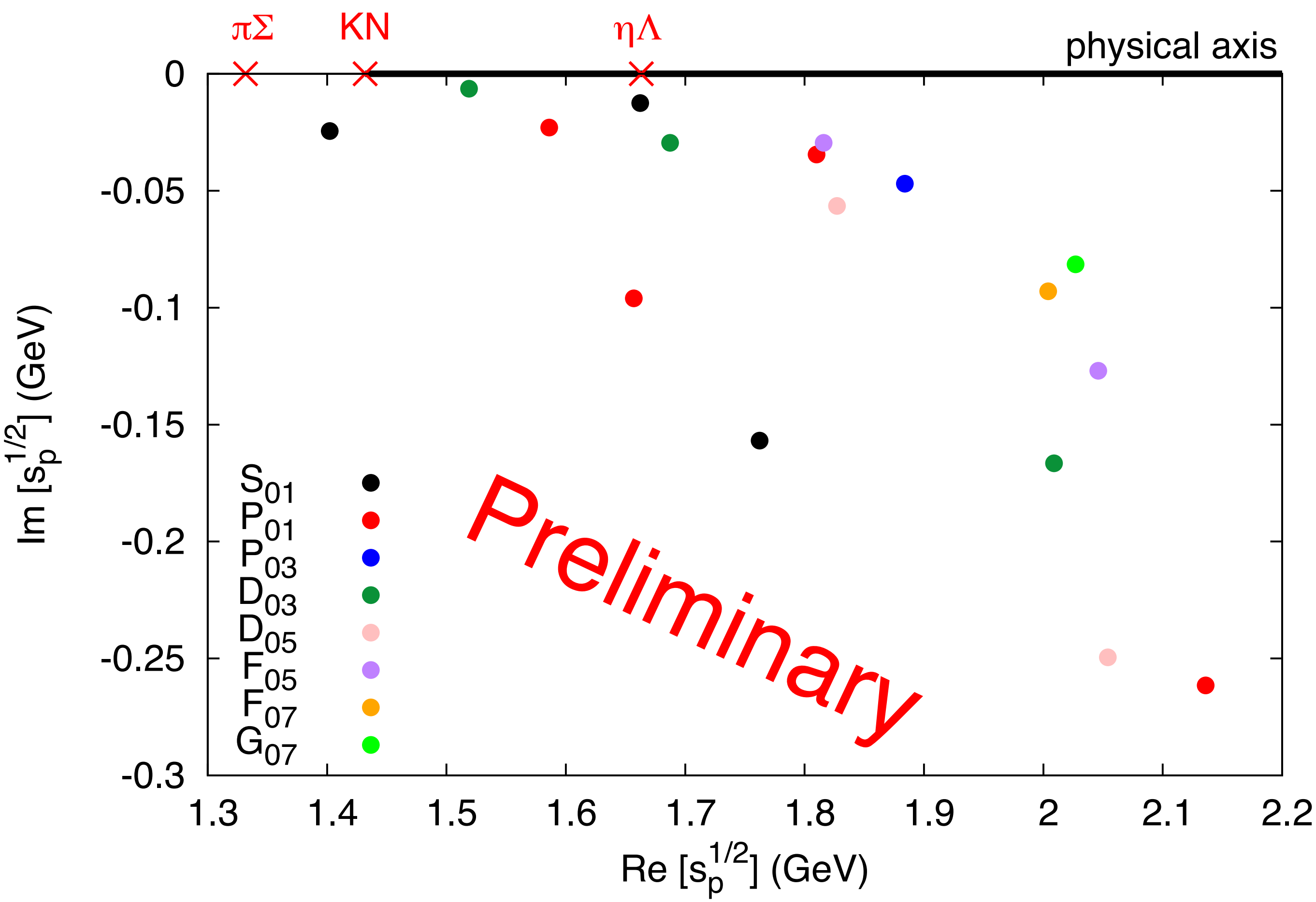


# $D_{15}$

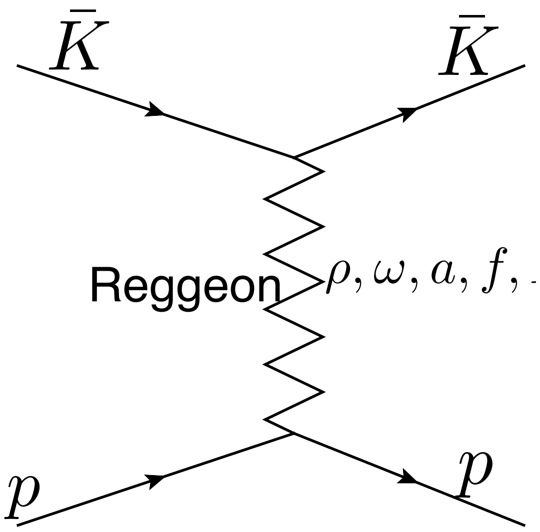
- Closest poles to the physical axis
- Errors computed through Bootstrap technique

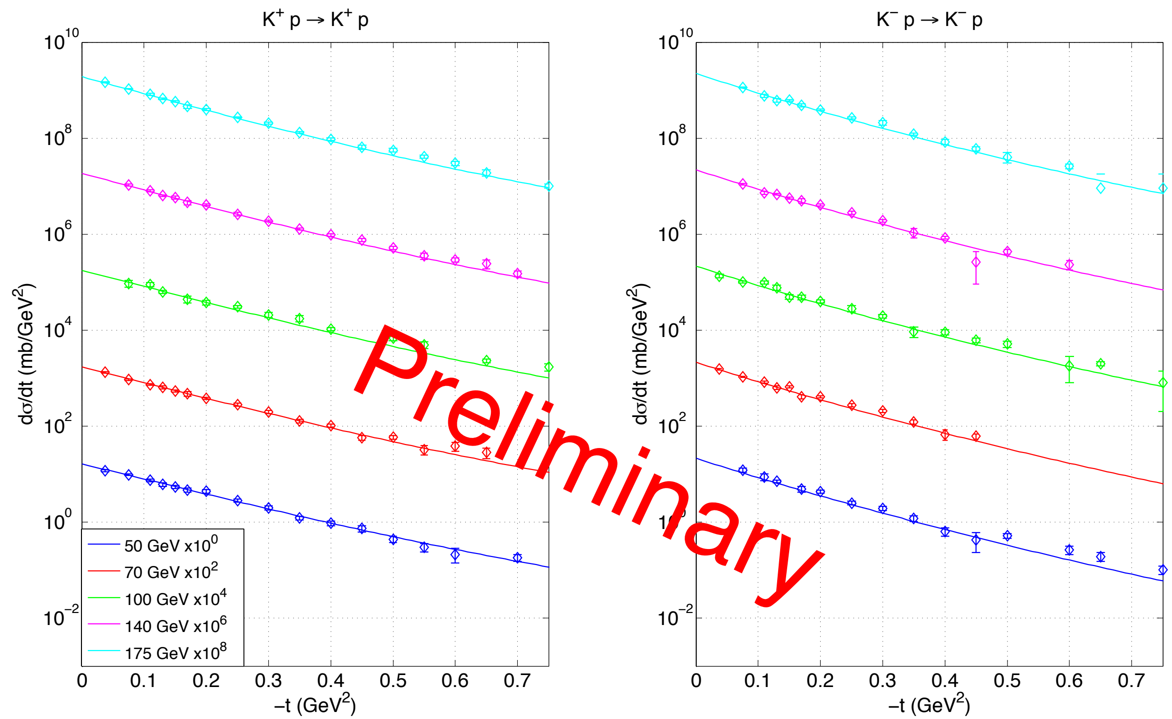


# Hyperons from KN scattering



# KN scattering in high energy region

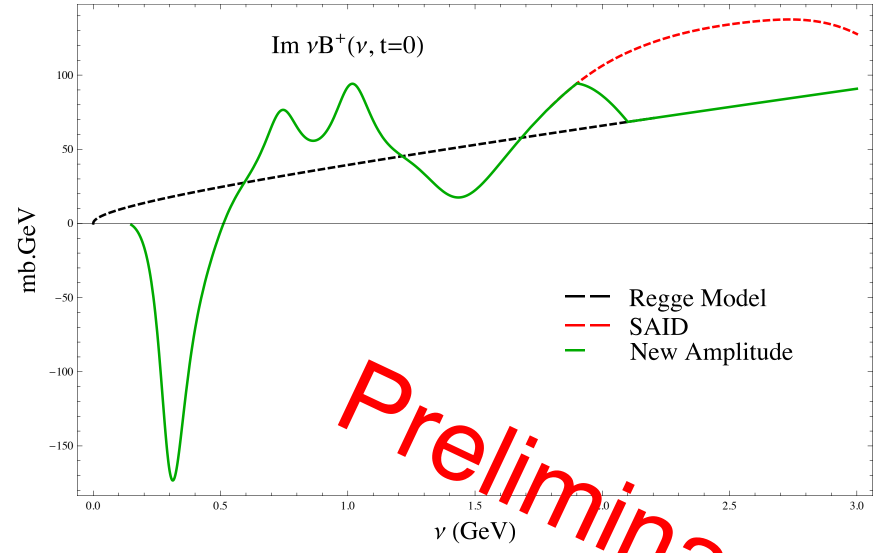


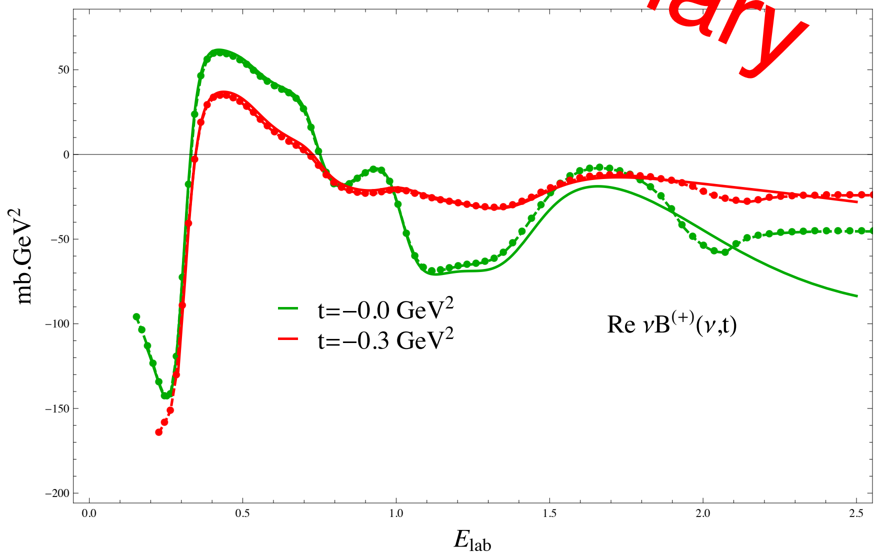



# Connecting high and low energies

- Let's use πN scattering as a playground
- Dispersion theory (Finite energy sum rules)
- We use high-energy to constrain low-energy
- Construct Im A from 0 to infinity via FESR
- Reconstruct amplitude from dispersion relations

$$\text{Re }\nu B^{(+)}(\nu,t) = \frac{g_r^2}{2m}\frac{2\nu^2}{\nu_m^2-\nu^2} + \frac{2\nu^2}{\pi}\text{P}\int_{\nu_0}^{\infty}\frac{\text{Im }B^{(+)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$



# Connecting high and low energies

- This is as far as I can go... we are working on finalizing both the high-energy and the low-energy models as well as the the pion-nucleon case
- Next steps will be to connect high and low energy in KN, build the full two kaon photoproduction amplitude and compare to data
- For B5 the next step is to compute the single Regge limit (associated to hyperon excitations)

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- Many interesting questions remain open regarding hadrons, QCD and confinement
- Hadron spectroscopy has been and remains as a prime tool
- Lattice is making big steps forward
- A lot of experimental data are going to come during the next years: JLAB, COMPASS, LHC, MAMI, ELSA, PANDA, KLOE, BES, J-PARC
- Amplitude analysis is a critical part of all this effort

# Summary

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- Hadron spectroscopy has been and remains as a prime tool
- Lattice is making big steps forward
- A lot of experimental data are going to come during the next years: JLAB, COMPASS, LHC, MAMI, ELSA, PANDA, KLOE, BES, J-PARC
- Amplitude analysis is a critical part of all this effort
  - ✓ No solid amplitude analysis ⇒ No reliable data interpretation

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  - ✓ At JPAC we are doing as much as we can in close collaboration with experimentalists