Lepton flavor mixing and CP symmetry

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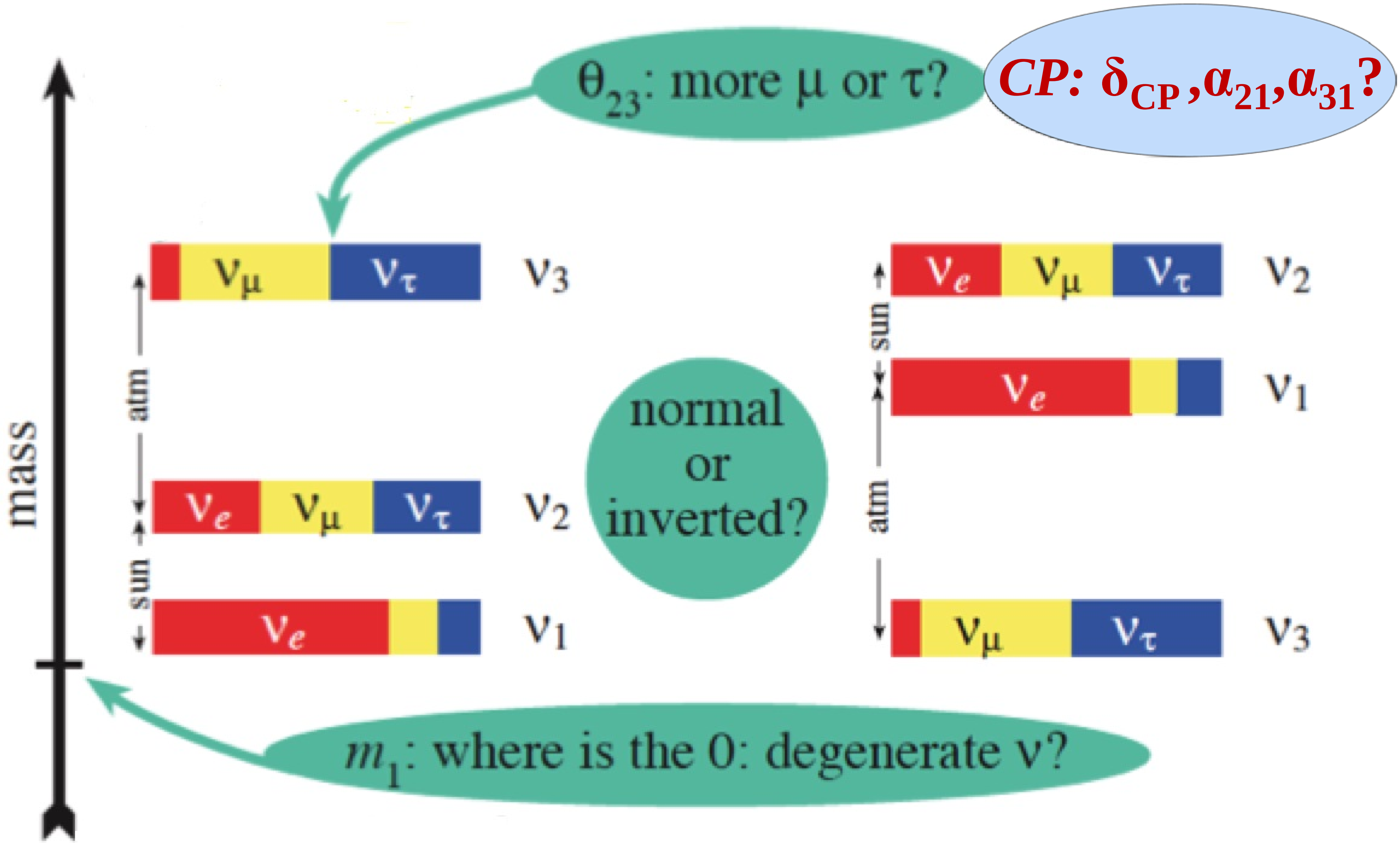
Based on Phys.Rev. D91 (2015) 3, 033003, arXiv:1412.8352
In collaboration with Peng Chen, Cai-Chang Li

FLASY15, June 29th-July 2nd, 2015, Manzanillo, Colima, Mexico

Outline:

- Motivation and remnant symmetries of neutrino mass matrix
- Reconstruction of PMNS matrix from remnant CP transformations
- Phenomenological implications
- Summary

What we know and unknow?



Classification of U_{PMNS} from finite flavor symmetries

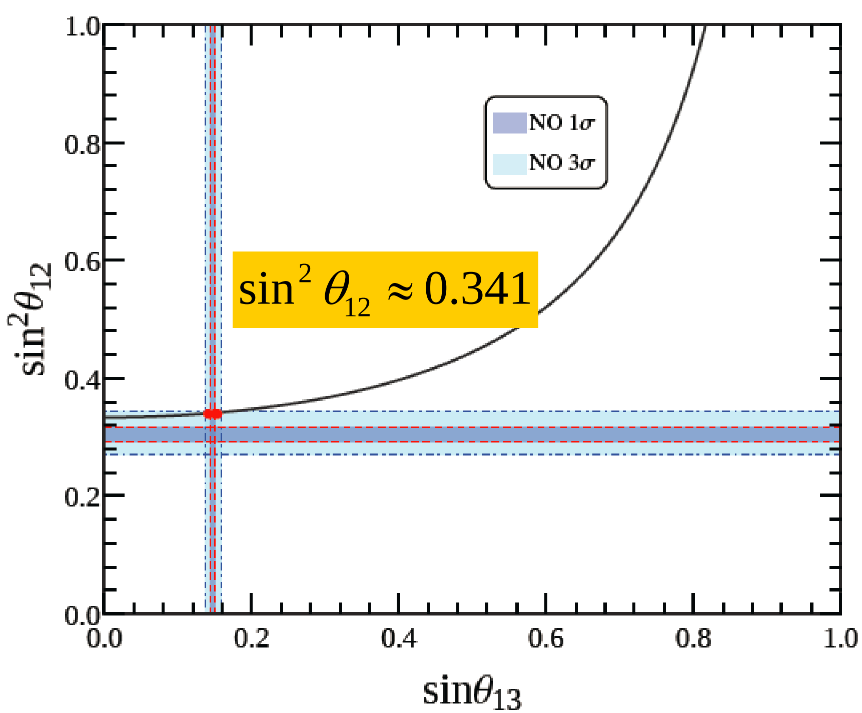
The PMNS matrix can take 16 discrete patterns or the trimaximal form.

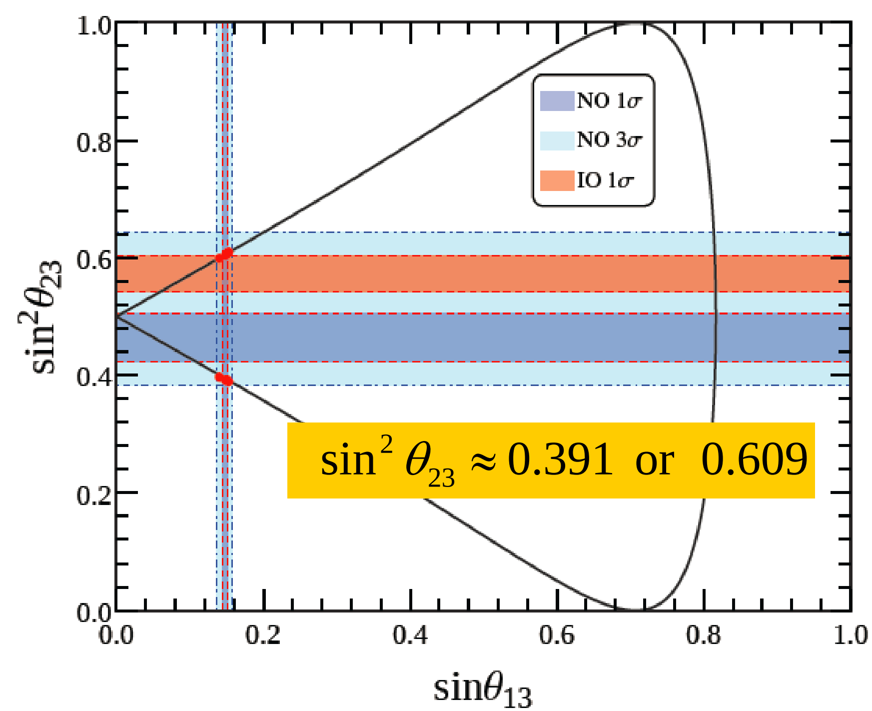
- Only trimaximal mixing can be compatible with data

$$U_{\rm PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2}\cos\vartheta & 1 & -\sqrt{2}\sin\vartheta \\ -\sqrt{2}\cos\left(\vartheta - \frac{\pi}{3}\right) & 1 & \sqrt{2}\sin\left(\vartheta - \frac{\pi}{3}\right) \\ -\sqrt{2}\cos\left(\vartheta + \frac{\pi}{3}\right) & 1 & \sqrt{2}\sin\left(\vartheta + \frac{\pi}{3}\right) \end{pmatrix},$$ ϑ/π is a rational number!

[R. M. Fonseca and W. Grimus, JHEP 1409, 033 (2014)]

Dirac CP phase can only be conserved : $\sin\delta_{CP}=0$





➢Underlying flavor symmetry group

$$G_f = \Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$$
$$\text{or} \quad G_f = \Delta'(18n^2) \cong (Z_n \times Z_{3n}) \rtimes S_3$$

Example:

G_f	$\Delta(600)$	$\Delta'(648)$	$\Delta(1536)$
ϑ	$\pm\pi/15$	$\pm\pi/18$	$\pm\pi/16$
$\sin^2\theta_{13}$	0.0288	0.0201	0.0254

The order of the flavor symmetry group is somewhat large!

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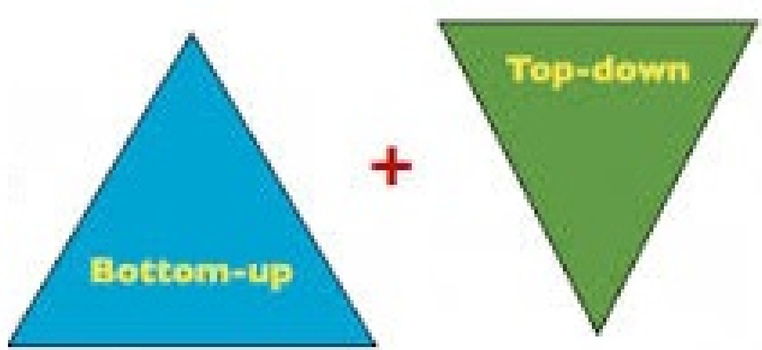
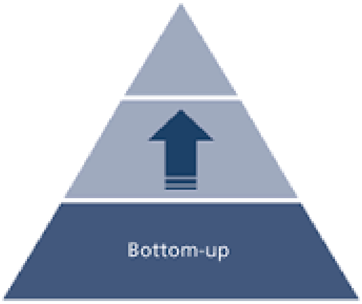
Q: do we have other approach of predicting lepton mixing parameters including CP phases ?

Our method:



Remnant symmetries of lepton mass matrices



Majorana neutrinos: $\mathcal{L}_{mass} = -\overline{l}_R m_l l_L + \frac{1}{2}\nu_L^T C^{-1} m_\nu \nu_L + h.c.$

Flavor basis: $m_l = \mathrm{diag}\left(m_e, m_\mu, m_\tau\right), \quad m_\nu = U_{PMNS}^* \mathrm{diag}\left(m_1, m_2, m_3\right) U_{PMNS}^\dagger$

- Residual **flavor** symmetries

Invariant under $\nu_L \to G_\nu \nu_L$

$$m_\nu \to G_\nu^T m_\nu G_\nu = m_\nu$$

$$G_\nu = U_{PMNS} \mathrm{diag}\left(\pm1, \pm1, \pm1\right) U_{PMNS}^\dagger$$

- Residual **CP** symmetries

Invariant under $\nu_L(x) \to i X_\nu \gamma^0 C \overline{\nu}_L^T(x_P)$

$$m_\nu \to X_\nu^T m_\nu X_\nu = m_\nu^*$$

$$X_\nu = U_{PMNS} \mathrm{diag}\left(\pm1, \pm1, \pm1\right) U_{PMNS}^T$$

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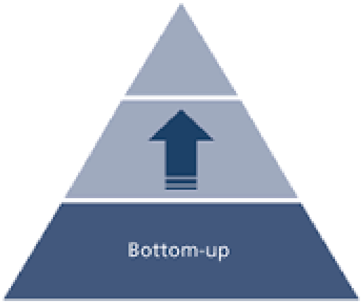

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●Disregarding an overall "-1" factor, there are essentially four solutions for G_ν and X_ν

$$G_i = U_{PMNS}\, d_i U_{PMNS}^\dagger,$$ [C. S. Lam, Phys. Lett. B 656, 193 (2007)]

$$X_i = U_{PMNS}\, d_i U_{PMNS}^T, \quad i = 1, 2, 3, 4$$ [F. Feruglio et al., JHEP 1307, 027 (2013)]

with

$$d_1 = \text{diag}\left(1, -1, -1\right),\ d_2 = \text{diag}\left(-1, 1, -1\right), d_3 = \text{diag}\left(-1, -1, 1\right),\ d_4 = \text{diag}\left(1, 1, 1\right)$$

$$G_i^2 = 1,\ \ G_i G_j = G_j G_i = G_k \ \text{ for } i \neq j \neq k \neq 4 \mapsto Klein\ group$$

- Only **three** of X_v are independent

$$X_i = X_i^T \mapsto symmetric, \qquad X_i = X_j X_m^* X_n, \quad i \neq j \neq m \neq n$$

$$G_i^2 = 1, \quad G_i G_j = G_j G_i = G_k \quad \text{for } i \neq j \neq k \neq 4 \mapsto \textit{Klein group}$$

- Only **three** of X_ν are independent

$$X_i = X_i^T \mapsto \textit{symmetric}, \qquad X_i = X_j X_m^* X_n, \quad i \neq j \neq m \neq n$$

- Consistency relations between residual flavor and CP symmetries

$$X_i G_j^* = G_j X_i \quad \text{for} \quad i, j = 1, 2, 3, 4$$

- Generating residual flavor symmetry from residual CP symmetry

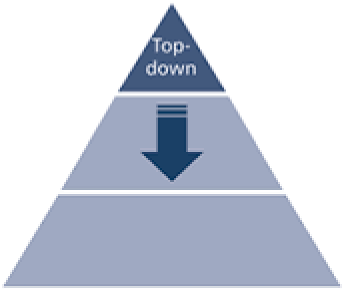
$$\nu_L(x) \xrightarrow{CP} iX_i \gamma^0 C \bar{\nu}_L^T(x_P) \xrightarrow{CP} X_i X_j^* \nu_L(x)$$

$$\begin{cases} X_2 X_3^* = X_3 X_2^* = X_4 X_1^* = X_1 X_4^* = G_1 \\ X_1 X_3^* = X_3 X_1^* = X_4 X_2^* = X_2 X_4^* = G_2 \\ X_1 X_2^* = X_2 X_1^* = X_4 X_3^* = X_3 X_4^* = G_3 \\ X_1 X_1^* = X_2 X_2^* = X_3 X_3^* = X_4 X_4^* = G_4 = 1 \end{cases}$$

CP symmetry is more general than flavor symmetry, and it can constrain the values of the Majorana phases.

Reconstruction of PMNS matrix from remnant CP symmetries

➢ **Four (or three) remnant CP transformations preserved**



Both lepton mixing angles and CP phases are completely determined by the assumed remnant CP.

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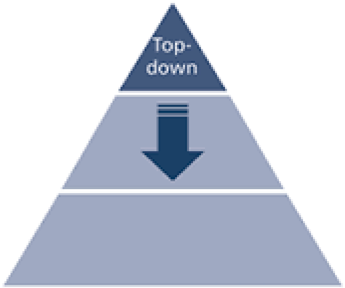

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Both lepton mixing angles and CP phases are completely determined by the assumed remnant CP.

➢**Two remnant CP transformations preserved**

$$X^T_{R1} = X_{R1}, \quad X^T_{R2} = X_{R2}, \quad X_{R1}X^*_{R2} = X_{R2}X^*_{R1}, \quad \left(X_{R1}X^*_{R2}\right)^2 = 1$$

● Z_2 residual flavor symmetry:

$$G_R \equiv X_{R1}X^*_{R2} = X_{R2}X^*_{R1} \Rightarrow X_{R1,R2}G^*_R = G_R X_{R1,R2}$$

● parameterization: G_R fixes one column of the PMNS matrix

$$v_1 = \begin{pmatrix} \cos\varphi \\ \sin\varphi\cos\phi \\ \sin\varphi\sin\phi \end{pmatrix} \Longrightarrow G_R = 2v_1v_1^\dagger - 1_{3\times3}$$

[D.Hernandez, A.Smirnov,Phys.Rev.D86 (2012)]

$$X_{R1} = e^{i\kappa_1} v_1 v_1^T + e^{i\kappa_2} v_2 v_2^T + e^{i\kappa_3} v_3 v_3^T$$

$$X_{R2} = G_R X_{R1} = e^{i\kappa_1} v_1 v_1^T - e^{i\kappa_2} v_2 v_2^T - e^{i\kappa_3} v_3 v_3^T$$

with

$$v_2 = \begin{pmatrix} \sin\varphi\cos\rho \\ -\sin\phi\sin\rho - \cos\varphi\cos\phi\cos\rho \\ \cos\phi\sin\rho - \cos\varphi\sin\phi\cos\rho \end{pmatrix}, \quad v_3 = \begin{pmatrix} \sin\varphi\sin\rho \\ \sin\phi\cos\rho - \cos\varphi\cos\phi\sin\rho \\ -\cos\phi\cos\rho - \cos\varphi\sin\phi\sin\rho \end{pmatrix}$$

- PMNS matrix: $X^T_{R1,R2} m_\nu X_{R1,R2} = m_\nu^*$

$$\boxed{U^{-1}_{PMNS} X_{R1,R2} U^*_{PMNS} = \mathrm{diag}\left(\pm 1, \pm 1, \pm 1\right)}$$

$$U_{PMNS} = \Sigma \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} Q_\nu$$

The lepton mixing matrix and therefore all mixing parameters depend on only one free parameter θ, which can be determined by θ_{13}.

up to permutations of rows and columns.

$$\Sigma = \left(v_1, v_2, v_3\right) \mathrm{diag}(e^{i\kappa_1/2}, e^{i\kappa_2/2}, e^{i\kappa_3/2}), \qquad X_{R1} = \Sigma\Sigma^T \mapsto \text{Takagi factorization}$$

$$Q_\nu = \mathrm{diag}\left(\sqrt{\pm 1}, \sqrt{\pm 1}, \sqrt{\pm 1}\right)$$

➢ One remnant CP transformation preserved

$$X_{R1}^{T} = X_{R1}$$

- parameterization: $X_{R1} = e^{i\kappa_1} v_1 v_1^T + e^{i\kappa_2} v_2 v_2^T + e^{i\kappa_3} v_3 v_3^T$

Takagi factorization: $X_{R1} = \Sigma\Sigma^T, \ \Sigma = \left(v_1, v_2, v_3\right) \mathrm{diag}(e^{i\kappa_1/2}, e^{i\kappa_2/2}, e^{i\kappa_3/2})$

- PMNS matrix : $X_{R1}^{T} m_\nu X_{R1} = m_\nu^{*}$

$$\Rightarrow \quad U_{PMNS}^{-1} X_{R1} U_{PMNS}^{*} = \mathrm{diag}\left(\pm 1, \pm 1, \pm 1\right)$$

$$\Rightarrow \quad \boxed{U_{PMNS} = \Sigma O_{3\times 3} Q_\nu}$$

where $O_{3\times 3}$ is a generic real orthogonal matrix.

The lepton mixing matrix is determined up to an arbitrary real orthogonal matrix which can be parameterized by three rotation angles.

Phenomenological implications

➢ **First column fixed for two remnant CP**

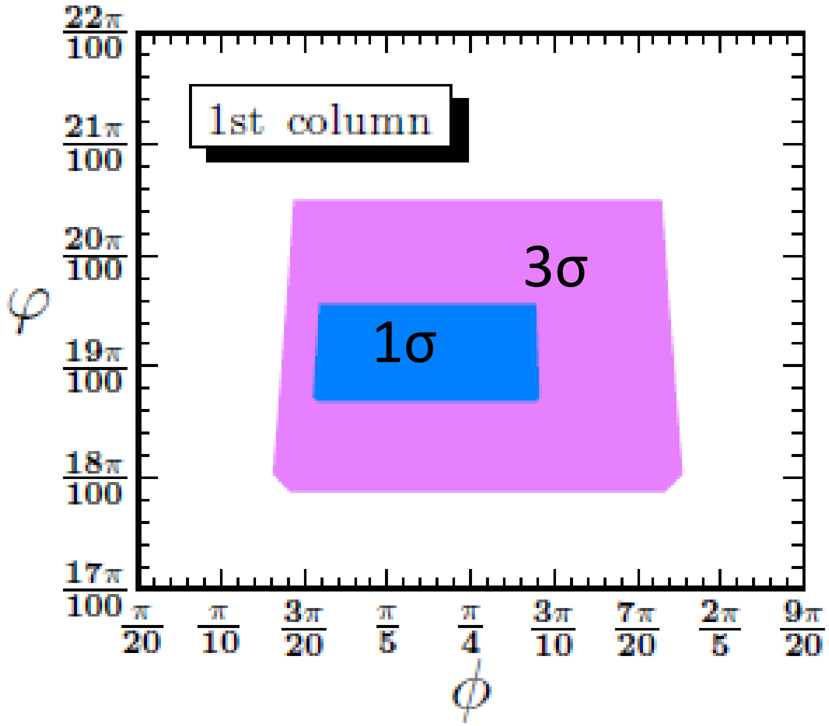
$$U_{PMNS}=\begin{pmatrix}\boxed{\begin{matrix}\cos\varphi\\ \sin\varphi\cos\phi\\ \sin\varphi\sin\phi\end{matrix}} & \begin{matrix}\sin\varphi\\ -\cos\varphi\cos\phi\\ -\cos\varphi\sin\phi\end{matrix} & \begin{matrix}0\\ \sin\phi\\ -\cos\phi\end{matrix}\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\rho & \sin\rho\\ 0 & -\sin\rho & \cos\rho\end{pmatrix}$$

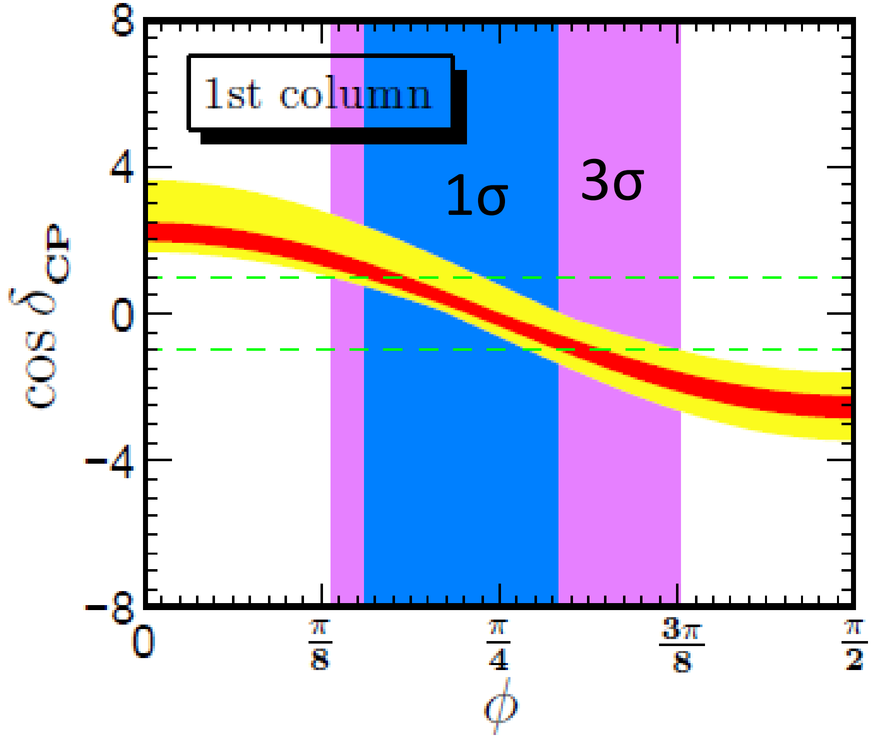
$$\times\begin{pmatrix}e^{i\kappa_1/2} & 0 & 0\\ 0 & e^{i\kappa_2/2} & 0\\ 0 & 0 & e^{i\kappa_3/2}\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{pmatrix}Q_\nu$$

● Correlation among mixing parameters

$$\cos^2\theta_{12}\cos^2\theta_{13}=\cos^2\varphi$$

$$\cos\delta_{CP}=\frac{\cos 2\theta_{23}(-\sin^2\theta_{12}+\cos^2\theta_{12}\sin^2\theta_{13})+\cos 2\phi(\sin^2\theta_{12}+\cos^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}}$$





- Condition of vanishing or maximal Dirac CP

$$\kappa_2 = \kappa_3 \quad \text{or} \quad \theta = 0, \pi/2, \pi, 3\pi/2 \Leftrightarrow \sin\delta_{CP} = 0$$

For $\kappa_3 = \kappa_2 \pm \pi,\ \phi = \dfrac{\pi}{4},\ \rho = 0, \dfrac{\pi}{2}, \pi \text{ or } \dfrac{3\pi}{2}$

$$\sin^2\theta_{12} = \frac{\cos^2\theta\sin^2\varphi}{1-\sin^2\theta\sin^2\varphi}, \qquad \sin^2\theta_{13} = \sin^2\theta\sin^2\varphi,$$

$$\sin^2\theta_{23} = \frac{1}{2}, \qquad \cos\delta_{CP} = 0, \qquad \tan\alpha_{21} = \tan\alpha_{31} = \tan(\kappa_2 - \kappa_1)$$

➢Second column fixed for two remnant CP

$$U_{PMNS}=\begin{pmatrix} 0 & \boxed{\begin{matrix}\cos\varphi \\ \sin\varphi\cos\phi \\ \sin\varphi\sin\phi\end{matrix}} & \begin{matrix}\sin\varphi \\ -\cos\varphi\cos\phi \\ -\cos\varphi\sin\phi\end{matrix} \end{pmatrix}$$

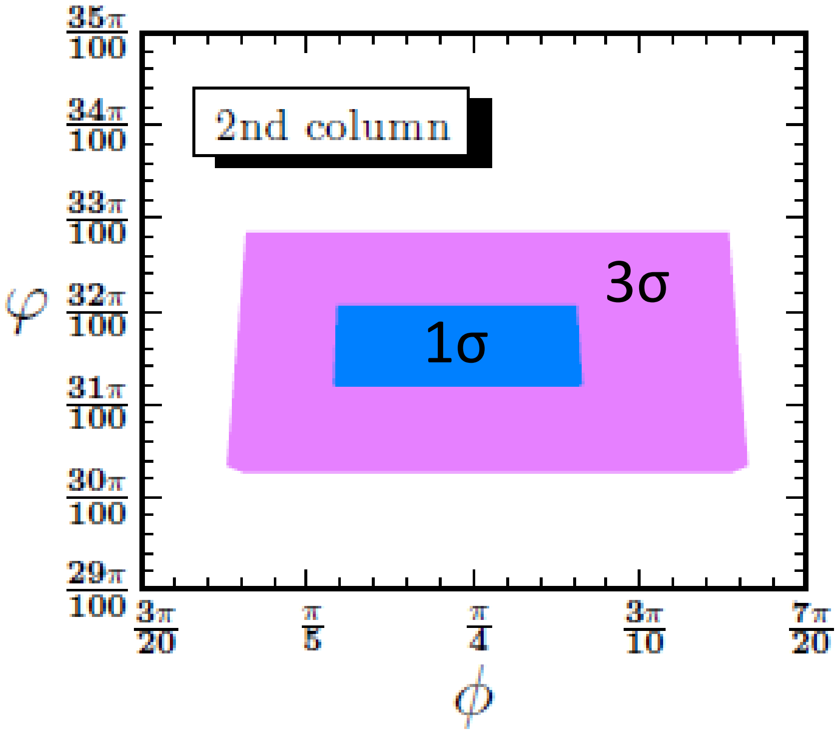
$$U_{PMNS}=\begin{pmatrix} 0 & \cos\varphi & \sin\varphi \\ \sin\phi & \sin\varphi\cos\phi & -\cos\varphi\cos\phi \\ -\cos\phi & \sin\varphi\sin\phi & -\cos\varphi\sin\phi \end{pmatrix}\begin{pmatrix} \cos\rho & 0 & -\sin\rho \\ 0 & 1 & 0 \\ \sin\rho & 0 & \cos\rho \end{pmatrix}$$

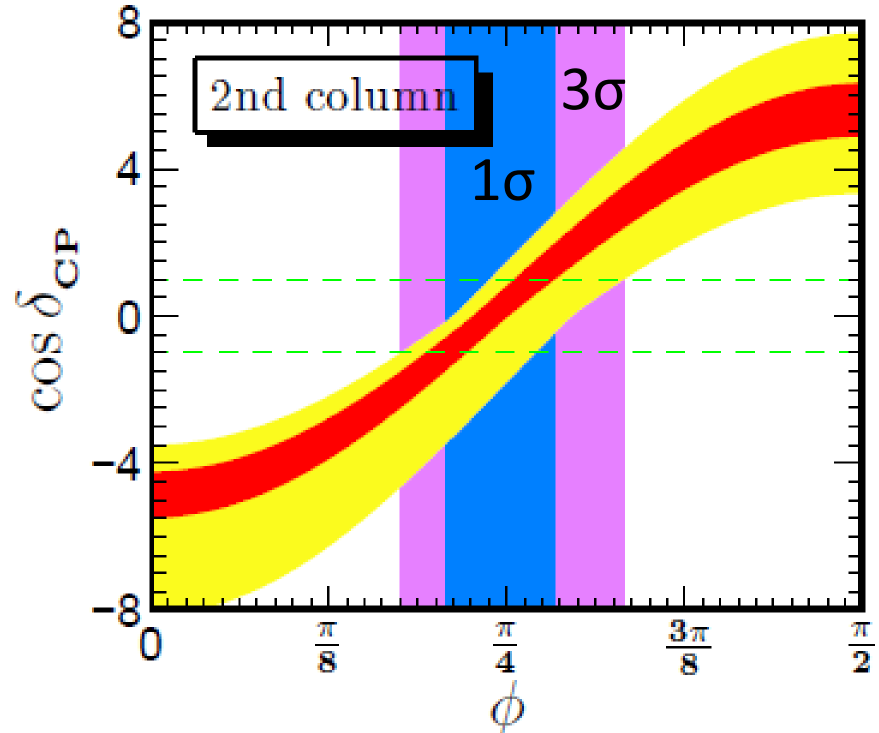
$$\times\begin{pmatrix} e^{i\kappa_3/2} & 0 & 0 \\ 0 & e^{i\kappa_1/2} & 0 \\ 0 & 0 & e^{i\kappa_2/2} \end{pmatrix}\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}Q_\nu$$

- Correlation among mixing parameters

$$\sin^2\theta_{12}\cos^2\theta_{13}=\cos^2\varphi$$

$$\cos\delta_{CP}=\frac{\cos 2\theta_{23}(\cos^2\theta_{12}-\sin^2\theta_{12}\sin^2\theta_{13})-\cos 2\phi(\cos^2\theta_{12}+\sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}}$$





●Condition of vanishing or maximal Dirac CP

$$\kappa_2 = \kappa_3 \quad \text{or} \quad \theta = 0, \pi/2, \pi, 3\pi/2 \Leftrightarrow \sin\delta_{CP} = 0$$

For $\kappa_3 = \kappa_2 \pm \pi, \ \phi = \dfrac{\pi}{4}, \ \rho = 0, \dfrac{\pi}{2}, \pi \text{ or } \dfrac{3\pi}{2}$

$$\sin^2\theta_{12} = \frac{\cos^2\varphi}{1-\cos^2\theta\sin^2\varphi}, \qquad \sin^2\theta_{13} = \cos^2\theta\sin^2\varphi,$$

$$\sin^2\theta_{23} = \frac{1}{2}, \qquad \cos\delta_{CP} = \sin\alpha_{31} = 0, \qquad \tan\alpha_{21} = -\tan(\kappa_2 - \kappa_1)$$

➢Third column fixed for two remnant CP

$$U_{PMNS}=\begin{pmatrix} \sin\varphi & 0 & \boxed{\begin{matrix}\cos\varphi \\ \sin\varphi\cos\phi \\ \sin\varphi\sin\phi\end{matrix}} \end{pmatrix}$$

$$U_{PMNS}=\begin{pmatrix} \sin\varphi & 0 & \cos\varphi \\ -\cos\varphi\cos\phi & \sin\phi & \sin\varphi\cos\phi \\ -\cos\varphi\sin\phi & -\cos\phi & \sin\varphi\sin\phi \end{pmatrix}\begin{pmatrix} \cos\rho & \sin\rho & 0 \\ -\sin\rho & \cos\rho & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\times\begin{pmatrix} e^{i\kappa_2/2} & 0 & 0 \\ 0 & e^{i\kappa_3/2} & 0 \\ 0 & 0 & e^{i\kappa_1/2} \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}Q_\nu$$

$$\sin^2\theta_{13}=\cos^2\varphi,\quad \sin^2\theta_{23}=\cos^2\phi \Rightarrow 0.449\pi\le\varphi\le0.456\pi,\ \ 0.204\pi\le\phi\le0.287\pi$$

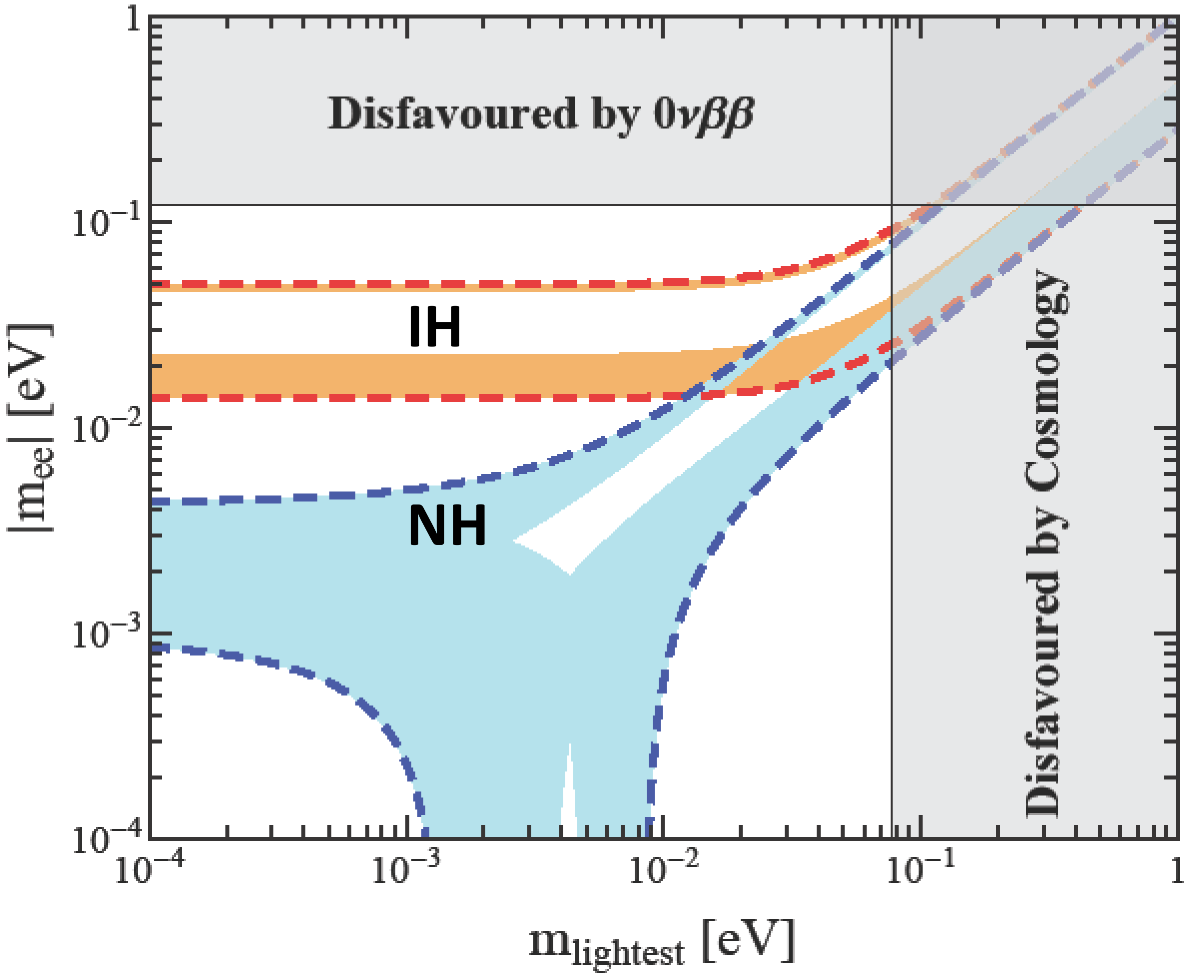
●Condition of maximal Dirac CP

For $\kappa_3=\kappa_2\pm\pi,\quad \rho=0,\dfrac{\pi}{2},\pi \text{ or } \dfrac{3\pi}{2}$

$$\cos\delta_{CP}=\sin\alpha_{21}=0,\qquad \tan\alpha_{31}=-\tan(\kappa_2-\kappa_1)$$

Predictions for neutrinoless double decay :

$$\left|m_{ee}\right| = \left|m_1 \cos^2\theta_{12} \cos^2\theta_{13} + m_2 \sin^2\theta_{12} \cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31} - 2\delta_{CP})}\right|$$



The predictions for the IH case are within the sensitivity of future 0νββ decay experiments.

➢Single remnant CP

$$U_{PMNS}=(v_1,v_2,v_3)\,\mathrm{diag}\left(e^{i\kappa_1/2},e^{i\kappa_2/2},e^{i\kappa_3/2}\right)O_{3\times3}\,Q_\nu$$

Usually the measured values of the lepton mixing angles can be easily obtained by choosing the three parameters in O_{3x3}.

- Conserved Dirac CP

$$X_R=\begin{pmatrix} e^{i\kappa_j} & 0 & 0 \\ 0 & e^{i\kappa_m} & 0 \\ 0 & 0 & e^{i\kappa_n} \end{pmatrix}\Rightarrow \sin\delta_{CP}=\sin\alpha_{21}=\sin\alpha_{31}=0$$

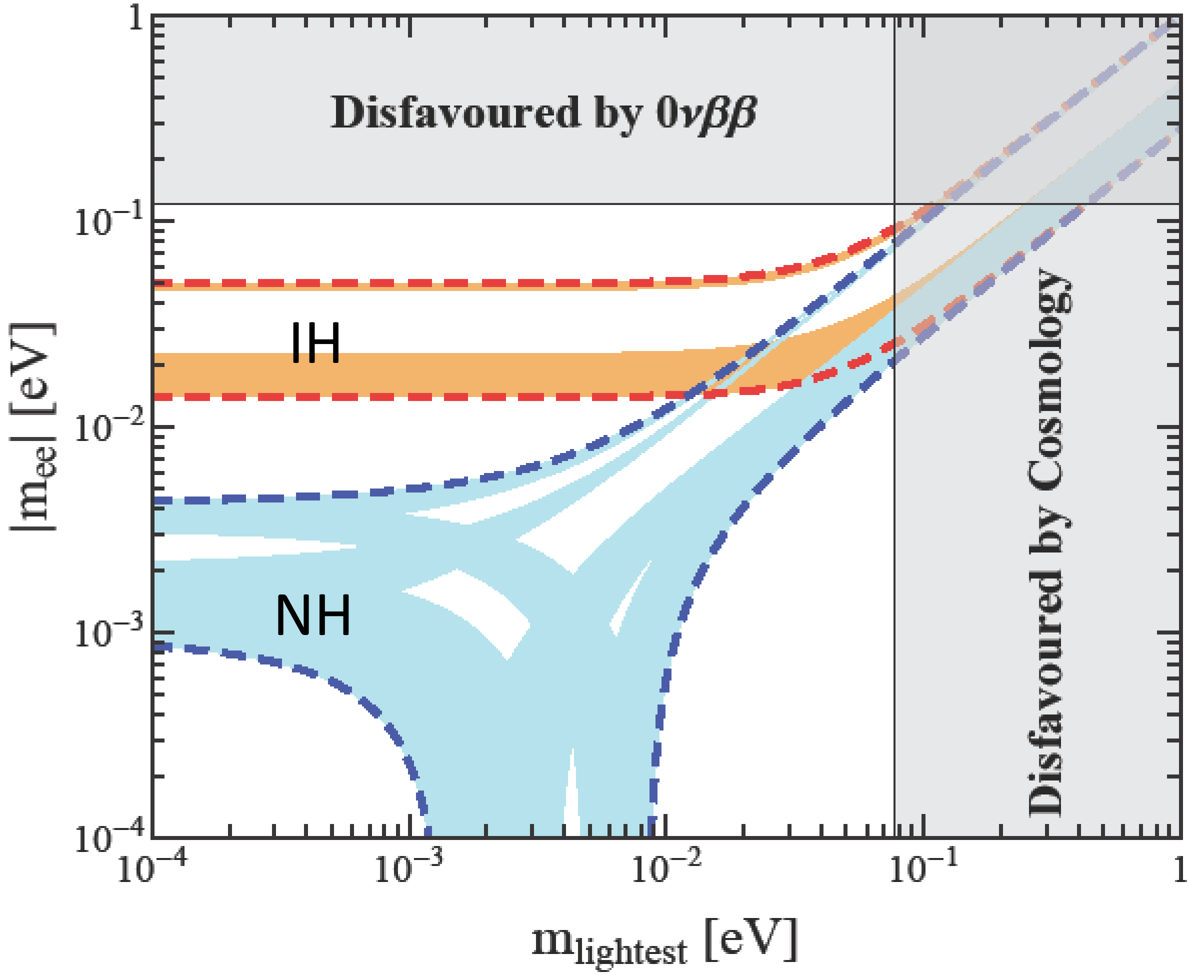
- Maximal Dirac CP

$$X_R=\begin{pmatrix} e^{i\kappa_a} & 0 & 0 \\ 0 & 0 & e^{i\kappa_b} \\ 0 & e^{i\kappa_b} & 0 \end{pmatrix}\Rightarrow \sin^2\theta_{23}=\frac{1}{2},\quad \cos\delta_{CP}=\sin\alpha_{21}=\sin\alpha_{31}=0$$

[P. Harrison, W.Scott, Phys.Lett. B547(2002) 219;
W. Grimus, L. Lavoura,Phys.Lett. B579 (2004)113]

This is the μ− τ reflection symmetry.

Predictions for neutrinoless double decay :



The predictions for IH case can be tested in near future.

Summary:

● The neutrino mass matrix generally admits four (three independent) remnant CP transformations which can be derived from the measured lepton mixing parameters, and vice versa lepton mixing matrix can be reconstructed from the remnant CP symmetries.

●

# of remnant CP transformations	# of free parameters in U_{PMNS}
3, 4	0
2	1
1	3

● CP symmetry can constrain the lepton flavor mixing more efficiently than flavor symmetry.

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Thank you for your attention!