

Partículas en un espacio de espín extendido: conexión Lagrangiana

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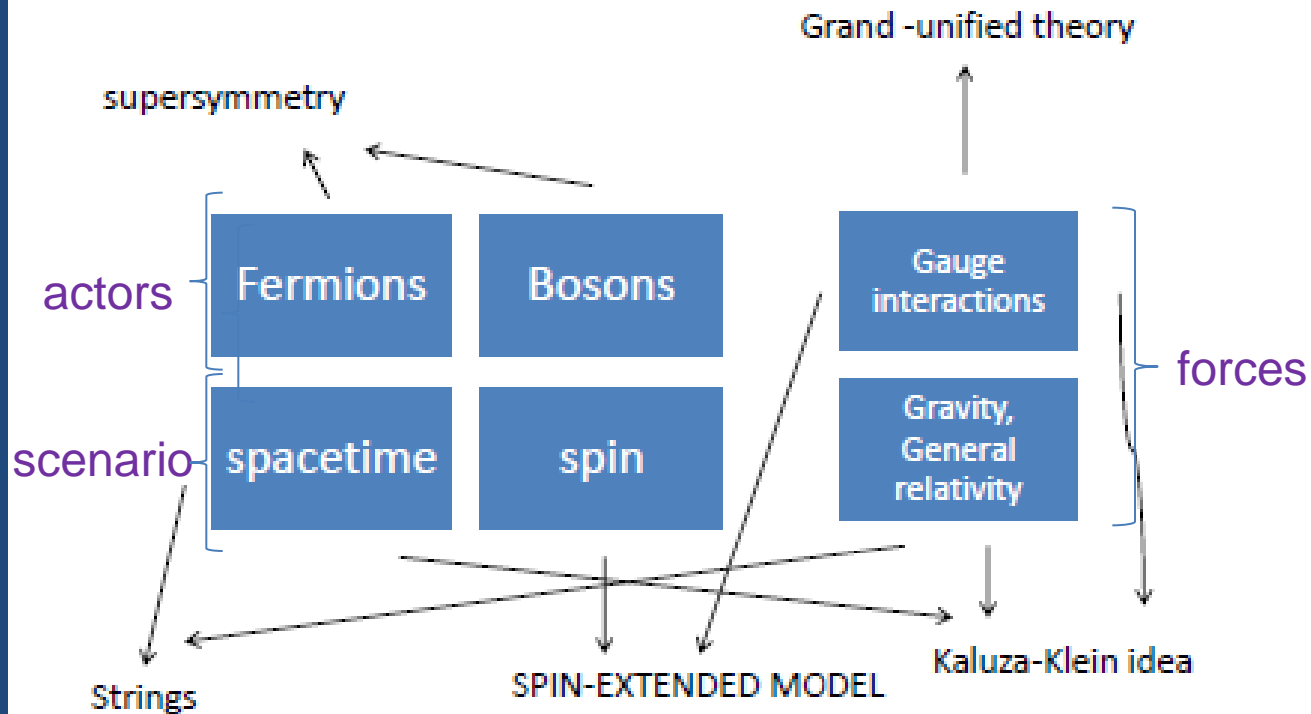
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Contents

- Spin-extended model within unification and SM extensions.
- Model's symmetry groups and representations: chiral elements.
- Lagrangian construction.
- fermions and scalars in $(7+1)$ -d, and heavy-quark mass **hierarchy** effect.

Standard-model extensions

Unification examples



LORENTZ AND MAXIMAL SCALAR SYMMETRY AT

D DIMENSION

$$\underbrace{\gamma_0 \ \gamma_1 \ \gamma_2 \ \gamma_3}_{\text{4-D Lorentz symmetry}} \quad \underbrace{\gamma_4, \dots, \gamma_{D-1}}_{\text{Scalar symmetry}}$$

$$J_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \quad \mu, \nu = 0, \dots, 3 \quad \gamma_a \quad a = 4, \dots, D-1$$

4-D Lorentz symmetry \otimes Scalar symmetry
 unitary: $U(2^{(D-4)/2})$

$$[J_{\mu\nu}, \gamma_a] = 0$$

$$[\tilde{\gamma}_5, \gamma_a] = 0 \quad \tilde{\gamma}_5 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3$$

$$[H, \gamma_a] = 0 \quad H = i \gamma_0 \bar{\nabla} \cdot \bar{\gamma}$$

maximal scalar symmetry

$$U_R \otimes U_L \quad U_R = \frac{1}{2} (1 + \tilde{\gamma}_5) U(2^{(D-4)/2})$$

$$U_L = \frac{1}{2} (1 - \tilde{\gamma}_5) U(2^{(D-4)/2})$$

Coleman-Mandula OK

Two physical interpretations

- Kaluza-Klein type of framework, for in higher than (3+1)-dimensions, only the spin component in

$$\mathcal{P}_P \left[\frac{1}{2} \sigma_{\mu\nu} + i(x_\mu \partial_\nu - x_\nu \partial_\mu) \right] \quad \mu=4, \dots, N-1, \nu=4, \dots, N-1$$

remains as symmetry operator; thus, spatial components are frozen.

- Elementary discrete degree-of-freedom matrix construction.

(a) Symmetry groups (b) Representations

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Table 1. (a) Shows the arrangement of symmetry operators U in matrix space of arbitrary dimension N , after projection over \mathcal{S}_P , with left-handed and right-handed operators subspaces;⁶ (*) represents the matrix subspace containing the projector $1 - \mathcal{P}_S = 1 - \mathcal{P}_P$; its choice within the right-handed symmetry components is arbitrary. (b) Shows the arrangement of matrix solutions Ψ in the extended-spin model which is divided into four $\frac{N}{2} \times \frac{N}{2}$ matrix blocks, containing fermion (F), vector (and axial-) (V), scalar (and pseudo-) and antisymmetric (S, A) terms.

(*)		
	$\mathcal{S}_{(N-4)R} \otimes \mathcal{C}_4$	
		$\mathcal{S}_{(N-4)L} \otimes \mathcal{C}_4$

(a)

(*)	F	F
F	V	S, A
F	S, A	V

(b)

(5+1)-d spin-extended model

Following the Kaluza-Klein idea, one seeks constraints on symmetries extending the spin space. For example, the Clifford algebra on 5+1 dimensions contains the usual Lorentz generators, and scalar ones.

Lorentz

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \quad \mu, \nu = 0, \dots, 3$$

$$\gamma_a, \quad a = 4, \dots, N-1$$

Scalars

$$U_{Le}(1) \times U_Y(1) \times SU_L(2)$$

$$L = \frac{3}{4} - \frac{i}{4}(1 + \tilde{\gamma}_5)\gamma^5\gamma^6 - \frac{1}{4}\tilde{\gamma}_5$$

$$Y = -1 + \frac{i}{2}(1 + \tilde{\gamma}_5)\gamma^5\gamma^6$$

$$I_1 = \frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^5$$

$$I_2 = -\frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^6$$

$$I_3 = -\frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^5\gamma^6$$

Standard-model heavy particles

	Masses (GeV)	Spin	Weak I^2	Hypercharge Y
• $W^{+/-}$	80.4	1	1	0
• Z	91.2	1	0	0
• H	126	0	$\frac{1}{2}$	-1
• t	173	$\frac{1}{2}$	$\frac{1}{2}, 0$	$\frac{1}{3}, \frac{4}{3}$
• b	4.6	$\frac{1}{2}$	$\frac{1}{2}, 0$	$\frac{1}{3}, -\frac{2}{3}$

Composite multiplet suggested

INTERACTIVE THEORY FOR FIELDS IN AN EXTENDED SPIN SPACE

Keep

Polarization basis for fields

vector $A^a \gamma_a$

scalar $\phi^a G_a$

transformation rule

$$\bar{\psi} \rightarrow u \bar{\psi} u^\dagger$$

drop

free-field generalized Dirac equation

There is an equivalence between
a field theory and its formulation in
an extended spin space

Spin-extended model equivalent Lagrangian terms

Fermion-vector

$$\frac{1}{N_f} \text{tr} \Psi^\dagger \{ [i\partial_\mu I_{\text{den}} + gA_\mu^a(x)I_a] \gamma_0 \gamma^\mu - M\gamma_0 \} \Psi P_f ,$$

Projection operator

$$P_f = \frac{1}{\sqrt{2}} (\tilde{\gamma}^5 - \gamma^0 \gamma^1)$$

(7+1)-d
fermions

Table 3. (a) Massless left-handed quark weak isospin doublet and (b) right-handed singlets, with momentum along $\pm\hat{z}$.

(a)				
Hypercharge 1/3 left-handed doublet	I_3	Q	$\frac{3i}{2}B\gamma^1\gamma^2$	
$Q_L^1 = \begin{pmatrix} U_L^1 \\ D_L^1 \end{pmatrix} = \begin{pmatrix} \frac{1}{16} (1 - \tilde{\gamma}_5) (\gamma^5 - i\gamma^6) (\gamma^7 + i\gamma^8) (\gamma^0 + \gamma^3) \\ \frac{1}{16} (1 - \tilde{\gamma}_5) (\gamma^5 - i\gamma^6) (1 - i\gamma^7\gamma^8) (\gamma^0 + \gamma^3) \end{pmatrix}$	1/2	2/3	1/2	
	-1/2	-1/3	1/2	
(b)				
$I_3 = 0$ right-handed singlet	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$	
$U_R^1 = \frac{1}{16} (1 + \tilde{\gamma}_5) (\gamma^5 - i\gamma^6) (\gamma^7 + i\gamma^8) \gamma^0 (\gamma^0 + \gamma^3)$	4/3	2/3	1/2	
$D_R^1 = \frac{1}{16} (1 + \tilde{\gamma}_5) (\gamma^5 - i\gamma^6) (1 - i\gamma^7\gamma^8) \gamma^0 (\gamma^0 + \gamma^3)$	-2/3	-1/3	1/2	

(7+1)-d
scalars

Table 4. Scalar Higgs-like pairs.

0 baryon-number scalar	I_3	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8} (1 - i\gamma^5\gamma^6) (\gamma^7 + i\gamma^8) \gamma^0 \\ \frac{1}{8} (1 - i\gamma^5\gamma^6) (1 + i\gamma^7\gamma^8\tilde{\gamma}_5) \gamma^0 \end{pmatrix}$	1/2	1	1	0
	-1/2	0	0	0
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8} (1 - i\gamma^5\gamma^6) (\gamma^7 + i\gamma^8) \tilde{\gamma}_5\gamma^0 \\ \frac{i}{8} (1 - i\gamma^5\gamma^6) (1 + i\gamma^7\gamma^8\tilde{\gamma}_5) \gamma^7\gamma^8\gamma^0 \end{pmatrix}$	1/2	1	1	0
	-1/2	0	0	0

$SU(2)_L \times U(1)_Y$ -invariant term

$$\frac{1}{N_f} \text{tr} \left\{ \left[m_U \Psi_R^{U\dagger}(x) [\phi_1(x) + \phi_2(x)] \Psi_L^Q(x) \right. \right. \\ \left. \left. + m_D \Psi_L^{Q\dagger}(x) [\phi_1(x) - \phi_2(x)] \Psi_R^D(x) \right] P_f \right\} + \{cc\},$$

Higgs mechanism \rightarrow

$$H_v = a\phi_1^0 + b\phi_2^0 + a\phi_1^{0\dagger} + b\phi_2^{0\dagger}$$

$$H_v U_M^1 = m_U U_M^1, \quad H_v U_M^{c1} = -m_U U_M^{c1}$$

$$H_v D_M^1 = m_D D_M^1, \quad H_v D_M^{c1} = -m_D D_M^{c1}$$

Hierarchy effect:

$$m_U = \frac{(a+b)}{2}, \quad m_D = \frac{(a-b)}{2}$$

Main points

- Spin-extended model provides for a feasible SM extension.
- At given dimension, it constrains groups and representations.
- A Lagrangian construction produces a gauge invariant formulation.
- At $(7+1)$ -d, a predicted Higgs doublet induces a fermion mass **hierarchy** effect, suggestive of the bottom and top quark mass values.

Composite models

- 1961 Nambu Jona-Lasignio. Superconductivity model in which four-fermion interaction generates both fermion and boson masses.
- 1989 Nambu. Higgs from top quark condensate.
- Bardeen, Hill, Lindner, use fixed point in renormalization.
- Technicolor: Higgs composed of fermions alleviates fine-tuning problem.
- Spin extended model.

Can standard-model bosons be constructed in terms of fermions?

Higgs

$$\Phi \propto \frac{1}{2} \begin{pmatrix} \bar{b}^a (1 + \gamma_5) t^a \\ -\bar{t}^a (1 + \gamma_5) t^a \end{pmatrix} = i\tau_2 (\bar{t}_R \Psi_L)^{\dagger T},$$
$$\tilde{\Phi} \equiv i\tau_2 \Phi^{\dagger T} \propto \bar{t}_R \Psi_L.$$

$$Y = -1 \quad I_3 = 1/2 \text{ states: } H_t^0 = t_L^\dagger \bar{t}_L^\dagger + \bar{t}_R t_R.$$

W

$$W_\downarrow^+ = t_L^\dagger \bar{b}_R^\dagger$$

Z

left-handed helicity -1 hypercharge carrier is described by $A_L = \frac{2}{3}(t_L^\dagger \bar{t}_R^\dagger + b_L^\dagger \bar{b}_R^\dagger)$

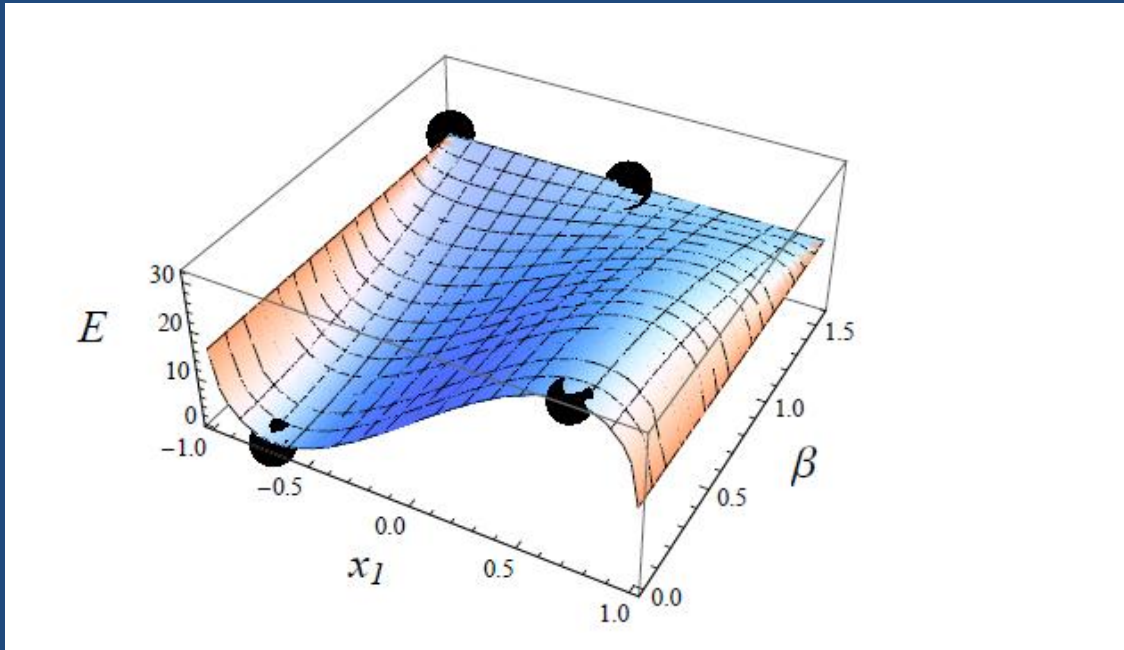
Standard-model extensions

Phenomenological prediction Conceptual basis

Hamiltonian model: known mesons

- Fermion Hamiltonian of the form:

$H=H(\psi_t, \psi_b, m_t, m_b)$; variational calculation



toponium and bottomium: masses $2m_t$ $2m_b$