Scalar-fermion and vector-fermion vertices connection within spin-extended model

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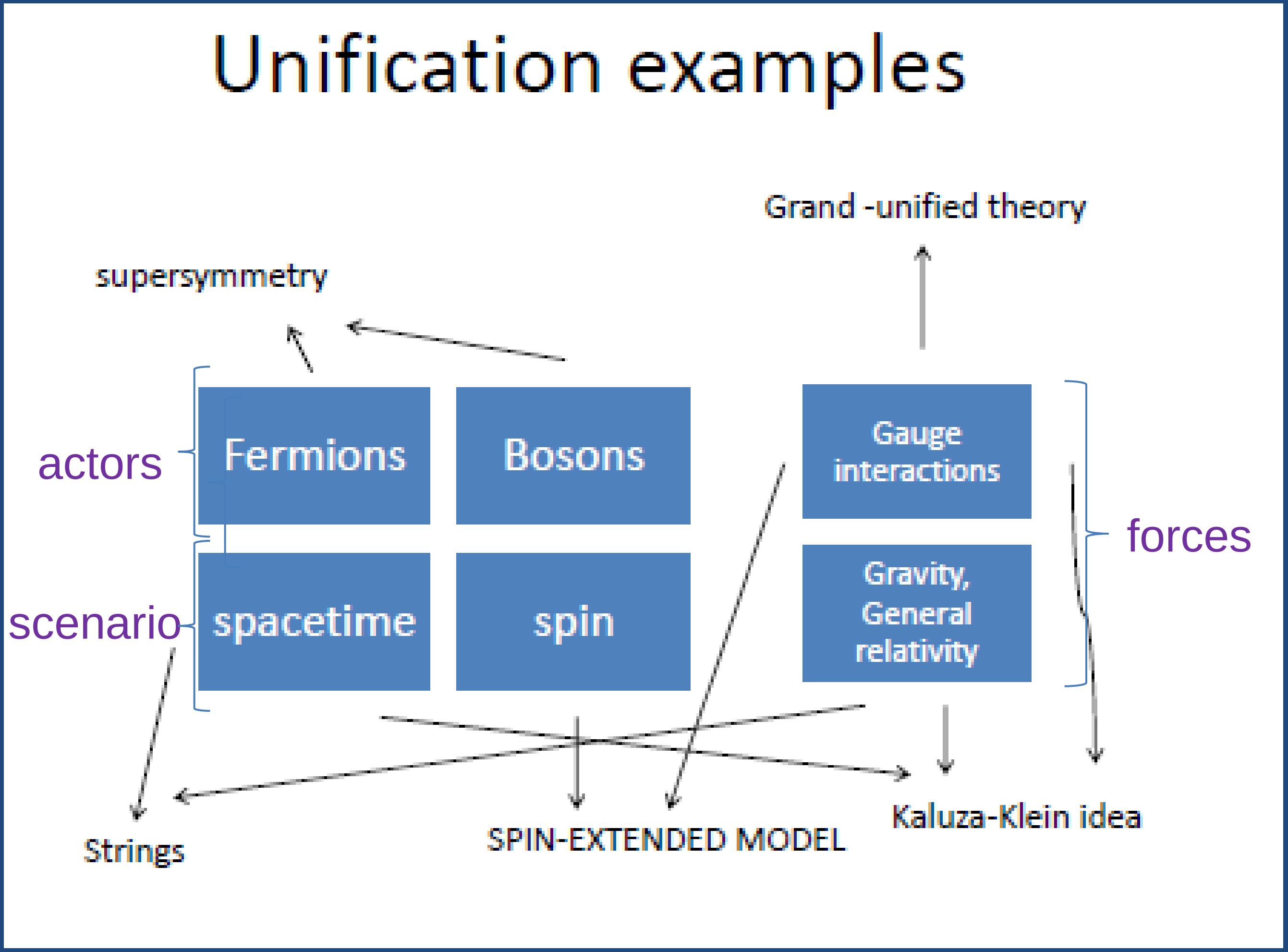
Contents

- Spin-extended model within unification and SM extensions.
- Model's symmetry groups and representations: chiral elements.
- Lagrangian construction.
- fermions and scalars in (7+1)-d, and heavy-quark mass hierarchy effect.

Standard-model extensions



LORENTZ AND MAXIMAL SCALAR SYMMETRY AT D DIMENSION

$\underbrace{\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3}$ $\quad \underbrace{\gamma_4, \dots \gamma_{D-1}}$

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \quad \mu,\nu = 0,..,3 \qquad \gamma_a \quad a = 4,..,D-1$

4-D Lorentz symmetry $\otimes$ Scalar symmetry

unitary: $U(2^{(D-4)/2})$

$[\sigma_{\mu\nu}, \gamma_a] = 0$

$[\tilde{\gamma}_5, \gamma_a] = 0 \qquad \tilde{\gamma}_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$

$[H, \gamma_a] = 0 \qquad H = i\gamma_0 \vec{\nabla}\cdot\vec{\gamma}$

maximal scalar symmetry

$U_R \otimes U_L \qquad U_R = \frac{1}{2}(1+\tilde{\gamma}_5)\, U(2^{(D-4)/2})$

$U_L = \frac{1}{2}(1-\tilde{\gamma}_5)\, U(2^{(D-4)/2})$

Coleman-Mandula OK

Two physical interpretations

- Kaluza-Klein type of framework, for in higher than (3+1)-dimensions, only the spin component in

$$\mathcal{P}_P\left[\frac{1}{2}\sigma_{\mu\nu} + i(x_\mu \partial_\nu - x_\nu \partial_\mu)\right]$$

μ=4,…,N-1, ν=4,…,N-1

  remains as symmetry operator; thus, spatial componets are frozen.

- Elementary discrete degree-of- freedom matrix construction.

(a) Symmetry groups (b) Representations

Table 1. (a) Shows the arrangement of symmetry operators U in matrix space of arbitrary dimension N, after projection over S_P, with left-handed and right-handed operators subspaces;[6] (*) represents the matrix subspace containing the projector $1-\mathcal{P}_S = 1-\mathcal{P}_P$; its choice within the right-handed symmetry components is arbitrary. (b) Shows the arrangement of matrix solutions Ψ in the extended-spin model which is divided into four $\frac{N}{2}\times\frac{N}{2}$ matrix blocks, containing fermion (F), vector (and axial-) (V), scalar (and pseudo-) and antisymmetric (S, A) terms.

(*)		
	$S_{(N-4)R} \bigotimes C_4$	
		$S_{(N-4)L} \bigotimes C_4$

(a)

(*)	F	F
F	V	S, A
F	S, A	V

(b)

(5+1)-d spin-extended model

Following the Kaluza-Klein idea, one seeks constraints on symmetries extending the spin space. For example, the Clifford algebra on 5+1 dimensions contains the usual Lorentz generators, and scalar ones.

Lorentz

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

$\mu, \nu = 0, ..., 3,$

$\gamma_a,\ a = 4, ..., N-1$

Scalars

$U_{Le}(1) \times U_Y(1) \times SU_L(2)$

$$L = \frac{3}{4} - \frac{i}{4}(1 + \tilde{\gamma}_5)\gamma^5\gamma^6 - \frac{1}{4}\tilde{\gamma}_5$$

$$Y = -1 + \frac{i}{2}(1 + \tilde{\gamma}_5)\gamma^5\gamma^6$$

$$I_1 = \frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^5$$

$$I_2 = -\frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^6$$

$$I_3 = -\frac{i}{2}(1 - \tilde{\gamma}_5)\gamma^5\gamma^6$$

Standard-model heavy particles

	Masses (GeV)	Spin	Weak I^2	Hypercharge Y
$W^{+/-}$	80.4	1	1	0
Z	91.2	1	0	0
H	126	0	½	-1
t	173	½	½ ,0	1/3, 4/3
b	4.6	½	½ ,0	1/3, -2/3

Composite multiplet suggested

INTERACTIVE THEORY FOR FIELDS IN AN EXTENDED SPIN SPACE

Keep

Polarization basis for fields

vector $A^{4a} \gamma_4 G_a$

scalar $\phi^a G'_a$

transformation rule

$\bar{\Psi} \rightarrow u \bar{\Psi} u^{+}$

drop

free-field generalized Dirac equation

There is an equivalence between a field theory and its formulation in an extended spin space

Spin-extended model equivalent Lagrangian terms

Fermion-vector

$$\frac{1}{N_f}\mathrm{tr}\,\Psi^\dagger\{[i\partial_\mu I_{\mathrm{den}} + gA^a_\mu(x)I_a]\gamma_0\gamma^\mu - M\gamma_0\}\Psi P_f\,,$$

Projection operator

$$P_f = \frac{1}{\sqrt{2}}(\tilde{\gamma}^5 - \gamma^0\gamma^1)$$

Projection operator

$$P_f = \tilde{g}_5\tilde{\gamma}_5 + g_I I + g_{01}\gamma^0\gamma^1 + g_{02}\gamma^0\gamma^2 + g_{03}\gamma^0\gamma^3 + g_{12}\gamma^1\gamma^2 + g_{13}\gamma^1\gamma^3 + g_{23}\gamma^2\gamma^3 + (g_{\tilde{5}56}\tilde{\gamma}_5 + g_{I56} I + g_{0156}\gamma^0\gamma^1 + g_{0256}\gamma^0\gamma^2 + g_{0356}\gamma^0\gamma^3 + g_{1256}\gamma^1\gamma^2 + g_{1356}\gamma^1\gamma^3 + g_{2356}\gamma^2\gamma^3)\gamma^5\gamma^6.$$

For the extended-spin model with $\Psi_L^l(x)$, the coefficient of the e_L^1 associated term $(\psi_L^1(x))^2$ is

$$(A - B)[W_0^3(x) - W_3^3(x)]$$

$$A = \frac{1}{2}(g_I + g_{\tilde{5}} - ig_{\tilde{5}56} - ig_{56})$$

$$B = -\frac{1}{2}(g_{03} - ig_{12} - ig_{0356} - g_{1256}).$$

For the conventional term with $\Psi_l(x)$, the e_L^1 coefficient is

$$\frac{1}{2}[W_0^3(x) - W_3^3(x)].$$

We conclude the choice $A = 1/2$, $B = 0$

(7+1)-d fermions

Table 3. (a) Massless left-handed quark weak isospin doublet and (b) right-handed singlets, with momentum along $\pm\hat{\mathbf{z}}$.

(a)

Hypercharge 1/3 left-handed doublet	I_3	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$Q_L^1 = \begin{pmatrix} U_L^1 \\ D_L^1 \end{pmatrix} = \begin{pmatrix} \frac{1}{16}\left(1-\tilde{\gamma}_5\right)\left(\gamma^5-i\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\left(\gamma^0+\gamma^3\right) \\ \frac{1}{16}\left(1-\tilde{\gamma}_5\right)\left(\gamma^5-i\gamma^6\right)\left(1-i\gamma^7\gamma^8\right)\left(\gamma^0+\gamma^3\right) \end{pmatrix}$	$1/2$ $-1/2$	$2/3$ $-1/3$	$1/2$ $1/2$

(b)

$I_3 = 0$ right-handed singlet	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$U_R^1 = \frac{1}{16}\left(1+\tilde{\gamma}_5\right)\left(\gamma^5-i\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\gamma^0\left(\gamma^0+\gamma^3\right)$	$4/3$	$2/3$	$1/2$
$D_R^1 = \frac{1}{16}\left(1+\tilde{\gamma}_5\right)\left(\gamma^5-i\gamma^6\right)\left(1-i\gamma^7\gamma^8\right)\gamma^0\left(\gamma^0+\gamma^3\right)$	$-2/3$	$-1/3$	$1/2$

(7+1)-d scalars

Table 4. Scalar Higgs-like pairs.

0 baryon-number scalar	I_3	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\gamma^0 \\ \frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(1+i\gamma^7\gamma^8\tilde{\gamma}_5\right)\gamma^0 \end{pmatrix}$	$1/2$ $-1/2$	1	1 0	0
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\tilde{\gamma}_5\gamma^0 \\ \frac{i}{8}\left(1-i\gamma^5\gamma^6\right)\left(1+i\gamma^7\gamma^8\tilde{\gamma}_5\right)\gamma^7\gamma^8\gamma^0 \end{pmatrix}$	$1/2$ $-1/2$	1	1 0	0

SU(2)$_L$xU(1)$_Y$-invariant term

$$\frac{1}{N_f}\mathrm{tr}\Big\{\Big[m_U \Psi_R^{U\dagger}(x)[\phi_1(x)+\phi_2(x)]\Psi_L^Q(x)$$
$$+ m_D \Psi_L^{Q\dagger}(x)[\phi_1(x)-\phi_2(x)]\Psi_R^D(x)\Big]P_f\Big\} + \{\mathrm{cc}\}\,,$$

Higgs mechanism →

$$H_v = a\phi_1^0 + b\phi_2^0 + a\phi_1^{0\dagger} + b\phi_2^{0\dagger}$$

$$H_v U_M^1 = m_U U_M^1\,, \quad H_v U_M^{c1} = -m_U U_M^{c1}$$
$$H_v D_M^1 = m_D D_m^1\,, \quad H_v D_M^{c1} = -m_D D_M^{c1}$$

Hierarchy effect:

$$m_U = \frac{(a+b)}{2}, \quad m_D = \frac{(a-b)}{2}$$

Main points

- Spin-extended model provides for a feasible SM extension.
- At given dimension, it constrains groups and representations.
- A Lagrangian construction produces a gauge invariant formulation.
- At (7+1)-d, a predicted Higgs doublet induces a fermion mass hierarchy effect, suggestive of the bottom and top quark mass values.

Composite models

- 1961 Nambu Jona-Lasignio. Superconductivity model in which four-fermion interaction generates both fermion and boson masses.
- 1989 Nambu. Higgs from top quark condensate.
- Bardeen, Hill, Lindner, use fixed point in renormalization.
- Technicolor: Higgs composed of fermions alleviates fine-tuning problem.
- Spin extended model.

Can standard-model bosons be constructed in terms of fermions?

Higgs

$$\Phi \propto \frac{1}{2}\begin{pmatrix} \overline{b^a}(1+\gamma_5)t^a \\ -\overline{t^a}(1+\gamma_5)t^a \end{pmatrix} = i\tau_2(\overline{t}_R\Psi_L)^{\dagger T},$$

$$\tilde{\Phi} \equiv i\tau_2\Phi^{\dagger T} \propto \overline{t}_R\Psi_L\,.$$

$$Y=-1\ I_3=1/2 \text{ states: } H_t^0 = t_L^\dagger \bar{t}_L^\dagger + \bar{t}_R t_R$$

W

$$W_\downarrow^+ = t_L^\dagger \bar{b}_R^\dagger$$

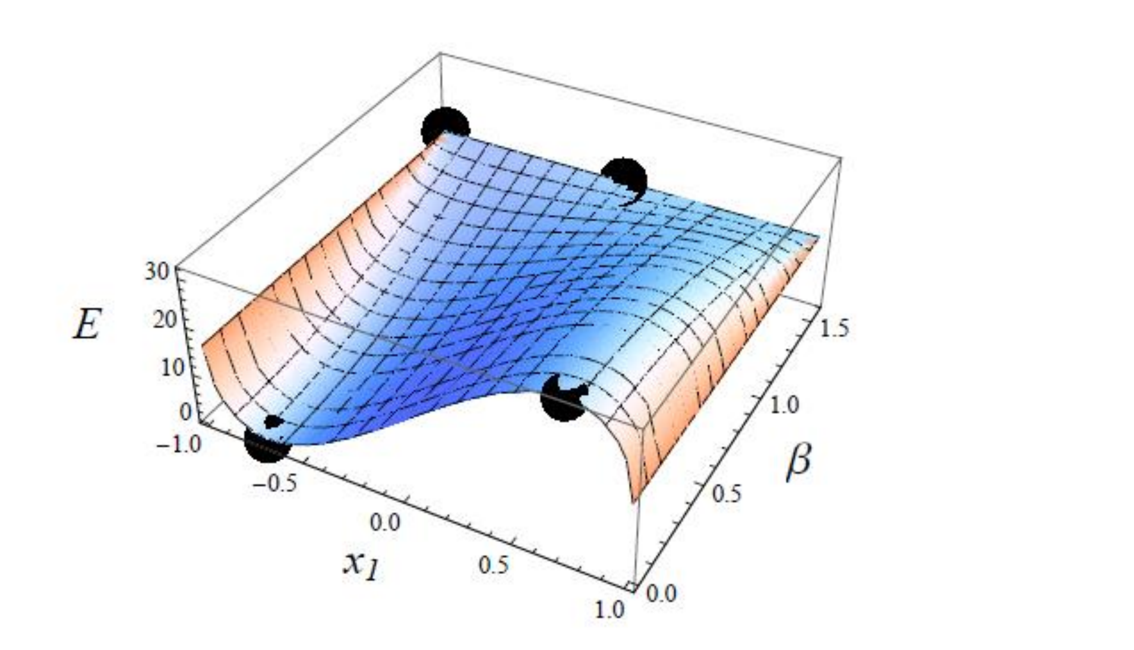
Z

left-handed helicity -1 hypercharge carrier is described by $A_L = \frac{2}{3}(t_L^\dagger \bar{t}_R^\dagger + b_L^\dagger \bar{b}_R^\dagger)$

Hamiltonian model: known mesons

- Fermion Hamiltonian of the form:

$H=H(\psi_t, \psi_{b,} m_t, m_b)$; variational calculation



toponium and bottomium: masses $2m_t$ $2m_b$

Standard-model extensions

Phenomenological prediction   Conceptual basis