Gauged Flavor Symmetries

Benjamín Grinstein
with
M. Redi (Firenze)
and
G. Villadoro (ICTP, Trieste)

JHEP 1011 (2010) 067

Ernest Ma

Ernest Ma

In his honor I will talk about model building

Outline

1. Minimal Flavor Violation

2. Minimal Lepton Flavor Violation

3. Gauge Flavor (quark sector)

Minimal Flavor Violation

Motivation, rationale, MFV

- New physics is required to explain the fine tuning puzzle.
- It must involve new dynamics that become relevant at an energy scale Λ. This new dynamics likely involves new fields, new particles and new interactions.
- The new dynamics scale Λ must not be much higher than the electroweak scale (the higher the scale the more severe the fine tuning).
- Quarks and leptons contribute to quadratic divergence in higgs mass
  - divergences depend on quark/lepton masses
  - new dynamics must have flavor dependence
- New flavor dependent dynamics at a scale Λ_F not far above the electroweak scale is a disaster: either
  - $\Lambda_F \sim 10^{6\text{-}7}$ GeV, or
  - fine tune coefficients of dangerous operators (those giving large flavor changing neutral "currents")
- Unless: avoid large FCNC automatically "$\Rightarrow$" Minimal Flavor Violation

- MFV is NOT the only possibility
  - *e.g.*, NMFV and generally theories with quark mass suppression

- But MFV is fairly minimal
  - good if you want to estimate the minimal effect of this new physics in flavor changing processes
  - more predictive, patterns

- Let' s gain some understanding by example. Consider $K_L \to \pi\nu\nu$

In the SM:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

In the SM:

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

1 loop factor,
$X \sim 1$

In the SM:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

CKM factor

1 loop factor,
$X \sim 1$

In the SM:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

1 loop factor,
$X \sim 1$

CKM factor

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In the SM:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

CKM factor

1 loop factor,
$X \sim 1$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\text{CKM}} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda(1+iA^2\lambda^4\bar{\eta}) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2(1+i\lambda^2\bar{\eta}) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

In the SM:

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

1 loop factor, $X \sim 1$

CKM factor

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda(1+iA^2\lambda^4\bar{\eta}) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2(1+i\lambda^2\bar{\eta}) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

$$\lambda \simeq 0.22 \qquad |V_{ts}V_{td}| \sim \lambda^5$$

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

New physics

$$\mathcal{H}_{\rm eff} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell_{\rm new} \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

Assume sensitivity to fractional deviation *r* from SM rate, with $\mathcal{C}^\ell_{\rm new} \sim 1$

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2$$

For example, $r = 4\%$ gives sensitivity to $\Lambda_F \sim 10^3$ TeV

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

New physics

$$\mathcal{H}_{\rm eff} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} \mathcal{C}_{\rm new}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

Assume sensitivity to fractional deviation *r* from SM rate, with $\mathcal{C}_{\rm new}^\ell \sim 1$

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2$$

For example, $r = 4\%$ gives sensitivity to $\Lambda_F \sim 10^3$ TeV

But if new physics has same CKM factors in $\mathcal{C}_{\rm new}^\ell$, then

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{1/(16\pi^2)} \right|^2$$

And now $r = 4\%$ gives sensitivity to $\Lambda_F \sim 10^{1-2}$ TeV

Minimal Flavor Violation (MFV)

Chivukula and Georgi, Phys.Lett. B188 (1987) 99
D'Ambrosio et al Nucl.Phys. B645 (2002) 155-187

- Premise: Unique source of flavor braking
- Quark sector in SM, in absence of masses has large flavor (global) symmetry:

$$G_F = SU(3)^3 \times U(1)^3$$

- In SM, symmetry is only broken by Yukawa interactions, parametrized by couplings Y_U and Y_D

$$\begin{aligned} -\mathcal{L}_{\text{Yuk}} &= H\bar{q}_L Y_U u_R + \tilde{H}\bar{q}_L Y_D d_R \\ &= \epsilon_U H\bar{q}_L \hat{Y}_U u_R + \epsilon_D \tilde{H}\bar{q}_L \hat{Y}_D d_R \end{aligned}$$

  Normalize: $\mathrm{tr}(\hat{Y}_U^\dagger \hat{Y}_U) = \mathrm{tr}(\hat{Y}_D^\dagger \hat{Y}_D) = 1$
  Breaking of $U(1)^2$ characterized by ϵ_U, ϵ_D

- MFV: all breaking of G_F must transform as these
- When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic
- Approach: via effective field theory: at low energies only SM fields

How does this work?
Consider $K_L \to \pi\nu\bar{\nu}$

Recall, G_F breaking from: $-\mathcal{L}_{\rm Yuk} = \epsilon_U H \bar{q}_L \hat{Y}_U u_R + \epsilon_D \tilde{H} \bar{q}_L \hat{Y}_D d_R$

Implications of G_F? use ***spurion*** method:

$$q_L \to e^{i\theta_q} V_L q_L \qquad \hat{Y}_U \to V_L \hat{Y}_U V_u^\dagger \qquad \epsilon_U \to e^{i(\theta_q - \theta_u)} \epsilon_U$$
$$u_R \to e^{i\theta_u} V_u u_R \qquad \hat{Y}_D \to V_L \hat{Y}_D V_d^\dagger \qquad \epsilon_D \to e^{i(\theta_q - \theta_d)} \epsilon_D$$
$$d_R \to e^{i\theta_d} V_d d_R$$

Effective lagrangian $\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2} \sum C_i O_i$

among the operators have, for example

$$O = \bar{q}_L (\hat{Y}_U \hat{Y}_U^\dagger) \gamma_\mu q_L \, \bar{\nu}_L \gamma^\mu \nu_L$$

In mass basis $\Rightarrow \left(\sum_{q=u,c,t} V_{qs}^* V_{qd} \frac{m_q^2}{v^2} \right) \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L \gamma^\mu \nu_L$

As needed it includes the factor

$$|V_{ts}^* V_{td}| m_t^2 / v^2 \approx A^2 \lambda^5 \approx 5 \times 10^{-4}$$

Digression:

Minimal Lepton Flavor Violation and Lepton (non)-universality

Minimal Lepton Flavor Violation

- Extension of MFV to lepton sector
- Need assumption on origin of neutrino masses: Dirac vs Majorana
- In charged lepton sector

$$-\mathcal{L}_{\rm Yuk} = \epsilon_E \tilde{H} \bar{\ell}_L \hat{Y}_E e_R$$

$$G_F = SU(3)^2 \times U(1)^2$$

$$\ell_L \to e^{i\theta_\ell} V_\ell \ell_L \qquad \hat{Y}_E \to V_\ell \hat{Y}_E V_e^\dagger$$

$$e_R \to e^{i\theta_e} V_e e_R \qquad \epsilon_E \to e^{i(\theta_\ell - \theta_e)} \epsilon_E$$

- Ignoring neutrino masses (small!), a symmetry transformation

$$\hat{Y}_E \to V_\ell \hat{Y}_E V_e^\dagger = \frac{\sqrt{2}}{v|\epsilon_E|} \mathrm{diag}(m_e, m_\mu, m_\tau)$$

- Unbroken symmetry

$$U(3)^2 \to U(1)_e \times U(1)_\mu \times U(1)_\tau$$

Flavor conservation without universality! (caveat, up to neutrino “Yukawas”)

Application: R_K anomaly:

LHCb: $$R_K \equiv \frac{\mathcal{B}\left(B^+ \to K^+\mu\mu\right)}{\mathcal{B}\left(B^+ \to K^+ee\right)} = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst}).$$ $q^2 \in [1,\, 6]\,\mathrm{GeV}^2$

LHCb: PRL113, 151601 (2014)

There are claims that violation to lepton universality implies (unacceptably large) lepton flavor violation

Glashow, Guadagnoli & Lane, PRL114, 091801 (2015)

With MLFV lepton flavor violation is controlled by neutrino "Yukawas" (much as in SM+neutrinos) while lepton flavor violation is controlled by charged lepton Yukawas

Alonso, BG, Martin Camalich, arXiv:1505.05164

4-fermion operators inducing $b \to sll$

$$\begin{aligned} Q_{\ell q}^{(1)} &= (\bar{q}\gamma^\mu q_L)(\bar{\ell}\gamma_\mu \ell_L) & Q_{\ell q}^{(3)} &= (\bar{q}\vec{\tau}\gamma^\mu q_L)\cdot(\bar{\ell}\vec{\tau}\gamma_\mu \ell_L) \\ Q_{\ell d} &= (\bar{d}\gamma^\mu d_R)(\bar{\ell}\gamma_\mu \ell_L) & Q_{qe} &= (\bar{q}\gamma_\mu q_L)(\bar{e}\gamma^\mu e_R) \\ Q_{ed} &= (\bar{d}_R\gamma^\mu d_R)(\bar{e}\gamma_\mu e_R) & Q_{\ell edq} &= (\bar{\ell}_L e_R)(\bar{d}_R q) + \mathrm{h.c.} \end{aligned}$$

Coefficients constrained by MFV+MFLV

$$\begin{aligned} C_{\ell q}^{(1)} &= c_{\ell q}^{(1)}\, \hat{Y}_u \hat{Y}_u^\dagger \otimes \hat{Y}_e \hat{Y}_e^\dagger, & C_{\ell q}^{(3)} &= c_{\ell q}^{(3)}\, \hat{Y}_u \hat{Y}_u^\dagger \otimes \hat{Y}_e \hat{Y}_e^\dagger, \\ C_{qe} &= c_{qe}\, \hat{Y}_u \hat{Y}_u^\dagger \otimes \hat{Y}_e^\dagger \hat{Y}_e, & C_{\ell edq} &= c_{\ell eqd}\, \varepsilon_e \varepsilon_d^*\, \hat{Y}_d^\dagger \hat{Y}_u \hat{Y}_u^\dagger \otimes \hat{Y}_e. \end{aligned}$$

Scalar operator additionally suppressed! More details $\to$ Jorge Martin Camalich tomorrow

Gauging Flavor

Issues

- Black holes: No global symmetry (other than accidental)
- If we insist: how do we make sense of transforming Yukawas?
  - Spurions: VEVs of fields:

    under $G_F = SU(3)_q \times SU(3)_u \times SU(3)_d$ introduce new fields

    $$Y_U = (\bar{3}, 3, 1)$$
    $$Y_D = (\bar{3}, 1, 3)$$

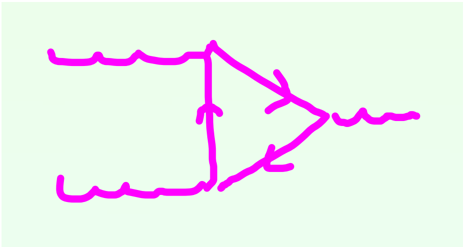
    and Yukawa coupling constants are $\langle Y_U \rangle, \langle Y_D \rangle,$
  - New Problems

    1. Goldstone' s theorem ⇒ 8+8+8 Nambu-Goldstone Bosons ⇒ FCNC disaster
    2. Renormalizability? $H\bar{q}_L Y_U u_R, \tilde{H}\bar{q}_L Y_D d_R,$ are operators of dimension 5
- Solution to problem 1: gauge G_F
  - New Problems:

    i. Anomalies: $G_F{}^3$ and $G_F{}^2 \times U(1)_Y$

    ii. Invisibility (high scale): next slide

    iii. Renormalizability (problem 2) still

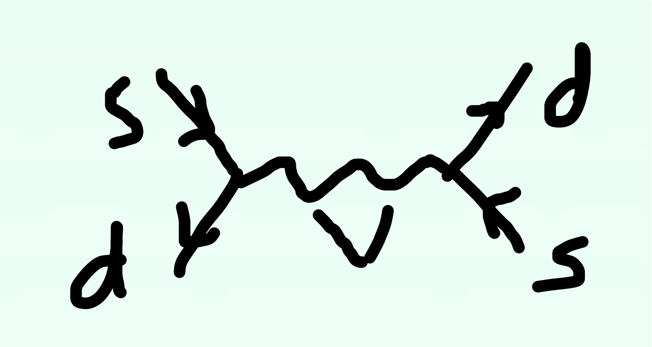


“Invisibility”

Massive vector bosons mediate FCNC

Masses: $M_V \sim g\langle Y_{U,D}\rangle$

K^0-mixing:



$$\sim \frac{1}{\langle Y_{U,D}\rangle^2}(\bar{s}d)(\bar{s}d)$$

$$\Rightarrow \langle Y_{U,D}\rangle \gtrsim 10^5 \text{ TeV}$$

And this is for the light generations. Expect much higher scales for heavy generations!

Hence “invisible.”

And then a miracle happens...

The minimal anomaly free extension of the SM gives

1. Renormalizable couplings

2. Inverted hierarchy $M_V \sim \frac{1}{y_{U,D}}$

where $y_{\mathrm{U,D}}$ are the usual Yukawa couplings

so that if $M_V \sim 10^5$ TeV for mediators among light generations, we can have

$$M_V \sim \frac{m_u}{m_t} 10^5 \text{ TeV} \sim \text{few TeV}$$

for mediators among heaviest generations

I am going to show you a model as a table of fields and their transformation properties

I am going to show you a model as a table of fields and their transformation properties

When I see this in talks it induces this response

I am going to show you a model as a table of fields and their transformation properties

When I see this in talks it induces this response

I am going to show you a model as a table of fields and their transformation properties

When I see this in talks it induces this response

I promise it is not so bad...

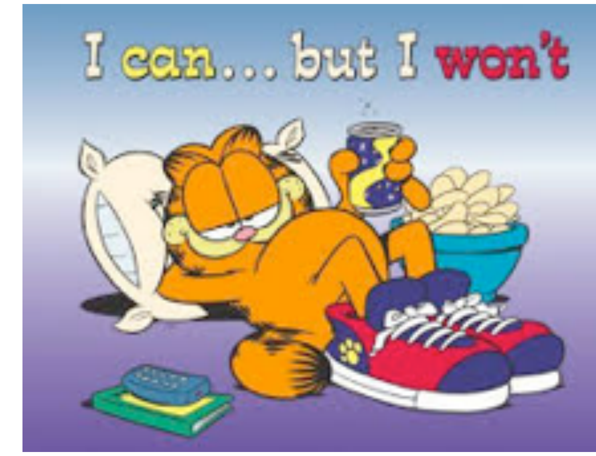
The Model

	$SU(3)_{Q_L}$	$SU(3)_{U_R}$	$SU(3)_{D_R}$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	1	1	3	2	1/6
U_R	1	3	1	3	1	2/3
D_R	1	1	3	3	1	-1/3
Ψ_{uR}	3	1	1	3	1	2/3
Ψ_{dR}	3	1	1	3	1	-1/3
Ψ_u	1	3	1	3	1	2/3
Ψ_d	1	1	3	3	1	-1/3
Y_u	$\bar{3}$	3	1	1	1	0
Y_d	$\bar{3}$	1	3	1	1	0
H	1	1	1	1	2	1/2

$$\begin{aligned}\mathcal{L} =&\mathcal{L}_{kin} - V(Y_u, Y_d, H)+ \\ &\left(\lambda_u \overline{Q}_L \tilde{H} \Psi_{uR} + \lambda'_u \overline{\Psi}_u Y_u \Psi_{uR} + M_u \overline{\Psi}_u U_R + \right. \\ &\left. \lambda_d \overline{Q}_L H \Psi_{dR} + \lambda'_d \overline{\Psi}_d Y_d \Psi_{dR} + M_d \overline{\Psi}_d D_R + h.c.\right) ,\end{aligned}$$

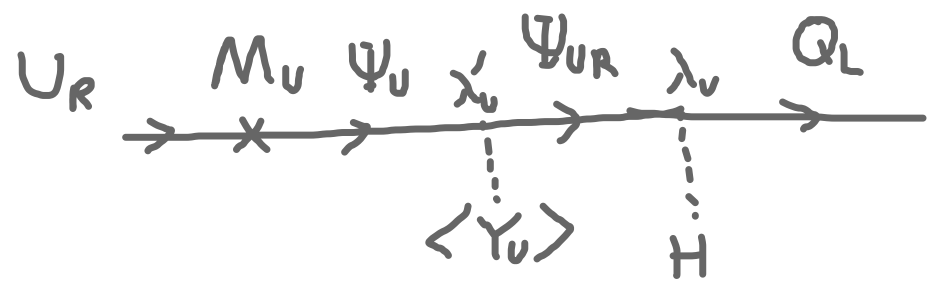
Note: all λ's and M's are 1×1 matrices

$$
\begin{aligned}
\mathcal{L} =&\mathcal{L}_{kin} - V(Y_u, Y_d, H) + \\
&\left(\lambda_u \overline{Q}_L \tilde{H} \Psi_{uR} + \lambda_u' \overline{\Psi}_u Y_u \Psi_{uR} + M_u \overline{\Psi}_u U_R + \right.\\
&\left.\lambda_d \overline{Q}_L H \Psi_{dR} + \lambda_d' \overline{\Psi}_d Y_d \Psi_{dR} + M_d \overline{\Psi}_d D_R + h.c.\right) ,
\end{aligned}
$$



For example:

With $Y_{u,d} \gg M_{u,d}$ get see-saw:



$$\Rightarrow \quad y_u = \frac{\lambda_u M_u}{\lambda_u' \langle Y_u \rangle}$$

and similarly $y_d = \dfrac{\lambda_d M_d}{\lambda_d' \langle Y_d \rangle}$

But still $M_V \sim g \langle Y_{u,d} \rangle \quad \Rightarrow \quad M_V \sim \dfrac{1}{y_{u,d}}$

1st generation flavor change $\leftrightarrow$ heaviest vectors

3rd generation $\leftrightarrow$ lightest, light enough for LHC?

Example

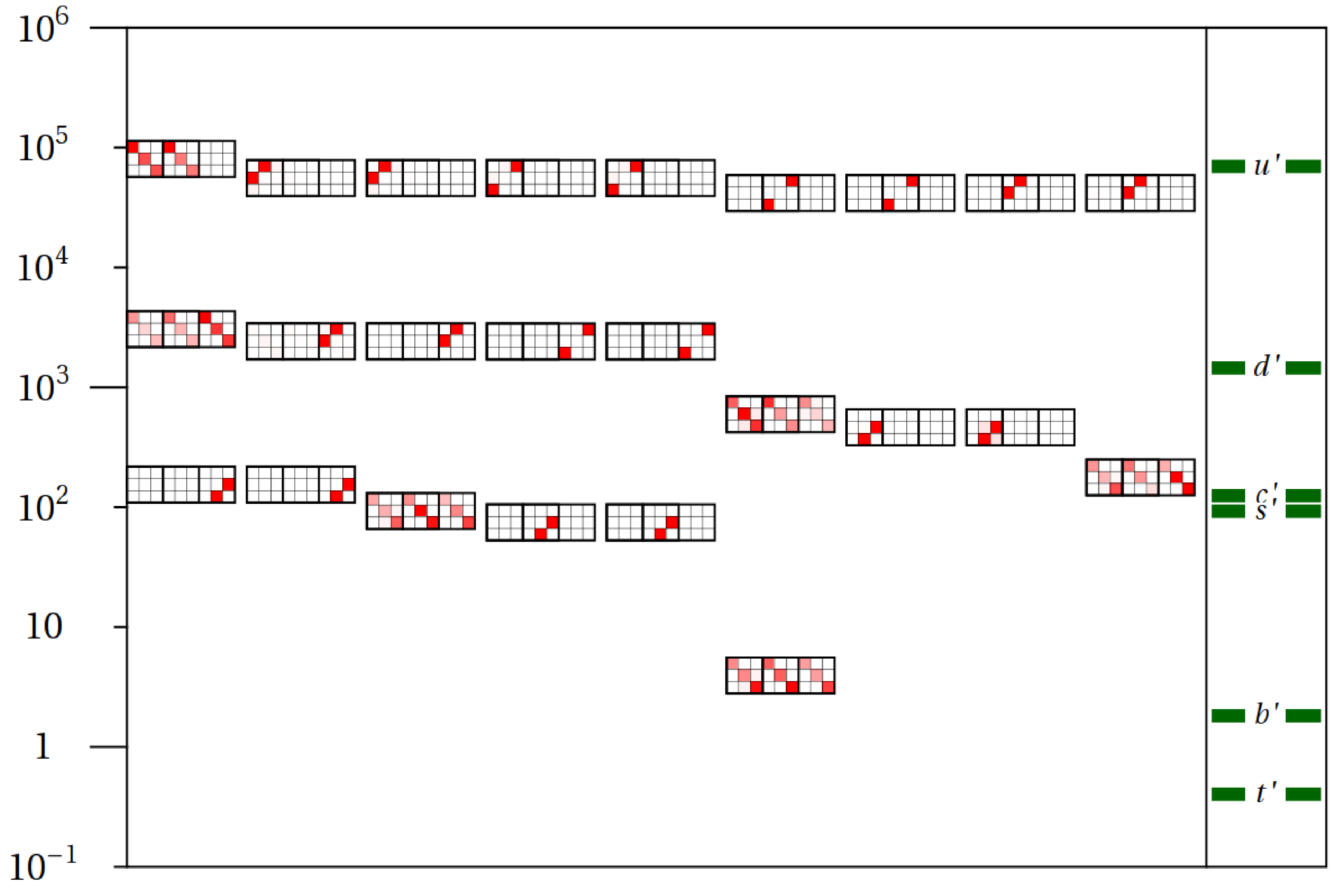
Choose

M_u (GeV)	M_d (GeV)	λ_u	λ'_u	λ_d	λ'_d	g_Q	g_U	g_D
400	100	1	0.5	0.25	0.3	0.4	0.3	0.5

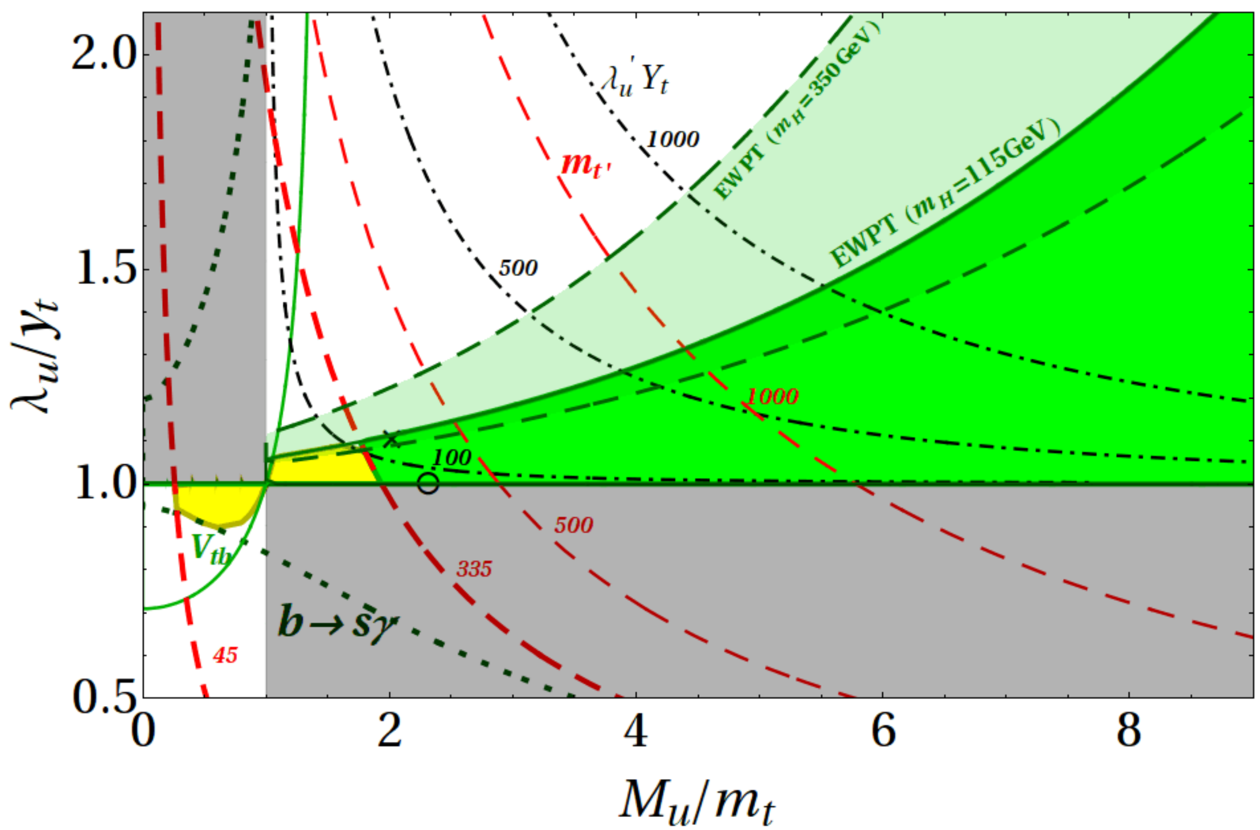
Compute

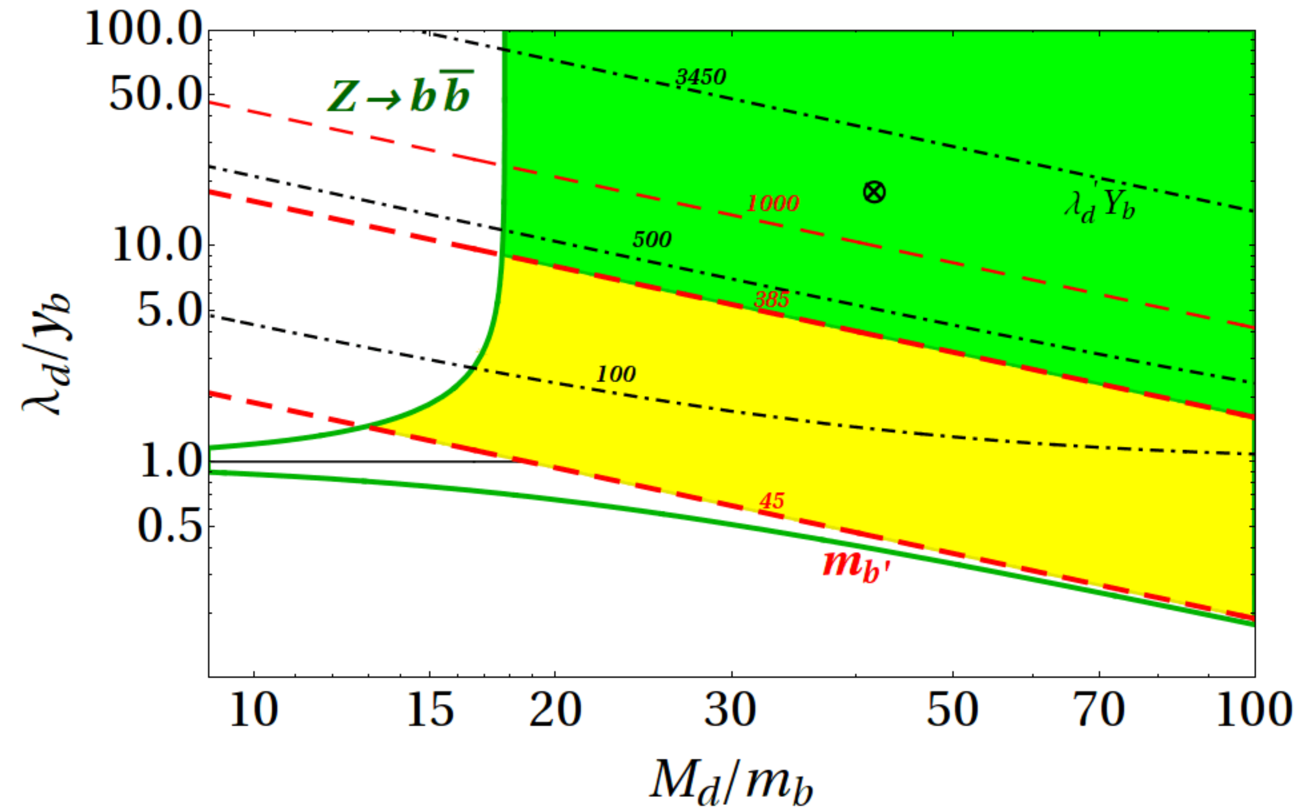
$$Y_u \approx \mathrm{Diag}\left(1\cdot 10^5\,,\ 2\cdot 10^2\,,\ 8\cdot 10^{-2}\right)\cdot V\ \ \mathrm{TeV}\,,$$
$$Y_d \approx \mathrm{Diag}\left(5\cdot 10^3\,,\ 3\cdot 10^2\,,\ 6\right)\ \ \mathrm{TeV}\,,$$

Spectrum:



Excluded/allowed regions of parameter space





Dirty laundry:

Can minimizing a G_F-invariant potential give the desired values of Yukawas?
See: R. Alonso et al, JHEP 1311 (2013) 187 arXiv:1306.5927

Orbit of enhanced symmetry are always extrema.
So the natural outcome would be not fully broken G_F.

Example: SU(3) with scalar field in adjoint, A. Two independent invariants, Tr(A^2) and det(A)

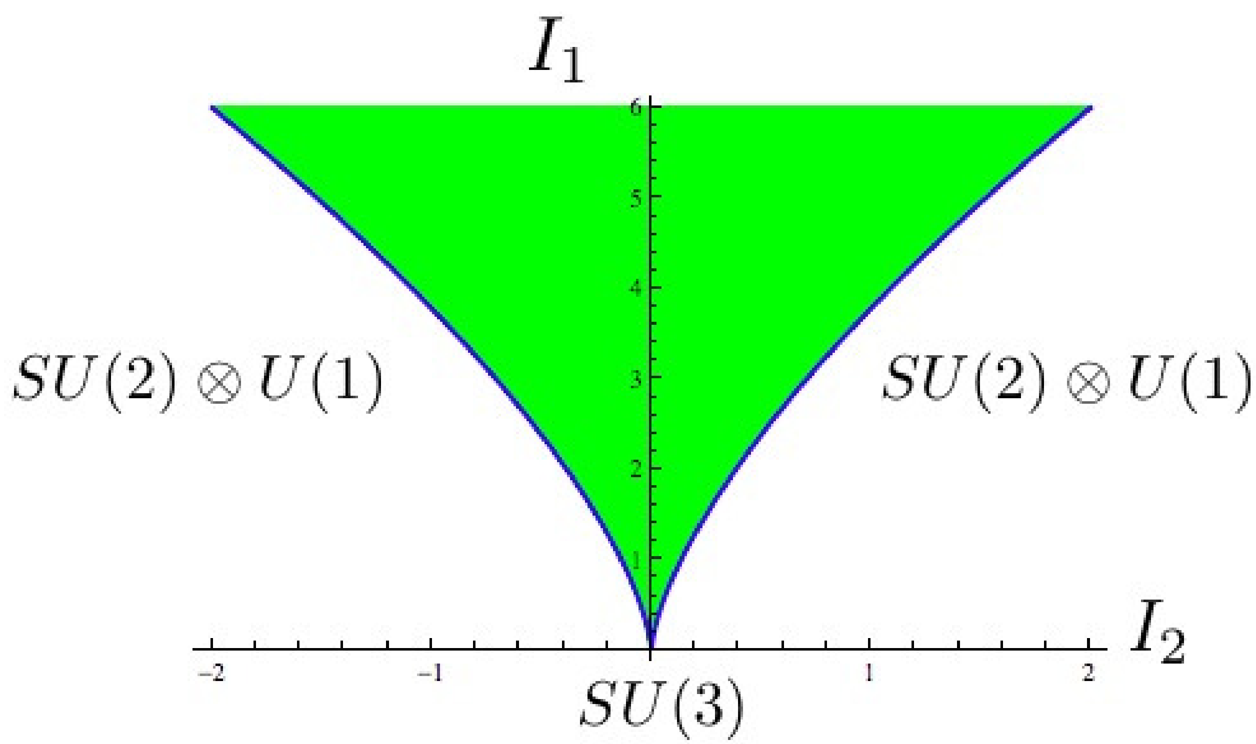


Figure 1: Manifold $\mathcal{M}$ of the $SU(3)$ invariants constructed from x=octet=hermitian, 3×3, traceless matrix (green region). Each point of $\mathcal{M}$ represents the orbit of x, namely the set of points in octet space given by: $x_g = gxg^{-1}$, when g runs over $SU(3)$. Boundaries of $\mathcal{M}$ are represented by Eq. (3.1). The little groups of the elements of different boundaries are indicated.

Summary and Conclusions

- MFV
  - Works very well when the SM Yukawas are the unique source of flavor breaking
    - Need not be spurions (VEVs of fields), ie, spurions as accounting device
    - Example: MSSM
  - But SM Yukawas may be derived from more fundamental source of G_F breaking
    - Then Flavor breaking matrices are proportional (but not equal) to SM Yukawas
    - This can give interesting phenomenology. Examples (Martin Camalich tomorrow):
      - MLFV can give breaking to lepton universality
        - R_K
        - τ in B decays
- Taking spurions seriously: gauge flavor (or some subgroup!)
  - Minimal anomaly free model gives inverted hierarchy of vector bosons
  - Can give interesting low energy phenomenology for heavier generations
  - Does not give an MFV model, but safe by small quark masses (not by angles)
  - MLFV version in progress. Open questions: other subgroups, GUTs, ...

The End