UNIVERSITY OF Southampton
School of Physics and Astronomy

invisibles
neutrinos, dark matter & dark energy physics

Unified Models of Neutrinos, Flavour and CP violation

Steve King

Manzanillo,
2nd July, 2015

Lepton Mixing Matrix

Standard Model states

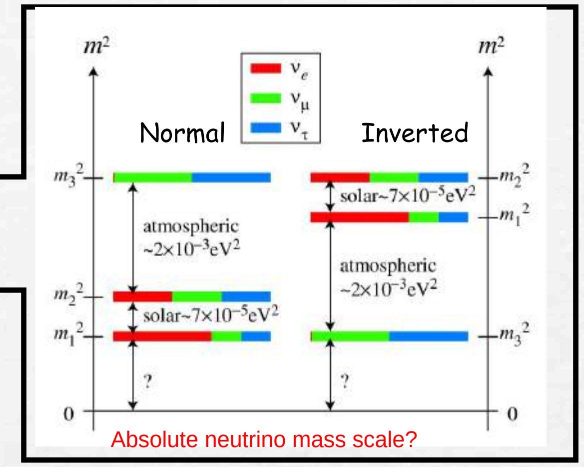
$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$$

PMNS matrix

Neutrino mass states

Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



$$U_{PMNS} = \begin{pmatrix}1 & 0 & 0\\ 0 & c^l_{23} & s^l_{23}\\ 0 & -s^l_{23} & c^l_{23}\end{pmatrix}\begin{pmatrix}c^l_{13} & 0 & s^l_{13}e^{-i\delta^l}\\ 0 & 1 & 0\\ -s^l_{13}e^{i\delta^l} & 0 & c^l_{13}\end{pmatrix}\begin{pmatrix}c^l_{12} & s^l_{12} & 0\\ -s^l_{12} & c^l_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \frac{\alpha_{21}}{2} & 0\\ 0 & 0 & \frac{\alpha_{31}}{2}\end{pmatrix}$$

$s^l_{ij} = \sin(\theta^l_{ij})$

$c^l_{ij} = \cos(\theta^l_{ij})$

Atmospheric | Reactor | Solar | Majorana

Oscillation phase δ^l

Majorana phases α_{21}, α_{31}

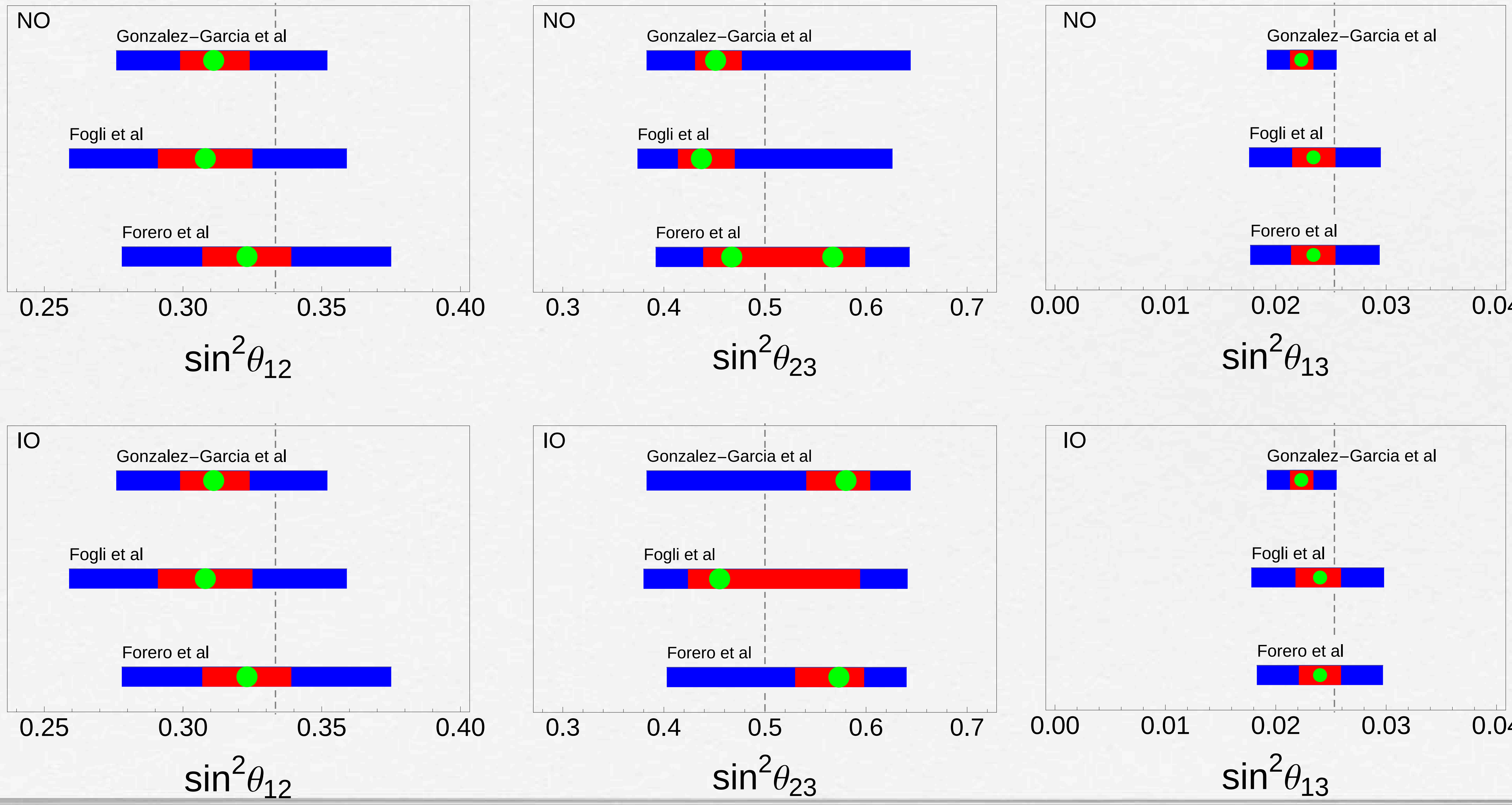
3 masses + 3 angles + 3 phases =
9 new parameters for SM

Gonzalez-Garcia et al = Gonzalez-Garcia, Maltoni, Salvado, Schwetz
Fogli et al = Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo
Forero et al = Forero, Tortola, Valle

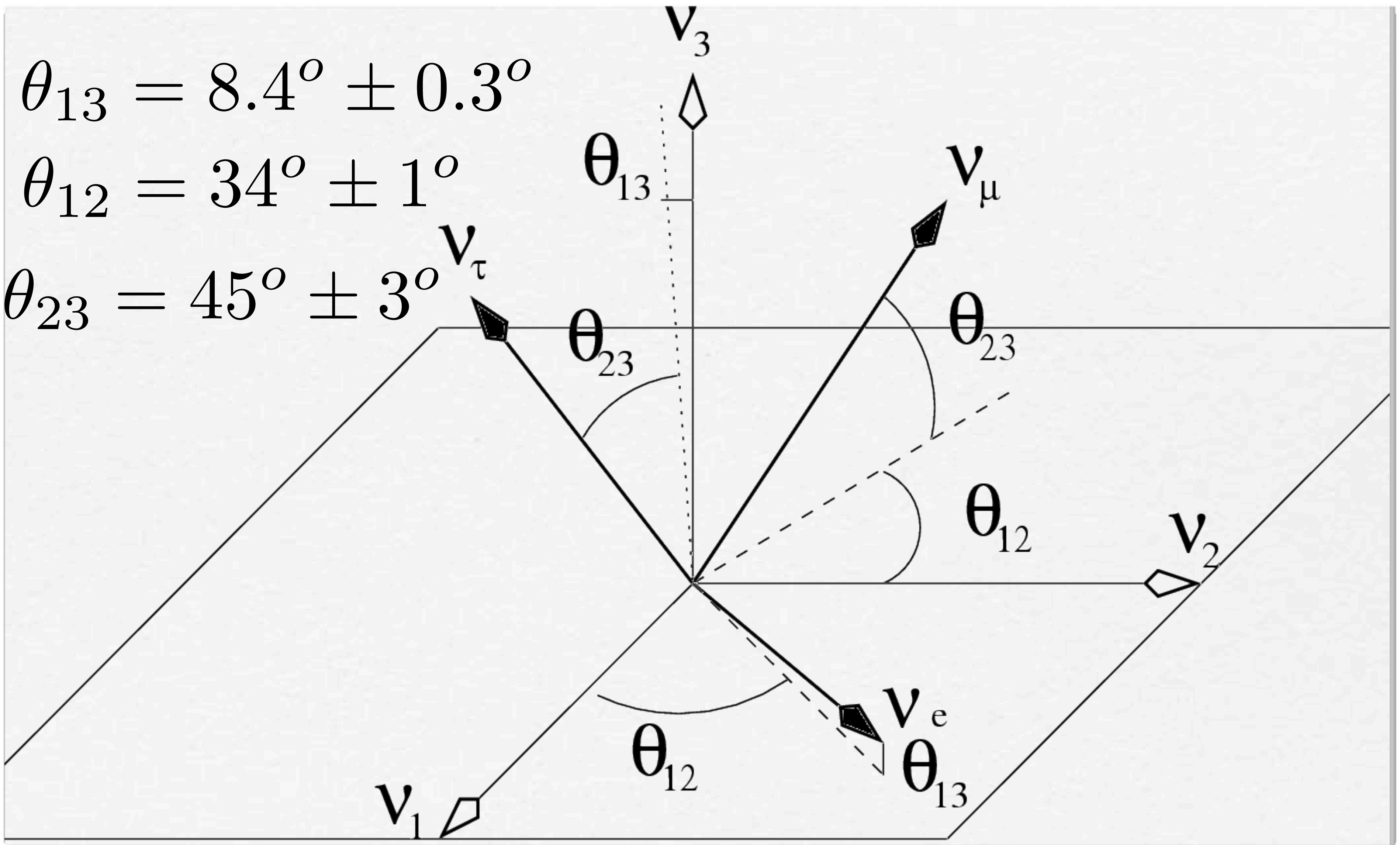
Global Fits 2014

$$\theta_{13} = \frac{\theta_C}{\sqrt{2}}$$

35° 45° → 9.2°



Lepton Mixing Angles (approx.)



$$\theta_{13} = 8.4^o \pm 0.3^o$$

$$\theta_{12} = 34^o \pm 1^o$$

$$\theta_{23} = 45^o \pm 3^o$$

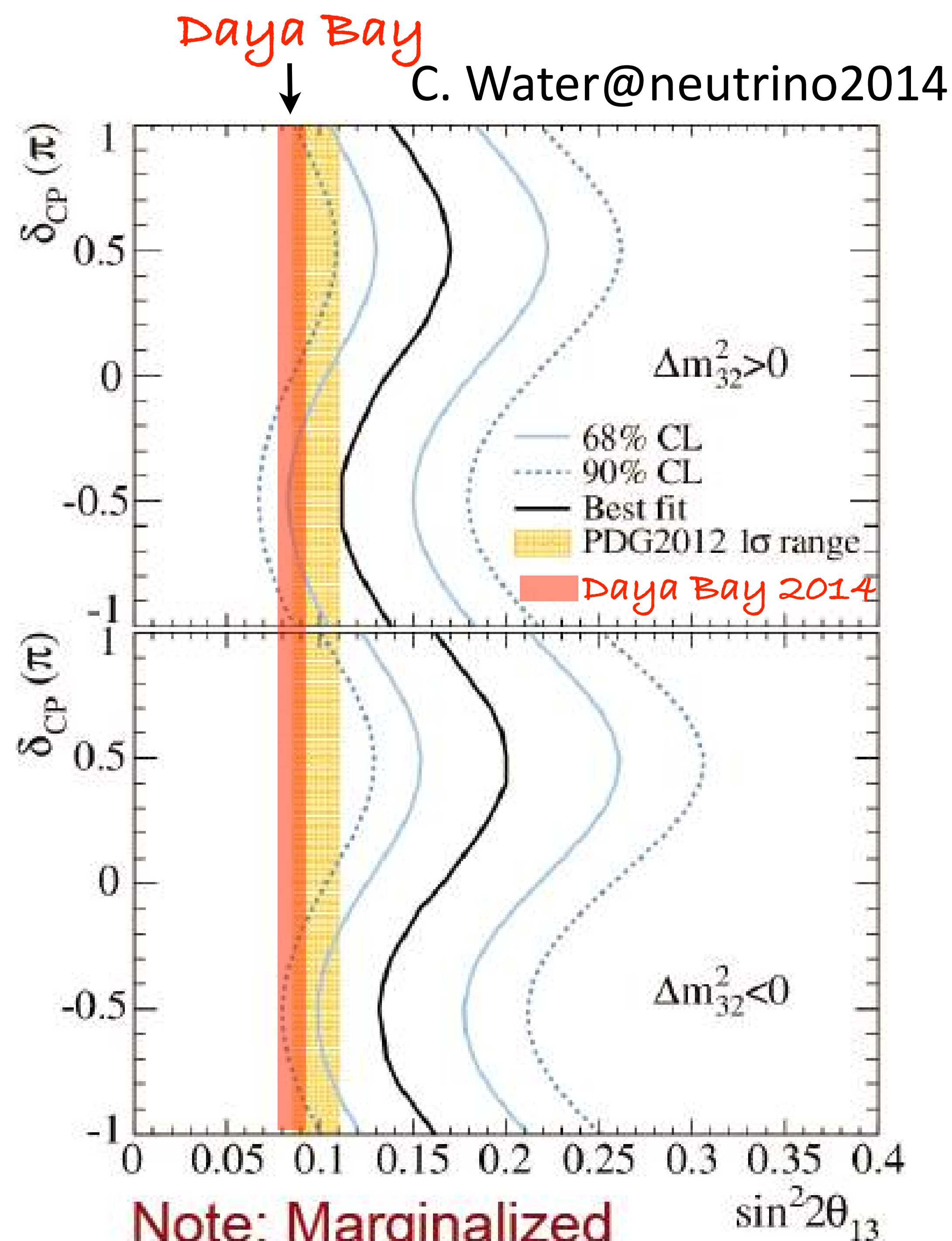
Daya bay $\sin^2 2\theta_{13} = 0.084^{+0.005}_{-0.005}$ ⟷ $\theta_{13} = 8.4^o \pm 0.3^o$

Note the magic number 8.4!!

CP Phase is known ?

- Taking reactor θ_{13} results, CP phase is constrained to be close to $-\pi/2$
- This is a very lucky value for NOVA and other accelerator experiments
- Mass hierarchy and CP phase will be known soon ?

Hint for $\delta_{CP} \approx -90^\circ$ and NH



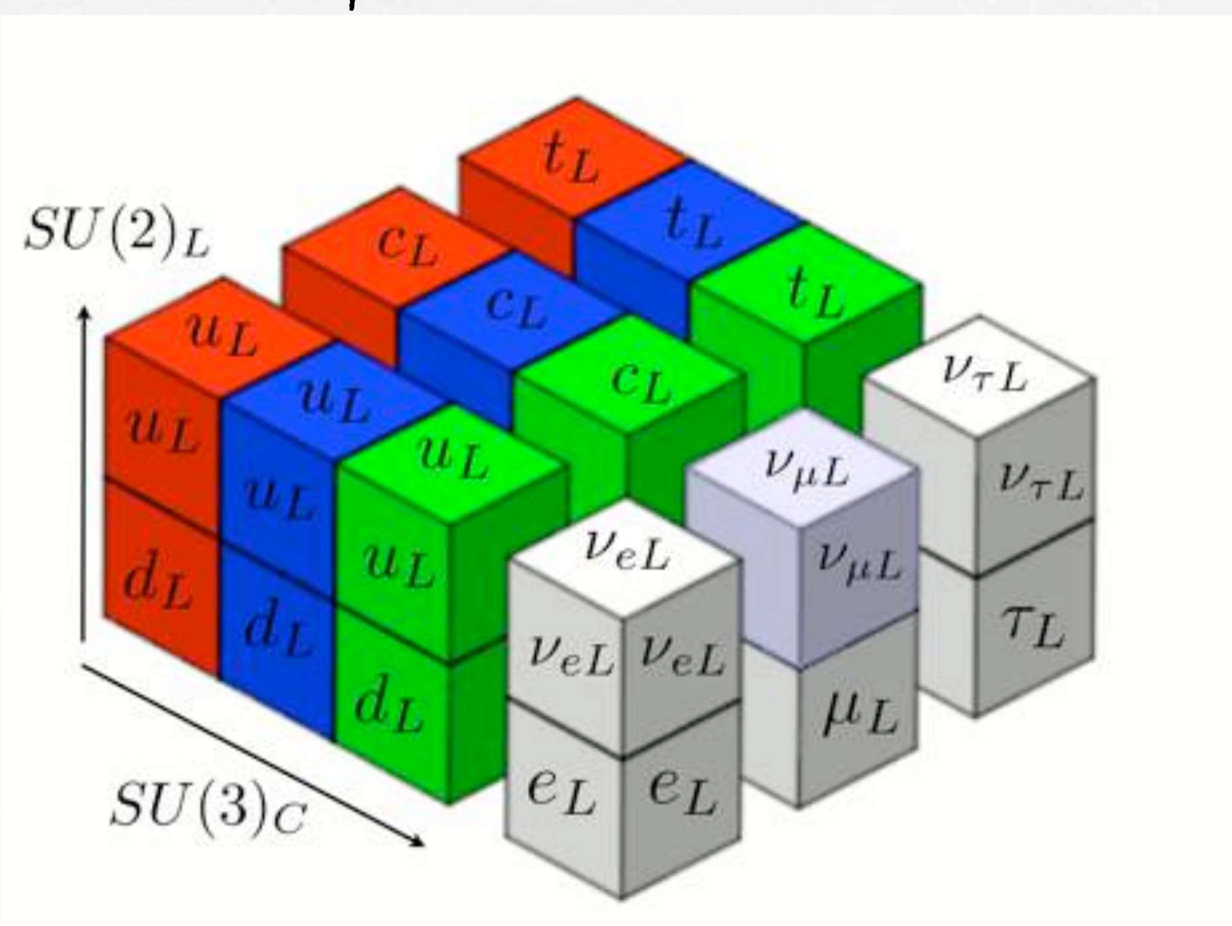
Note: Marginalized over θ_{23} and Δm^2_{32}

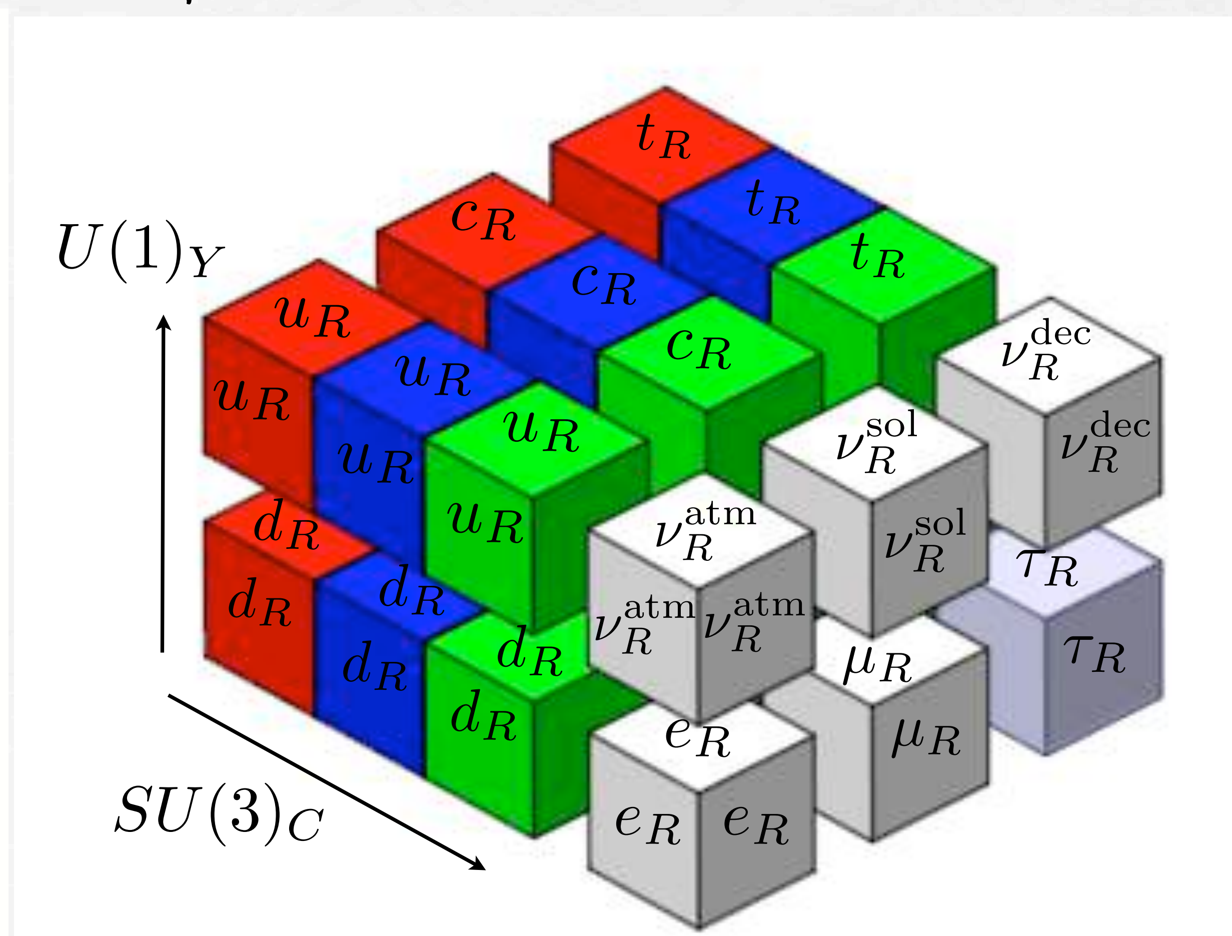
Seesaw motivates Standard Model with right-handed neutrinos

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

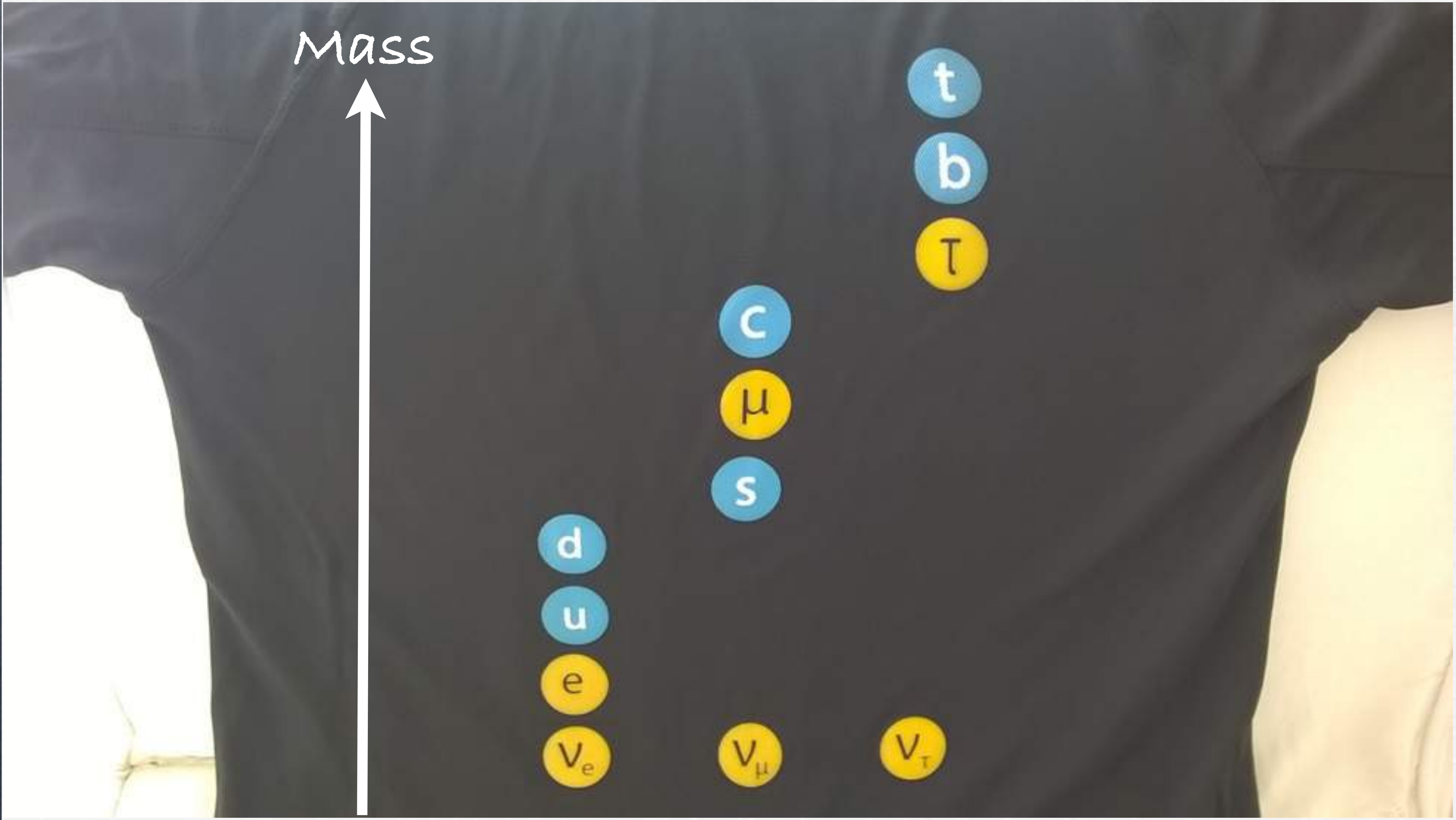
Left-handed quarks and leptons (active neutrinos)

Right-handed quarks and leptons (sterile neutrinos)

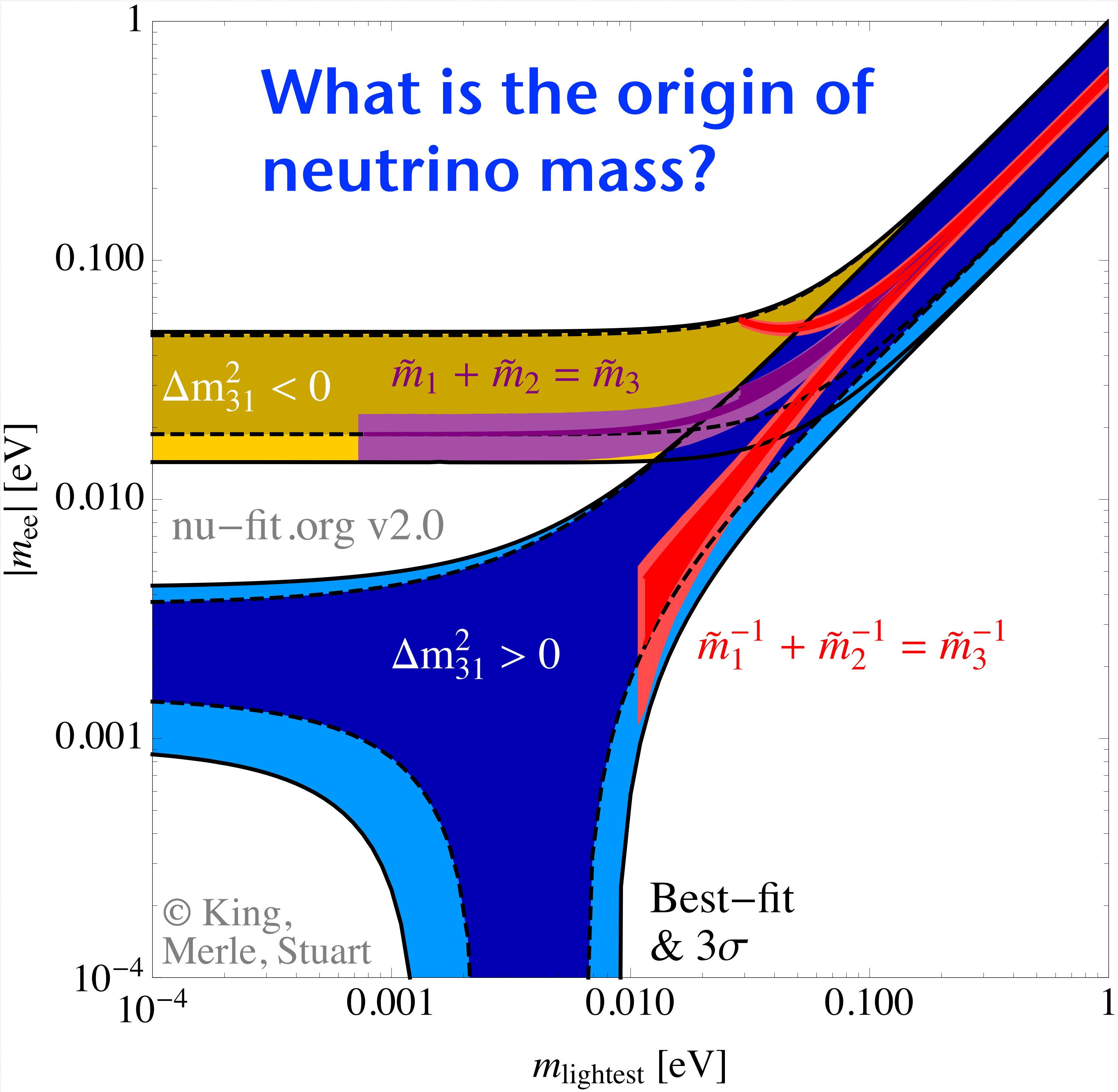


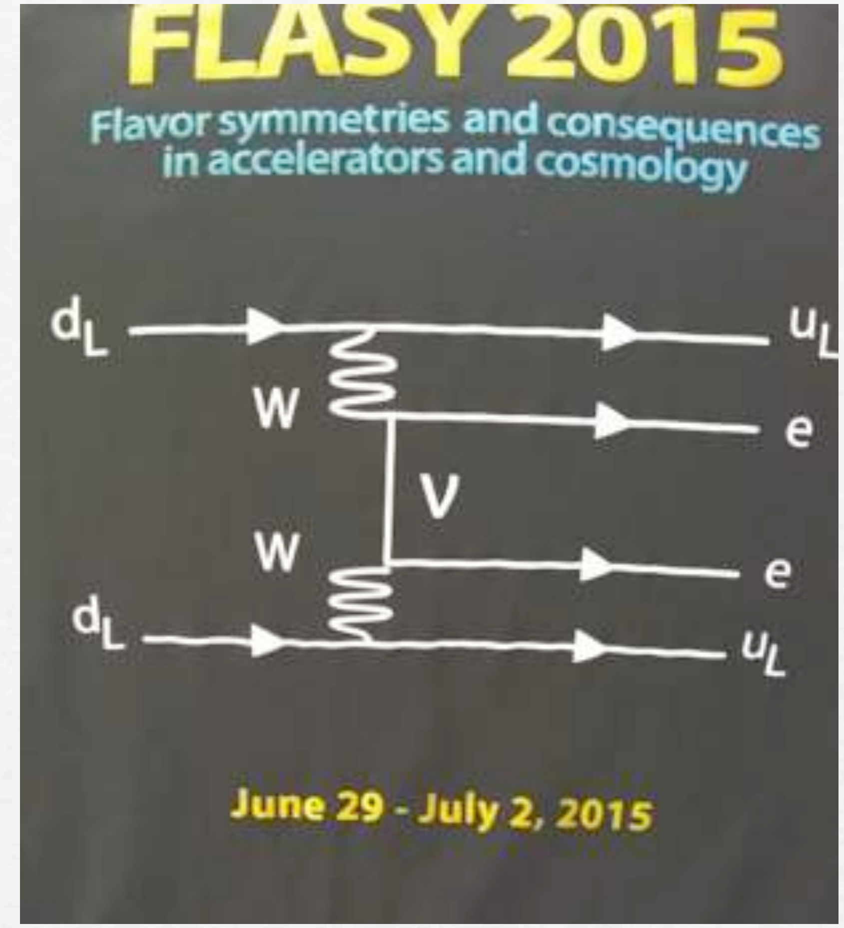


What is the origin of quark and charged lepton masses?



What is the origin of neutrino mass?



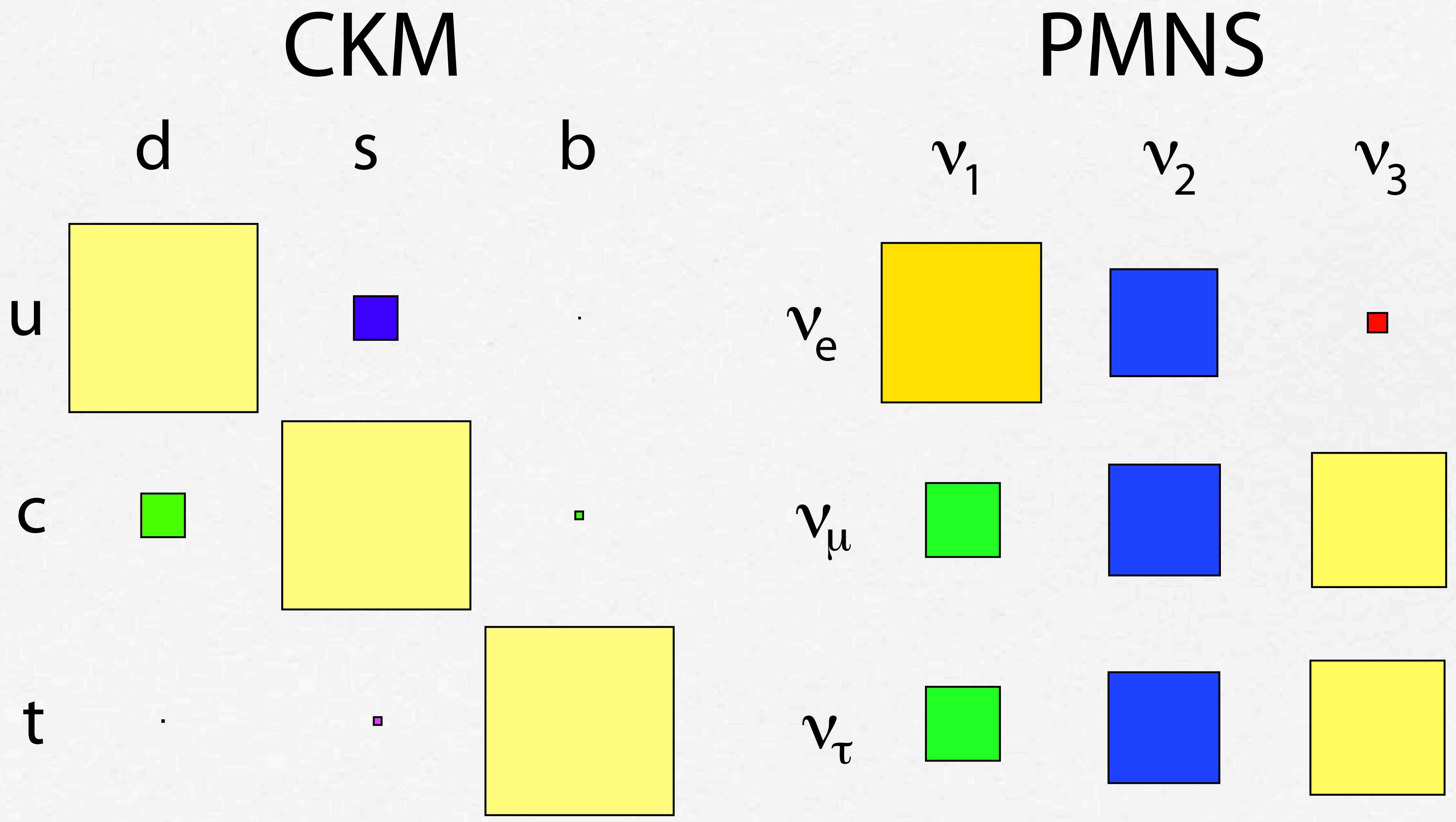


See talk by Valle

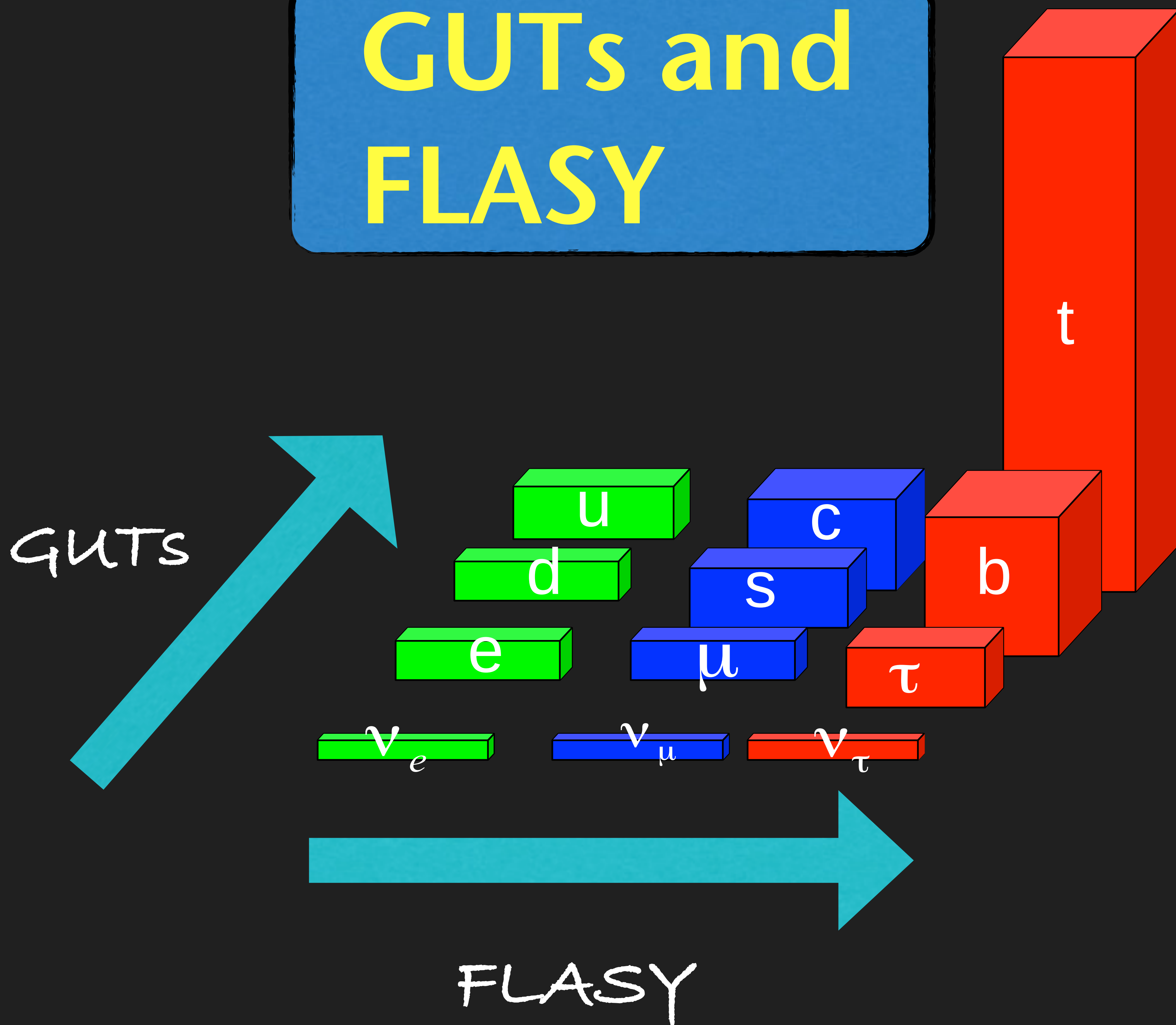
But maybe neutrinos are Dirac...

Aranda, Bonilla, Morisi, Peinado, Valle, arXiv:1307.3553

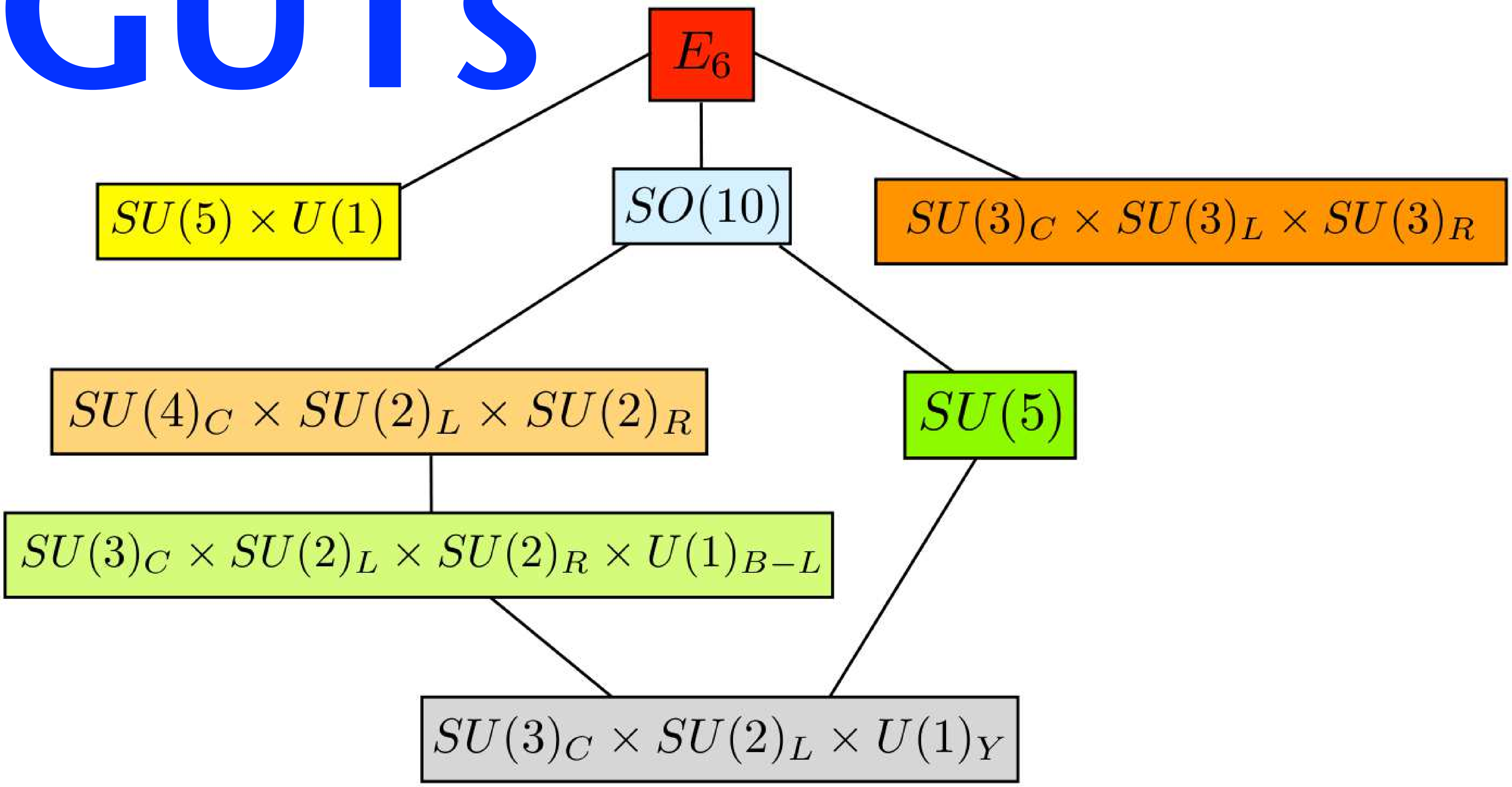
What is the origin of Quark and Lepton Mixing?



GUTs and FLASY



GUTs

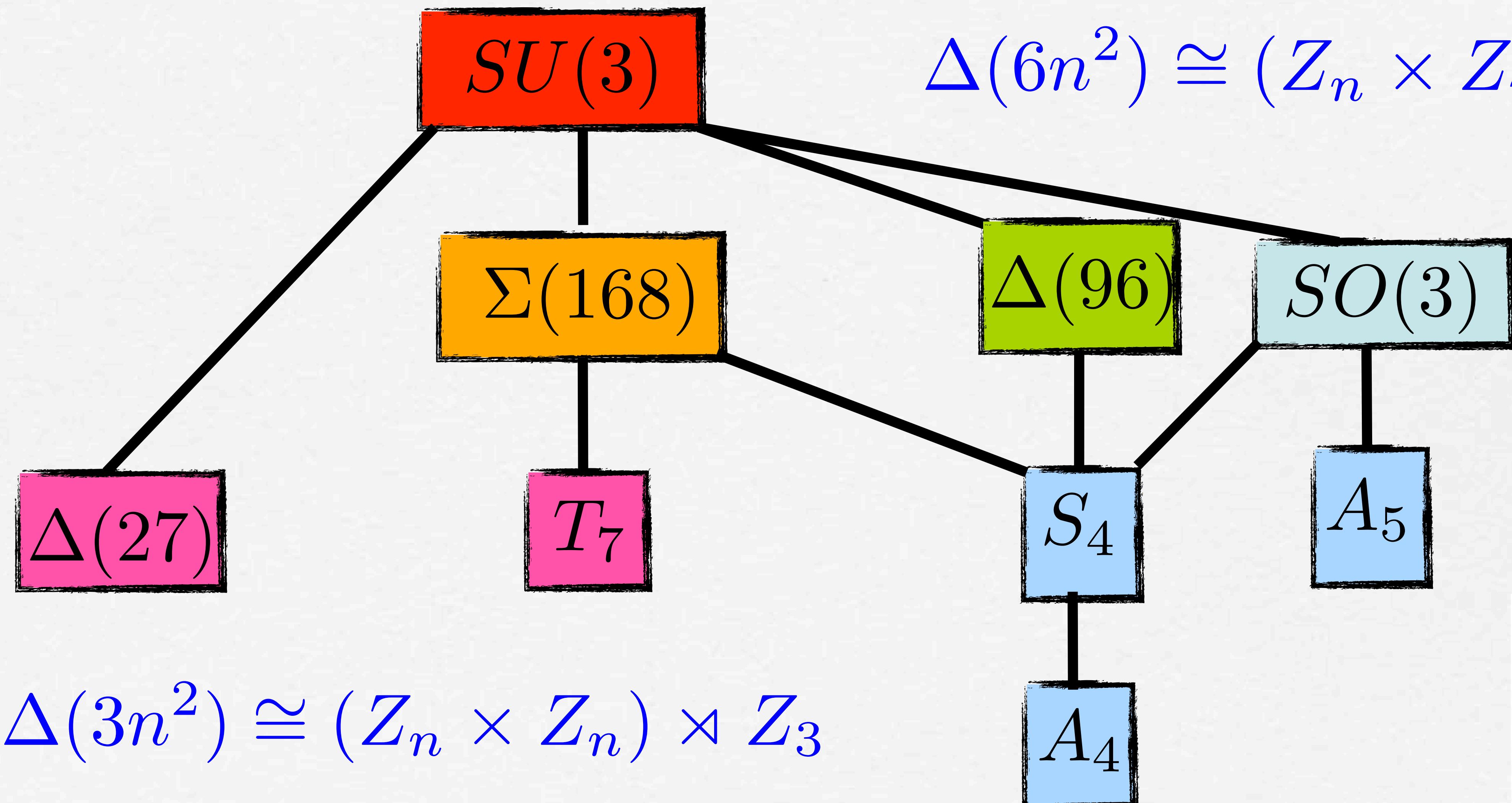


E_6
$SU(5) \times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_C \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Flavour Symmetry (FLASY)

Escobar, Luhn

$$\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$$



$$\Delta(3n^2) \cong (Z_n \times Z_n) \rtimes Z_3$$

Luhn, Nasri, Ramond

Ma, Rajasekaran

The Klein Symmetry

Felix Klein

Phase symmetry of diagonal charged lepton mass matrix

$$T^\dagger(M_e^\dagger M_e)T = M_e^\dagger M_e \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2i\pi/n}$$

Symmetry of Majorana matrix depends on PMNS

$$m_\nu = S^T m_\nu S \qquad m_\nu = U^T m_\nu U$$

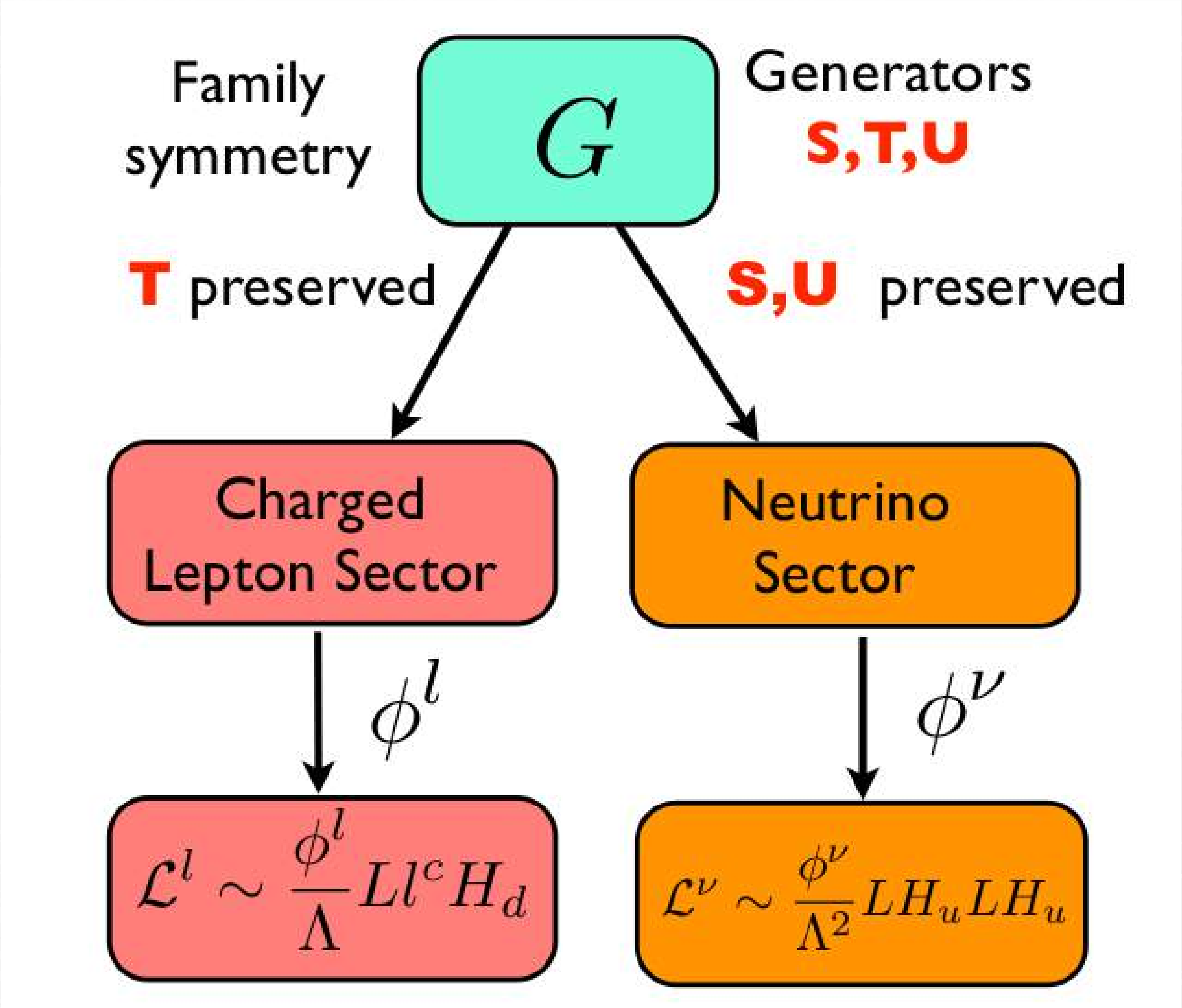
$$\left.\begin{aligned} S &= U^*_{\rm PMNS}\ {\rm diag}(+1,-1,-1)\ U^T_{\rm PMNS} \\ U &= U^*_{\rm PMNS}\ {\rm diag}(-1,+1,-1)\ U^T_{\rm PMNS} \\ SU &= U^*_{\rm PMNS}\ {\rm diag}(-1,-1,+1)\ U^T_{\rm PMNS} \end{aligned}\right\}$$

Klein Symmetry

$$\mathcal{K} = \{1, S, U, SU\}$$

$$Z_2 \times Z_2$$

Direct Models



Klein symmetry S,U and T are each identified as subgroups of some family symmetry

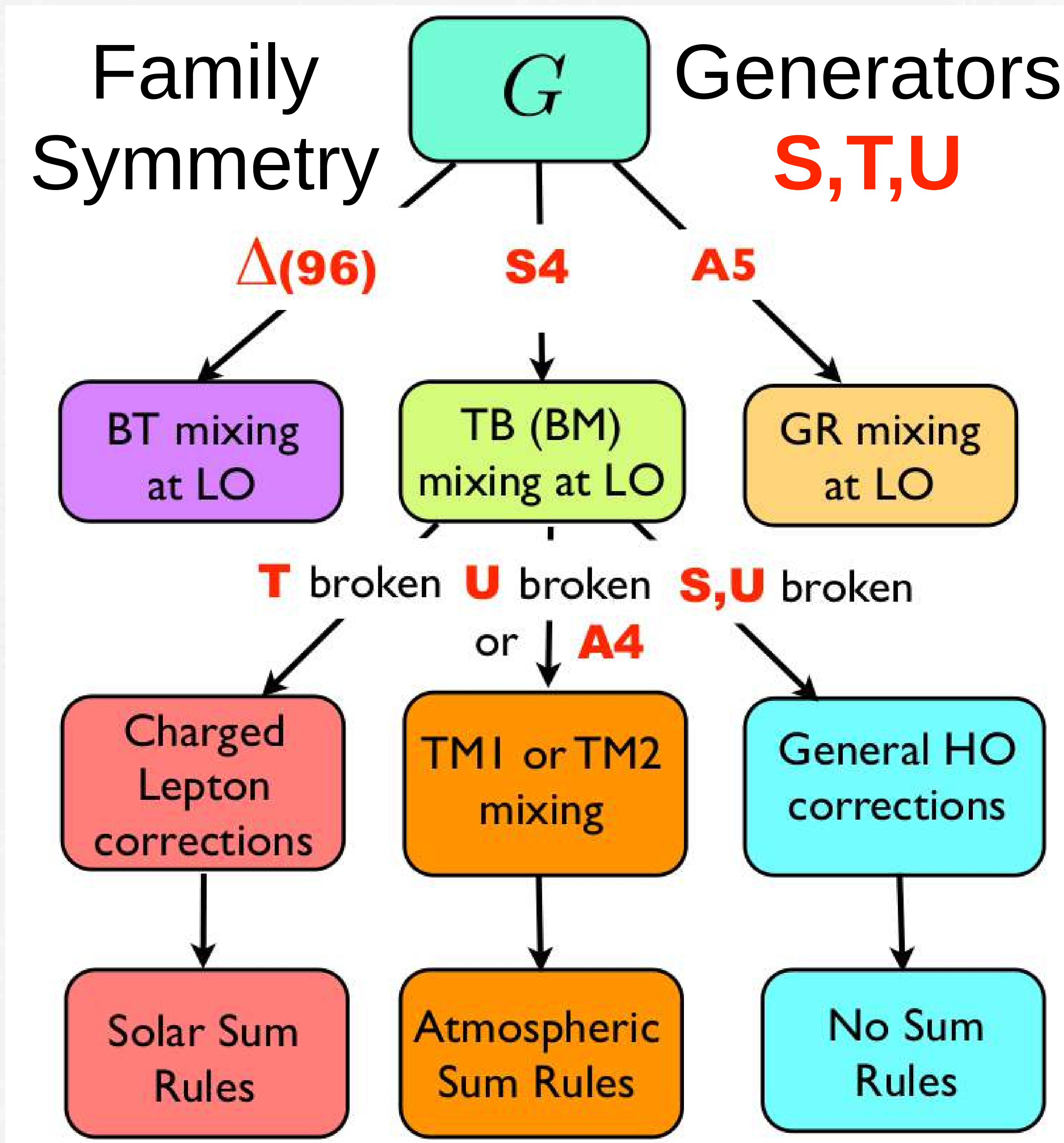
$$\Delta(6n^2)$$

is the only viable symmetry class - predicts zero Dirac CPV but non-zero Majorana phases

Holthausen, Lim, Lindner;
SK, Neder, Stuart;
Lavoura, Ludl;
Fonseca, Grimus

Semi-Direct Models

Klein symmetry and T are partly preserved as subgroups of some family symmetry



Reviews:

S.F.K.,Luhn 1301.1340;

S.F.K.,Merle, Morisi,Shimizu, Tanimoto, 1402.4271

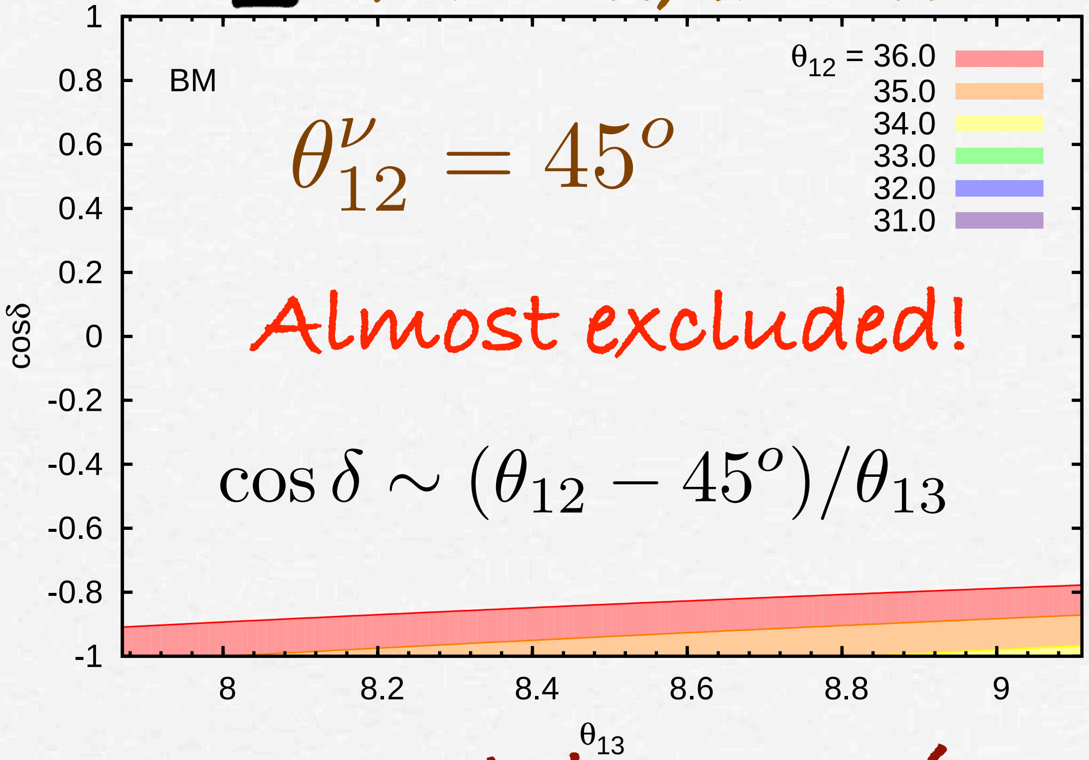
Antusch,S.F.K. 0506297/0508044

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

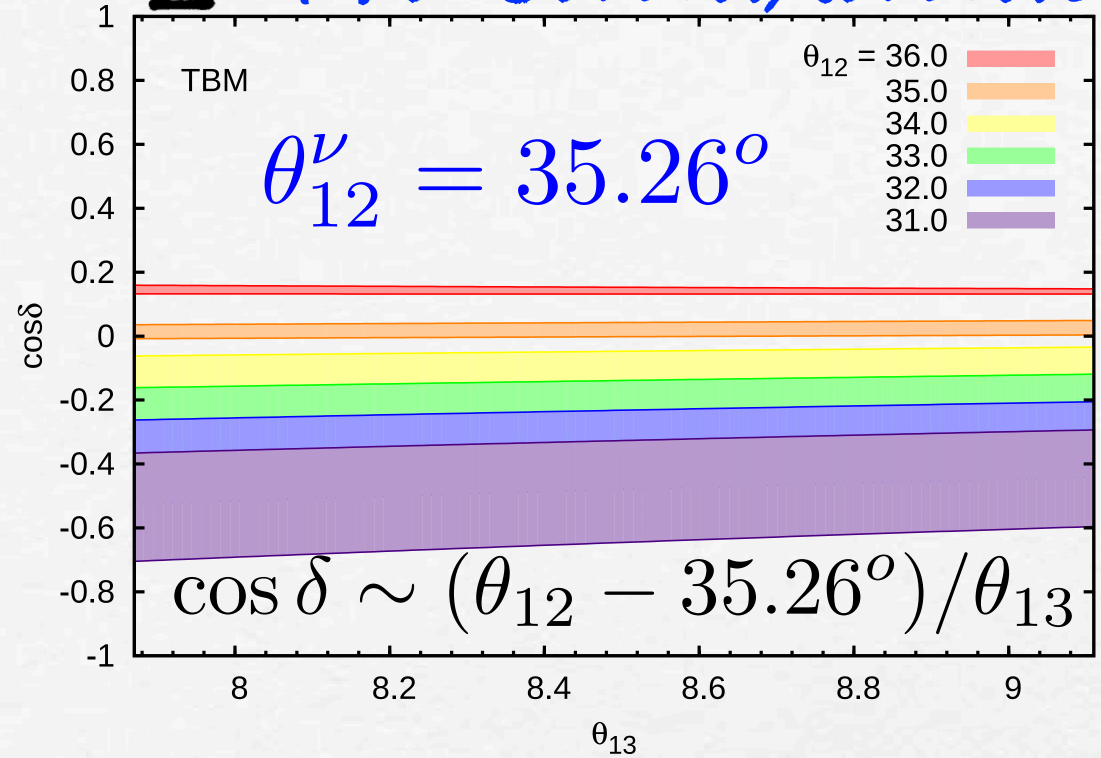
	θ_{13}^{ν}	θ_{23}^{ν}	θ_{12}^{ν}
TB	$0°$	$45°$	$35.3°$
BM	$0°$	$45°$	$45°$
GR	$0°$	$45°$	$31.7°$
BT	$12.2°$	$36.2°$	$36.2°$
TM	$\neq 0°$	$\neq 45°$	$35.3°$

$$\theta_{12} \approx \theta_{12}^{\nu} + \theta_{13} \cos\delta$$

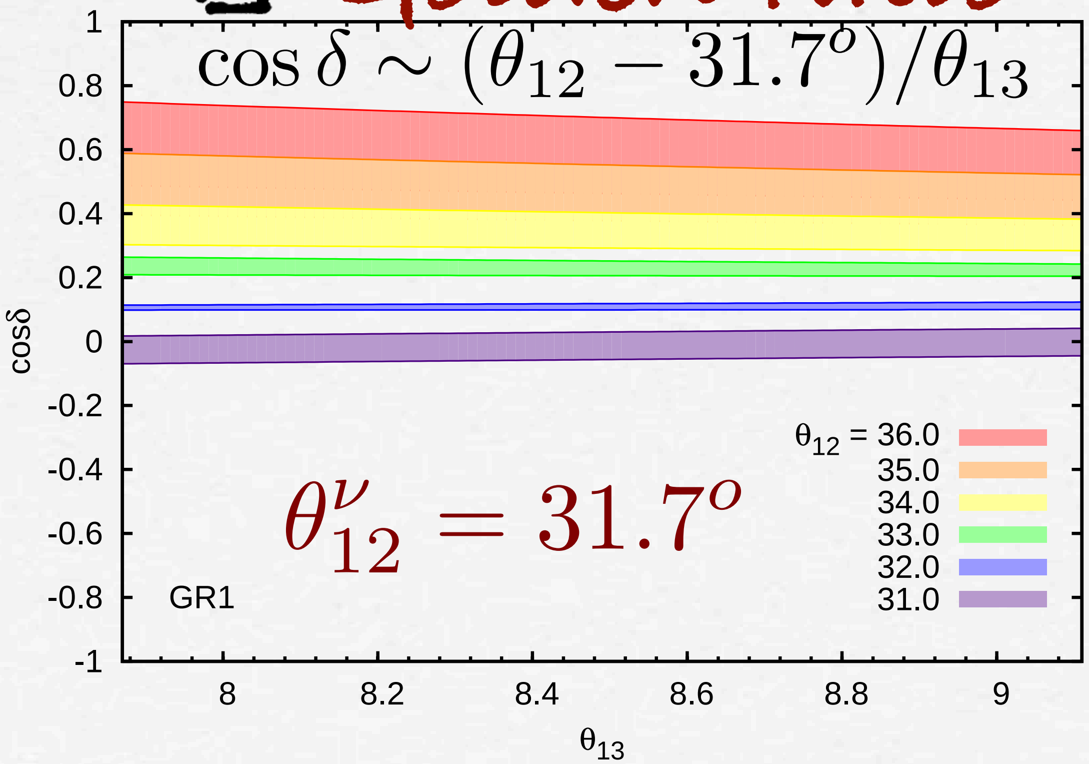
☐ Bimaximal



☐ Tri-bimaximal



☐ Golden ratio



Solar Sum Rule

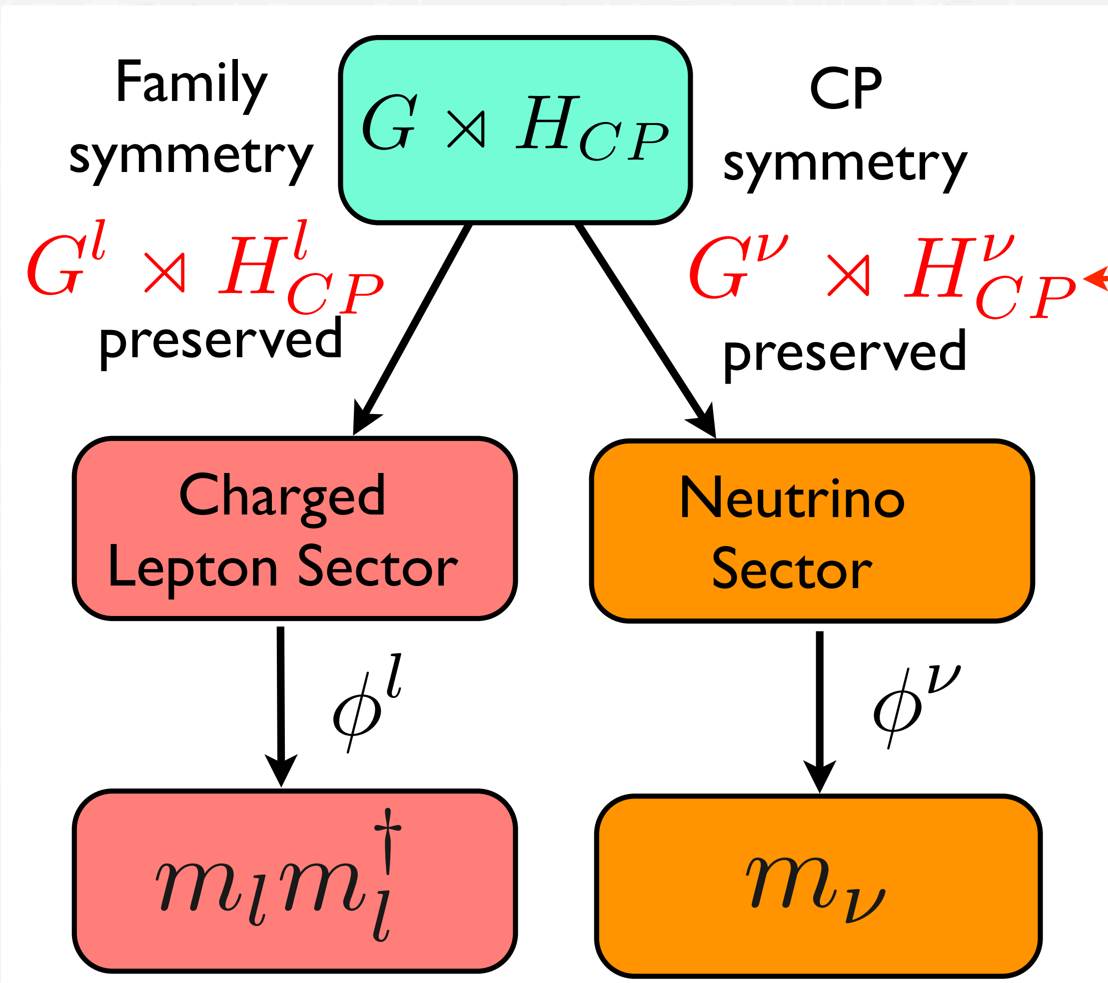
$$\cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - s^{\nu 2}_{12}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12} s_{13}}.$$

$\cos\delta \sim (\theta_{12} - \theta^\nu_{12})/\theta_{13}$ $\quad |\Delta(\cos\delta)| \lesssim 0.1$ for TB

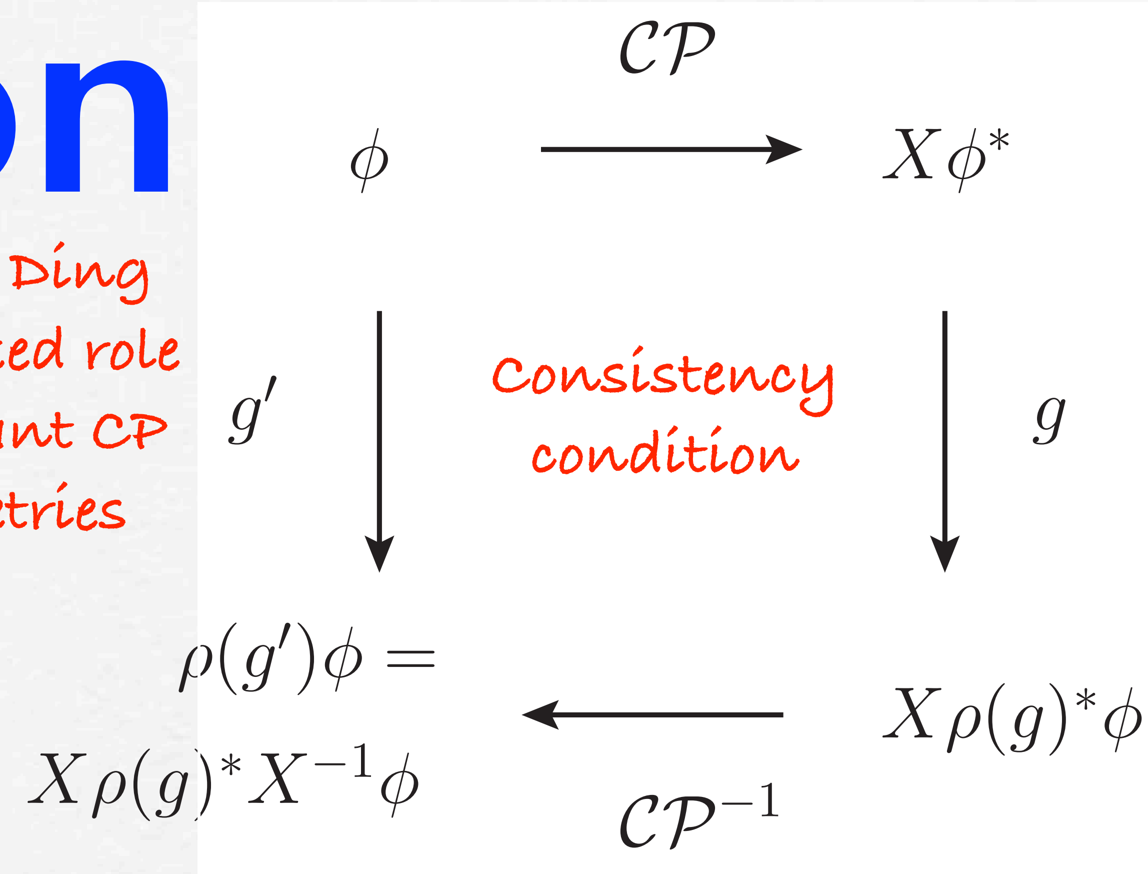
Ballett, SK, Luhn, Pascoli, Schmidt 1410.7573
Girardi, Petcov, Titov 1410.8056

See talk by Girardi for other references

CP violation



Talk by Ding emphasized role of remnant CP symmetries



Equivalently use CP invariants of the Lagrangian

$$I_1 \equiv \mathrm{Tr}\,[H_\nu, H_l]^3 = 0$$

Branco, de Medeiros Varzielas, S.F.K., 1502.03105 1505.06165

$$H_\nu \equiv m_\nu m_\nu^\dagger \text{ and } H_l \equiv m_l m_l^\dagger$$

- S_4 and A_4 models with CP symmetry are constructed, all the possible cases following from the model-independent analysis can be realized. **Dirac CP phase is predicted to be trivial or maximal.**

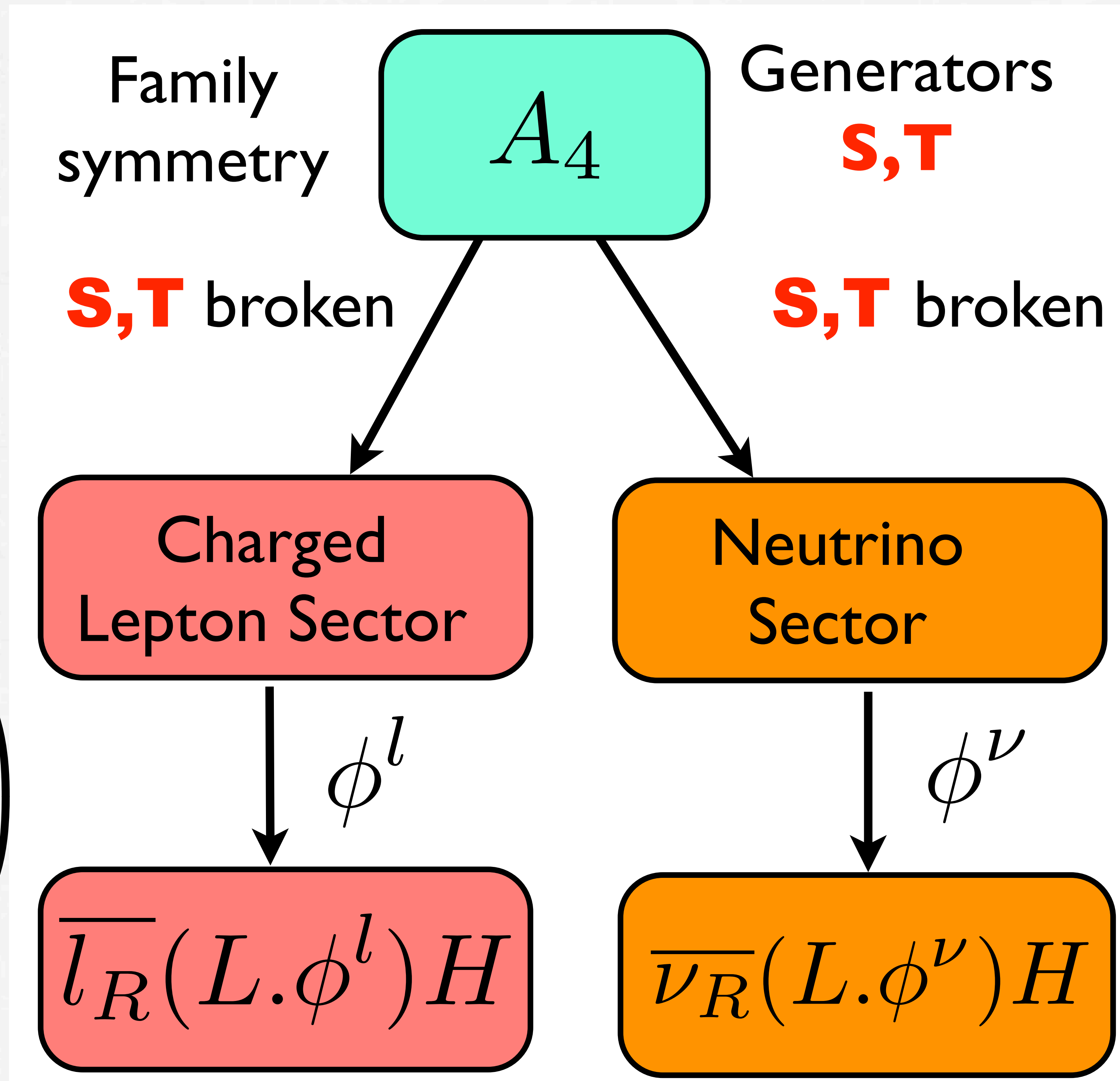
Feruglio, Hagedorn; Holthausen, Lindner Schmidt; Ding, SFK, Luhn, Stuart; Nishi, Xing; Hagedorn, Meroni, Molinaro; Ding, SFK, Neder; see also talk by Chen

Indirect Models

Main advantage: highly predictive

Diagonal alignments

$$\begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$



Family symmetry is fully broken in each sector by orthogonal A_4 alignments

CSD(n) alignments Motivated by neutrino data

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

$$n = 3, 4$$

Minimal Predictive Seesaw models

S.F.K. 1304.6264,1305.4846
Björkeroth and S.F.K. 1412.6996

Two right-handed neutrinos ("minimal")

$M_1 = M_{\rm atm}$ and $M_2 = M_{\rm sol}$

$$H(L.\phi_{\rm atm})N^c_{\rm atm} + H(L.\phi_{\rm sol})N^c_{\rm sol} + M_{\rm atm}N^c_{\rm atm}N^c_{\rm atm} + M_{\rm sol}N^c_{\rm sol}N^c_{\rm sol}$$

$$\langle\phi_{\rm atm}\rangle = v_{\rm atm}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\phi_{\rm sol}\rangle = v_{\rm sol}\begin{pmatrix}1\\n\\n-2\end{pmatrix}$$

CSD(n) ("predictive")

$$\lambda^\nu = \begin{pmatrix}0 & b\\ a & nb\\ a & (n-2)b\end{pmatrix}, \quad M^c = \begin{pmatrix}M_1 & 0\\ 0 & M_2\end{pmatrix}$$

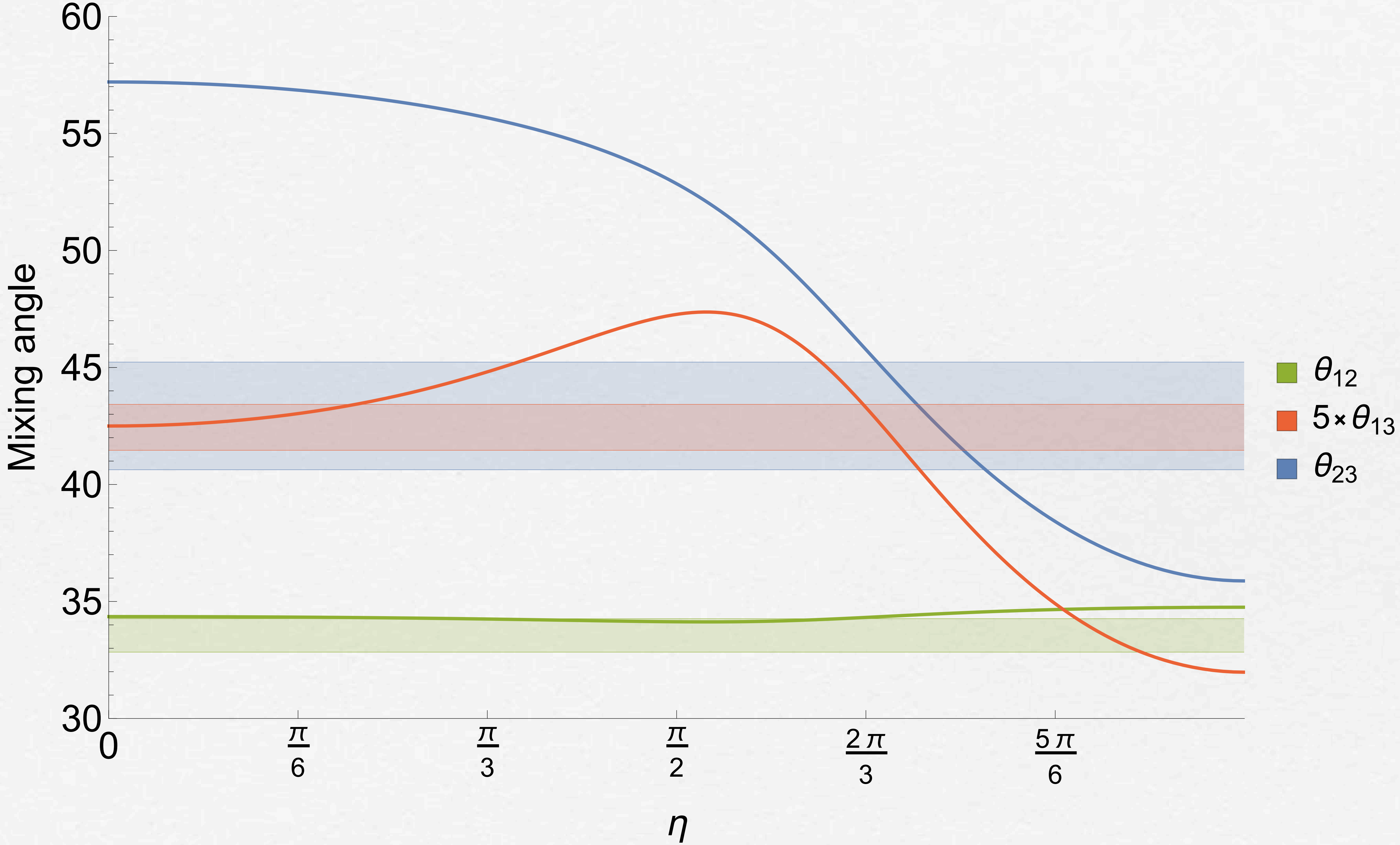
Seesaw matrices

PMNS fixed by one free phase parameter

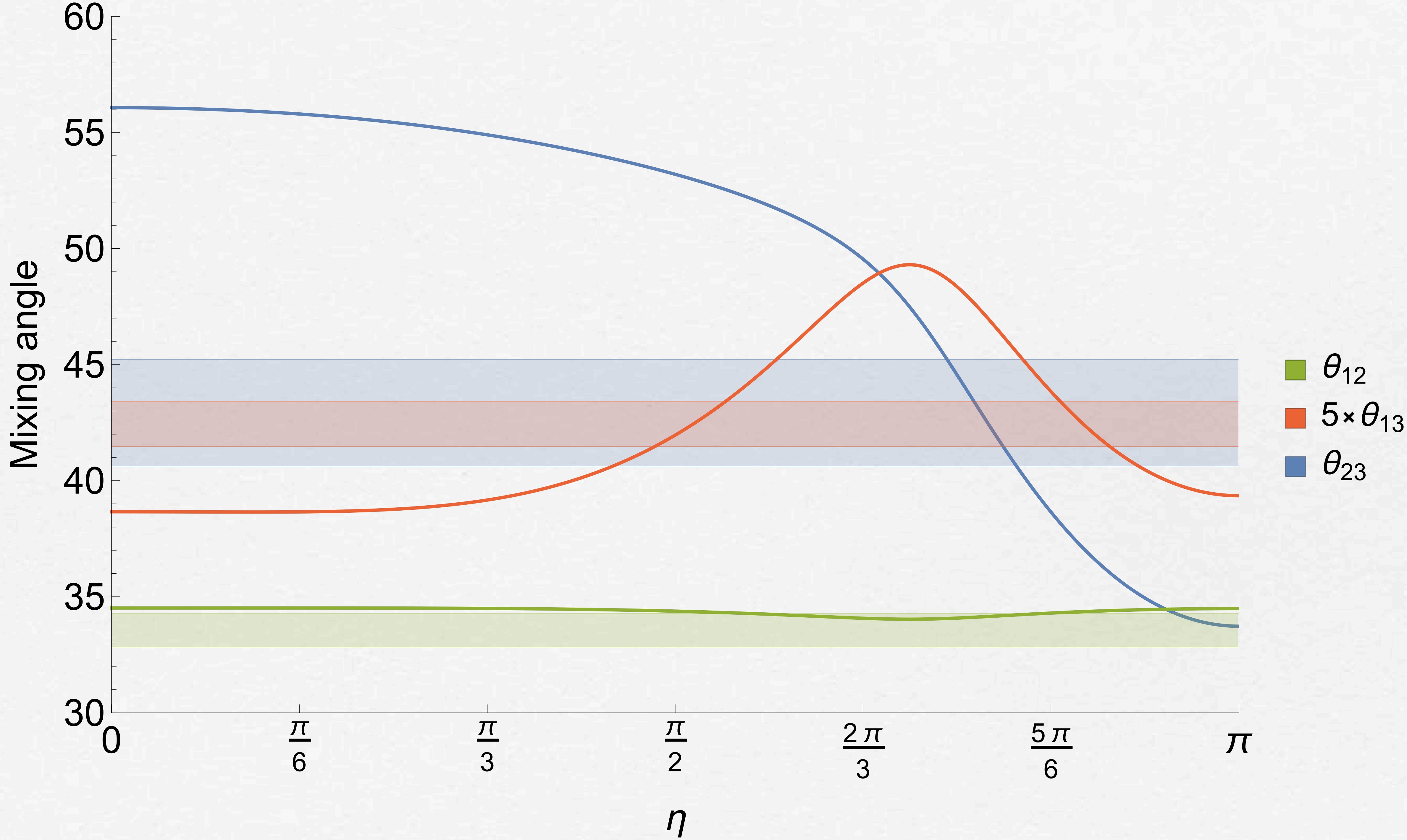
$$m^\nu_{(n)} = m_a\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + m_b e^{i\eta}\begin{pmatrix}1 & n & n-2\\ n & n^2 & n(n-2)\\ n-2 & n(n-2) & (n-2)^2\end{pmatrix}$$

Neutrino mass matrix

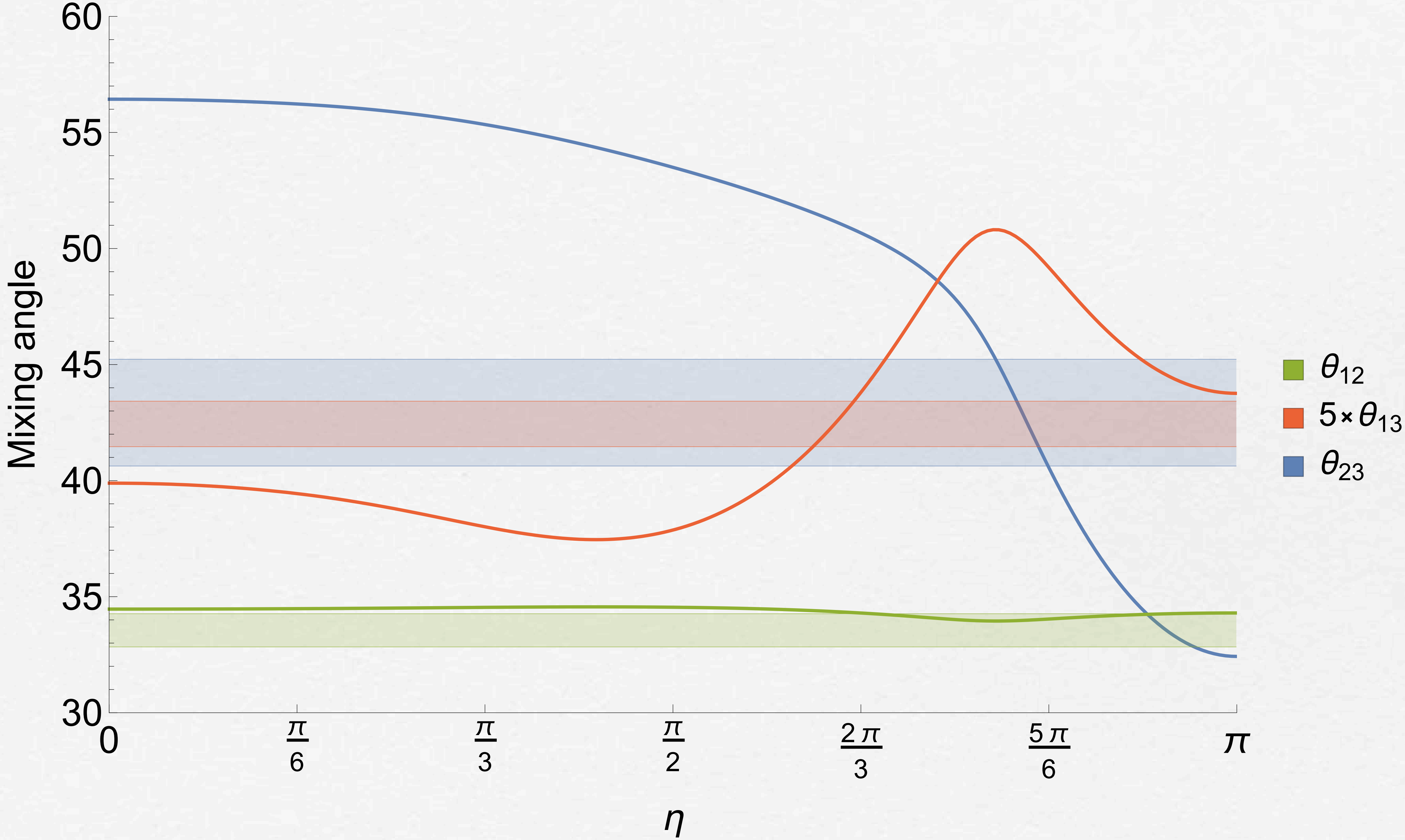
m_a and m_b are fixed by the neutrino mass squared differences



(a) CSD(3)

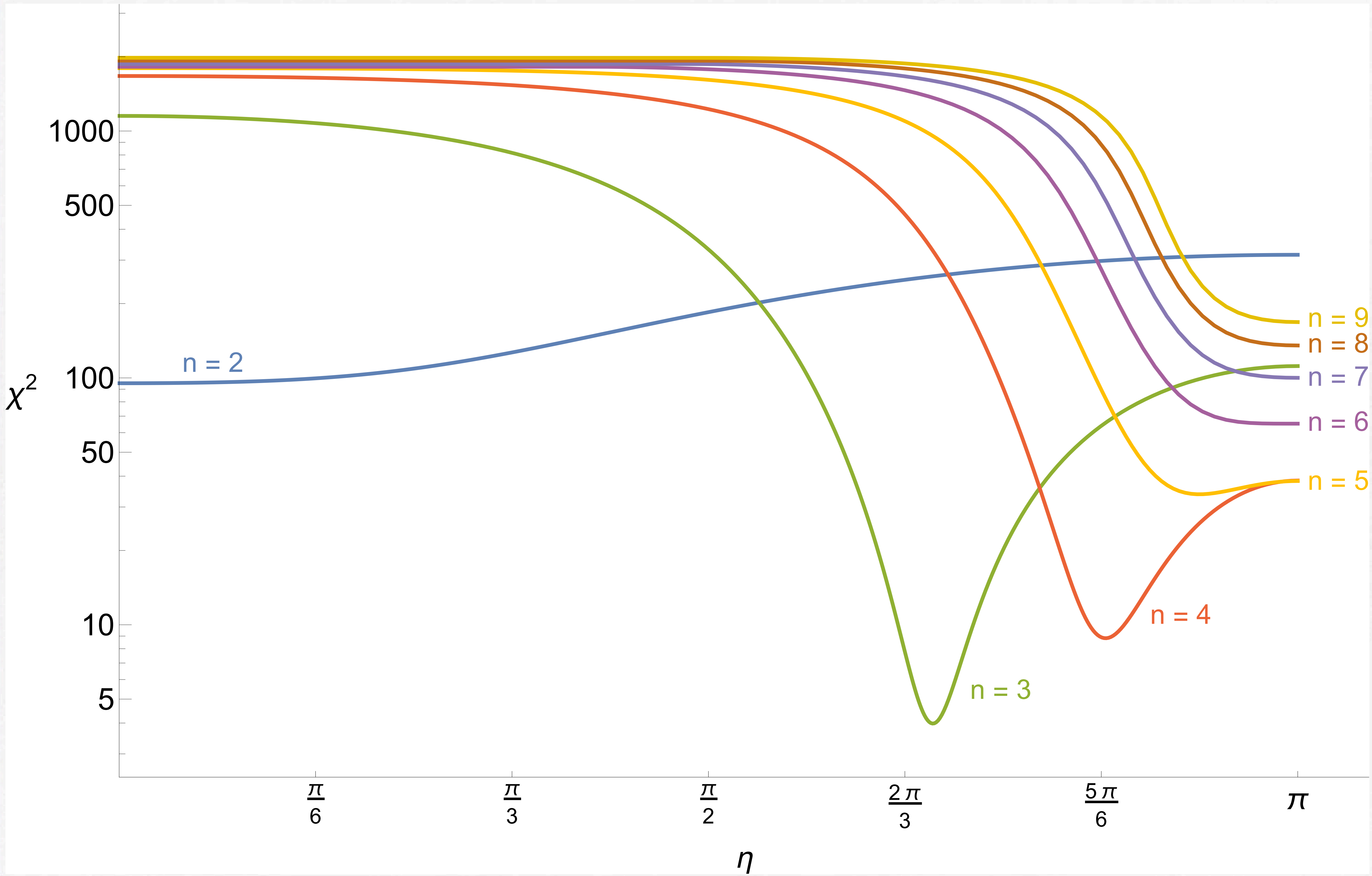

60
55
50
45
40
35
30
Mixing angle
0
π/6
π/3
π/2
2π/3
5π/6
π
η
θ_{12}
$5\times\theta_{13}$
θ_{23}

(b) CSD(4)



Mixing angle
60
55
50
45
40
35
30
0
π/6
π/3
π/2
2π/3
5π/6
π
η
θ12
5×θ13
θ23

(c) CSD(5)



1000
500
100
50
10
5
χ²
n = 2
n = 3
n = 4
n = 5
n = 6
n = 7
n = 8
n = 9
π/6
π/3
π/2
2π/3
5π/6
π
η

Minimum χ^2 predictions

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	$\|\delta_{CP}\|$ (°)	m_2 (meV)	m_3 (meV)	χ^2
1	24.8	2.89	3.14	35.3	0	45.0	0	8.66	49.6	485
2	19.7	3.66	0	34.5	7.65	56.0	0	8.85	48.8	95.1
3	27.3	2.62	2.17	34.4	8.39	44.5	92.2	8.69	49.5	3.98
4	36.6	1.95	2.63	34.3	8.72	38.4	120	8.61	49.8	8.82
5	45.9	1.55	2.88	34.2	9.03	34.4	142	8.53	50.0	33.8
6	55.0	1.29	3.13	34.2	9.30	31.6	179	8.46	50.2	65.2
7	63.0	1.12	3.14	34.1	9.68	31.0	180	8.35	50.6	100
8	71.0	0.984	3.14	34.0	9.96	30.6	180	8.25	50.8	135
9	79.0	0.880	3.14	33.9	10.2	30.3	180	8.17	51.0	168

Note: if eta is positive then delta_CP is negative (consistent with the -90 deg hint!)

Björkeroth, de Anda, de Medeiros Varzielas and S.F.K. 1505.05504

Leptogenesis in Minimal Predictive Seesaw models

3 inputs 6 outputs (incl. CP phase)

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	δ_{CP} (°)	m_2 (meV)	m_3 (meV)	χ^2
3	27.3	2.62	2.17	34.4	8.39	44.5	-92.2	8.69	49.5	3.98
4	36.6	1.95	2.63	34.3	8.72	38.4	-120	8.61	49.8	8.82
5	45.9	1.55	2.88	34.2	9.03	34.4	-142	8.53	50.0	33.8

BAU $$Y_B = \frac{675}{31\pi^5 g_*}\frac{M_1 m_b}{v_u^2}\eta_{1,\mu}(n-1)^2\sin\eta$$

leptogenesis phase

Washouts depend on m_a

$$\eta_{1,\mu} = (0.0236, 0.0166, 0.0126)$$

$$\text{CSD}(3): \quad Y_B \sim 2.2\times 10^{-11}\left[\frac{M_1}{10^{10}\text{ GeV}}\right] \quad \Rightarrow \quad M_1 \sim 4.0\times 10^{10}\text{ GeV}$$

$$\text{CSD}(4): \quad Y_B \sim 1.5\times 10^{-11}\left[\frac{M_1}{10^{10}\text{ GeV}}\right] \quad \Rightarrow \quad M_1 \sim 5.8\times 10^{10}\text{ GeV}$$

$$\text{CSD}(5): \quad Y_B \sim 0.86\times 10^{-11}\left[\frac{M_1}{10^{10}\text{ GeV}}\right] \quad \Rightarrow \quad M_1 \sim 10\times 10^{10}\text{ GeV}$$

Note the correlations:

- Positive BAU
- positive lepto phase
- negative CP phase

Björkeroth, de Anda, de Medieoros Varzielas and S.F.K. 1503.03306

Towards a complete A_4xSU(5) SUSY GUT

See talk by de Anda for full discussion

Quite complete model!

- Renormalisable at GUT scale, SU(5) breaking potential, spontaneously broken CP.
- The MSSM is reproduced with R-parity emerging from a discrete Z_4^R .
- Doublet-triplet splitting is achieved through the Missing Partner mechanism.
- mu term is generated at the correct scale.
- Proton decay is sufficiently suppressed.
- It solves the strong CP problem through the Nelson-Barr mechanism .
- Explains quark mass hierarchies, mixing angles and the CP phase.
- Reproduces minimal predictive seesaw model via A4 vacuum alignments with CSD(3).

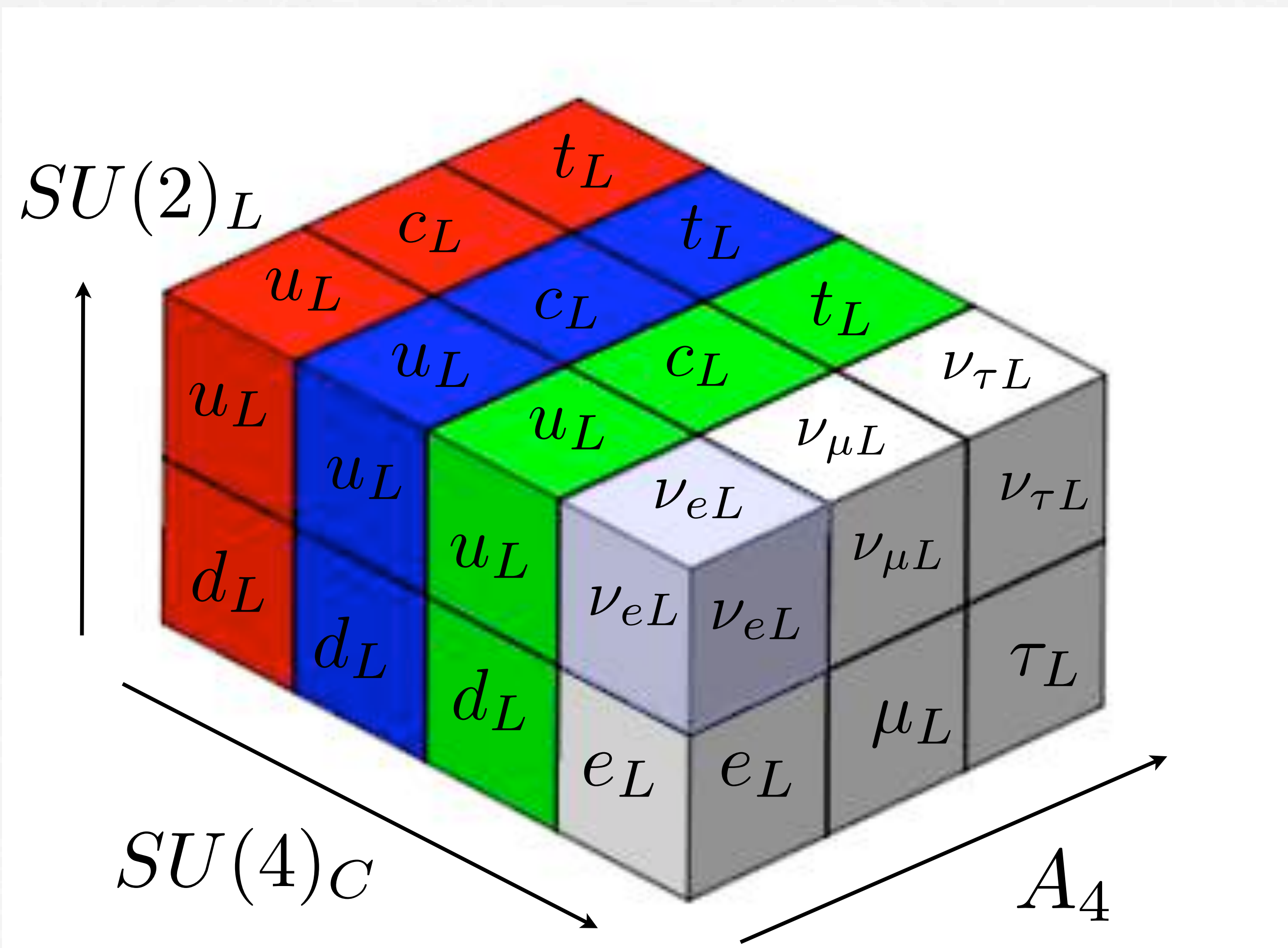
S.F.K. 1406.7005

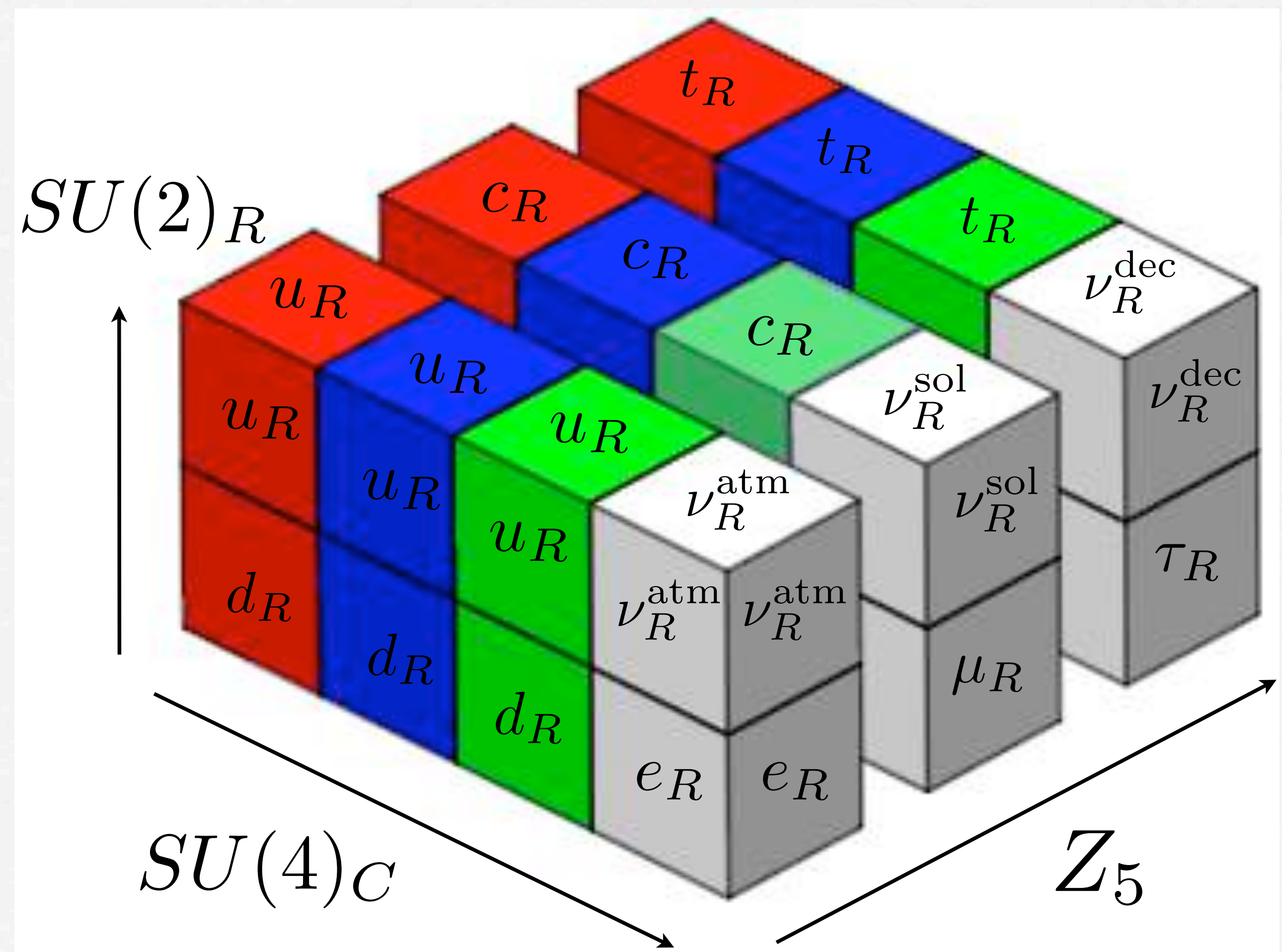
A to Z of Flavour with Pati-Salam

$$A_4 \times Z_5 \times SU(4)_C \times SU(2)_L \times SU(2)_R$$

Left-handed quarks and leptons triplets of A_4

Right-handed quarks and leptons distinguished by Z_5





S.F.K. 1406.7005

A to Z of Flavour with Pati-Salam

$$Y^u = Y^\nu = \begin{pmatrix} 0 & be^{-i3\pi/5} & \epsilon c \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & c \end{pmatrix} \quad Y^d = \begin{pmatrix} y_d^0 e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & y_s^0 e^{-i2\pi/5} & Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 + Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

$$M_R \approx \begin{pmatrix} M_1 e^{8i\pi/5} & 0 & 0 \\ 0 & M_2 e^{4i\pi/5} & 0 \\ 0 & 0 & M_3 \end{pmatrix} \quad Y^e = \begin{pmatrix} -(y_d^0/3) e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & -4.5 y_s^0 e^{-i2\pi/5} & -3Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 - 3Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

SO(10)-like diagonal RHN masses $M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$

Physical neutrino masses in a normal hierarchy CSD(4)

Explains the Cabibbo angle $\theta_C \approx 1/4$ or $\theta_C \approx 14°$

All CP phases are fifth roots of unity due to Z_5

A to Z of Flavour with Pati-Salam

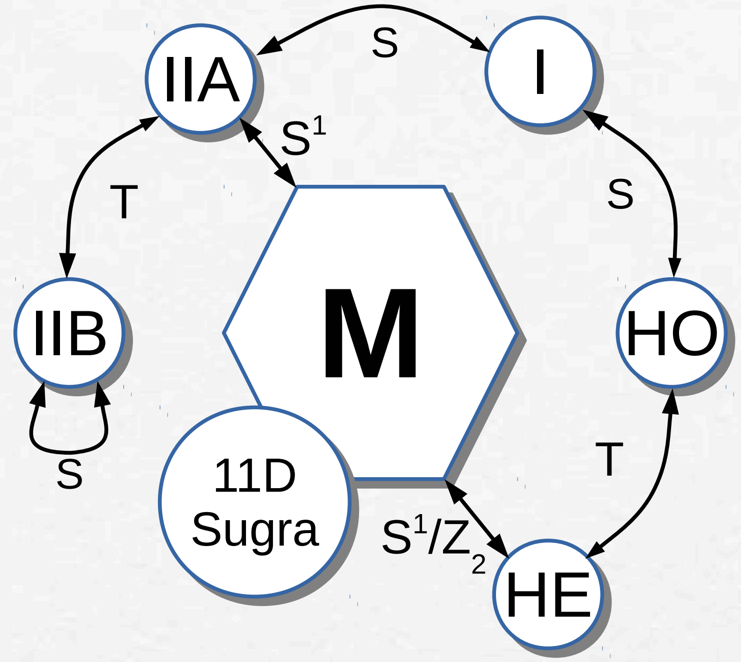
15 inputs　　20 outputs　　$\chi^2 = 12.7$

Björkeroth, S.F.K. (to appear)

$\tan\beta$	Input		Output			
10	a	4.528×10^{-6}	y_u	$2.88\ \times10^{-6}$	θ^q_{12}	$13.027°$
	b	3.446×10^{-4}	y_c	$1.41\ \times10^{-3}$	θ^q_{13}	$0.1802°$
	c	5.229×10^{-1}	y_t	$5.20\ \times10^{-1}$	θ^q_{23}	$2.054°$
	y^0_d	5.690×10^{-5}	y_d	$4.85\ \times10^{-6}$	δ^q	$69.21°$
	y^0_s	8.864×10^{-4}	y_s	$9.60\ \times10^{-5}$		
	y^0_b	-7.345×10^{-2}	y_b	$7.38\ \times10^{-3}$		
	M_1	1.793×10^{4}	Δm^2_{21}	$7.50\ \times10^{-5}$ eV^2	θ^l_{12}	$34.3°$
	M_2	1.793×10^{9}	Δm^2_{31}	$2.46\ \times10^{-3}$ eV^2	θ^l_{13}	$8.67°$
	M_3	2.436×10^{16}			θ^l_{23}	$45.8°$
	ϵ	-2.221×10^{-3}	y_e	$1.98\ \times10^{-6}$	δ^l	$-86.7°$
	A	11.5	y_μ	$4.19\ \times10^{-4}$		
	B	6.93	y_τ	$7.15\ \times10^{-3}$		
	C	46.2				
	x	4.76	$m_1 \approx 0$			

PMNS predictions

M-theory GUTs



- M-theory → 11d SUGRA → 4d N=1 SUGRA
- Compactified 7d → G_2 manifold

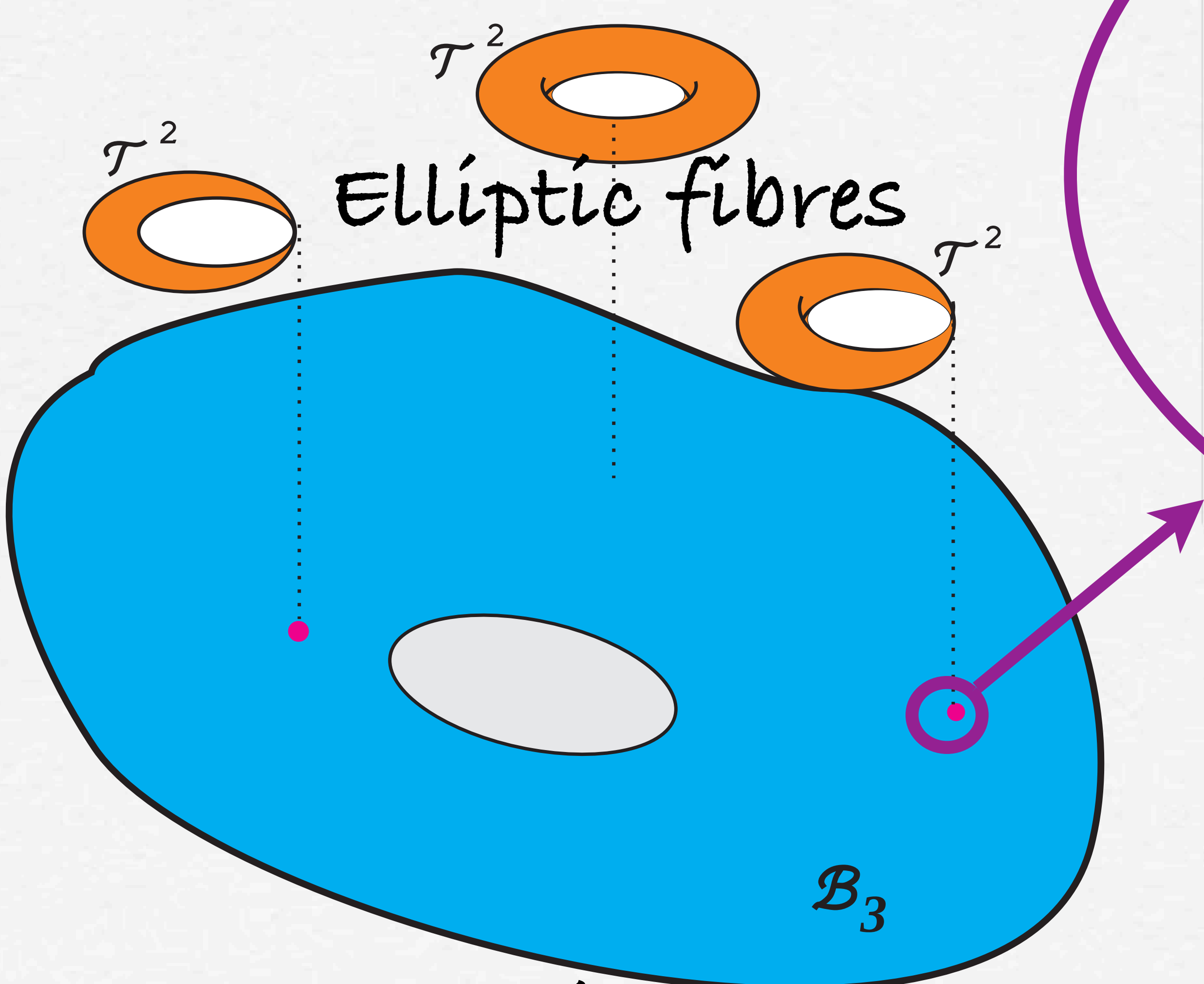
Witten, Acharya, Kane,...

- Gauge fields on dominant volume 3d submanifold
- SU(5) GUT
- SO(10) GUT
- main prediction: extra 16+16bar at TeV scale

Acharya, Bozek, M.C.Romao, S.F.K. and Pongkitivanichkul 1502.01727

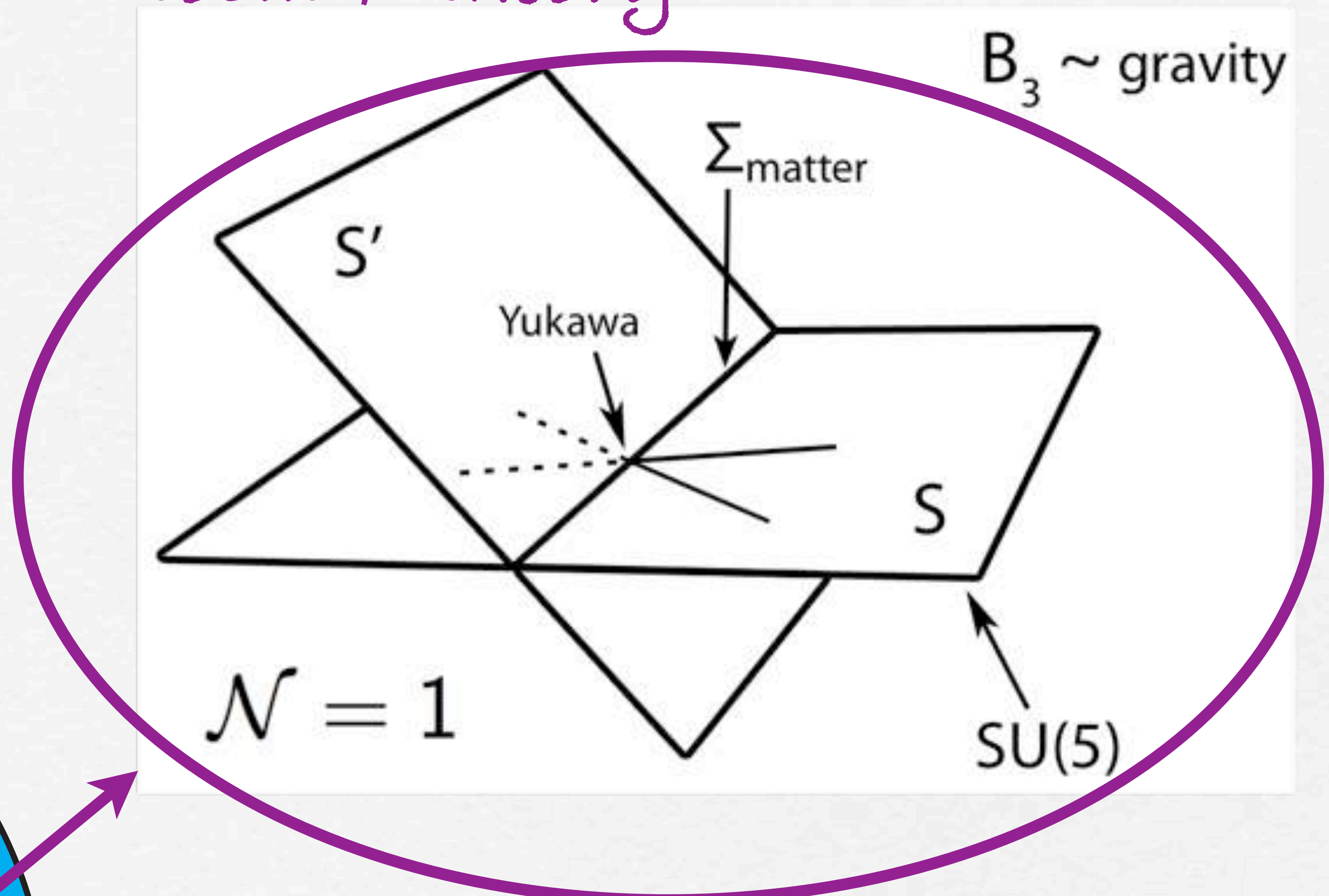
F-theory GUTs

G.K. Leontaris,
``Aspects of F-Theory GUTs,''
PoS CORFU {\bf 2011} (2011) 095
[arXiv:1203.6277 [hep-th]].



Type IIB string theory

local F-theory

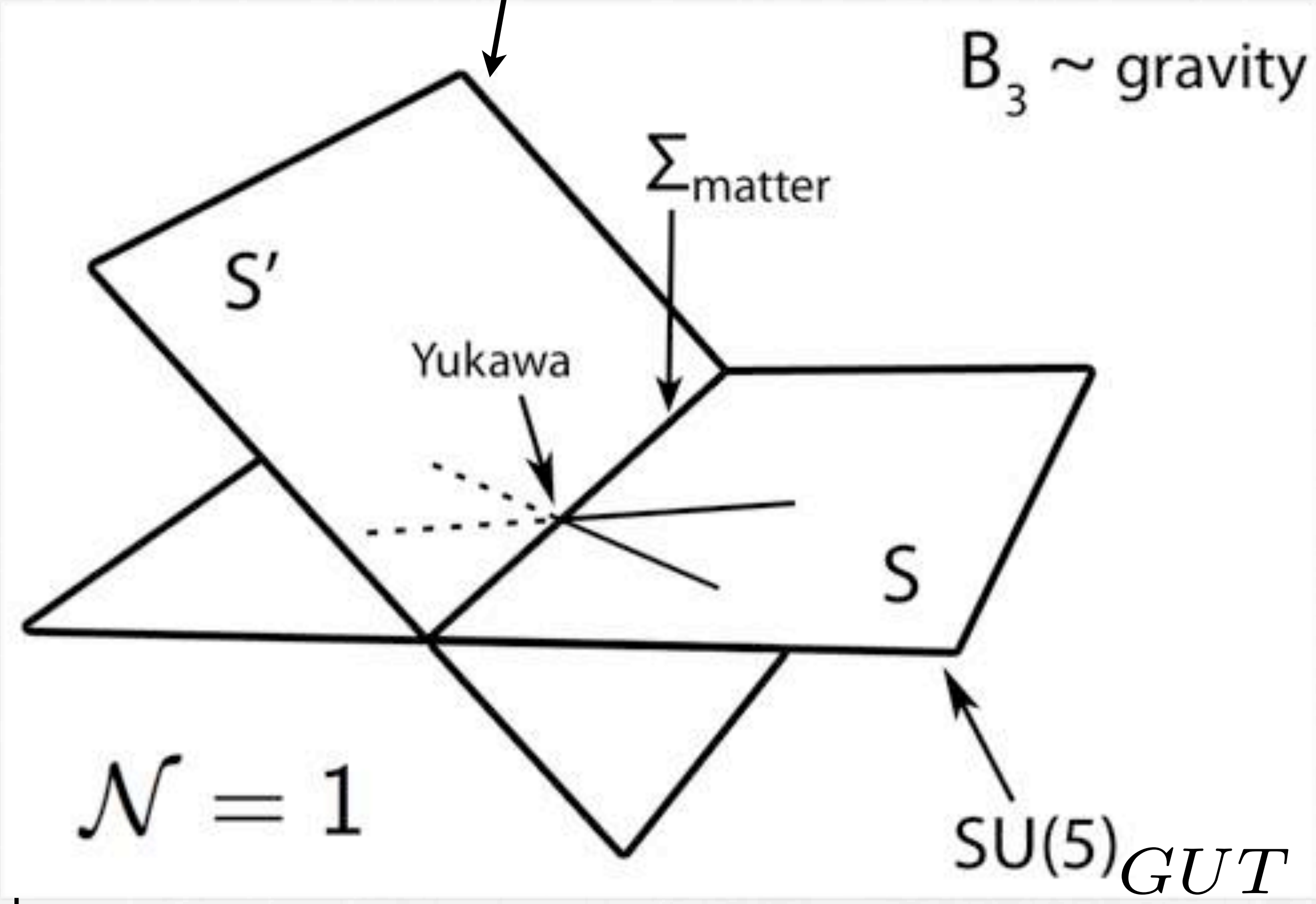


dim.	internal dim.	feature
10	$6 = \dim(B_3)$	gravity
8	$4 = \dim(S)$	gauge fields
6	$2 = \dim(S \cap S')$	matter
4	$0 = \dim(S \cap S' \cap S'')$	interactions

F-theory SU(5)

G.K.Leontaris, ``The origin of discrete symmetries in F-theory models,'' arXiv:1501.06499 [hep-th].

$SU(5)_\perp$



$$E_8 \to SU(5)_{\rm GUT} \times SU(5)_\perp$$

Conventionally — Heckman and Vafa

$$SU(5)_\perp \to U(1)_\perp^4$$

New possibilities — Antoniadis and Leontaris

$$SU(5)_\perp \to S_4 \times U(1)_\perp$$
$$SU(5)_\perp \to A_4 \times U(1)_\perp$$
$$SU(5)_\perp \to D_4 \times U(1)_\perp$$

S_4, A_4, D_4

Karozas, S.F.K.,Leontaris and Meadowcroft, 1505.00937, 1406.6290

Identified as discrete family symmetries

Dimou, Hagedorn, S.F.K., Luhn (to appear)

Testing SUSY flavour models

semi direct model	Matter fields				Higgs fields			Flavon fields								
	T_3	T	F	ν^c	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$	ϕ^u_2	$\widetilde{\phi}^u_2$	ϕ^d_3	$\widetilde{\phi}^d_3$	ϕ^d_2	$\phi^\nu_{3'}$	ϕ^ν_2	ϕ^ν_1	η
$SU(5)$	**10**	**10**	$\overline{\mathbf{5}}$	**1**	**5**	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
S_4	**1**	**2**	**3**	**3**	**1**	**1**	**1**	**2**	**2**	**3**	**3**	**2**	$\mathbf{3'}$	**2**	**1**	**1**
$U(1)$	0	5	4	−4	0	0	1	−10	0	−4	−11	1	8	8	8	7

$$\delta^u_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{LR} \sim \begin{pmatrix} \lambda^8 & 0 & \lambda^7 \\ 0 & \lambda^4 & \lambda^6 \\ 0 & \lambda^7 & 1 \end{pmatrix},$$

Mimics Minimal Flavour Violation (MFV) due to high powers of $\lambda \approx 0.22$

$$\delta^d_{LL} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda^2 \end{pmatrix},$$

$$\delta^e_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{RR} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}.$$

$$(\delta^f_{LL})_{ij} = \frac{(m^2_{\tilde{f}_{LL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LL}}$$

$$(\delta^f_{RR})_{ij} = \frac{(m^2_{\tilde{f}_{RR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RR}}$$

$$(\delta^f_{LR})_{ij} = \frac{(m^2_{\tilde{f}_{LR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LR}}$$

Dimou, Hagedorn, S.F.K., Luhn (to appear)

Testing SUSY flavour models

B_s

Parameter	Our naive expectation	Our range	Exp. bound
$\lvert(\delta^d_{LL})_{23}\rvert$	$\mathcal{O}\left(\frac{2\,R_q\,\eta\,\lambda^2}{1+6.5\,x}\rvert_{b_{01}=b_{02}} \approx 4\times 10^{-3}\right)$	$\mathcal{O}(10^{-5}, 5\times 10^{-2})$	$\mathcal{O}(10^{-2}, 10^{-1})$
$\lvert(\delta^d_{RR})_{23}\rvert$	$\mathcal{O}\left(\frac{\lambda^4}{1+6.1\,x} \approx 4\times 10^{-4}\right)$	$\mathcal{O}(10^{-5}, 10^{-2})$	$\mathcal{O}(10^{-1}, 1)$
$\lvert(\delta^d_{LR})_{23}\rvert$	$\mathcal{O}\left(\frac{v_d\,A_0\,\lambda^4}{m_0^2(1+6\,x)} \approx 10^{-6}\right)$	$\mathcal{O}(10^{-9}, 5\times 10^{-4})$	$\mathcal{O}(10^{-3}, 10^{-2})$
$\lvert(\delta^d_{RL})_{23}\rvert$	$\mathcal{O}\left(\frac{v_d\,A_0\,\lambda^6}{m_0^2(1+6\,x)} \approx 5\times 10^{-8}\right)$	$\mathcal{O}(10^{-11}, 5\times 10^{-6})$	$\mathcal{O}(10^{-2})$

$\mu \to e\gamma$

Parameter	Our naive expectation	Our range	Exp. bound
$\lvert(\delta^e_{LL})_{12}\rvert$	$\mathcal{O}\left(\frac{1+\frac{2\,R_l\,\eta_N}{1+0.5\,x}}{1+0.5\,x}\lambda^4 \approx 10^{-3}\right)$	$\mathcal{O}(10^{-6}, 5\times 10^{-2})$	$\mathcal{O}(10^{-5}, 10^{-4})$
$\lvert(\delta^e_{LL})_{23,13}\rvert$			$\mathcal{O}(10^{-2}, 10^{-1})$
$\lvert(\delta^e_{RR})_{12}\rvert$	$\mathcal{O}\left(\frac{2}{3}\,\frac{\lambda^3}{1+0.15\,x} \approx 6\times 10^{-3}\right)$	$\mathcal{O}(10^{-5}, 5\times 10^{-2})$	$\mathcal{O}(10^{-3}, 10^{-2})$
$\lvert(\delta^e_{RR})_{23}\rvert$	$\mathcal{O}\left(3\,\frac{\lambda^2}{1+0.15\,x} \approx 10^{-1}\right)$	$\mathcal{O}(10^{-3}, 10^{-1})$	$\mathcal{O}(10^{-1}, 1)$

$\tau \to \mu\gamma$

Conclusions

- GUT x Discrete Family Symmetry very predictive framework
- Direct models: Klein and T from Delta($6n^2$), zero Dirac phase
- Semi-direct models: partial symmetry S or SU, allows smaller groups, lepton mixing sum rules, possible CP phase predictions
- Indirect models: allows A_4 with CSD alignments, gives minimal predictive seesaw with CSD(3) being most successful
- A4xSU(5) SUSY GUT based on CSD(3), quite complete
- A to Z Pati-Salam based on CSD(4), unifies RH neutrinos
- Good motivation for discrete symmetries from string/F-theory
- SUSY flavour models mimic MFV but with testable deviations