Discrete FLAvor SYmmetries and Origin of CP Violation

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Work done in collaboration with
Maximilian Fallbacher, K.T. Mahanthappa, Michael Ratz, Andreas Trautner, Nucl. Phys. B883 (2014) 267

CP Violation in Nature

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
  - SM: CKM matrix for the quark sector
    - experimentally established δ_{CKM} as major source of CP violation
    - not sufficient for observed cosmological matter-antimatter asymmetry
- Search for new source of CP violation:
  - CP violation in neutrino sector
  - if found $\Rightarrow$ phase in PMNS matrix
- Discrete family symmetries:
  - suggested by large neutrino mixing angles
  - neutrino mixing angles from group theoretical CG coefficients

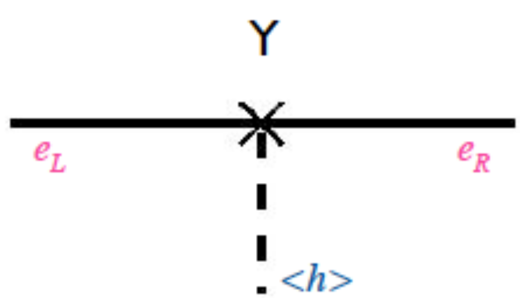
Discrete (family) symmetries $\Leftrightarrow$ Physical CP violation

Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



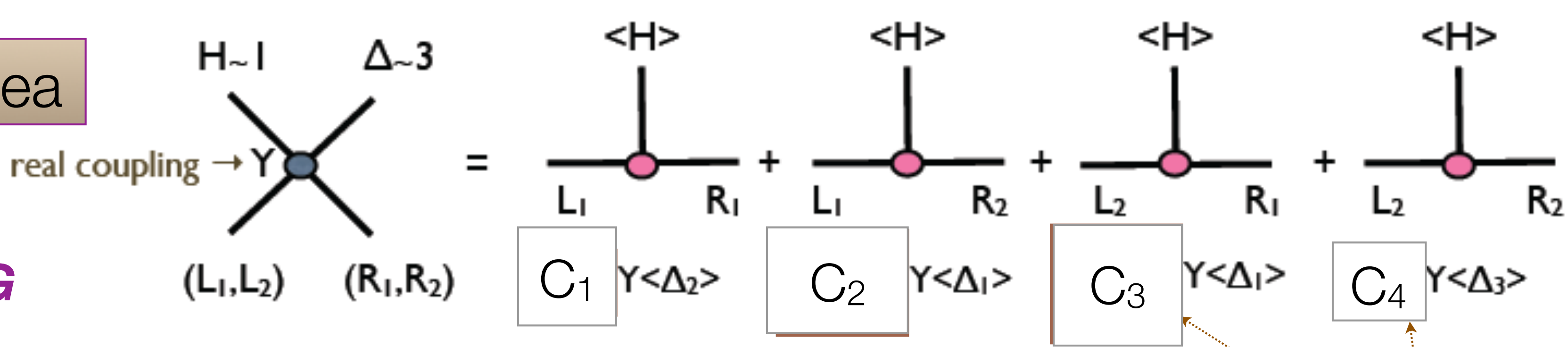
A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- Complex CG coefficients in certain discrete groups ⇒ explicit CP violation
  - Real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - No additional parameters needed ⇒ extremely predictive model!

Basic idea

Discrete symmetry ***G***



- Scalar potential: if Z_3 symmetric ⇒ $\langle \Delta_1 \rangle = \langle \Delta_2 \rangle = \langle \Delta_3 \rangle \equiv \langle \Delta \rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$\begin{array}{cc} & \begin{array}{cc} L_1 & L_2 \end{array} \\ M = & \begin{pmatrix} C_1 & C_3 \\ C_2 & C_4 \end{pmatrix} Y \langle \Delta \rangle \end{array} \quad \begin{pmatrix} R_1 & R_2 \end{pmatrix}$$

$C_{1,2,3,4}$: complex CG coefficients of ***G***

CP Transformation

- Canonical CP transformation

$$\phi(x) \xmapsto{\mathcal{CP}} \eta_{\mathcal{CP}} \; \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \xmapsto{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}} \, \Phi^*(\mathcal{P} \; x)$$

unitary matrix

Generalized CP Transformation

- setting w/ discrete symmetry G **G and CP transformations do not commute**
- generalized CP transformation Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)
- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➥ need generalized CP transformation $\widetilde{CP}$: $\phi \xmapsto{\widetilde{CP}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{CP}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{CP}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Constraints on generalized CP transformations

- generalized CP transformation

$$\Phi(x) \quad \xrightarrow{\widetilde{CP}} \quad U_{\mathrm{CP}}\,\Phi^*(\,\mathcal{P}\;x)$$

- consistency condition

Holthausen, Lindner, and Schmidt (2013)

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\;\rho(g)^*\,U_{\mathrm{CP}}{}^{\dagger} \quad \forall\, g \in G$$

- further properties:

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^{\dagger} \quad \forall\, g \in G \text{ and } \forall\, i \quad \Longleftrightarrow \quad \boxed{\text{physical CP transformations}}$$

- u has to be class–inverting
- in all known cases, u is equivalent to an automorphism of order two

bottom–line:

u has to be a class–inverting (involutory) automorphism of G

The Bickerstaff-Damhus automorphism (BDA)

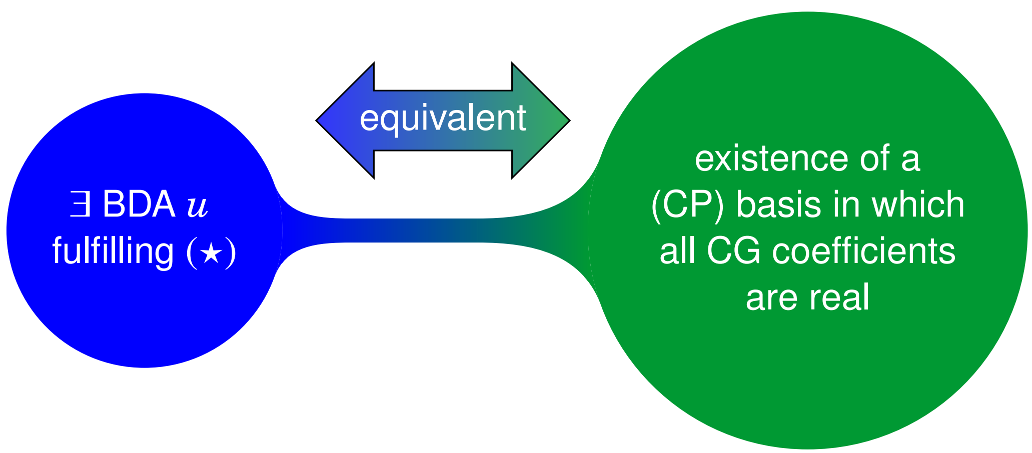
- Bickerstaff-Damhus automorphism (BDA) $\boldsymbol{u}$

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(\boldsymbol{u}(\boldsymbol{g})) = U_{\boldsymbol{r}_i}\ \rho_{\boldsymbol{r}_i}(\boldsymbol{g})^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, \boldsymbol{g} \in G \text{ and } \forall\, \boldsymbol{i} \qquad (\star)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

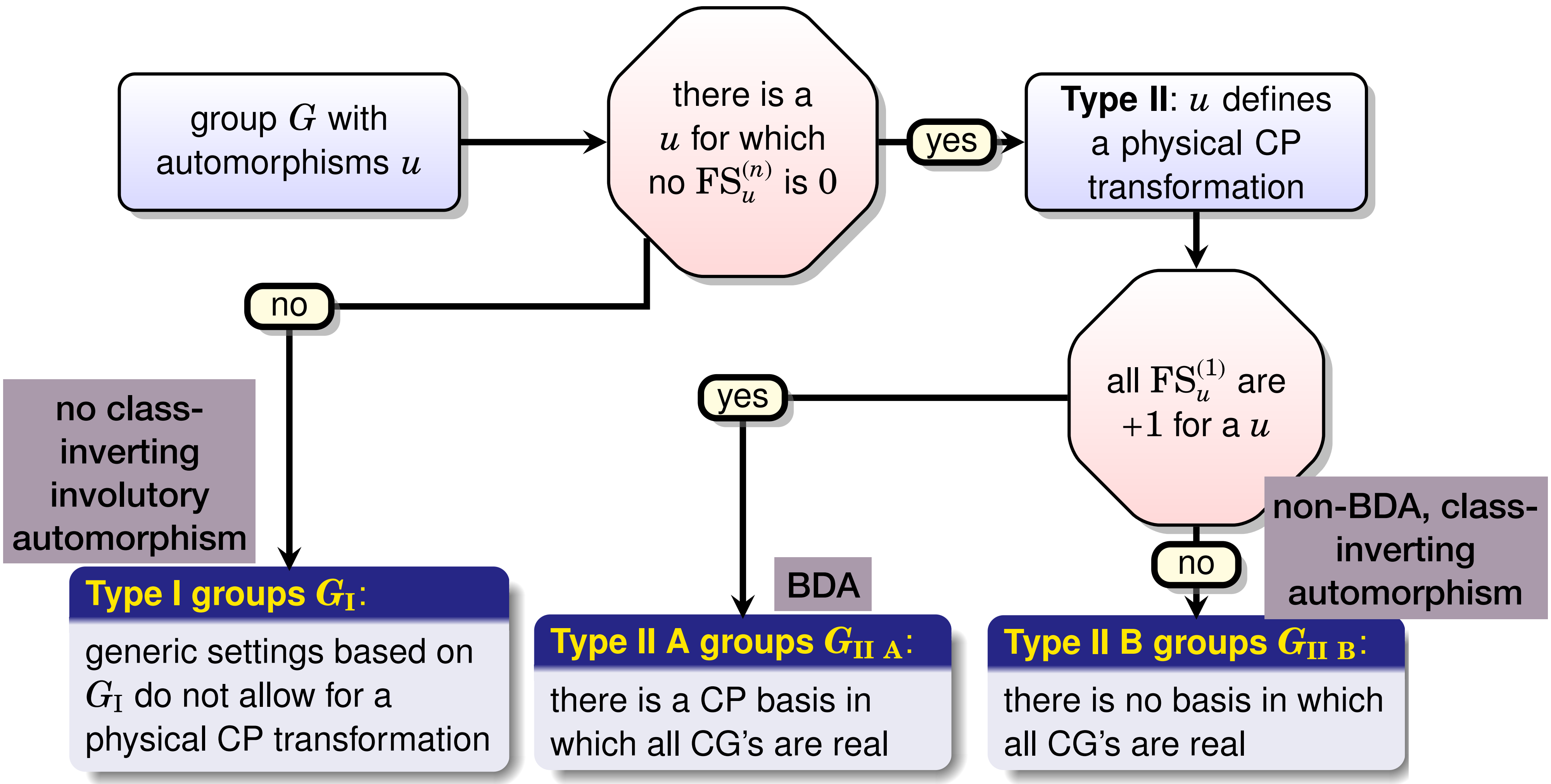
Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

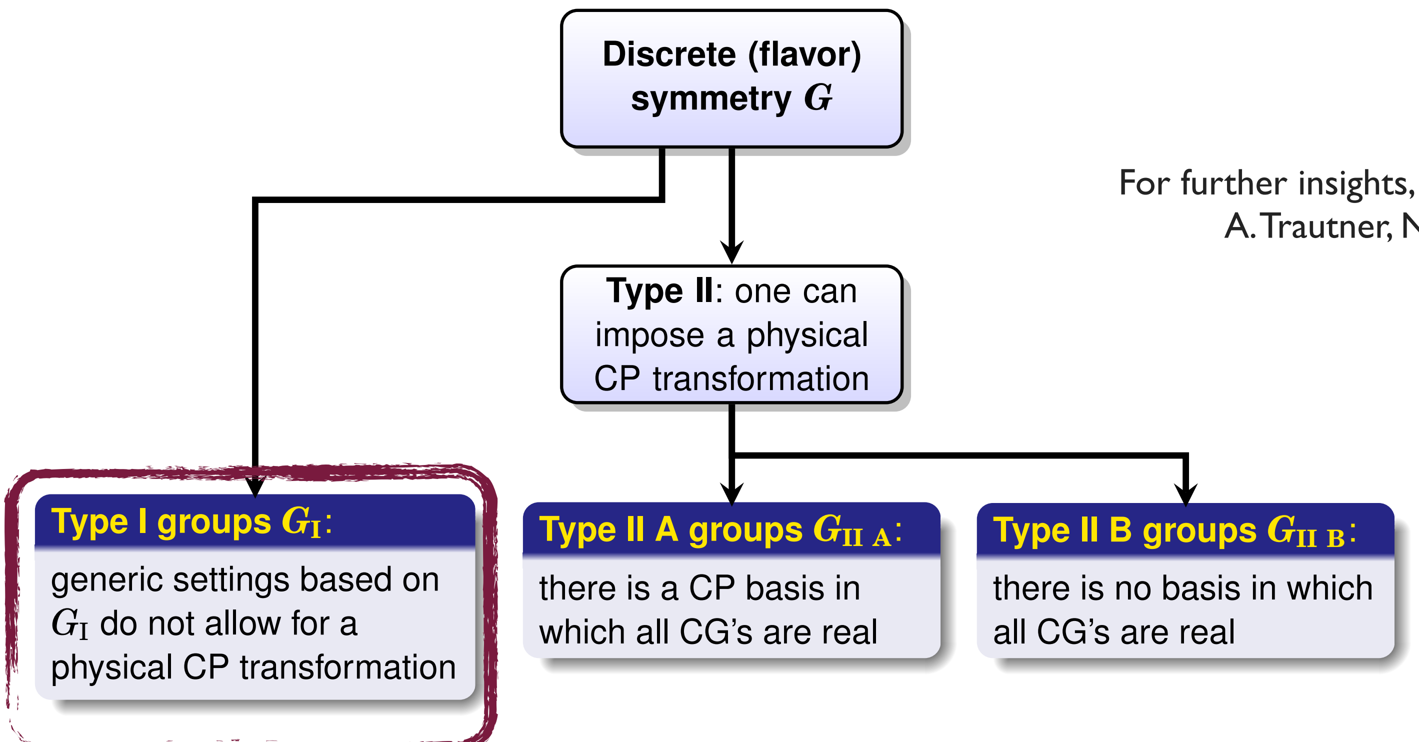


A Novel Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

- **For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ Physical CP violation**

CP Violation from Group Theory!



For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T′	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for a type I group:

$$\mathbf{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H_\Psi^{ij}\, Y\, \overline{\Psi}_i \Psi_j + H_\Sigma^{ij}\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\, \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$

"flavor" structures determined by (complex) CG coefficients

arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

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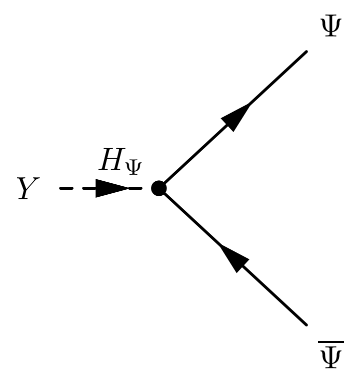
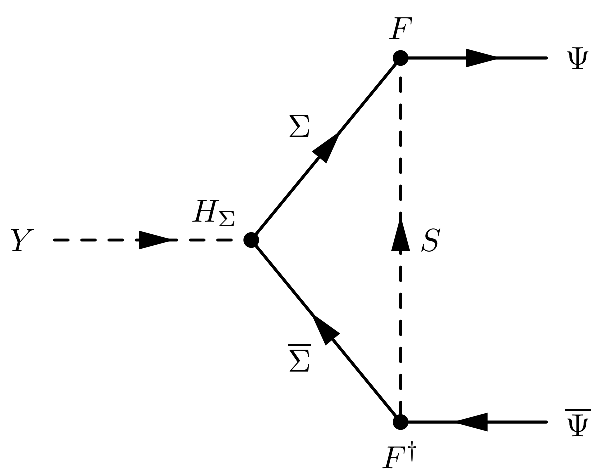
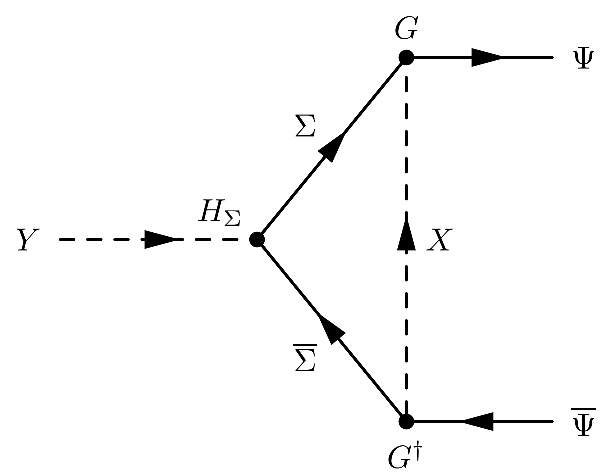
- Particle decay $Y \longrightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi)-\Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi)+\Gamma(Y^*\to\overline{\Psi}\Psi)}$$

$$\propto \ \mathrm{Im}\left[I_S\right]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\left[I_X\right]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\left[I_S\right]\ \mathrm{Im}\left[h_\Psi\, h_\Sigma^*\right] + |g|^2\ \mathrm{Im}\left[I_X\right]\ \mathrm{Im}\left[\omega\, h_\Psi\, h_\Sigma^*\right]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi\, h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega\, h_\Psi\, h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi\, h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$$\Delta(27) \subset \mathrm{SG}(54,5): \begin{cases} (X,Z) & : \text{ doublet} \\ (\Psi,\Sigma^C) & : \text{ hexaplet} \\ \phi & : \text{ non–trivial 1–dim. representation} \end{cases}$$

- non–trivial $\langle\phi\rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$

- allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}}\langle\phi\rangle \left(|X|^2 - |Z|^2\right) + \text{h.c.}\right]$

CG coefficient of $\mathrm{SG}(54,5)$

- CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2\,|h_\Psi|^2\ \mathrm{Im}\left[\,\omega\,\right]\ \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Example for a type II A group:

$$\mathbf{T'}$$

- CP basis and its complications
- generalized CP transformation

(Generalized) CP Transformation for T′

- unique outer automorphism

$$u \ : \ (S,T) \ \rightarrow \ (S^3, T^2) \qquad \curvearrowright \begin{cases} \mathbf{1}_i & \rightarrow \quad U_{\mathbf{1}_i}\, \mathbf{1}_i{}^* \\ \mathbf{2}_i & \rightarrow \quad U_{\mathbf{2}_i}\, \mathbf{2}_i{}^* \\ \mathbf{3} & \rightarrow \quad U_{\mathbf{3}}\, \mathbf{3}^* \end{cases}$$

- twisted Frobenius–Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}_0$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{3}$
$\mathrm{FS}_u(\boldsymbol{R})$	1	1	1	1	1	1	1

➥ u is a Bickerstaff–Damhus automorphism

➥ there is a basis in which all Clebsch–Gordan coefficients are real

basis can been found e.g. in Ishimori, Kobayashi, Ohki, Shimizu, Okada, et al. (2010)

- u defines a physical CP transformation

- invariance of $\mathscr{L}$ under u restricts the phases of the coupling coefficients

Issues with the CP basis and other bases

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- $\mathbf{3}$ of T' is a real representation

- however, in many T' bases (including the CP basis), $\mathbf{3}$ transforms with complex matrices

- need to describe a real $\mathbf{3}$–plet by complex field(s) and impose 'Majorana–like condition' $\phi^* = U\,\phi$

  with e.g. $U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ in the 'Feruglio basis' "Fefo basis" (2001)

  with the 'Feruglio basis' defined in Appendix A of Feruglio, Hagedorn, Lin, and Merlo (2007)

- problems do not appear in the T' extension of the 'Ma basis' for A_4

  A_4 basis can be found in Ma and Rajasekaran (2001)

- proper CP transformation

$$\mathbf{1}_i \xmapsto{\widetilde{\mathcal{CP}}} \mathbf{1}_i{}^* \,, \quad \mathbf{2}_i \xmapsto{\widetilde{\mathcal{CP}}} \mathbf{2}_i{}^* \,, \quad \mathbf{3} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \mathbf{3}^*$$

Example for a type II B group:

$$\boldsymbol{\Sigma(72)}$$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings

Example of a type IIB group: Σ(72)

☞ presentation of $\Sigma(72)$

$$M^4 = N^4 = P^3 = \left(M^2 P^{-1}\right)^2 = \mathbb{1}\,, \quad M^2 = N^2\,, \quad M^{-1} N = N M$$
$$P M P N^{-1} M P^{-1} N = \mathbb{1}\,, \quad N P M^{-1} P = M P N$$

☞ 6 inequivalent irreducible representations: $\mathbf{1}_{0-3}$, $\mathbf{2}$ and $\mathbf{8}$

☞ character table

	C_{1a}	C_{3a}	C_{2a}	C_{4a}	C_{4b}	C_{4c}
	1	8	9	18	18	18
$\Sigma(72)$	$\mathbb{1}$	P	M^2	MN	N	M
$\mathbf{1}_0$	1	1	1	1	1	1
$\mathbf{1}_1$	1	1	1	1	−1	−1
$\mathbf{1}_2$	1	1	1	−1	1	−1
$\mathbf{1}_3$	1	1	1	−1	−1	1
$\mathbf{2}$	2	2	−2	0	0	0
$\mathbf{8}$	8	−1	0	0	0	0

Example of a type IIB group: Σ(72)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- $\Sigma(72)$ is ambivalent, i.e. each conjugacy class contains with an element g also its inverse element g^{-1}
- identity is already class–inverting (and involutory)
- twisted Frobenius–Schur indicators of identity

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{2}$	$\mathbf{8}$
$\mathrm{FS}_{\mathrm{id}}(\boldsymbol{R})$	1	1	1	1	-1	1

no BDA

- ➥ there is no CP basis
- generalized CP transformation

$$\mathbf{1}_i \xrightarrow{\widetilde{CP}} \mathbf{1}_i{}^* \,, \quad \mathbf{2} \xrightarrow{\widetilde{CP}} U_{\mathbf{2}}\, \mathbf{2}^* \,,\ \mathbf{8} \xrightarrow{\widetilde{CP}} \mathbf{8}^*$$

$$U_{\mathbf{2}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Example of a type IIB group: Σ(72)

- generalized CP transformation

$$\mathbf{1}_i \xrightarrow{\widetilde{CP}} \mathbf{1}_i{}^* \;,\quad \mathbf{2} \xrightarrow{\widetilde{CP}} U_{\mathbf{2}}\ \mathbf{2}^* \;,\; \mathbf{8} \xrightarrow{\widetilde{CP}} \mathbf{8}^*$$

- imposing this CP transformation as a symmetry enlarges the flavor group by an additional $\mathbb{Z}_2$ factor to $\Sigma(72) \times \mathbb{Z}_2$

- additional symmetry generator acts trivially on all representations except for the $\mathbf{2}$ on which it acts as $V_{\mathbf{2}} = U_{\mathbf{2}}\, U_{\mathbf{2}}^* = -\mathbb{1}$

- this additional $\mathbb{Z}_2$ forbids all terms which contain an odd number of fields in the representation $\mathbf{2}$ such as

$$\mathscr{L} \supset c \;\; (\mathbf{2} \otimes (\mathbf{8} \otimes \mathbf{8})_{\mathbf{2}})_{\mathbf{1}_0}$$

forbidden by additional $\mathbb{Z}_2$

unusal feature of type II B groups:

CP may forbid couplings rather than restricting the phases!

Summary

Summary

- **NOT all outer automorphisms correspond to physical CP transformations**

- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^{\dagger} \quad \forall\, g \in G \text{ and } \forall\, i$$

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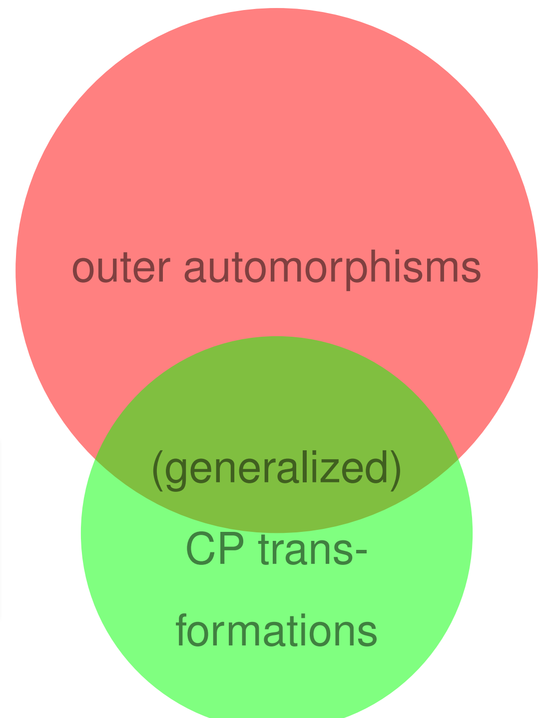
class inverting, involutory automorphisms $\Leftrightarrow$ physical CP transformations



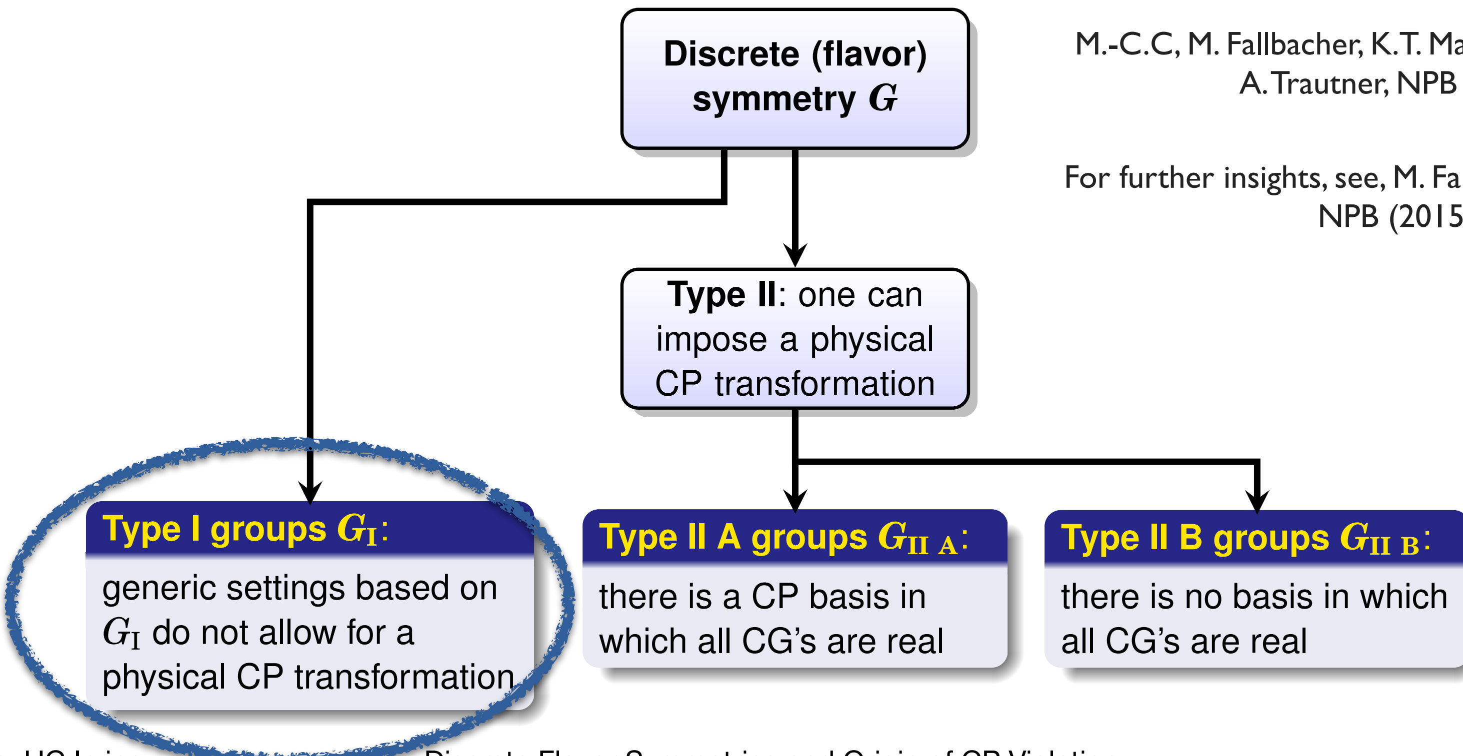
Summary

- **For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)



Summary

Three examples:

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

☞ Type I group: $\Delta(27)$

- generic settings based on $\Delta(27)$ violate CP!
- spontaneous breaking of type II A group SG$(54,5) \rightarrow \Delta(27)$
  ↷ prediction of CP violating phase from group theory!

☞ Type II A group: T'

- CP basis exists but has certain shortcomings
- advantageous to work in a different basis & impose generalized CP transformation
- CP constrains phases of coupling coefficients

☞ Type II B group: $\Sigma(72)$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings

Backup Slides

CP Conservation vs Symmetry Enhancement

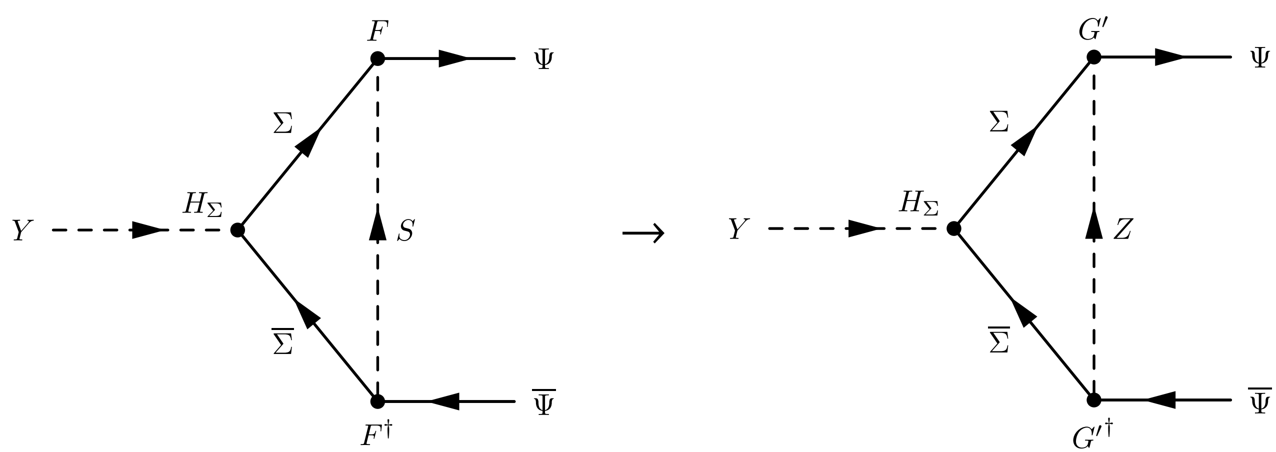
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ $\curvearrowright$ interaction

$$\mathscr{L}_{\text{toy}}^{Z} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij}\, Z\, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



CP Conservation vs Symmetry Enhancement

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8 \curvearrowright$ interaction

$$\mathscr{L}^{Z}_{\text{toy}} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij}\ Z\, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

- different contribution to decay asymmetry: $\varepsilon^{S}_{Y \to \overline{\Psi}\Psi} \to \varepsilon^{Z}_{Y \to \overline{\Psi}\Psi}$

- total CP asymmetry of the Y decay vanishes if $\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$

- relations (i)—(iii) can be due to an outer automorphism

$$X \overset{u_3}{\longleftrightarrow} Z\,, \quad Y \overset{u_3}{\longrightarrow} Y\,, \quad \Psi \overset{u_3}{\longrightarrow} U_{u_3}\, \Sigma^C \quad \& \quad \Sigma \overset{u_3}{\longrightarrow} U_{u_3}\, \Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

... BUT this enlarges $\Delta(27) \to \mathrm{SG}(54,5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

SG(54, 5): group name from GAP library

Some Outer Automorphisms of Δ(27)

- sample outer automorphisms of Δ(27)

$$u_1 \ : \ \mathbf{1}_1 \leftrightarrow \mathbf{1}_2 \ , \ \mathbf{1}_4 \leftrightarrow \mathbf{1}_5 \ , \ \mathbf{1}_7 \leftrightarrow \mathbf{1}_8 \ , \ \mathbf{3} \rightarrow U_{u_1}\,\mathbf{3}^*$$
$$u_2 \ : \ \mathbf{1}_1 \leftrightarrow \mathbf{1}_4 \ , \ \mathbf{1}_2 \leftrightarrow \mathbf{1}_8 \ , \ \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \ , \ \mathbf{3} \rightarrow U_{u_2}\,\mathbf{3}^*$$
$$u_3 \ : \ \mathbf{1}_1 \leftrightarrow \mathbf{1}_8 \ , \ \mathbf{1}_2 \leftrightarrow \mathbf{1}_4 \ , \ \mathbf{1}_5 \leftrightarrow \mathbf{1}_7 \ , \ \mathbf{3} \rightarrow U_{u_3}\,\mathbf{3}^*$$
$$u_4 \ : \ \mathbf{1}_1 \leftrightarrow \mathbf{1}_7 \ , \ \mathbf{1}_2 \leftrightarrow \mathbf{1}_5 \ , \ \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \ , \ \mathbf{3} \rightarrow U_{u_4}\,\mathbf{3}^*$$
$$u_5 \ : \ \mathbf{1}_i \leftrightarrow \mathbf{1}_i{}^* \ , \ \mathbf{3} \rightarrow U_{u_5}\,\mathbf{3}$$

- twisted Frobenius-Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{FS}_{u_1}(\boldsymbol{R})$	1	1	1	0	0	0	0	0	0	1	1
$\mathrm{FS}_{u_2}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_3}(\boldsymbol{R})$	1	0	0	0	0	1	0	1	0	1	1
$\mathrm{FS}_{u_4}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_5}(\boldsymbol{R})$	1	1	1	1	1	1	1	1	1	0	0

  - none of the u_i maps all representations to their conjugates
  - however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\boldsymbol{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \overline{\mathbf{3}}\}$
  - CP conservation possible in non-generic models
    - e.g. some well-known multiple Higgs model   Branco, Gerard, and Grimus (1984)

CP-like Symmetries

- outer automorphism u_5

$$X \to X^* \,, \quad Z \to Z^* \,, \quad Y \to Y^* \,, \quad \Psi \to U_{u_5}\,\Sigma \quad \& \quad \Sigma \to U_{u_5}\,\Psi$$

$$U_{u_5} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

- does **not** lead to a vanishing decay asymmetry

➥ in general, imposing an outer automorphism as a symmetry does not lead to physical CP conservation!

➥ CP–like symmetry