

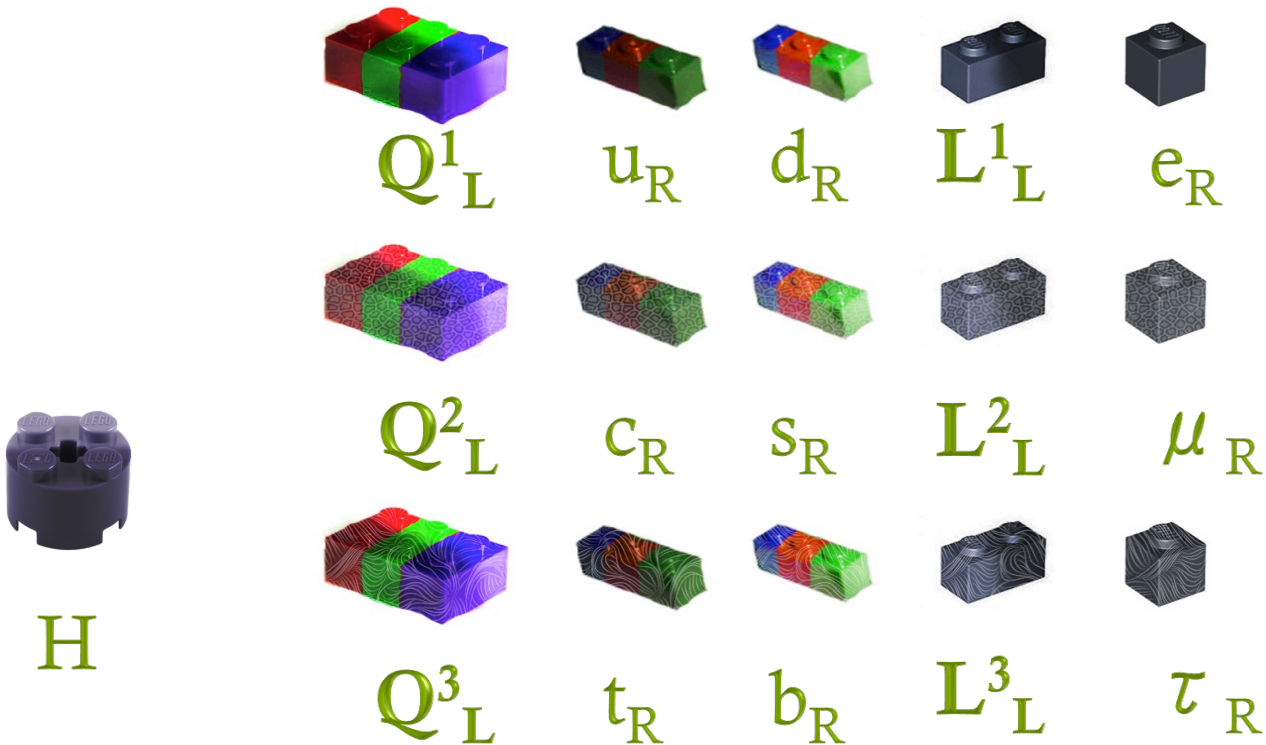
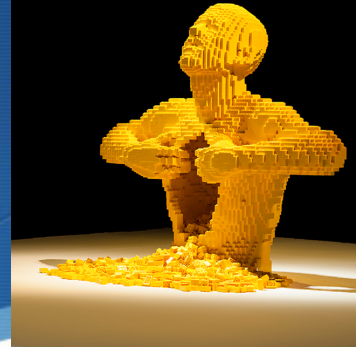
# Towards a complete A4 X SU(5) SUSY GUT

F. Börkeroth, **F. J. de Anda**, I. de Medeiros Varzielas, S. F. King



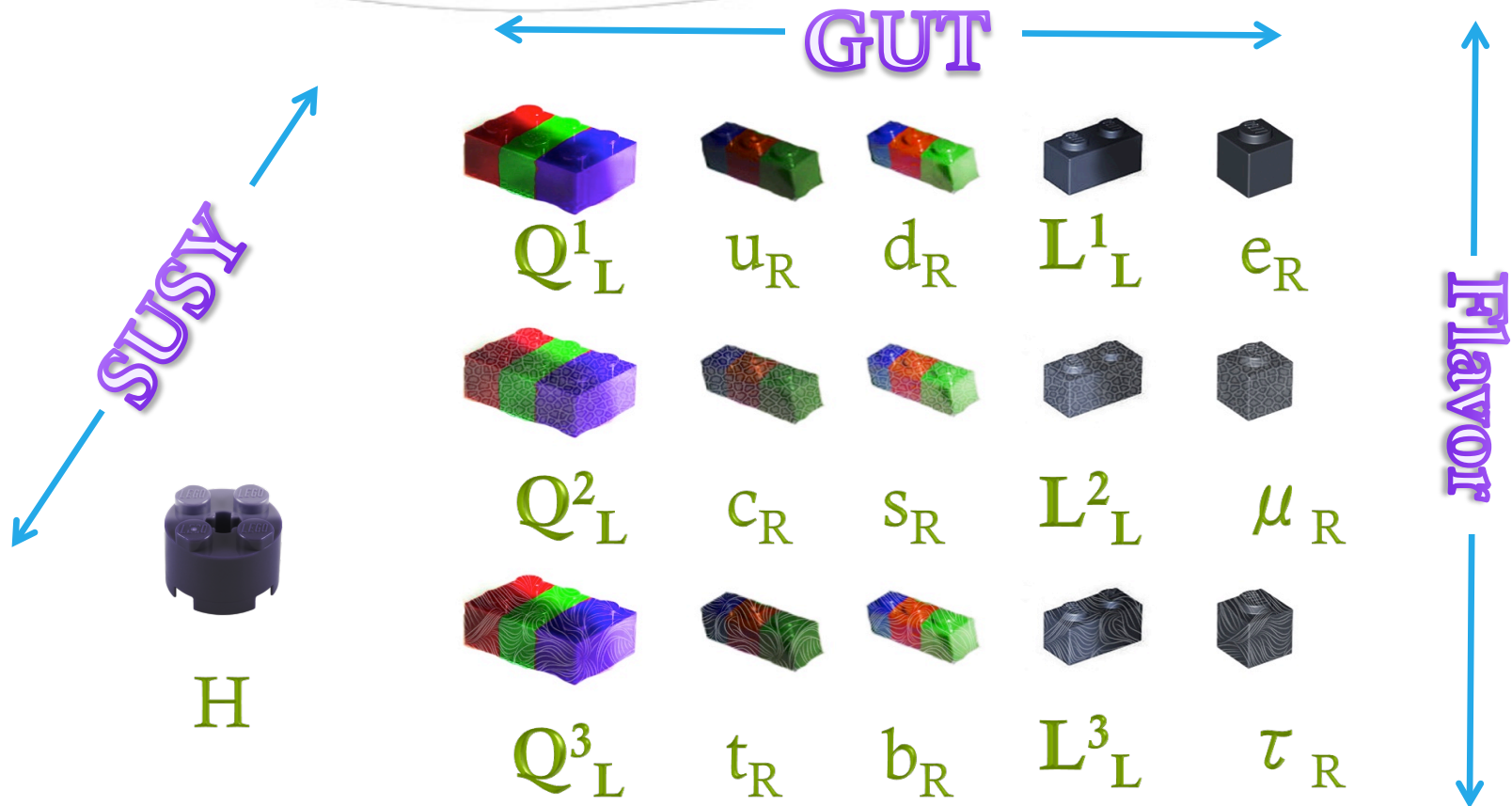
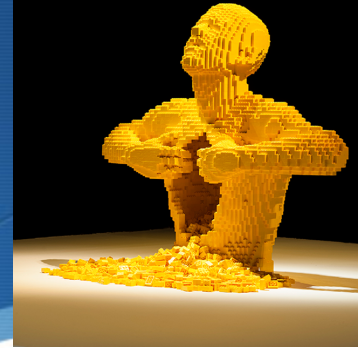
# Standard Model

Gauge theory  
 $SU(3)_C \times SU(2)_L \times U(1)_Y$



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# SM: Problems to solve

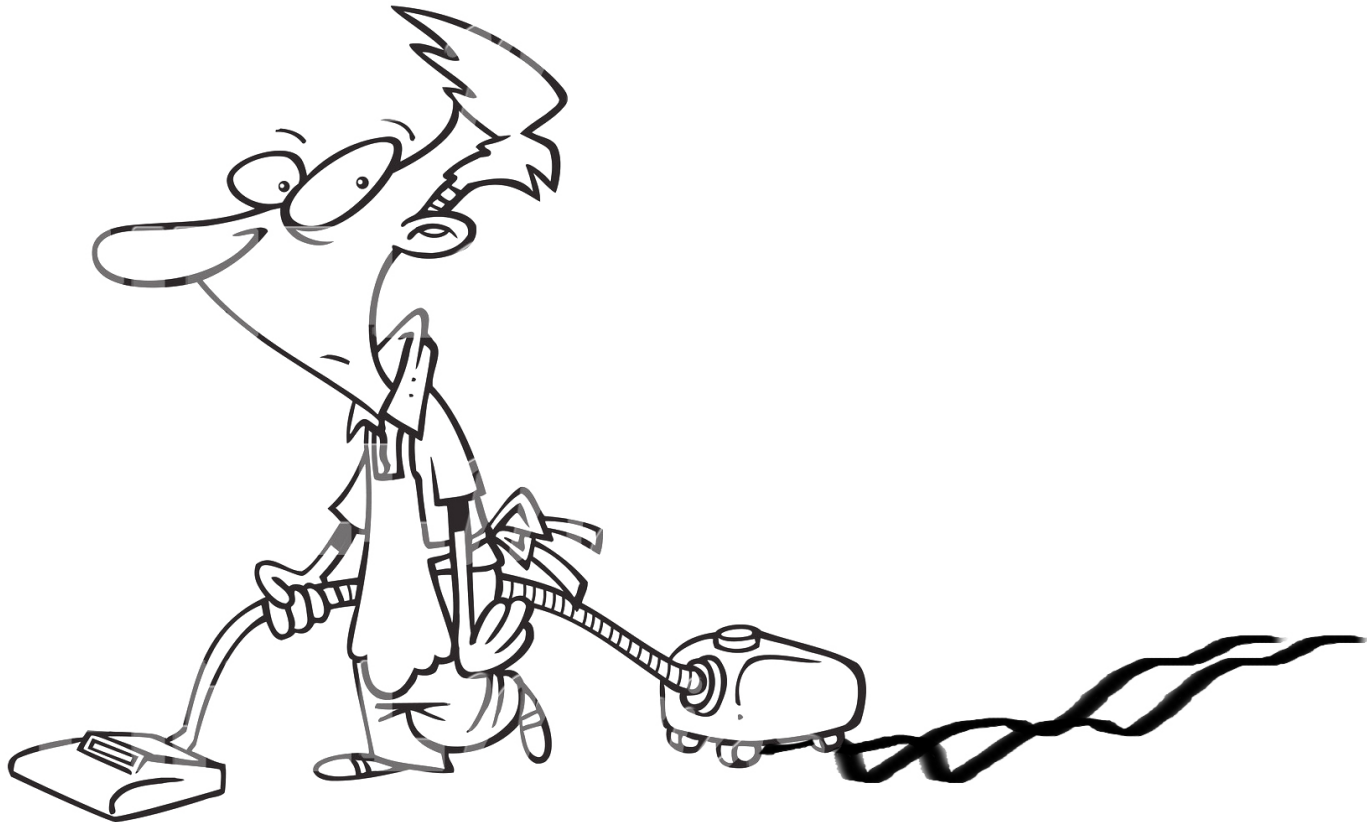
- ◆ Free gauge couplings.
- ◆ Free masses and mixings.
- ◆ Hierarchy problem.
- ◆ Not enough baryon asymmetry.
- ◆ Arbitrary charge quantization.
- ◆ No dark matter.
- ◆ No neutrino mass mechanism.
- ◆ Strong CP problem.

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**SUSY**  
**GUT**  
**FLAVOR**

# New problems



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- ◆ GUT breaking mechanism.
- ◆ Flavor symmetry breaking mechanism (alignment).
- ◆ FCNCs.
- ◆ Missing new particles (heavy masses, doublet-triplet splitting).
- ◆ SUSY breaking.
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- ◆ Proton decay.
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**How to fix them now?**



# New problems

- ◆ GUT breaking mechanism.
- ◆ Flavor symmetry breaking (misalignment).
- ◆ Lepton number violation ( $\psi$ ).
- ◆ Majorana mass generation (mixing).
- ◆ SUSY breaking.
- ◆  $\mu$  problem.
- ◆ Proton decay.

◆ Hard to obtain correct mass etc.

**There. It's fixed.**

# New problems

**IF IT CAN'T BE FIXED WITH  
DUCT TAPE**



**IT MEANS YOU'RE NOT  
USING ENOUGH DUCT TAPE**

# $A_4 \times SU(5)$ based theory

- ◆ No doublet-triplet splitting,  $\mu$  problem nor proton decay.
- ◆ CP spontaneously broken and no strong CP violation.
- ◆ Renormalizable at GUT scale.
- ◆ Correct baryon asymmetry.
- ◆ Neutrino mass mechanism (with right handed neutrinos).
- ◆ Reduces to MSSM at low energies (with R parity to have DM and no FCNC).
- ◆ All  $O(1)$  free parameters.
- ◆ Explicit GUT breaking.
- ◆ Explicit flavor breaking and alignment.

$$A_4 \times SU(5) \times Z_9 \times Z_6 \times Z_4^R$$

Flavor symmetry

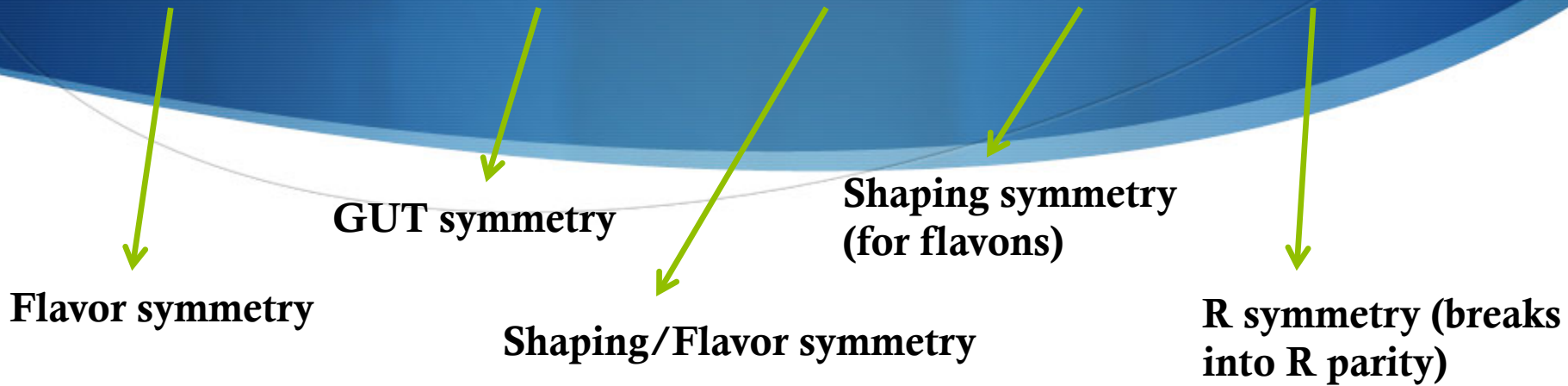
GUT symmetry

Shaping/Flavor symmetry

Shaping symmetry  
(for flavons)

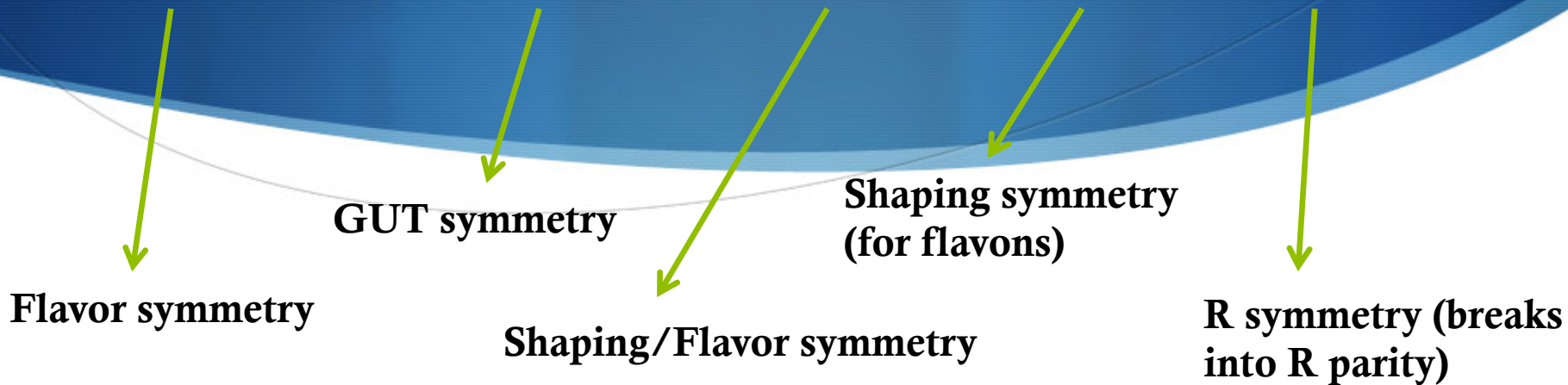
R symmetry (breaks  
into R parity)

$A_4 \times SU(5) \times Z_9 \times Z_6 \times Z_4^R$



- ◆ Superfields containing MSSM superfields (EW scale masses).
- ◆ Symmetry breaking superfields (masses around the GUT scale).
- ◆ Messenger superfields (renormalizable masses).

$A_4 \times SU(5) \times Z_9 \times Z_6 \times Z_4^R$



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**102 superfields in total.**

# MSSM containing superfields

Field		Representation				
		$A_4$	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
$L_L, d_R$	$F$	3	$\bar{5}$	0	0	1
	$T_1$	1	10	5	0	1
	$Q_R, u_R, e_R$	$T_2$	1	10	7	0
$T_3$		1	10	0	0	1
2 RH $\nu$ 's	$N_{\text{atm}}^c$	1	1	7	3	1
	$N_{\text{sol}}^c$	1	1	8	3	1
$H_u$	$H_5$	1	5	0	0	0
$H_d$	$H_{\bar{5}}$	1	$\bar{5}$	2	0	0
Flavons	$\xi$	1	1	2	0	0
	$\phi_i$	3	1	$\alpha_i$	$\beta_i$	0

# Flavon VEV alignment

New superfields A's ( $A_4$  triplets) and O's ( $A_4$  singlets) whose **F term** determine the flavon VEV alignment.



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$$W \sim A\phi_a\phi_a$$

$$\phi_a^1\phi_a^2 = 0$$

$$\phi_a^2\phi_a^3 = 0$$

$$\phi_a^3\phi_a^1 = 0$$

2 components must vanish.

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$$\phi_a^3\phi_a^1 = 0$$

2 components must vanish.

$$W \sim O\phi_a\phi_b$$

$$\phi_a \cdot \phi_b = 0$$

They must be orthogonal.

# Flavon VEV alignment

New superfields A's ( $A_4$  triplets) and O's ( $A_4$  singlets) whose **F term** determine the flavon VEV alignment.

$$\begin{aligned} W_{\text{align}} \sim & A_\mu \phi_\mu \phi_\mu + A_\tau \phi_\tau \phi_\tau + A_2(\phi_2 \phi_2 + \phi_2 \theta_1) \\ & + O_{e\mu} \phi_e \phi_\mu + O_{e\tau} \phi_e \phi_\tau + O_{\mu\tau} \phi_\mu \phi_\tau \\ & + O_{e3} \phi_e \phi_3 + O_{23} \phi_2 \phi_3 + O_{12} \phi_1 \phi_2 + O_{13} \phi_1 \phi_3 \\ & + O_{\mu 5} \phi_\mu \phi_5 + O_{25} \phi_2 \phi_5 + O_{\mu 6} \phi_\mu \phi_6 + O_{56} \phi_5 \phi_6 \\ & + O_{64} \phi_6 \phi_4 + O_{14} \phi_1 \phi_4. \end{aligned}$$

# Flavon VEV alignment

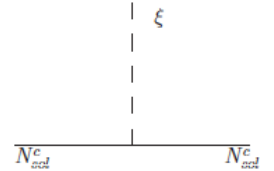
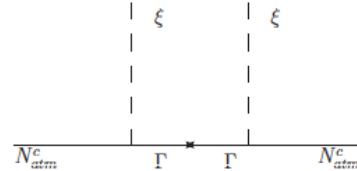
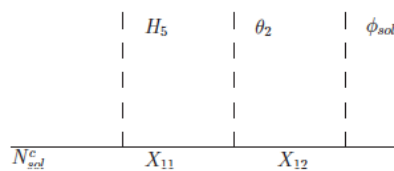
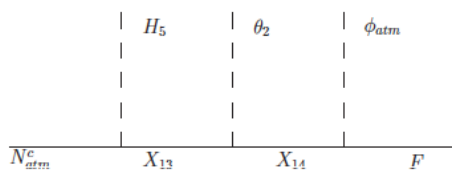
**Completely fixed flavon VEV alignments:**

$$\langle \phi_e \rangle = v_e \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \langle \phi_\mu \rangle = v_\mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \langle \phi_\tau \rangle = v_\tau \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\langle \phi_{\text{atm}} \rangle = v_{\text{atm}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_{\text{sol}} \rangle = v_{\text{sol}} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

# CSD(3) alignment for neutrinos

$$W_\nu = y_1 H_5 F \frac{\phi_{\text{atm}}}{\langle \theta_2 \rangle} N_{\text{atm}}^c + y_2 H_5 F \frac{\phi_{\text{sol}}}{\langle \theta_2 \rangle} N_{\text{sol}}^c + y_3 \frac{\xi^2}{M} N_{\text{atm}}^c N_{\text{atm}}^c + y_4 \xi N_{\text{sol}}^c N_{\text{sol}}^c.$$



# CSD(3) alignment for neutrinos

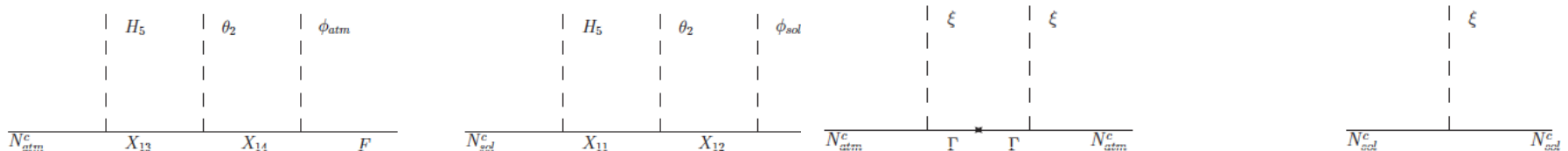
$$W_\nu = y_1 H_5 F \frac{\phi_{\text{atm}}}{\langle \theta_2 \rangle} N_{\text{atm}}^c + y_2 H_5 F \frac{\phi_{\text{sol}}}{\langle \theta_2 \rangle} N_{\text{sol}}^c + y_3 \frac{\xi^2}{M} N_{\text{atm}}^c N_{\text{atm}}^c + y_4 \xi N_{\text{sol}}^c N_{\text{sol}}^c.$$

- 2 right handed neutrinos with  $M_{\text{atm}} \ll M_{\text{sol}}$ .
- Seesaw mechanism.
- Normal hierarchy with one massless neutrino.

Dirac mass matrix:

$$\lambda^\nu = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}$$

Diagonal RH  $\nu$  mass matrix.



# Neutrino masses

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$\eta = \frac{2\pi n}{9}$ , with  $n \in \mathbb{Z}$ ,  
due to the  $\mathbb{Z}_9$  symmetry.  
We choose  $\eta = 2\pi/3$

We fit the  $\Delta m^2$  and choose  
the phase  $\eta$  (1 in 9)  
and the PMNS matrix  
is completely fixed.

(Negligible contributions from charged  
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	Experimental	Theory
$\theta_{12}^l$ ( $^\circ$ )	$33.48^{+0.78}_{-0.75}$	34.3
$\theta_{23}^l$ ( $^\circ$ )	$42.3^{+3.0}_{-1.6}$	45.8
$\theta_{13}^l$ ( $^\circ$ )	$8.5^{+0.20}_{-0.21}$	8.67
$\delta^l$ ( $^\circ$ )	$-54^{+39}_{-70}$	-86.7



# Leptogenesis

- ◆ Baryon asymmetry of the universe due to CP violation in leptons.
- ◆ Generated through decays of  $N_{\text{atm}}$  (lightest RH neutrino).
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- ◆ Decay violates lepton number and CP in this model.
- ◆ To obtain correct BAU we fix  $M_{\text{atm}} = 4 \times 10^{10} \text{ GeV}$ .

$$M_{\text{atm}} = y_3 \frac{\langle \xi \rangle^2}{M}$$

Taking  $M \rightarrow M_P$  fixes  $y_3 \simeq 0.3$ .

# Up-quark masses

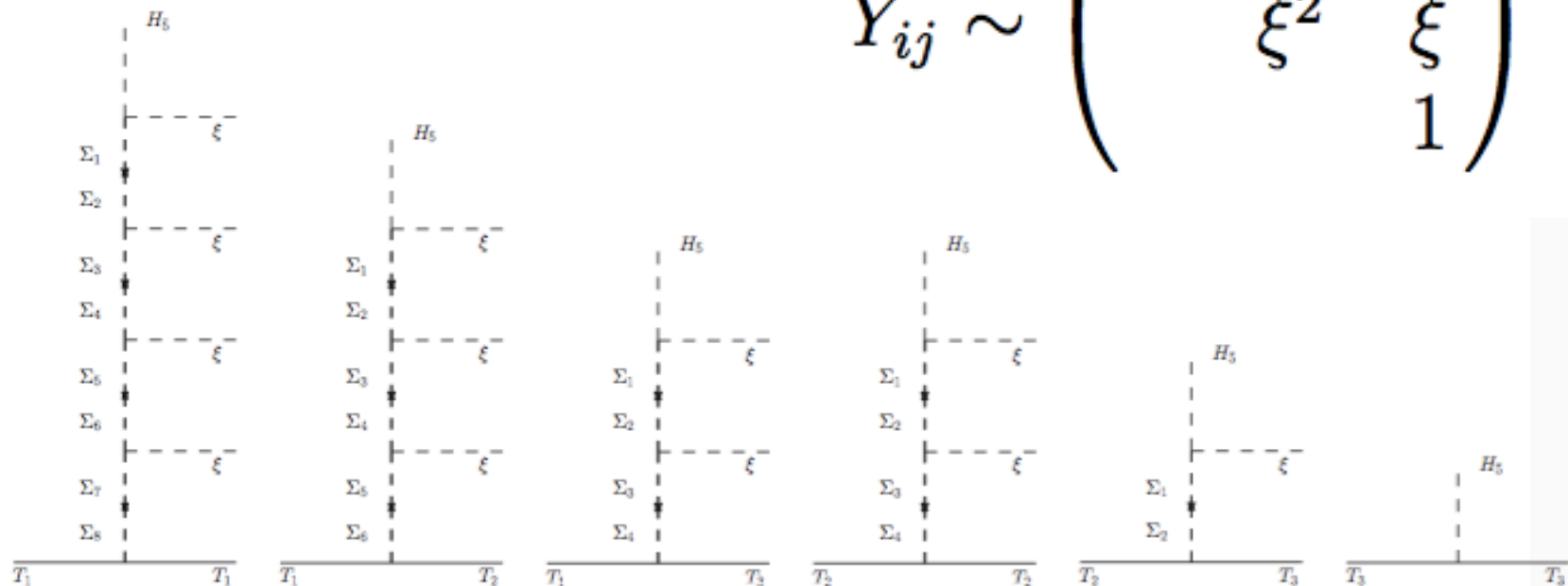
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# Up-quark masses

$$\tilde{\xi} = \frac{\langle \xi \rangle}{M} \simeq 0.06$$

$$W_{\text{up}} = Y_{ij} H_5 T_i T_j$$

$$Y_{ij} \sim \begin{pmatrix} \tilde{\xi}^4 & \tilde{\xi}^3 & \tilde{\xi}^2 \\ & \tilde{\xi}^2 & \tilde{\xi} \\ & & \tilde{\xi} \\ & & & 1 \end{pmatrix}$$



# Down-quarks and charged leptons masses

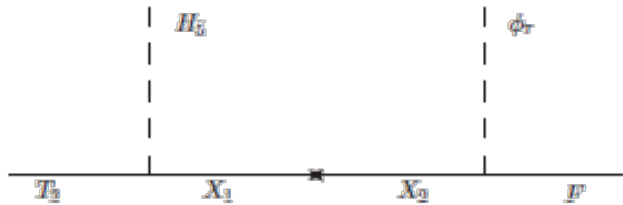
$$W_{d\ell} = Y_{\psi} H T_i \phi_j F$$

They come from the same terms so we need Georgi-Jarlskog relations (from a 45 and 24)

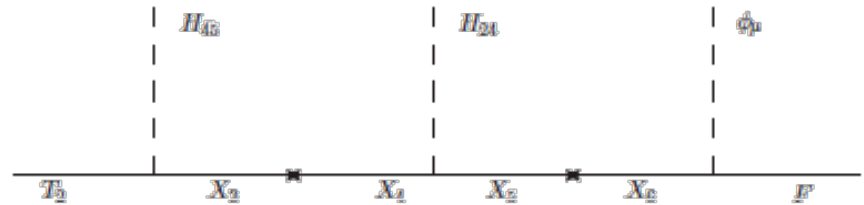
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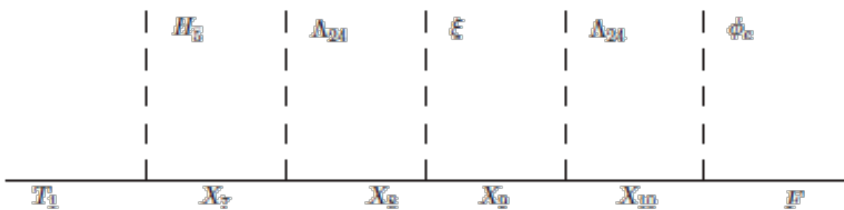
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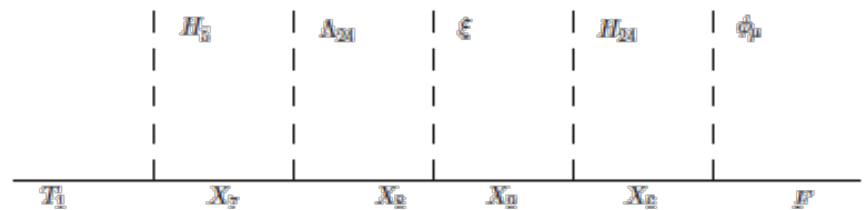
(a)



(b)



(c)

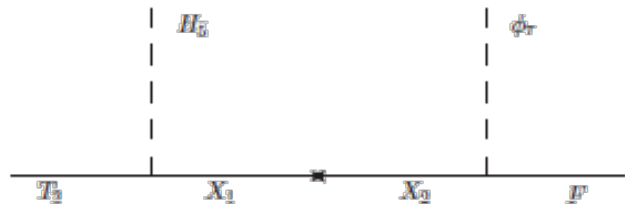


(d)

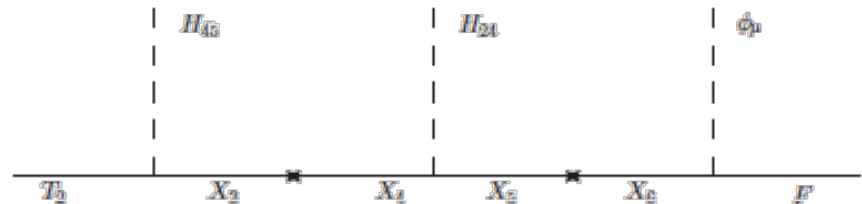
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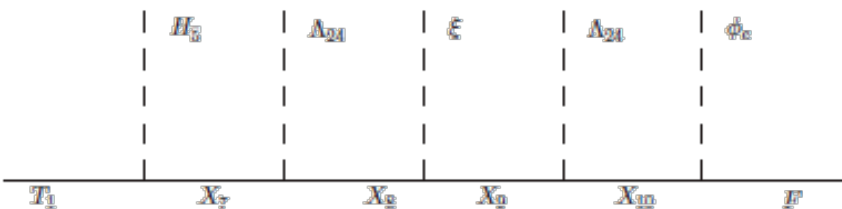
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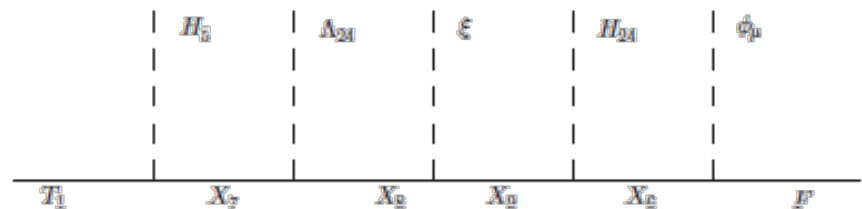
(a)



(b)



(c)



(d)

Experimental (at GUT and no threshold c.)

$$\frac{y_\mu}{y_s} \approx 4.36 \pm 0.23, \quad \frac{y_e}{y_d} \approx 0.41 \pm 0.06$$

Our model

$$\frac{y_{22}^e}{y_{22}^d} = \frac{9}{2} = 4.5, \quad \frac{y_{11}^e}{y_{11}^d} = \frac{4}{9} \approx 0.44$$

# Strong CP

CP is only spontaneously broken

$$\theta = \arg \det(Y^u Y^d)$$



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$$\theta = \arg \det(Y^u Y^d)$$

$$Y^u \in \mathbb{R} \quad Y_{LR}^d \sim \begin{pmatrix} * & * & e^{i\zeta} & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & 0 & * \end{pmatrix}$$

No strong CP violation: Nelson-Barr mechanism

# Parameter fit

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All dimensionless parameters  $y \in [0.32, 3.12]$

$$\chi^2 = \sum_{i=1}^N \left( \frac{P_i(\{x\}) - \mu_i}{\sigma_i} \right)^2 = 7.98$$

# More facts

- ◆ Doublet-triplet splitting due to the Missing Partner Mechanism.

Adding a 75 with a SM singlet VEV.

Adding a 50 with SU(3) triplets but no SU(2) doublets.

$$W_{DT} \sim H_{\bar{5}} \Omega_{50} \Pi_{75}$$

# More facts

- ◆ Doublet-triplet splitting due to the Missing Partner Mechanism.
- ◆ Higgs mixing generates  $\mu$  term.

$$W_\mu \sim M_{GUT} \begin{pmatrix} 0 & 1 \\ \tilde{\xi}^8 & 1 \end{pmatrix} \begin{matrix} \mathbf{2}(H_{\bar{5}}) \\ \mathbf{2}(H_{45}) \end{matrix}$$
$$\mu \sim \tilde{\xi}^8 M_{GUT} \sim 10^{-10} M_{GUT}$$

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- ◆ No dangerous (TTF) proton decay terms allowed.
- ◆ There's an explicit symmetry breaking potential.

Not shown, too large :-P

# Conclusion

- ◆ SUSY Flavor-GUT based on  $A_4 \times SU(5) \times Z_9 \times Z_6 \times Z_4^R$ .
- ◆ Reduces to MSSM at LE (Everything has GUT scale masses).
- ◆ Fixed PMNS matrix with great precision.
- ◆ Gives hierarchy to quark masses and G-J relations.
- ◆ Fits quarks and lepton masses and CKM with  $O(1)$  parameters.
- ◆ Generates BAU through leptogenesis.
- ◆ No strong CP violation and controlled proton decay.
- ◆ Generates doublet-triplet splitting and small  $\mu$  term.



# Conclusion

