Flavor-unified GUTs, maximal CP phase, and $\delta_{\rm PMNS} \simeq \pm\delta_{\rm CKM}$

Jihn E. Kim

Kyung Hee University &
Seoul National Univ.

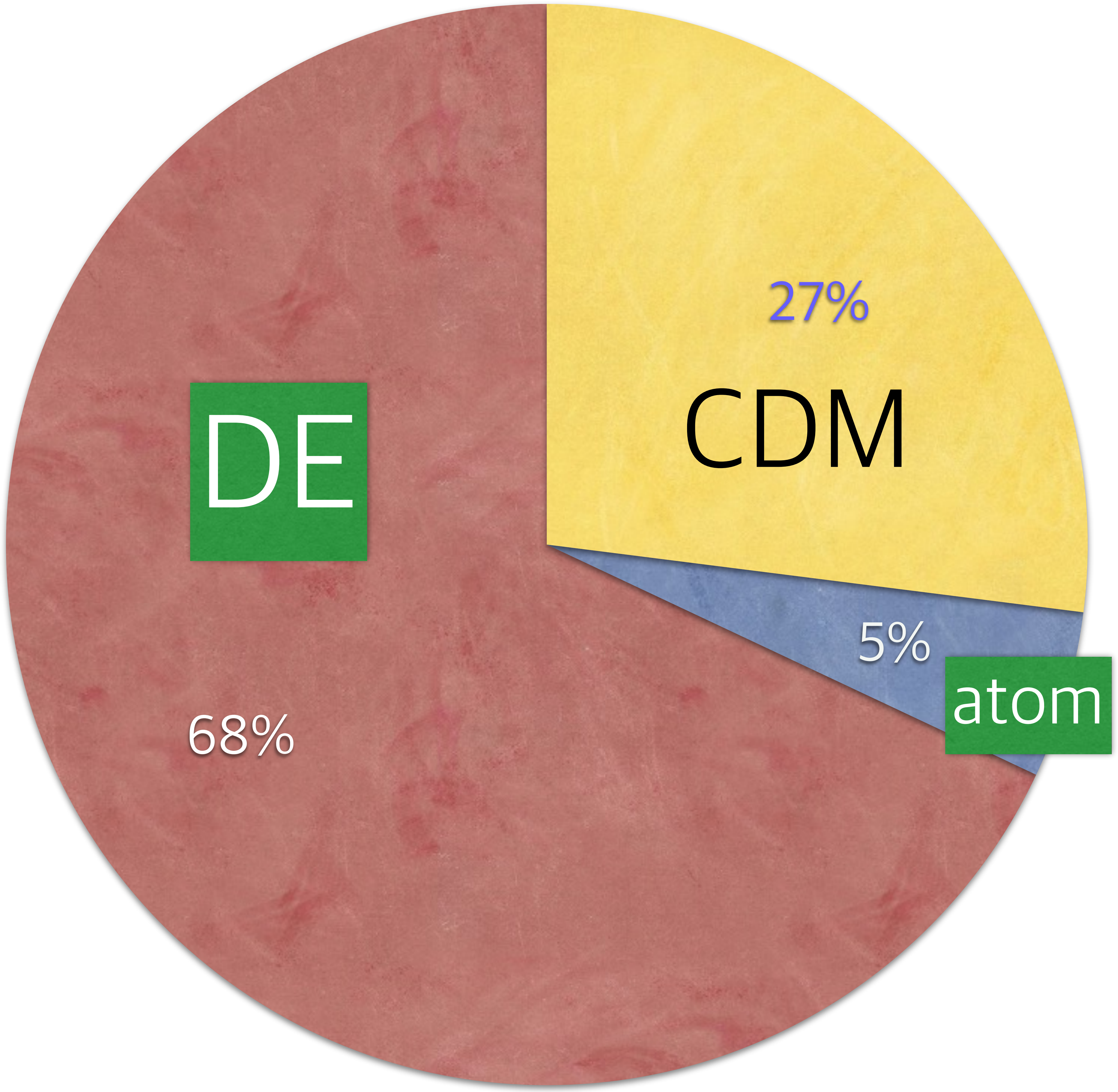
Manzannilo, Colimar,
Mexico, 2 July 2015

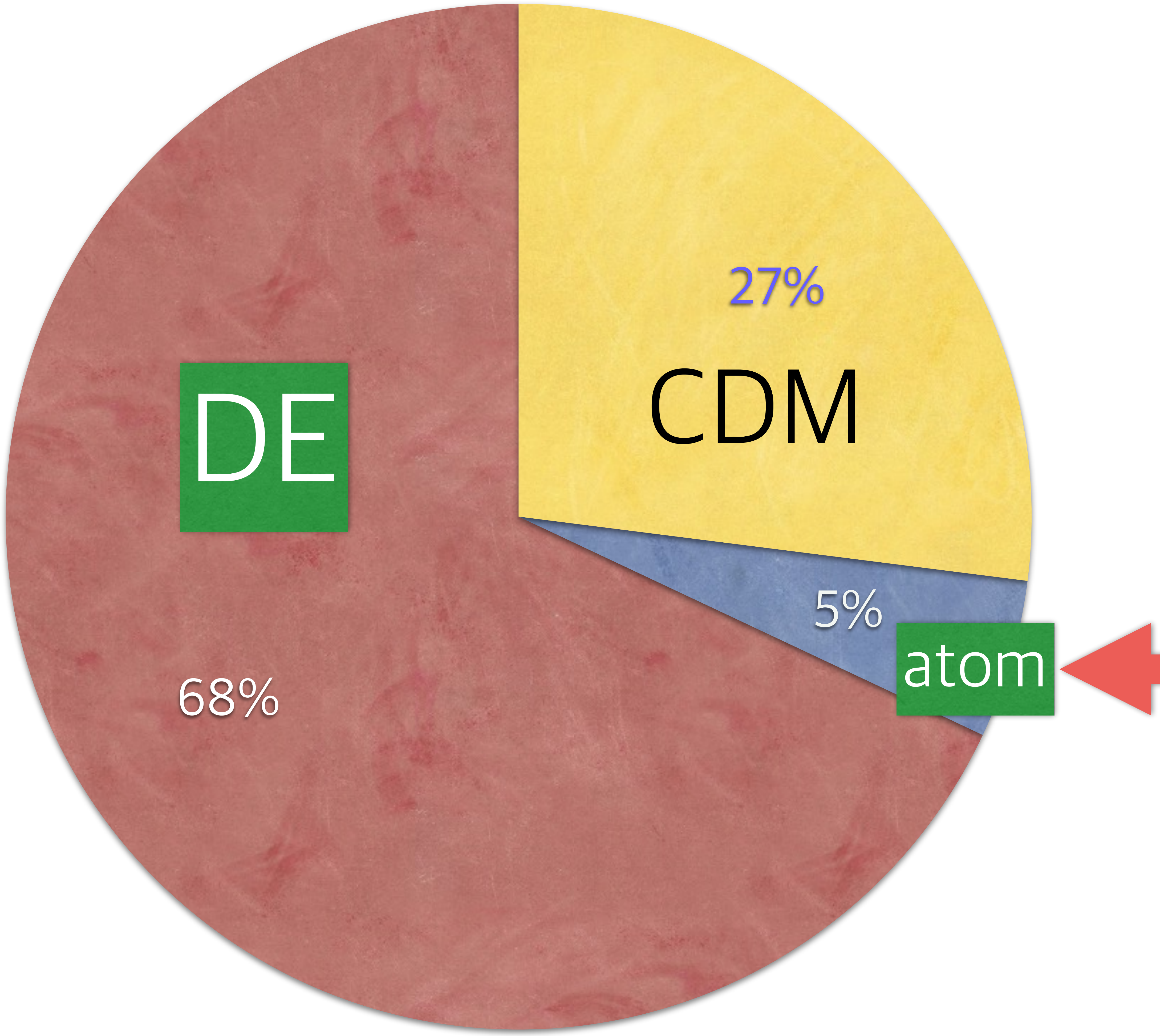
JEK+M-S Seo, arXiv:1105.3304

JEK+DY Mo+ S Nam, arXiv:1402.2978

JEK, JHEP 06 (2015) 114[1503.03104]

JEK-Nam, 1506. 08491; JEK-Mo-Seo, 1506.08984


27%
CDM
DE
5%
atom
68%



Chiral fields at GUT scale
SU(5), SU(7) GUTs

UGUTF:
Kim, PRL 45, 1916 (1980);
arXiv:1503.03104;
JEK, D.Y.Mo, S. Nam,
JKPS 66, 894 (2015) [arXiv:
1402.2978]

Flavor GUT

Flavor GUT

SU(11)

Georgi

Flavor GUT

SU(11)

Georgi

SU(7)

Flavor GUT

SU(11)

Georgi

SU(7)

No Grand Unification

With INQ

Flipped SU(5)

Flipped SU(5)

Flipped SU(5)

From Z(12-I) orbifold

From Z(12-I) orbifold

With Kang-Sin Choi

From Z(12-I) orbifold

With Kang-Sin Choi

With Bumseok Kyae

With Kang-Sin Choi

With Bumseok Kyae

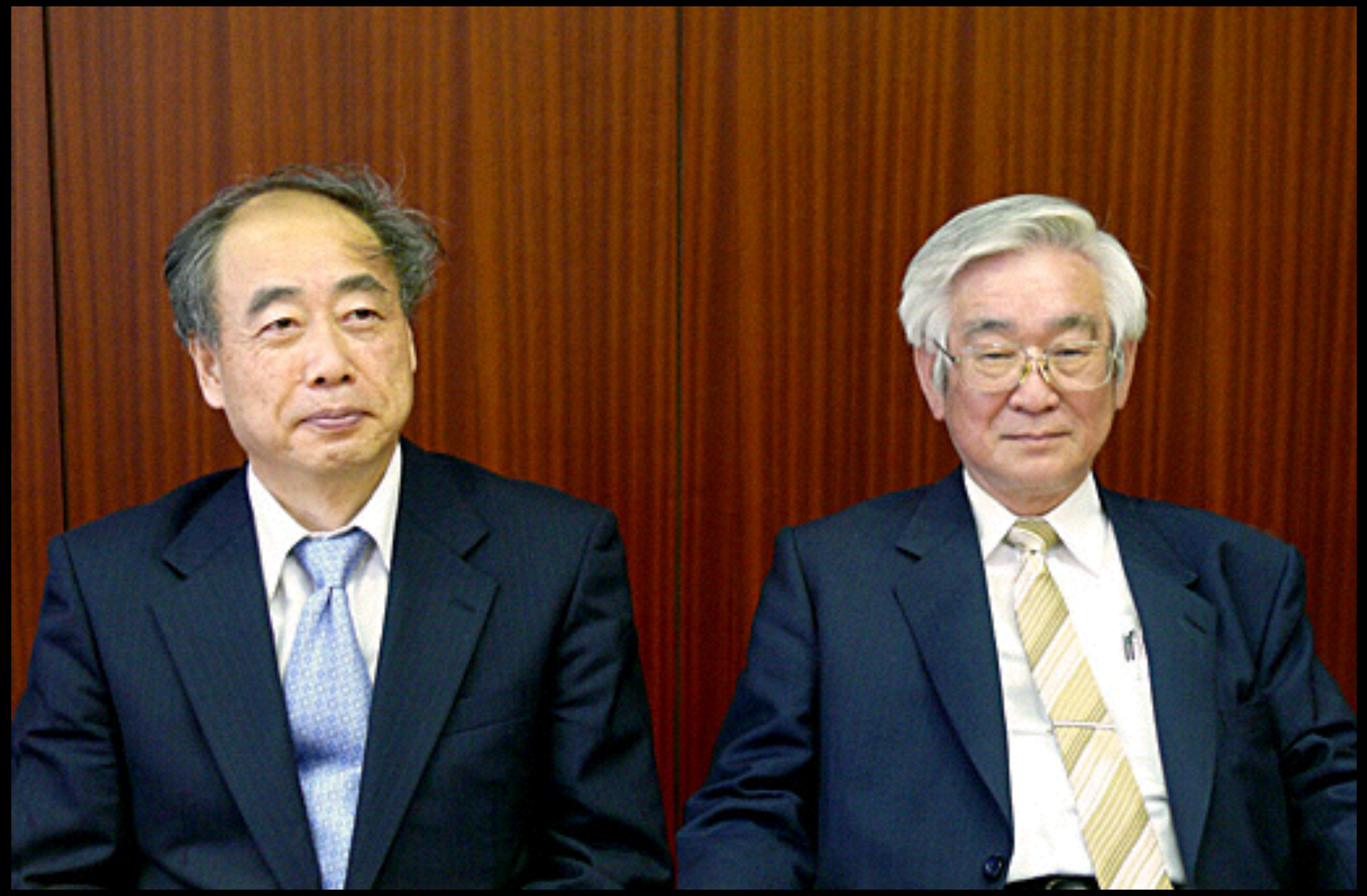
These are special cases of anti-SU(N)

With Kang-Sin Choi

With Bumseok Kyae

CKM matrix

CKM matrix



J determinant

J determinant as a phase in CKM matrix

J determinant as a phase in CKM matrix

With Min-Seok Seo & Doh Young Mo

1. Jarlskog phase in CKM matrix

To have physical effects of CP violation, the J must be non vanishing. Our form for the CKM matrix is, with the 1st row real,

$$\begin{pmatrix} c_1, & s_1 c_3, & s_1 s_3 \\ -c_2 s_1, & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3, & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2, & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta}, & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

To have physical effects of CP violation, the J must be non vanishing. Our form for the CKM matrix is, with the 1st row real,

$$\begin{pmatrix} c_1, & s_1 c_3, & s_1 s_3 \\ -c_2 s_1, & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3, & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2, & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta}, & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

The individual element of determinant is

$$\begin{aligned}
V_{11}V_{22}V_{33} &= c_1^2 c_2^2 c_3^2 + c_1^2 s_2^2 s_3^2 + 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{11}V_{23}V_{32} &= c_1^2 c_2^2 s_3^2 + c_1^2 s_2^2 c_3^2 - 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{12}V_{23}V_{31} &= s_1^2 s_2^2 c_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{12}V_{21}V_{33} &= s_1^2 c_2^2 c_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{13}V_{21}V_{32} &= s_1^2 c_2^2 s_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{13}V_{22}V_{31} &= s_1^2 s_2^2 s_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}.
\end{aligned}$$

To have physical effects of CP violation, the J must be non vanishing. Our form for the CKM matrix is, with the 1st row real,

$$\begin{pmatrix} c_1, & s_1 c_3, & s_1 s_3 \\ -c_2 s_1, & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3, & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2, & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta}, & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

The individual element of determinant is

$$\begin{aligned}
V_{11}V_{22}V_{33} &= c_1^2 c_2^2 c_3^2 + c_1^2 s_2^2 s_3^2 + 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{11}V_{23}V_{32} &= c_1^2 c_2^2 s_3^2 + c_1^2 s_2^2 c_3^2 - 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{12}V_{23}V_{31} &= s_1^2 s_2^2 c_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{12}V_{21}V_{33} &= s_1^2 c_2^2 c_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{13}V_{21}V_{32} &= s_1^2 c_2^2 s_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{13}V_{22}V_{31} &= s_1^2 s_2^2 s_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}.
\end{aligned}$$

Is Im(V_{11} V_{22} V_{33}) the Jarlskog determinant?

Is Im(V_{11} V_{22} V_{33}) the Jarlskog determinant?

The Jarlskog determinant is
$J=|\mathrm{Im}\, V_{11} V_{22} V_{12}^* V_{21}^*|$, or $|\mathrm{Im}\, V_{ii}V_{jj}V_{ij}^*V_{ji}^*|$

Is $\mathrm{Im}(V_{11} V_{22} V_{33})$ the Jarlskog determinant?

The Jarlskog determinant is
$J=|\mathrm{Im}\, V_{11} V_{22} V_{12}^* V_{21}^*|$, or $|\mathrm{Im}\, V_{ii} V_{jj} V_{ij}^* V_{ji}^*|$

Let J be
$J=\mathrm{Im}\, V_{11} V_{33} V_{13}^* V_{31}^*$. Then, on $1=\mathrm{Det}\, V$

Is Im(V11 V22 V33) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}* V_{21}*|, or |Im V_{ii}V_{jj}V_{ij}*V_{ji}*|

Let J be
J=Im V_{11} V_{33} V_{13}* V_{31}*. Then, on 1=Det V

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* &= |V_{22}|^2 V_{11} V_{33} V_{13}^* V_{31}^* - V_{11} V_{23} V_{32} V_{13}^* V_{31}^* V_{22}^* \\
&+ |V_{31}|^2 V_{12} V_{23} V_{13}^* V_{22}^* - V_{12} V_{21} V_{33} V_{13}^* V_{31}^* V_{22}^* \\
&+ |V_{13}|^2 V_{21} V_{32} V_{31}^* V_{22}^* - |V_{13} V_{22} V_{31}|^2.
\end{aligned}
$$

Is Im(V11 V22 V33) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}* V_{21}*|, or |Im V_{ii}V_{jj}V_{ij}*V_{ji}*|

Let J be
J=Im V_{11} V_{33} V_{13}* V_{31}*. Then, on 1=Det V

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* = |V_{22}|^2 V_{11} V_{33} V_{13}^* V_{31}^* - V_{11} V_{23} V_{32} V_{13}^* V_{31}^* V_{22}^* \\
+ |V_{31}|^2 V_{12} V_{23} V_{13}^* V_{22}^* - V_{12} V_{21} V_{33} V_{13}^* V_{31}^* V_{22}^* \\
+ |V_{13}|^2 V_{21} V_{32} V_{31}^* V_{22}^* - |V_{13} V_{22} V_{31}|^2.
\end{aligned}
$$

unitarity of V

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* = (1 - |V_{21}|^2) V_{11} V_{33} V_{13}^* V_{31}^* \\
+ V_{11} V_{23} V_{13}^* V_{21}^* |V_{31}|^2 + (1 - |V_{11}|^2) V_{12} V_{23} V_{13}^* V_{22}^* \\
+ |V_{13}|^2 (V_{12} V_{21} V_{11}^* V_{22}^* + V_{21} V_{32} V_{31}^* V_{22}^*) \\
- |V_{13} V_{22} V_{31}|^2.
\end{aligned}
$$

Is Im(V11 V22 V33) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}* V_{21}*|, or |Im V_{ii}V_{jj}V_{ij}*V_{ji}*|

Let J be
J=Im V_{11} V_{33} V_{13}* V_{31}*. Then, on 1=Det V

$$
\begin{aligned}
V_{13}^*V_{22}^*V_{31}^* = &|V_{22}|^2 V_{11}V_{33}V_{13}^*V_{31}^* - V_{11}V_{23}V_{32}V_{13}^*V_{31}^*V_{22}^* \\
&+ |V_{31}|^2 V_{12}V_{23}V_{13}^*V_{22}^* - V_{12}V_{21}V_{33}V_{13}^*V_{31}^*V_{22}^* \\
&+ |V_{13}|^2 V_{21}V_{32}V_{31}^*V_{22}^* - |V_{13}V_{22}V_{31}|^2.
\end{aligned}
$$

unitarity of V

$$
\begin{aligned}
V_{13}^*V_{22}^*V_{31}^* = &(1-|V_{21}|^2)V_{11}V_{33}V_{13}^*V_{31}^* \\
&+ V_{11}V_{23}V_{13}^*V_{21}^*|V_{31}|^2 + (1-|V_{11}|^2)V_{12}V_{23}V_{13}^*V_{22}^* \\
&+ |V_{13}|^2(V_{12}V_{21}V_{11}^*V_{22}^* + V_{21}V_{32}V_{31}^*V_{22}^*) \\
&- |V_{13}V_{22}V_{31}|^2.
\end{aligned}
$$

Is Im(V11 V22 V33) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}* V_{21}*|, or |Im V_{ii}V_{jj}V_{ij}*V_{ji}*|

Let J be
J=Im V_{11} V_{33} V_{13}* V_{31}*. Then, on 1=Det V

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* = {} & |V_{22}|^2 V_{11} V_{33} V_{13}^* V_{31}^* - V_{11} V_{23} V_{32} V_{13}^* V_{31}^* V_{22}^* \\
& + |V_{31}|^2 V_{12} V_{23} V_{13}^* V_{22}^* - V_{12} V_{21} V_{33} V_{13}^* V_{31}^* V_{22}^* \\
& + |V_{13}|^2 V_{21} V_{32} V_{31}^* V_{22}^* - |V_{13} V_{22} V_{31}|^2.
\end{aligned}
$$

unitarity of V

imaginary part of this is J

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* = {} & (1 - |V_{21}|^2) V_{11} V_{33} V_{13}^* V_{31}^* \\
& + V_{11} V_{23} V_{13}^* V_{21}^* |V_{31}|^2 + (1 - |V_{11}|^2) V_{12} V_{23} V_{13}^* V_{22}^* \\
& + |V_{13}|^2 (V_{12} V_{21} V_{11}^* V_{22}^* + V_{21} V_{32} V_{31}^* V_{22}^*) \\
& - |V_{13} V_{22} V_{31}|^2.
\end{aligned}
$$

Is Im(V11 V22 V33) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}* V_{21}*|, or |Im V_{ii}V_{jj}V_{ij}*V_{ji}*|

Let J be
J=Im V_{11} V_{33} V_{13}* V_{31}*. Then, on 1=Det V

imaginary part of this is J

$$V_{13}^* V_{22}^* V_{31}^* = |V_{22}|^2 V_{11} V_{33} V_{13}^* V_{31}^* - V_{11} V_{23} V_{32} V_{13}^* V_{31}^* V_{22}^* + |V_{31}|^2 V_{12} V_{23} V_{13}^* V_{22}^* - V_{12} V_{21} V_{33} V_{13}^* V_{31}^* V_{22}^* + |V_{13}|^2 V_{21} V_{32} V_{31}^* V_{22}^* - |V_{13} V_{22} V_{31}|^2.$$

unitarity of V

$$V_{13}^* V_{22}^* V_{31}^* = (1 - |V_{21}|^2) V_{11} V_{33} V_{13}^* V_{31}^* + V_{11} V_{23} V_{13}^* V_{21}^* |V_{31}|^2 + (1 - |V_{11}|^2) V_{12} V_{23} V_{13}^* V_{22}^* + |V_{13}|^2 (V_{12} V_{21} V_{11}^* V_{22}^* + V_{21} V_{32} V_{31}^* V_{22}^*) - |V_{13} V_{22} V_{31}|^2.$$

Similar considerations for other elements give the imaginary part as [(1-|V_{21}|^2) -|V_{31}|^2 +(1-|V_{11}|)^2]J=J

Kim-Seo form of J: $J=\mathrm{Im}\,(V_{31}^{*} V_{22}^{*} V_{13}^{*})$

Kim-Seo form of J: $J = \mathrm{Im}\,(V_{31}^* V_{22}^* V_{13}^*)$

JEK, M-S. Seo, PoS DSU2012 (2012) 009 [arXiv:1211.0357[hep-ph]]

JEK, D.Y. Mo, S. Nam, JKPS 66 (2015) 894 [arXiv:1402.2978[hep-ph]]

Kim-Seo form of J: $J=\mathrm{Im}\,(V_{31}^* V_{22}^* V_{13}^*)$

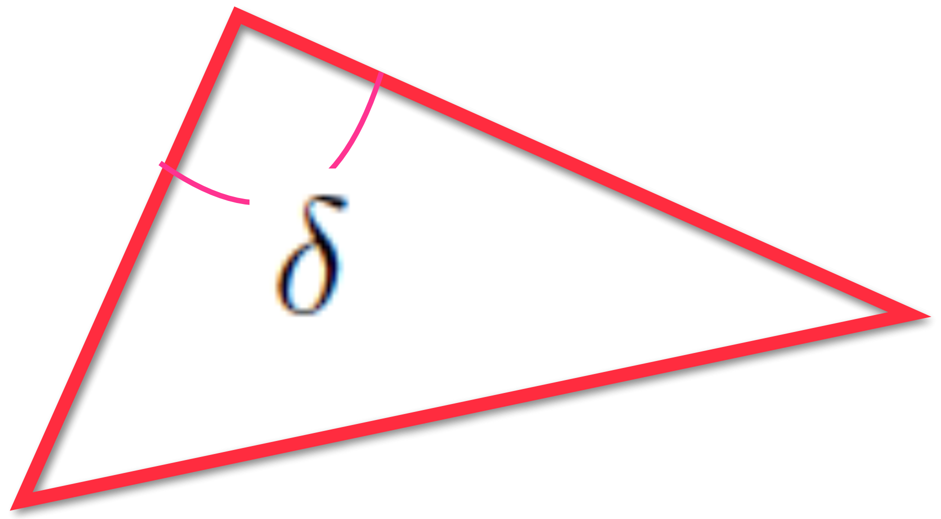
JEK, M-S. Seo, PoS DSU2012 (2012) 009 [arXiv:1211.0357[hep-ph]]

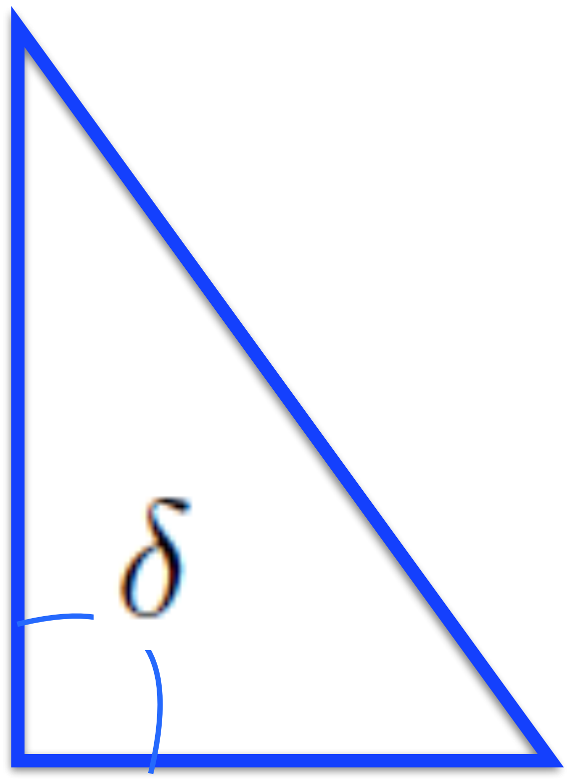
JEK, D.Y. Mo, S. Nam, JKPS 66 (2015) 894 [arXiv:1402.2978[hep-ph]]

By looking at the KS form of J, we can see the importance of physical CP violation effect.

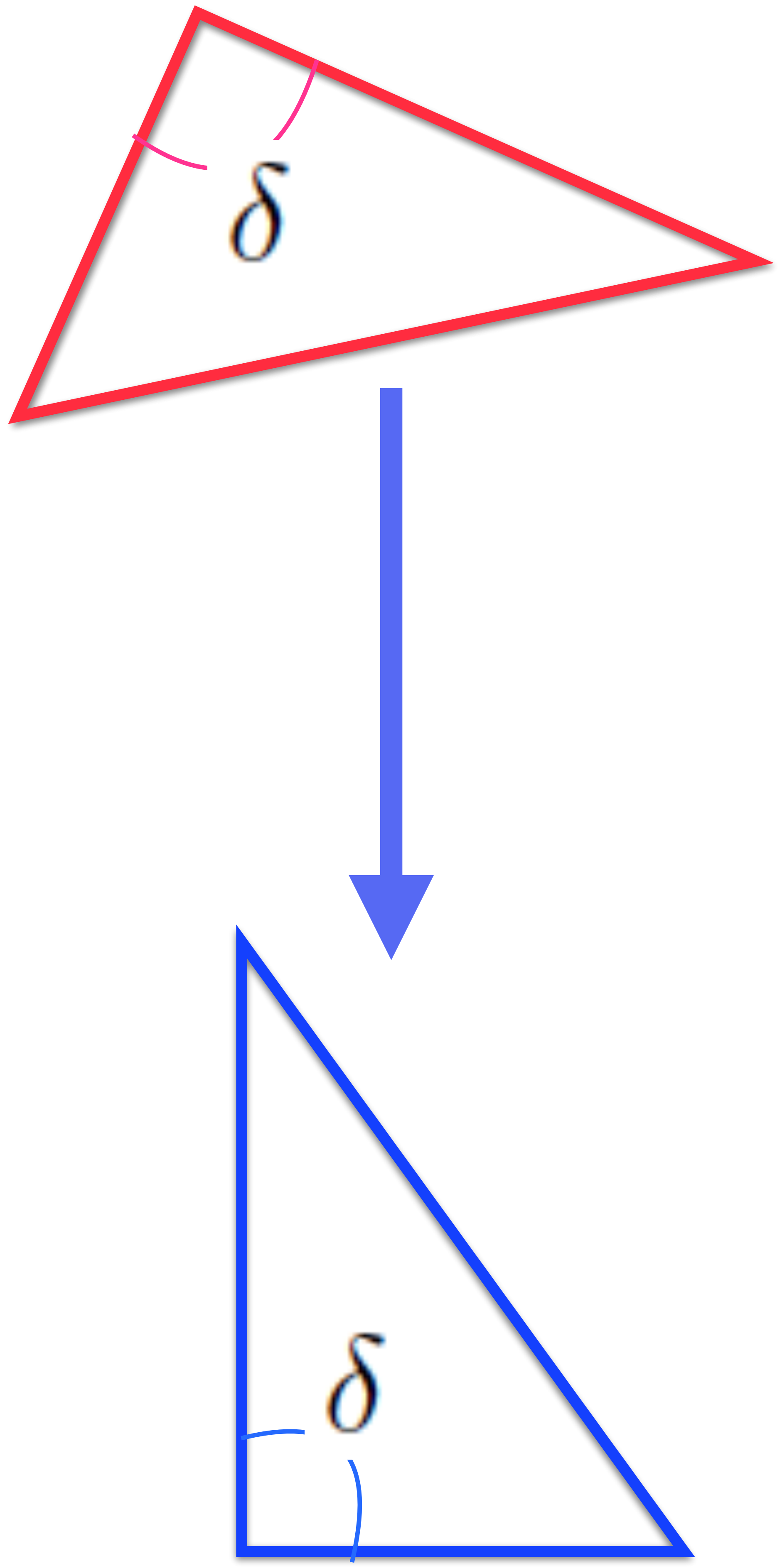
2. Maximal CP violation in quark sector

We used Det(V)=1. If it were not so, we can multiply a common phase to all q=2/3 quarks to make Det=1. It corresponds to a rotation of J triangle.



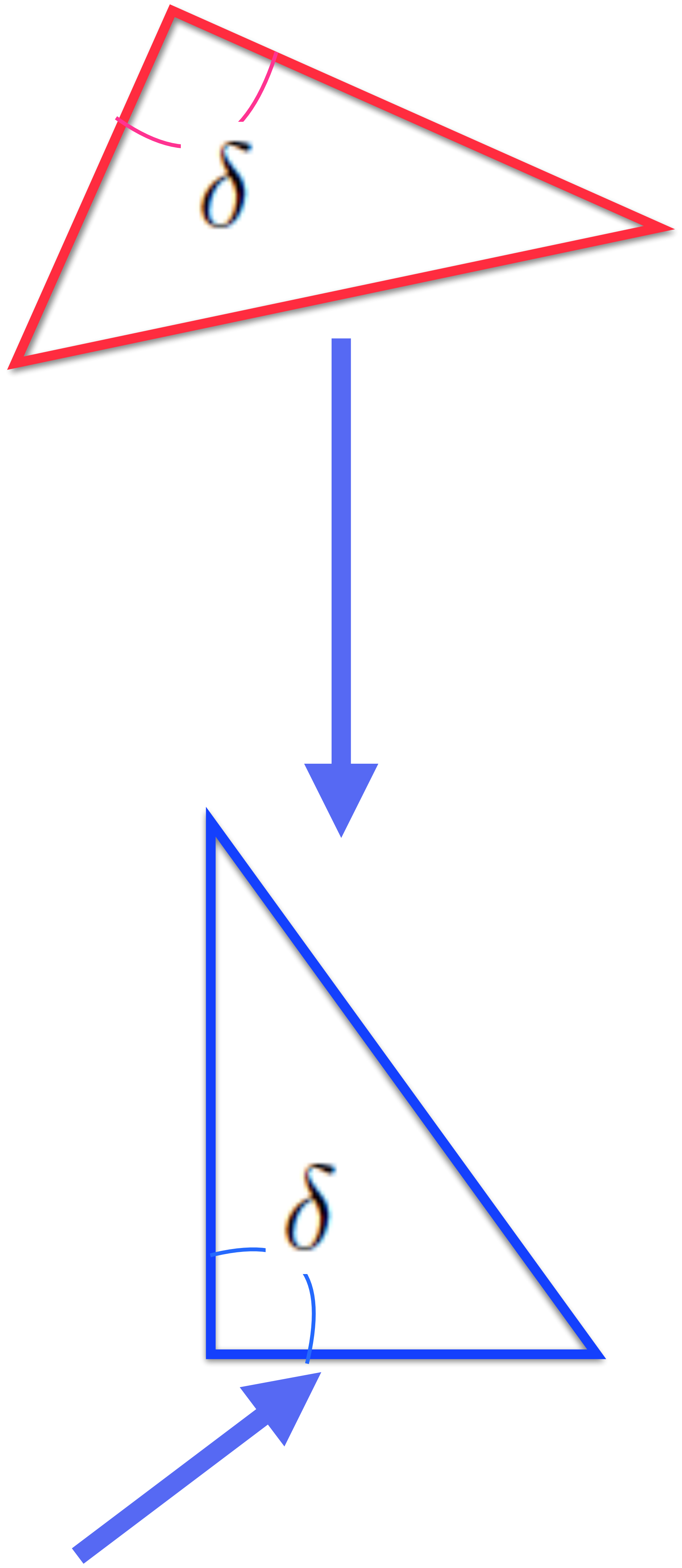


We used Det(V)=1. If it were not so, we can multiply a common phase to all q=2/3 quarks to make Det=1. It corresponds to a rotation of J triangle.



Making Det=real with 1st row real is rotating it such that the phase appears at origin.

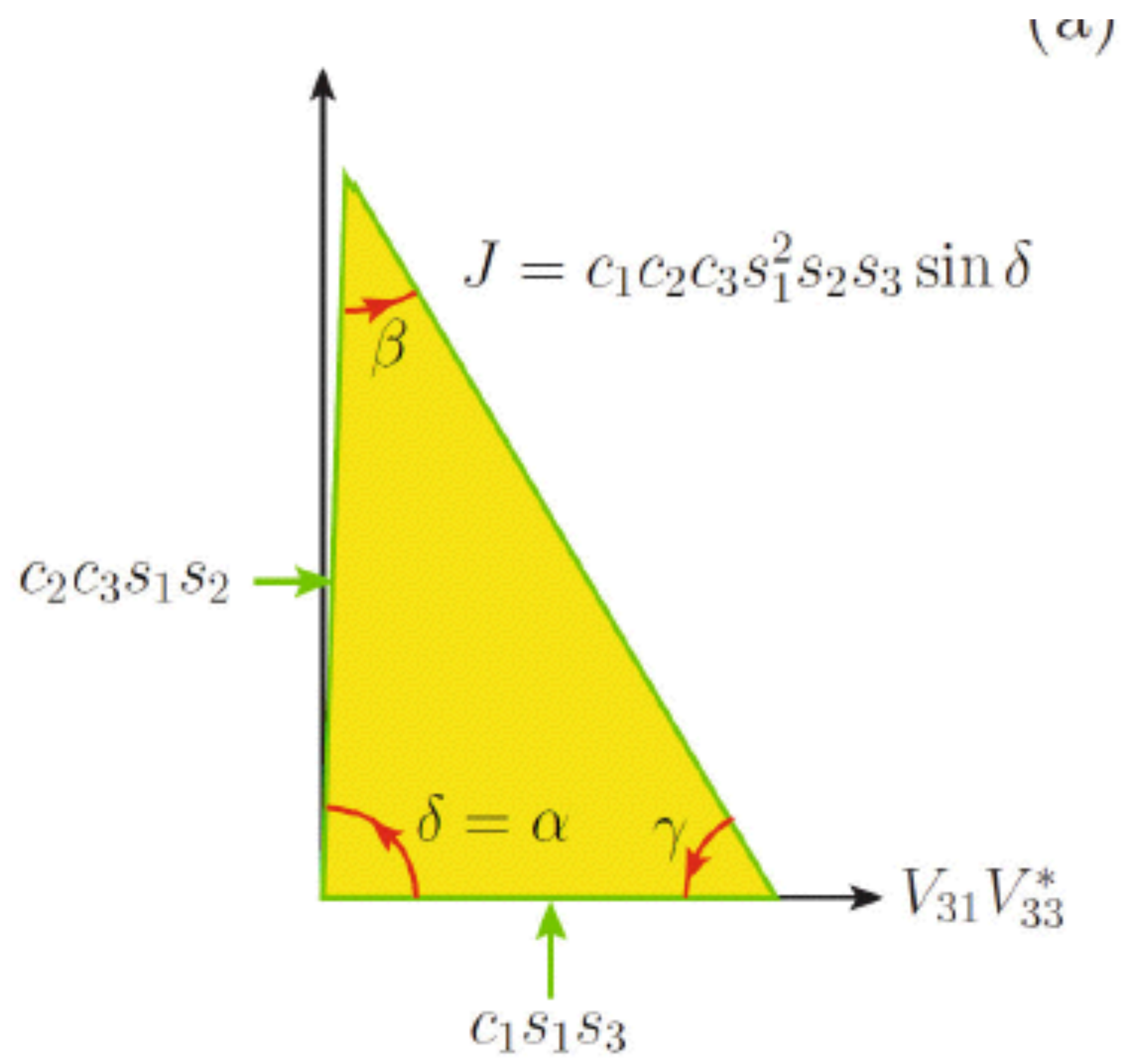
We used Det(V)=1. If it were not so, we can multiply a common phase to all q=2/3 quarks to make Det=1. It corresponds to a rotation of J triangle.



Making Det=real with 1st row real is rotating it such that the phase appears at origin.

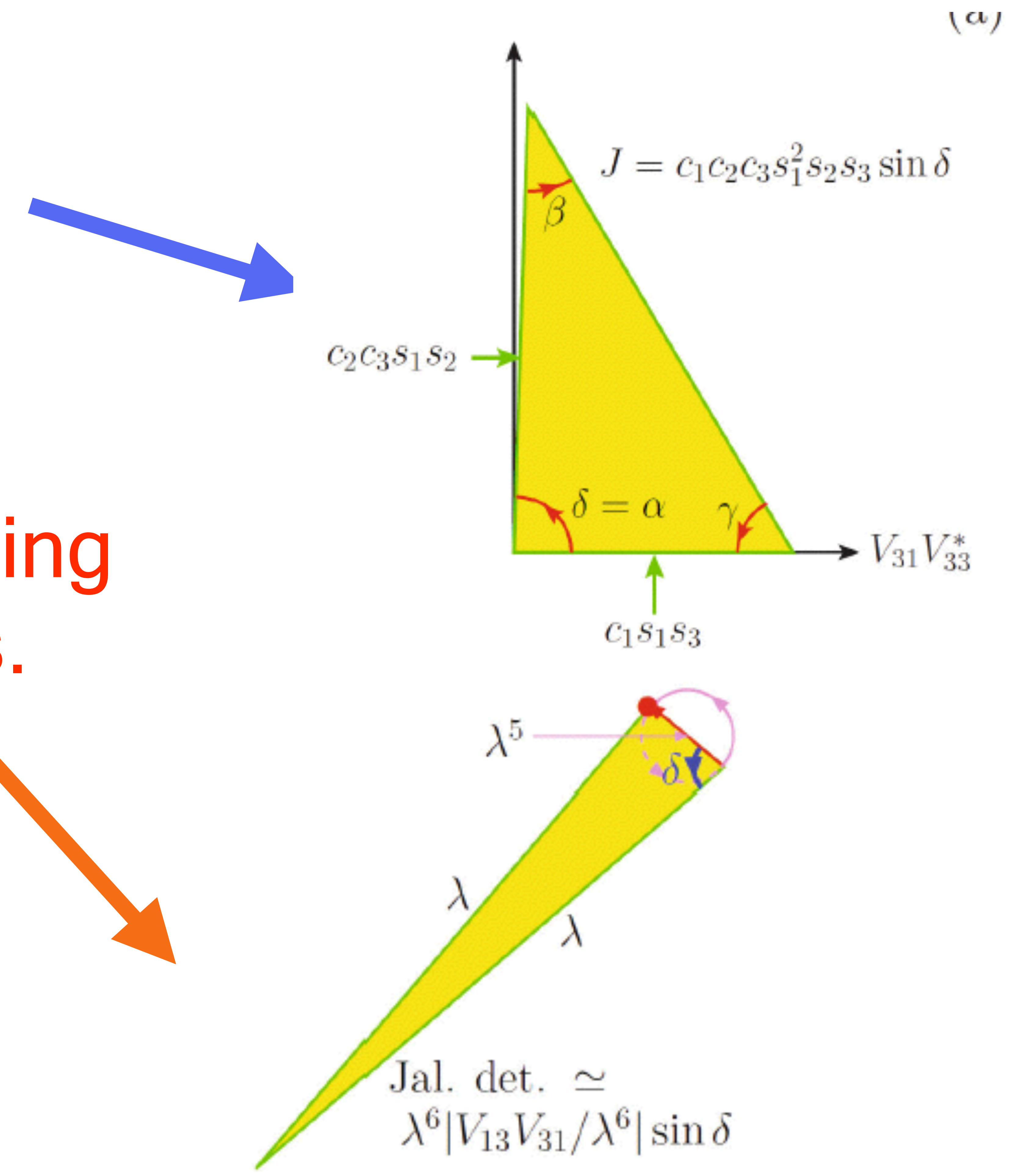
The 1st row is real.
One side becomes x-axis.

There are 6 Jarlskog triangles. One of them corresponds to B-meson decay to K.
PDG gives alpha or our delta almost 90 degrees.

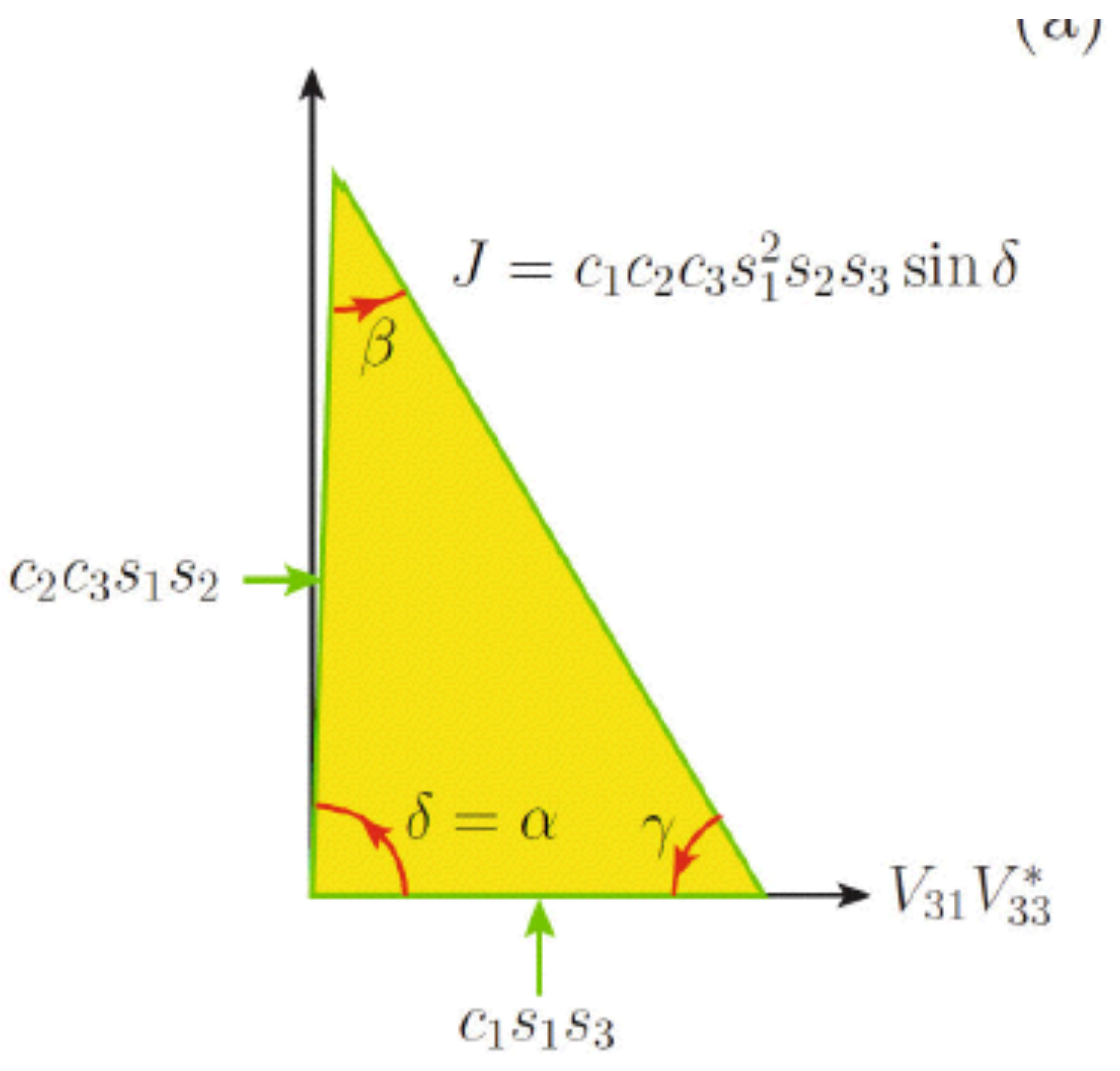


There are 6 Jarlskog triangles. One of them corresponds to B-meson decay to K. PDG gives alpha or our delta almost 90 degrees.

We can consider another J: B decaying to pi meson. This has two long sides.

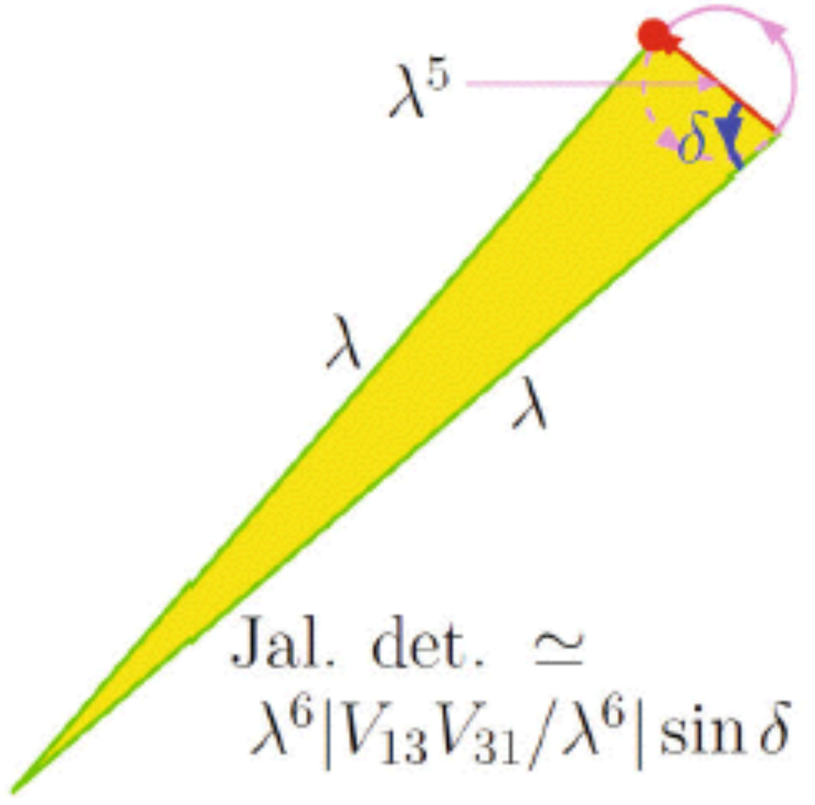


There are 6 Jarlskog triangles. One of them corresponds to B-meson decay to K. PDG gives alpha or our delta almost 90 degrees.



We can consider another J: B decaying to pi meson. This has two long sides.

So, delta=90 degrees is a maximal CP violation! in KS parametrization. In other parametrizations too.



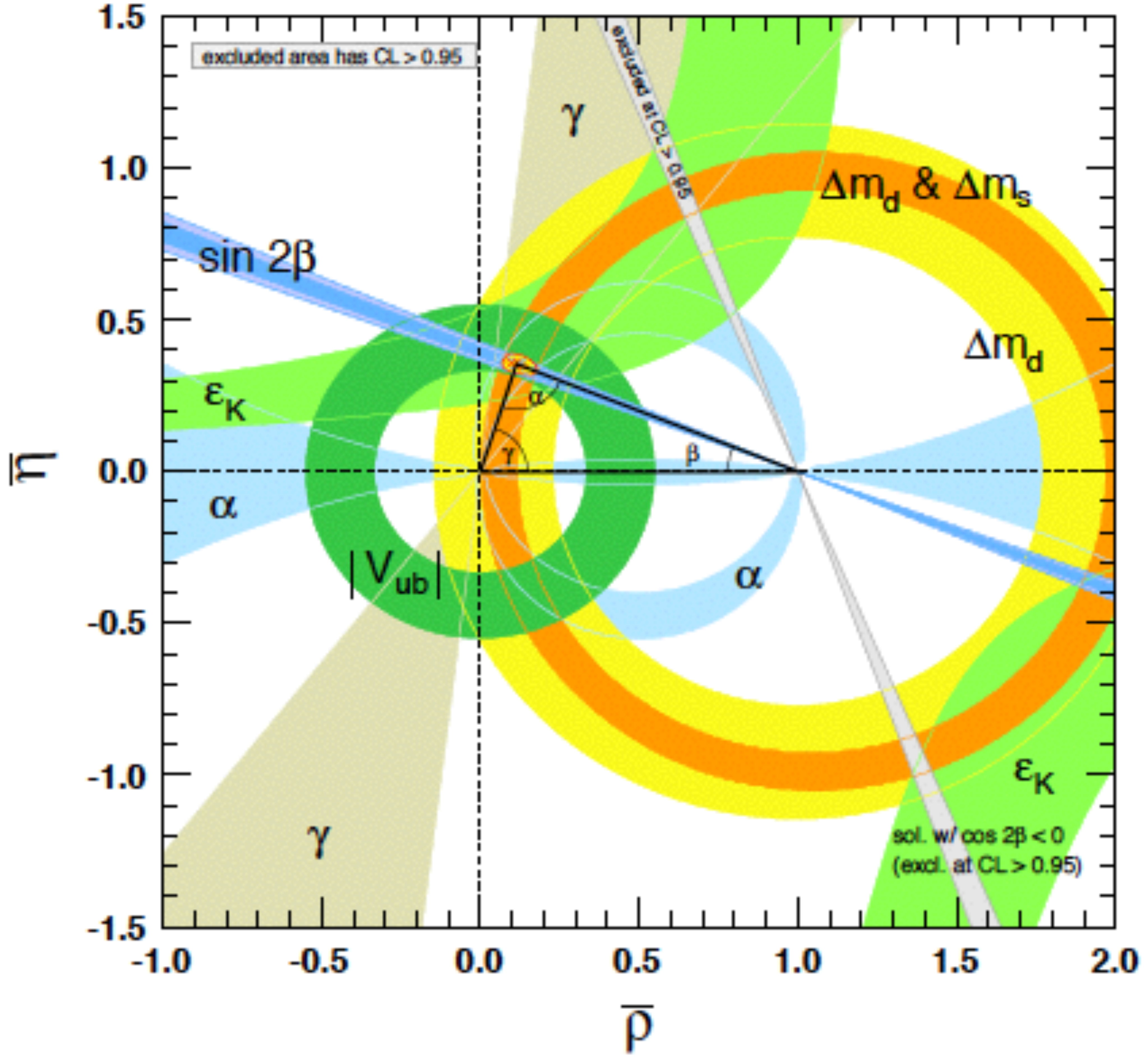


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

and the Jarlskog invariant is $J = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.

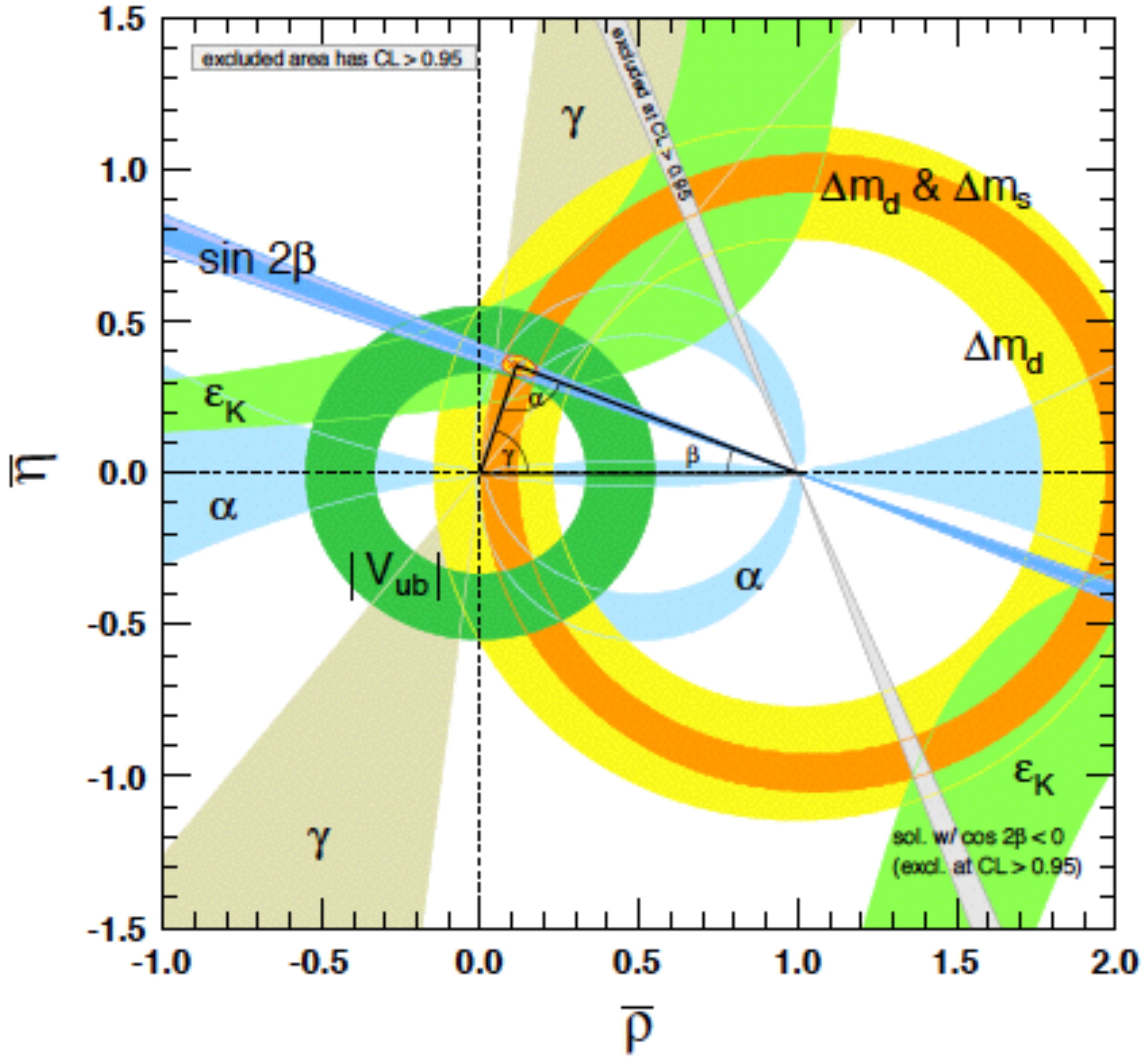


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

and the Jarlskog invariant is $J = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.

This is PDG compilation. α or ϕ_2 is our δ.

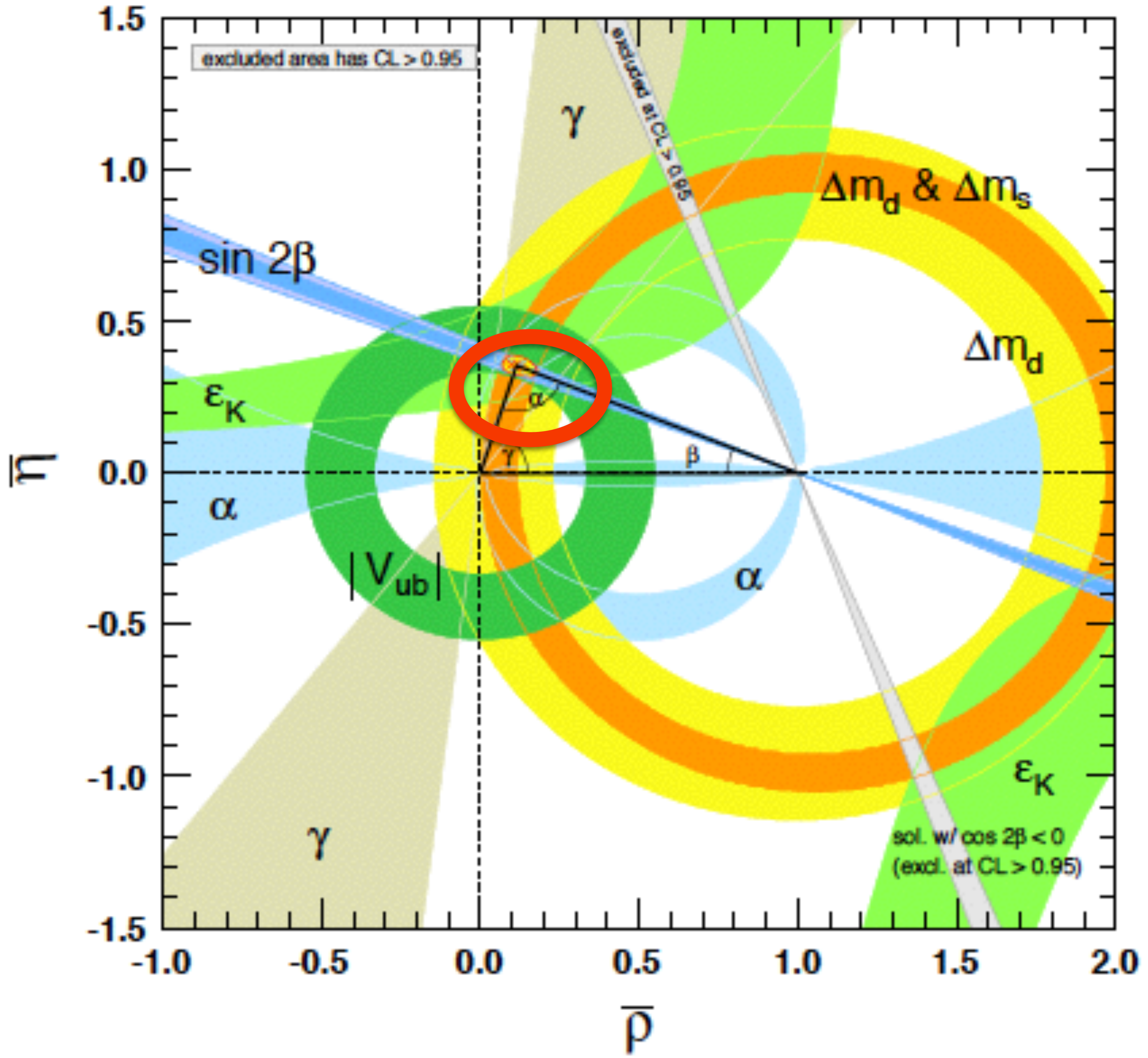


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

and the Jarlskog invariant is $J = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.

This is PDG compilation. α or ϕ_2 is our δ.

PDG determines

Combining the $B \to \pi\pi$, $\rho\pi$, and $\rho\rho$ decay modes [105], α is constrained as

$$\alpha = (85.4^{+3.9}_{-3.8})^\circ.$$

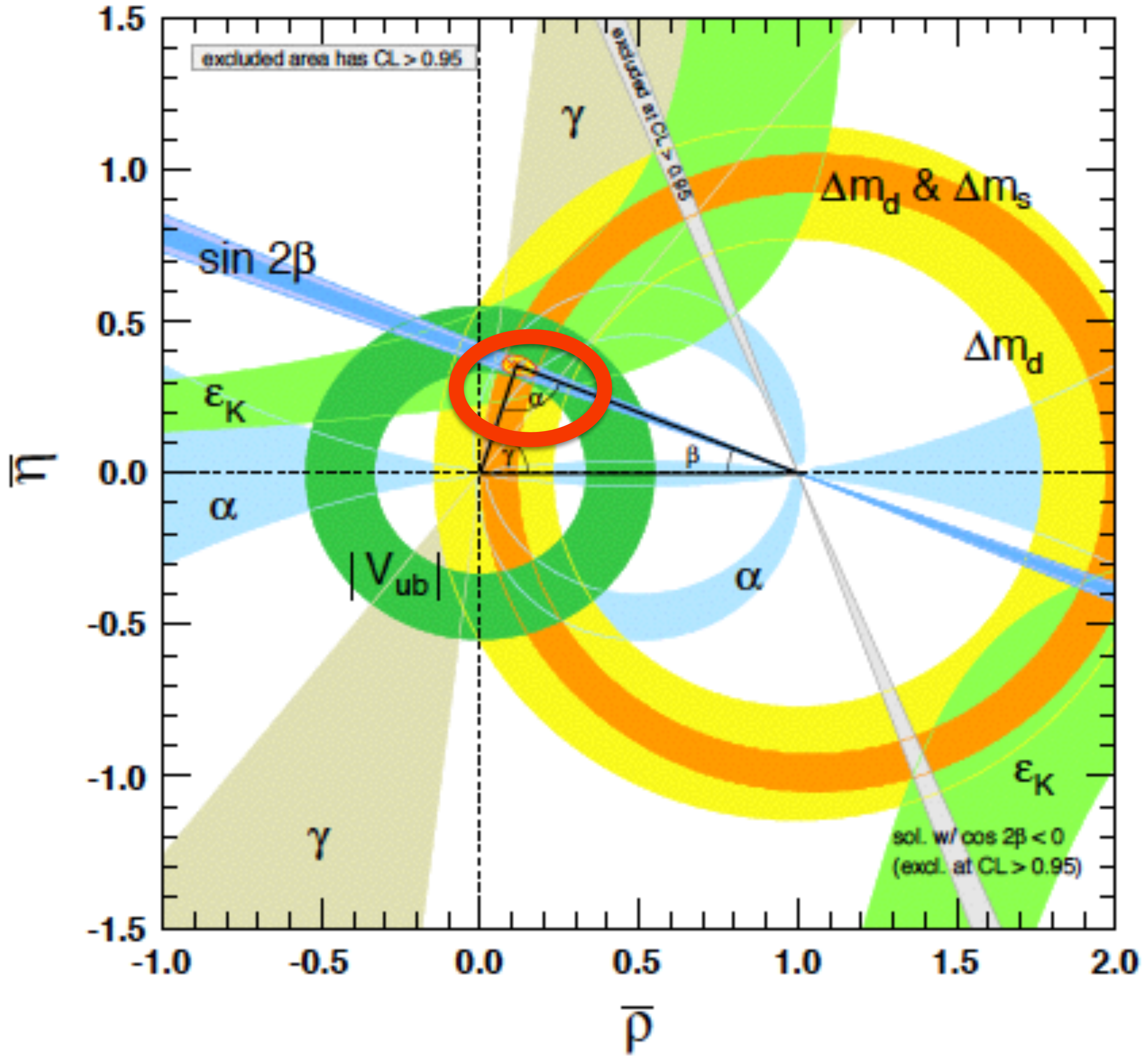


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

and the Jarlskog invariant is $J = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.

This is PDG compilation. α or ϕ_2 is our δ.

PDG determines

Combining the $B \to \pi\pi$, $\rho\pi$, and $\rho\rho$ decay modes [105], α is constrained as

$$\alpha = (85.4^{+3.9}_{-3.8})^\circ.$$

This implies that the weak CP violation in the quark sector is almost maximal with some real angles fixed. Here, the parametrization must allow 90 degrees.

In the quark sector, we can consider the leading CP violation term is maximal !! Also, simple in formulae. Further corrections may lower the value a bit.

Since we know the weak CP phase, the final state interaction phase can be estimated. D. Y. Mo has already talked about this: the first try in particle physics to calculate the phase shift analysis problem in quantum mechanics. The order is about -180(Delta I=1/2), and 27(penguin) degrees.

Maximal CP violation in lepton sector?

Maximal CP violation in lepton sector?

In the recent T2K experiment [Y. Oyama at Planck 2015, A. Fiorentini here], $delta_{PMNS}$ seems to be +- 90 degrees at 2 sigma level. Whether it is true or not, it is worthwhile to ask a question on it. See also, D.V. Forero-M. Tortola-J. Valle, arXiv:1405.7540.

Determination of $delta_{PMNS}$ may choose $delta_{CKM}$ in certain models.

3. Unification GUT Families

15 +1 chiral fields are grouped into

$$\begin{pmatrix} & \vdots & u & \vdots & \\ u^c & \vdots & & \vdots & e^+ \\ & \vdots & d & \vdots & \end{pmatrix}_L , \quad \begin{pmatrix} d^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L , \quad N_L^0$$

15 +1 chiral fields are grouped into

$$\begin{pmatrix} u^c & \begin{array}{|c|} u \\ \\ d \end{array} & e^+ \end{pmatrix}_L , \quad \begin{pmatrix} d^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L , \quad N_L^0$$

↑

GG model

15 +1 chiral fields are grouped into

$$\begin{pmatrix} u^c & \begin{array}{|c|} u \\ d \end{array} & e^+ \end{pmatrix}_L , \begin{pmatrix} d^c \\ \hline \nu_e \\ e \end{pmatrix}_L , N^0_L$$

GG model

$$\begin{pmatrix} d^c & \begin{array}{|c|} u \\ d \end{array} & N^0 \end{pmatrix}_L , \begin{pmatrix} u^c \\ \hline \nu_e \\ e \end{pmatrix}_L , e^+_L$$

Anti-SU(5) model

15 +1 chiral fields are grouped into

$$\begin{pmatrix} u^c & \begin{array}{c} u \\ d \end{array} & e^+ \end{pmatrix}_L, \quad \begin{pmatrix} d^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L, \quad N_L^0$$

$$\begin{pmatrix} d^c & \begin{array}{c} u \\ d \end{array} & N^0 \end{pmatrix}_L, \quad \begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L, \quad e_L^+$$

GG model

Anti-SU(5) model

N(N-1)/2+ N chiral fields are grouped into

$$[2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & {\color{red}\alpha_{45}} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right) \qquad [1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}$$

N(N-1)/2+ N chiral fields are grouped into

$$[2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}$$

↑ Anti-SU(N) model

N(N-1)/2+ N chiral fields are grouped into

$$[2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}$$

Anti-SU(N) model

N(N-1)/2+ N chiral fields are grouped into

$$[2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ . & . & . & . & . & . & . \\ . & . & . & \alpha_{45} & . & . & . \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ . & . & . & . & . & . & . \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ . \\ . \\ . \\ f_N \end{pmatrix}$$

Anti-SU(N) model

First example:
JEK, PRL 45,
1916 (1980)

An early example is a spinor of SO(14)

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

An early example is a spinor of SO(14)

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

- One SU(5) family

An early example is a spinor of SO(14)

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

- One SU(5) family
- Two SU(5) anti-family, and one SU(5) family

An early example is a spinor of SO(14)

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

- One SU(5) family
- Two SU(5) anti-family, and one SU(5) family
- Net result: 0 SU(5) family

Used these in JEK, PRL 45, 1916 (1980):
shifted hypercharges, for 2 SM q's & 3 l's

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \tau^+ \\ \overline{\nu}_\tau \end{pmatrix}_R , \begin{pmatrix} L^- \\ L^{--} \end{pmatrix}_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} Q^{5/3} \\ t \end{pmatrix}_R , \begin{pmatrix} b \\ Q^{-4/3} \end{pmatrix}_R$$

Used these in JEK, PRL 45, 1916 (1980):
shifted hypercharges, for 2 SM q's & 3 l's

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \tau^+ \\ \overline{\nu}_\tau \end{pmatrix}_R, \begin{pmatrix} L^- \\ L^{--} \end{pmatrix}_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} Q^{5/3} \\ t \end{pmatrix}_R, \begin{pmatrix} b \\ Q^{-4/3} \end{pmatrix}_R$$

Used these in JEK, PRL 45, 1916 (1980):
shifted hypercharges, for 2 SM q's & 3 l's

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \tau^+ \\ \overline{\nu}_\tau \end{pmatrix}_R, \begin{pmatrix} L^- \\ L^{--} \end{pmatrix}_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} Q^{5/3} \\ t \end{pmatrix}_R, \begin{pmatrix} b \\ Q^{-4/3} \end{pmatrix}_R$$

t can decay to b by scalar exchange, but not by W exchange, and

$$\sin^2\theta_W = \frac{3}{20}$$

not 3/8.

- Deadend of SO(4N+2).

- Family unification in SU(N):
  Georgi (1979): SU(11) model

We want to have 3 left-handed families

We want to have 3 left-handed families

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ,$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L$$

We want to have 3 left-handed families

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ,$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L$$

and

$$\sin^2 \theta_W = \frac{3}{8}$$

We want to have 3 left-handed families

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L,$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

and

$$\sin^2\theta_W = \frac{3}{8}$$

Family unified GUTs,
Unification of GUT families (UGUTF)

$$\mathrm{SU}(5): [2] \to n_f = 1$$
$$\mathrm{SU}(6): [3] \to n_f = 0,\ [2] \to n_f = 1$$
$$\mathrm{SU}(7): [3] \to n_f = 1,\ [2] \to n_f = 1$$
$$\mathrm{SU}(8): [4] \to n_f = 0,\ [3] \to n_f = 2,\ [2] \to n_f = 1$$
$$\mathrm{SU}(9): [4] \to n_f = 5,\ [3] \to n_f = 3,\ [2] \to n_f = 1$$
$$\mathrm{SU}(11): [5] \to n_f = -5,\ [4] \to n_f = 9,\ [3] \to n_f = 5,\ [2] \to n_f = 1$$

The anomaly units in SU(N) are

$$\mathcal{A}([m]) = \frac{(N-3)!(N-2m)}{(N-m-1)!(m-1)!},$$
$$\mathcal{A}([1]) = 1,\ \mathcal{A}([2]) = N-4,\ \mathcal{A}([3]) = \frac{(N-3)(N-6)}{2},\ \text{etc.}$$

$\mathrm{SU}(5) : [2] \to n_f = 1$

$\mathrm{SU}(6) : [3] \to n_f = 0,\ [2] \to n_f = 1$

$\mathrm{SU}(7) : [3] \to n_f = 1,\ [2] \to n_f = 1$

$\mathrm{SU}(8) : [4] \to n_f = 0,\ [3] \to n_f = 2,\ [2] \to n_f = 1$

$\mathrm{SU}(9) : [4] \to n_f = 5,\ [3] \to n_f = 3,\ [2] \to n_f = 1$

$\mathrm{SU}(11) : [5] \to n_f = -5,\ [4] \to n_f = 9,\ [3] \to n_f = 5,\ [2] \to n_f = 1$

The anomaly units in $\mathrm{SU}(N)$ are

$$\mathcal{A}([m]) = \frac{(N-3)!(N-2m)}{(N-m-1)!(m-1)!},$$

$$\mathcal{A}([1]) = 1,\ \mathcal{A}([2]) = N-4,\ \mathcal{A}([3]) = \frac{(N-3)(N-6)}{2}, \text{ etc.}$$

- The simplest case is SU(7) with

$$\mathrm{SU}(7)\ :\ [3]\oplus 2\,[2]\oplus 8\,[\bar{1}]\oplus n_1([1]\oplus[\bar{1}])\oplus n_2([2]\oplus[\bar{2}])+\cdots\,.$$

- The simplest case is SU(7) with

$$\mathrm{SU}(7)\ :\ [3]\oplus 2\,[2]\oplus 8\,[\bar{1}]\oplus n_1([1]\oplus[\bar{1}])\oplus n_2([2]\oplus[\bar{2}])+\cdots .$$

- For example, SU(8) with

$$\mathrm{SU}(8)\ :\ [3]\oplus[2]\oplus 9\,[\bar{1}]\oplus n_1([1]\oplus[\bar{1}])\oplus n_2([2]\oplus[\bar{2}])+\cdots .$$

contains more non-singlet fields.

After the heterotic string compactification, this field theoretic models were not considered much. String compactification contains the GUT breaking mechanism intrinsically.
But, the weak mixing angle problem is serious and it is better to have a GUT with

$$\sin^2\theta_W = \frac{3}{8}$$

After the heterotic string compactification, this field theoretic models were not considered much. String compactification contains the GUT breaking mechanism intrinsically.

But, the weak mixing angle problem is serious and it is better to have a GUT with

$$\sin^2\theta_W = \frac{3}{8}$$

Can we succeed in finding a UGUTF from string?

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2}\right)$$

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2}\right)$$

$$N = 3(\frac{-1}{3})^2 + 2(\frac{+1}{2})^2 = \frac{5}{6} \to 2N = \frac{5}{3}$$

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3},\frac{+1}{2},\frac{+1}{2}\right)$$

$$N = 3(\frac{-1}{3})^2 + 2(\frac{+1}{2})^2 = \frac{5}{6} \to 2N = \frac{5}{3}$$

$$\sin^2\theta_W = \frac{\tilde{g}'^{\,2}}{\tilde{g}^2+\tilde{g}'^{\,2}} = \frac{\frac{1}{2N}}{1+\frac{1}{2N}} \to \frac{3}{8}$$

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2}\right)$$

$$N = 3(\frac{-1}{3})^2 + 2(\frac{+1}{2})^2 = \frac{5}{6} \to 2N = \frac{5}{3}$$

$$\sin^2\theta_W = \frac{\tilde{g}'^2}{\tilde{g}^2 + \tilde{g}'^2} = \frac{\frac{1}{2N}}{1 + \frac{1}{2N}} \to \frac{3}{8}$$

SM is SO(10) subgroup with intermediate SU(5): Then,

$$\sin^2\theta_W = \frac{3}{8}$$

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2}\right)$$

$$N = 3(\frac{-1}{3})^2 + 2(\frac{+1}{2})^2 = \frac{5}{6} \to 2N = \frac{5}{3}$$

$$\sin^2\theta_W = \frac{\tilde{g}'^2}{\tilde{g}^2 + \tilde{g}'^2} = \frac{\frac{1}{2N}}{1 + \frac{1}{2N}} \to \frac{3}{8}$$

SM is SO(10) subgroup with intermediate SU(5): Then,

$$\sin^2\theta_W = \frac{3}{8}$$

This is true even for the flipped SU(5) if extra U(1) coupling is the same as that of SU(5).

Georgi-Quinn-Weinberg expression is

$$\sin^2\theta_W = \frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}\, Q_{\rm em}^2}$$

Georgi-Quinn-Weinberg expression is

$$\sin^2\theta_W = \frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}\, Q_{\mathrm{em}}^2}$$

Since there is no more funny particles beyond 16 of SO(10),

$$\sin^2\theta_W = \frac{3}{8}$$

Georgi-Quinn-Weinberg expression is

$$\sin^2\theta_W = \frac{\mathrm{Tr}\,T_3^2}{\mathrm{Tr}\,Q_{\rm em}^2}$$

Since there is no more funny particles beyond 16 of SO(10),

$$\sin^2\theta_W = \frac{3}{8}$$

UGUTF is the one for an acceptable weak mixing angle.

In early SM-like construction [Ibanez-Kim-Nilles-Quevedo(1987), Casas-Munoz(1988)], where the weak mixing angle problem could not be resolved. Only if GUT is somehow working at the compactification scale, then an appropriate weak mixing angle can be obtained.

Flipped SU(5) from heterotic string: JEK-Kyae,

Antoniadis-Ellis-Hagelin-Nanopoulos (1988)

Large extra dimensions: DESY group, Buchmueller et al.

In early SM-like construction [Ibanez-Kim-Nilles-Quevedo(1987), Casas-Munoz(1988)], where the weak mixing angle problem could not be resolved. Only if GUT is somehow working at the compactification scale, then an appropriate weak mixing angle can be obtained.

Flipped SU(5) from heterotic string: JEK-Kyae,
Antoniadis-Ellis-Hagelin-Nanopoulos (1988)

Large extra dimensions: DESY group, Buchmueller et al.

Standard models from non-prime orbifolds:
- Many papers by the Bonn-DESY-Ohio group
- JEK-Ji-hun Kim-Kyae

But these were not family unification models.

If we find different SO(10) subgroups at different fixed points, then there is a possibility that the weak mixing angle is 3/8. But it is not so obvious to me.

GUTs containing SU(5) is an automatic solution to the weak mixing angle problem.

In addition we want to unify families a la Georgi. So far, there has not been any model, from string, on the unification of GUT families.

Here, we must resolve the

doublet-triplet splitting problem.

existence of GUT Higgs to break the GUT group down to the SM.

bonus: simpifies fermion mass matrix testure

In addition we want to unify families a la Georgi. So far, there has not been any model, from string, on the unification of GUT families.

Here, we must resolve the

doublet-triplet splitting problem.

existence of GUT Higgs to break

the GUT group down to the SM.

bonus: simpifies fermion mass matrix testure

Compactification for UGUTF from heterotic string:

SM gauge group can be studied

with applicable phenomenologies

Unresolved issue: moduli stabilization: this may be found in other method.

Anti-SU(N)

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \color{red}{\alpha_{45}} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

Anti-SU(N)

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

Anti-SU(N)

N

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

Anti-SU(N)

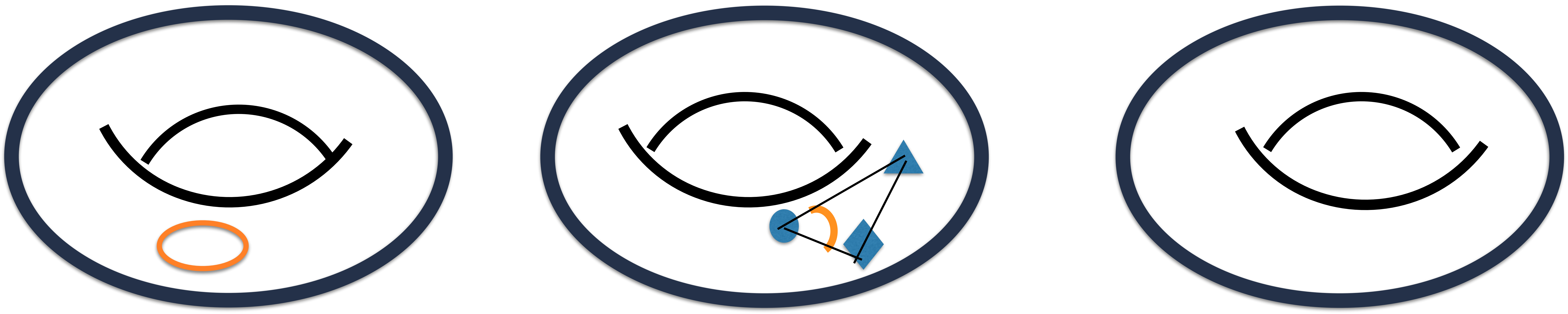
N

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

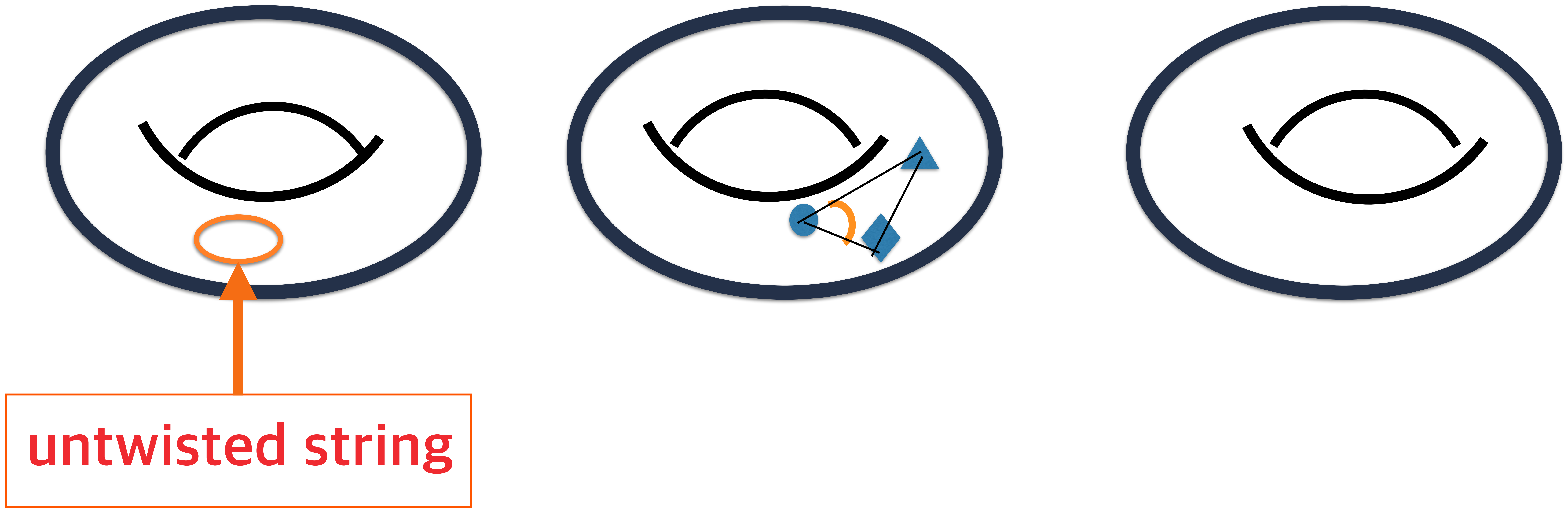
First paper: JEK, PRL45, 1916 (1980).

Flipped-SU(5): S. M. Barr, PLB 112, 219 (1982),
J. Derendinger, JEK, D. Nanopoulos, PLB139, 170 (1984).

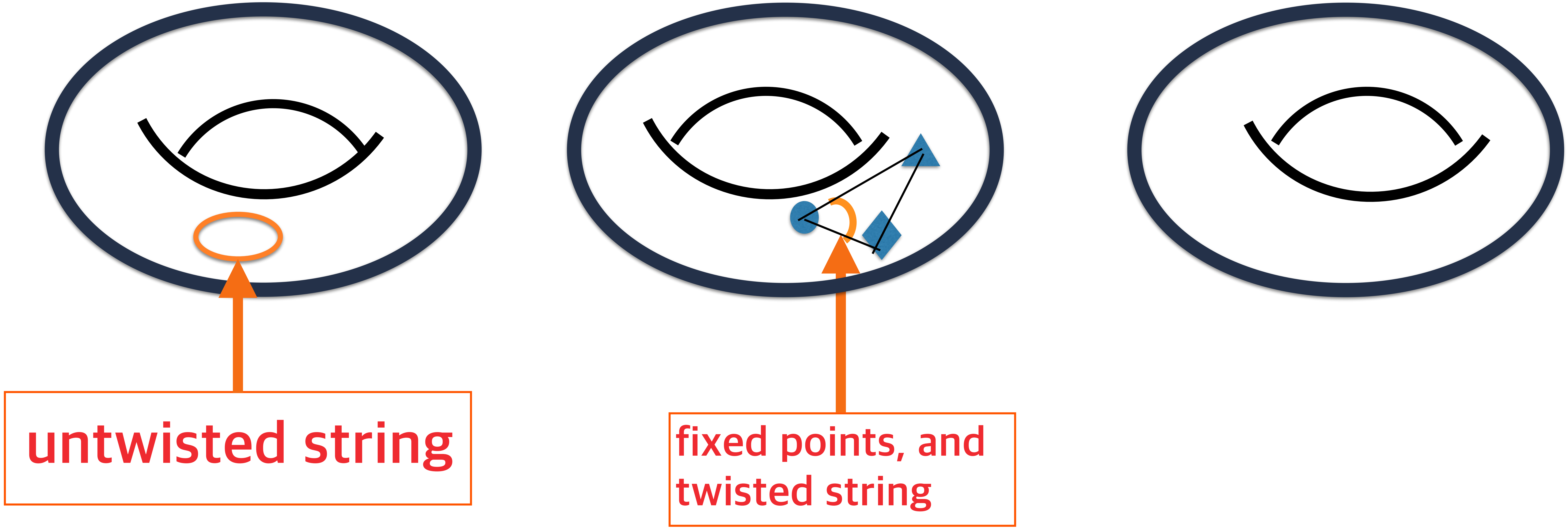
Three two-tori:



Three two-tori:



Three two-tori:



Z(12-I) model. Representation 35 is possible only in U.

U_i	Number of $\mathbf{10}$ s	Tensor form	Chirality	$[p_{\rm spin}]$ $(p_{\rm spin}\cdot\phi_s)$
$U_1\,(p\cdot V=\frac{5}{12})$	1	$\Psi^{[ABC]}$	R	$[\oplus;+++]$ $(\frac{+5}{12})$
$U_2\,(p\cdot V=\frac{4}{12})$	3	$\Psi^{[ABC]}$	L	$[\ominus;++-]$ $(\frac{+4}{12})$
$U_3\,(p\cdot V=\frac{1}{12})$	1	$\Psi^{[ABC]}$	L	$[\ominus;+-+]$ $(\frac{+1}{12})$

Z(12-I) model. Representation 35 is possible only in U.

U_i	Number of $\mathbf{10}$ s	Tensor form	Chirality	$[p_{\rm spin}]$ $(p_{\rm spin}\cdot\phi_s)$
$U_1\,(p\cdot V=\frac{5}{12})$	1	$\Psi^{[ABC]}$	R	$[\oplus;++\,+]$ $(\frac{+5}{12})$
$U_2\,(p\cdot V=\frac{4}{12})$	3	$\Psi^{[ABC]}$	L	$[\ominus;++-]$ $(\frac{+4}{12})$
$U_3\,(p\cdot V=\frac{1}{12})$	1	$\Psi^{[ABC]}$	L	$[\ominus;+-+]$ $(\frac{+1}{12})$

Z(12-I) model. Representation 35 is possible only in U.

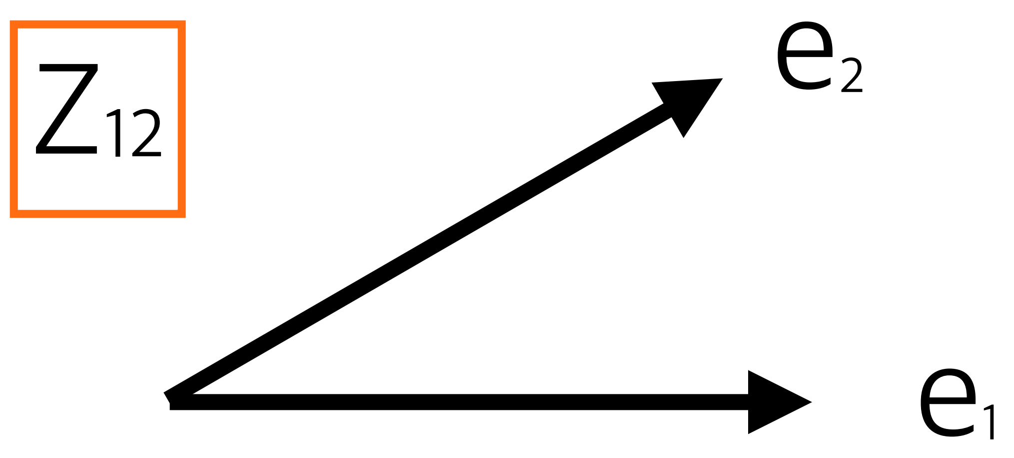
U_i	Number of $\mathbf{10}$ s	Tensor form	Chirality	$[p_{\rm spin}]$ $(p_{\rm spin}\cdot\phi_s)$
$U_1\,(p\cdot V=\frac{5}{12})$	1	$\Psi^{[ABC]}$	R	$[\oplus;++ +]$ $(\frac{+5}{12})$
$U_2\,(p\cdot V=\frac{4}{12})$	3	$\Psi^{[ABC]}$	L	$[\ominus;++-]$ $(\frac{+4}{12})$
$U_3\,(p\cdot V=\frac{1}{12})$	1	$\Psi^{[ABC]}$	L	$[\ominus;+-+]$ $(\frac{+1}{12})$

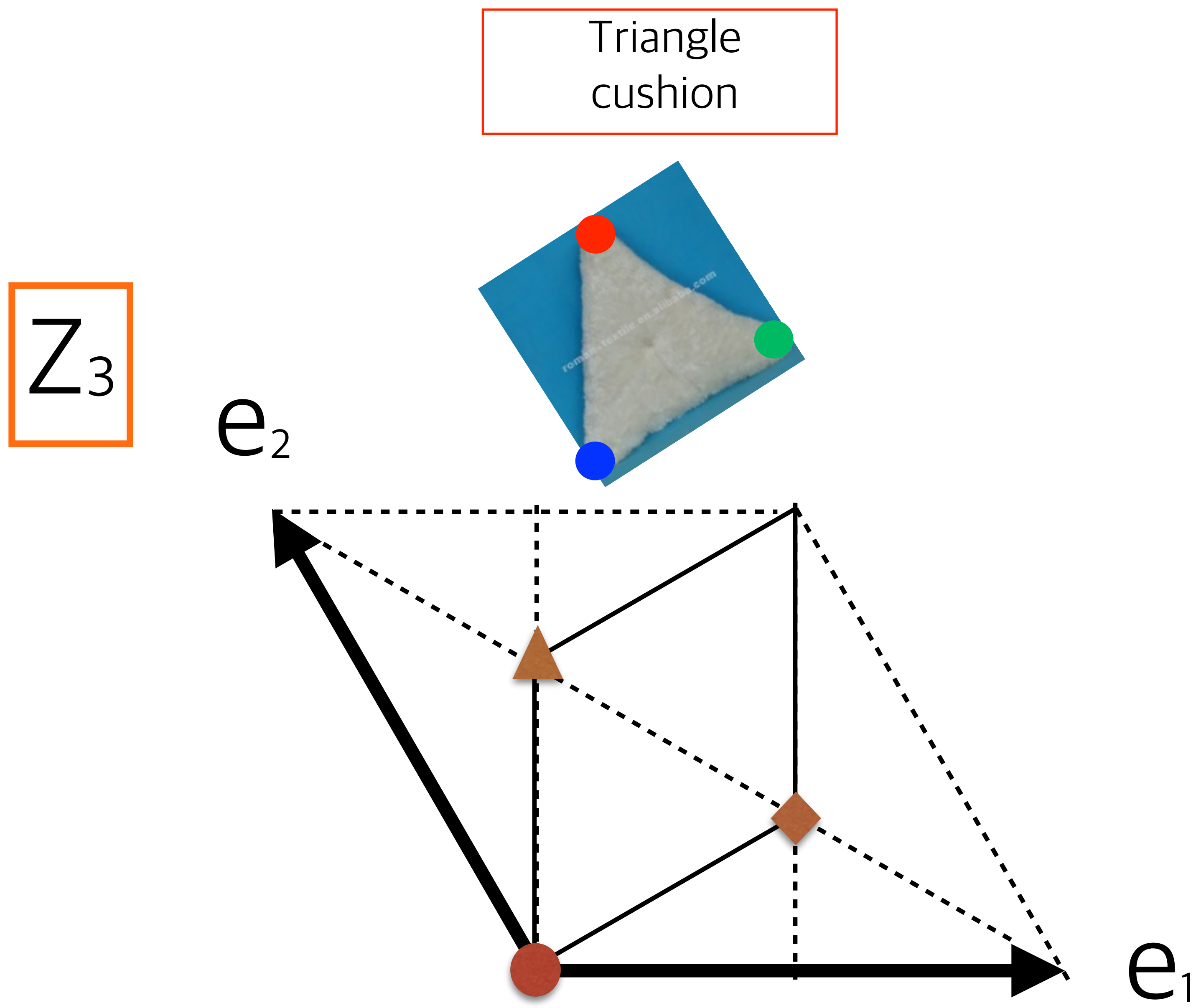
We require

Matter representation $\Psi^{[\rm ABC]}$ $({\rm A}=1,2,\cdots,7)$ must be present in the untwisted se

Matter $\Psi^{[\rm AB]}$ must not appear in the untwisted sector.

Matter $\Psi^{[\rm AB]}$ must be present in a twisted sector with the chirality that of $\Psi^{[\rm ABC]}$

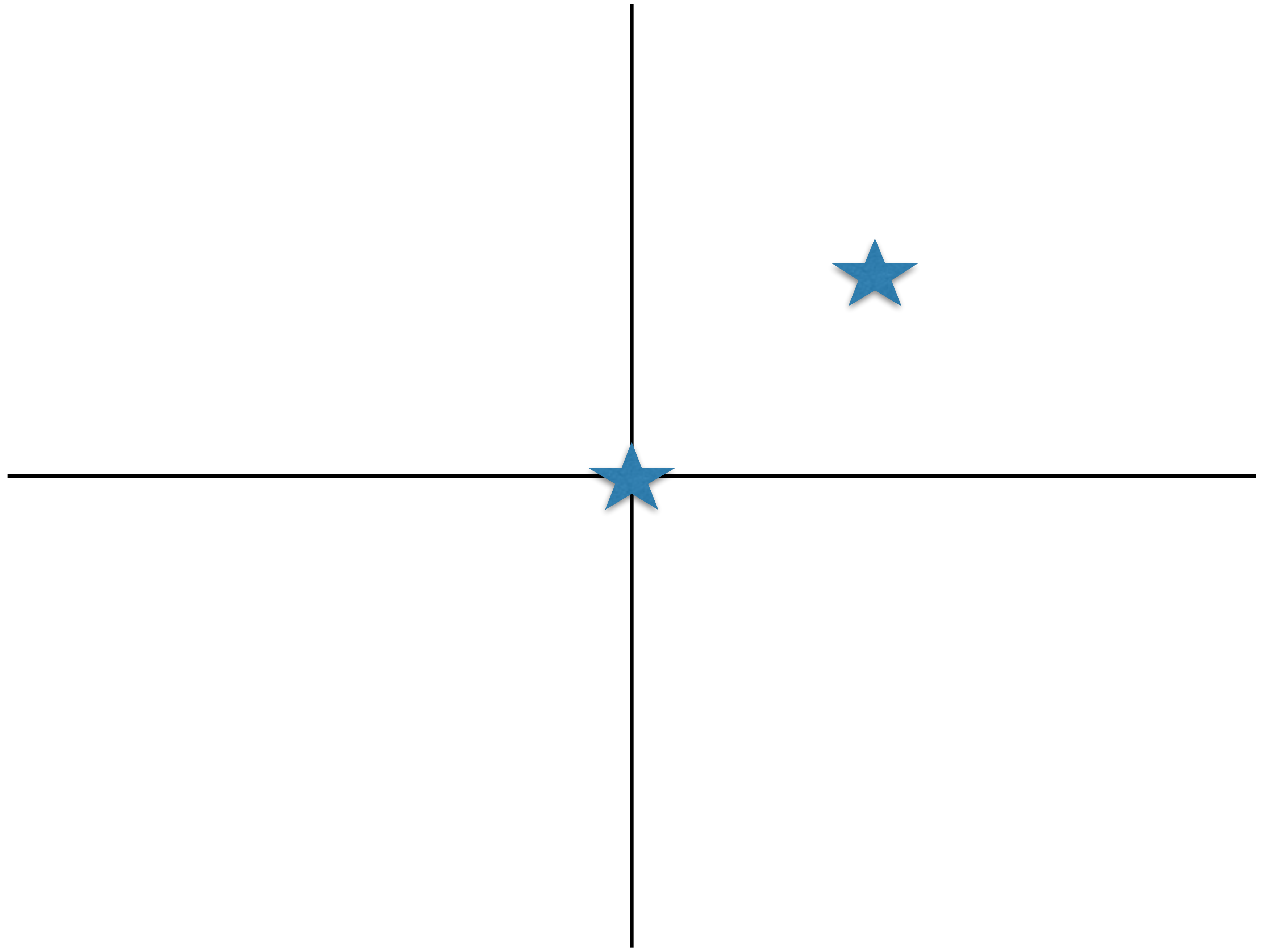




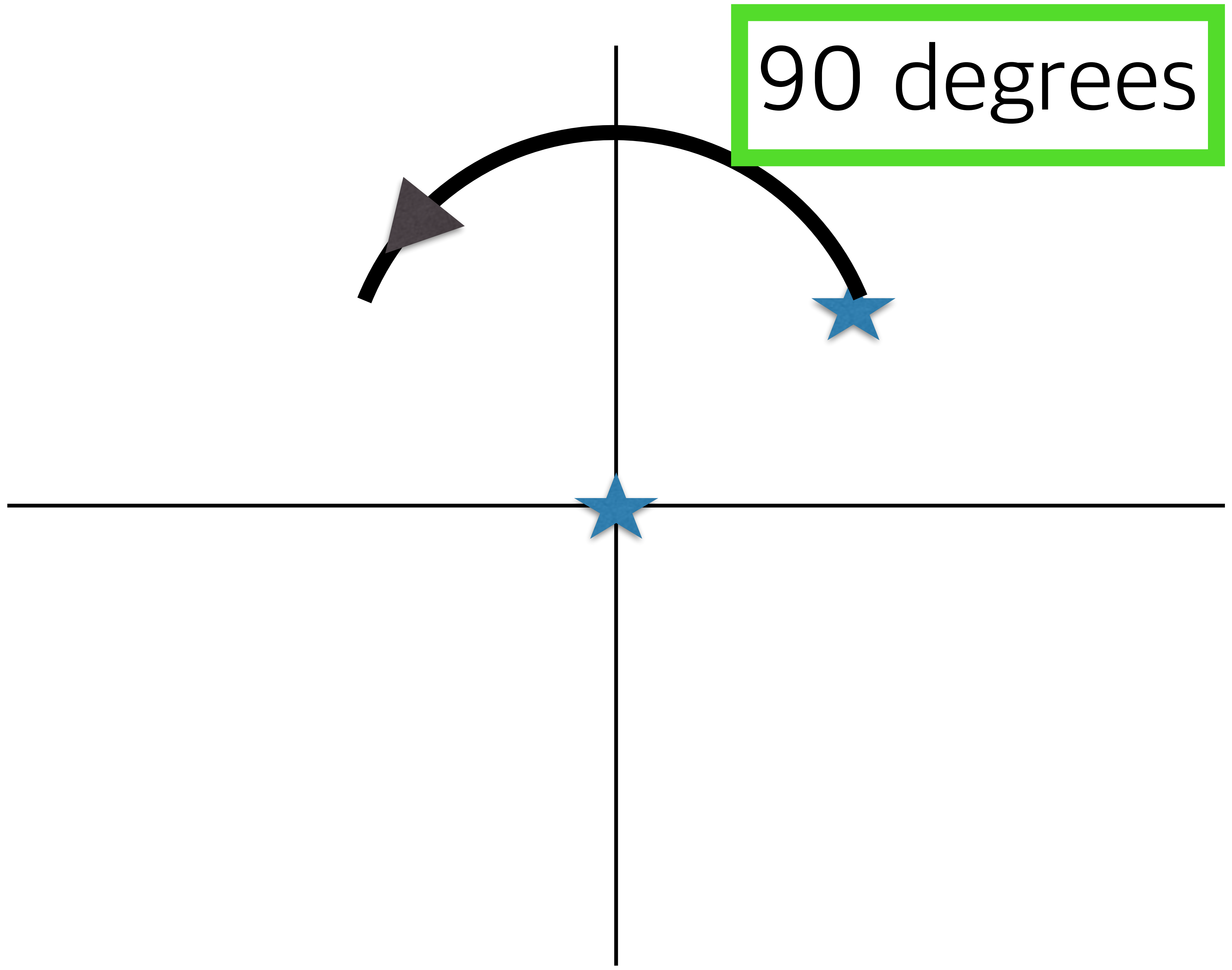
$$a_3 = \left(\frac{n_1}{3}\ \frac{n_2}{3}\ \cdots\right)\left(\frac{n'_1}{3}\ \frac{n'_2}{3}\ \cdots\right)'$$

So, $3a_3$ contains integers.
No Wilson line effect at
T3, and T6.

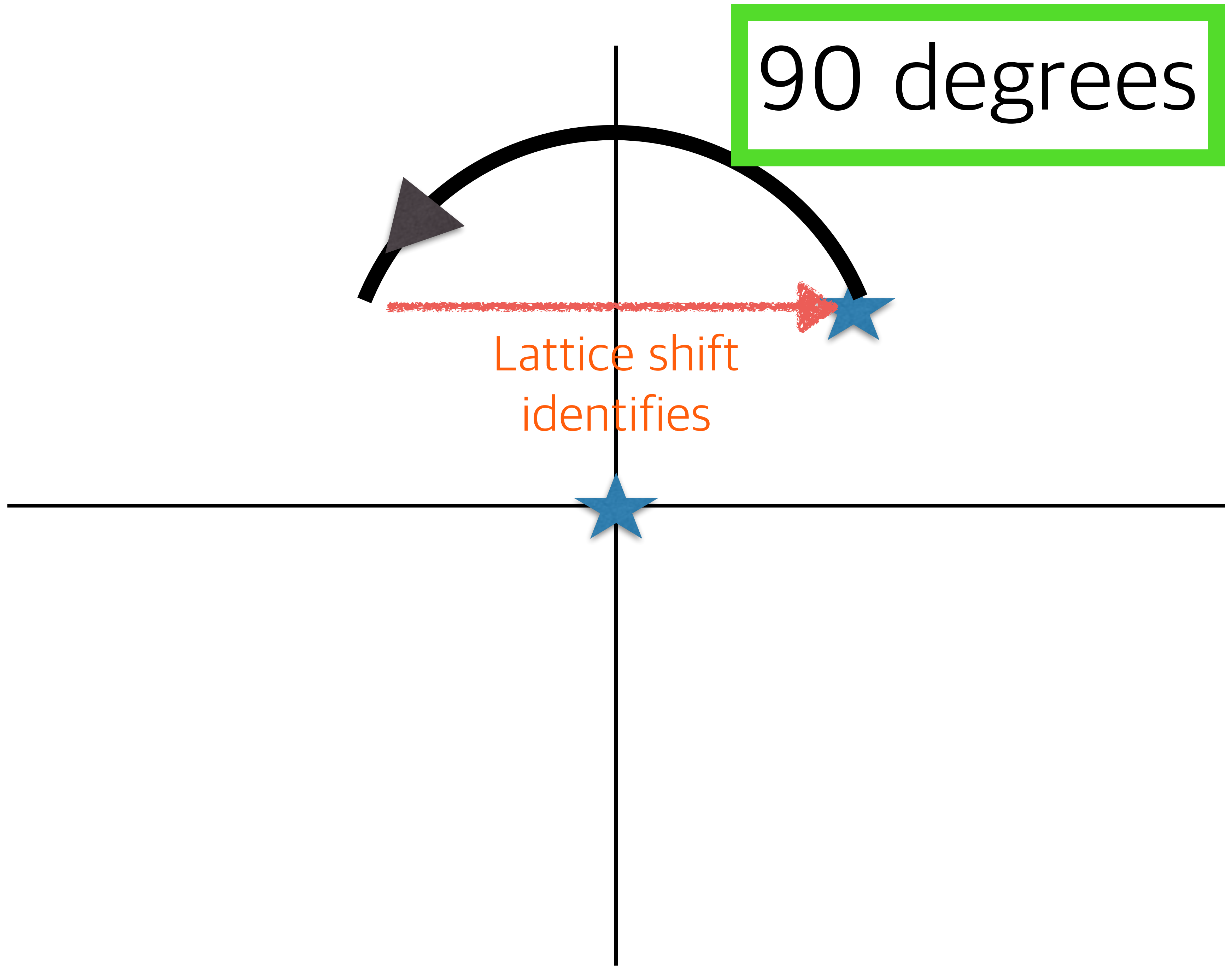
At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of 1/3. Z(12-I) has numbers of multiples of 1/12. So, numbers in 3V are multiples of 1/4.
At two-dimensional torus, Z4 has multiplicity 2.



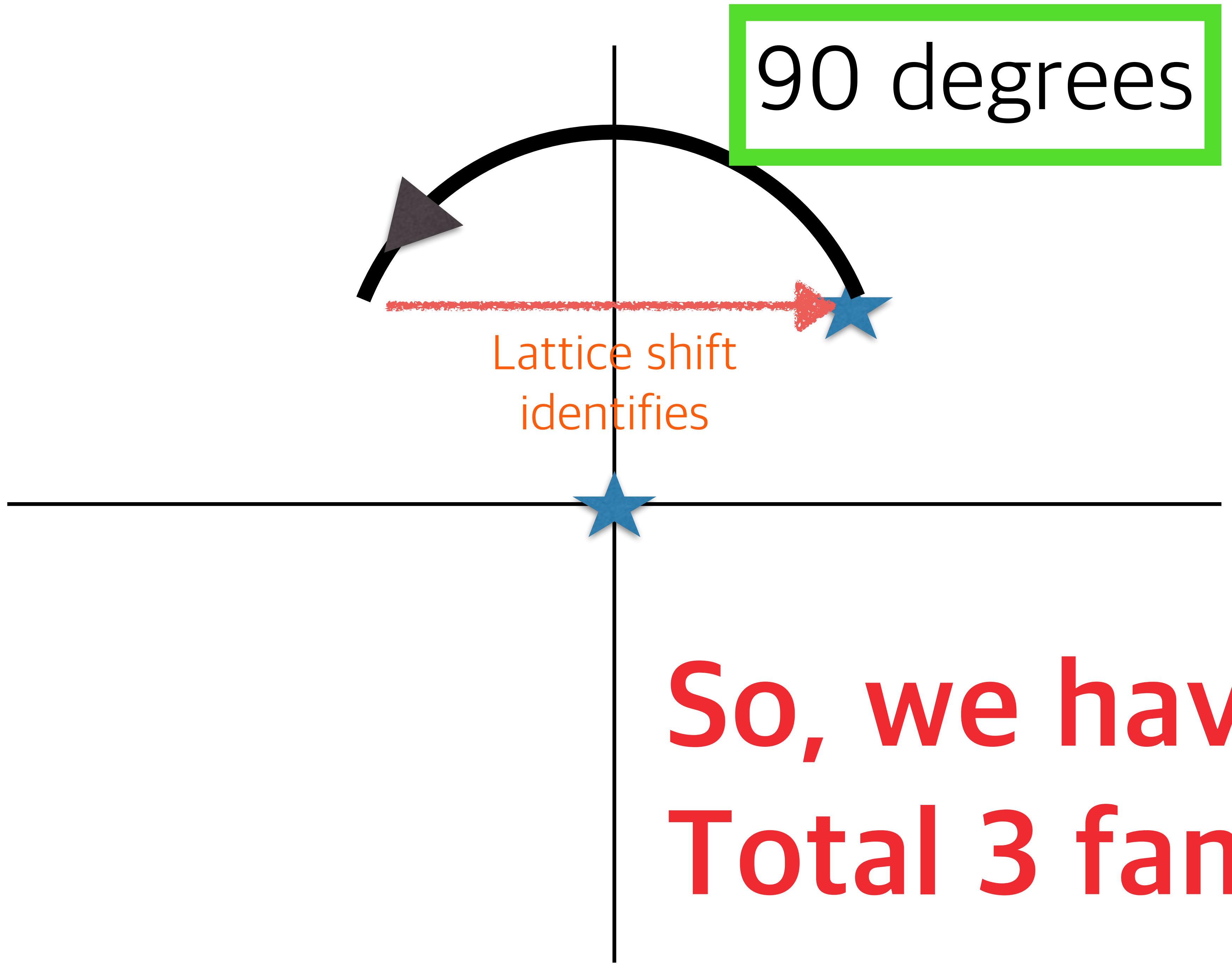
At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of 1/3. Z(12-I) has numbers of multiples of 1/12. So, numbers in 3V are multiples of 1/4.
At two-dimensional torus, Z4 has multiplicity 2.



At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of 1/3. Z(12-I) has numbers of multiples of 1/12. So, numbers in 3V are multiples of 1/4.
At two-dimensional torus, Z4 has multiplicity 2.



At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of 1/3. Z(12-I) has numbers of multiples of 1/12. So, numbers in 3V are multiples of 1/4.
At two-dimensional torus, Z4 has multiplicity 2.



So, we have two $\Psi^{[AB]}$ from T3.
Total 3 families.

	$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\frac{3}{4}\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1\,0\,0\,0};0\,0\,\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

A=2

A=6

	$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\frac{3}{4}\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

		$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
A=2	(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-\,-\,-\,-\,+\,+\,+};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
A=6	(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\tfrac{3}{4}\,\tfrac{3}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}};\tfrac{1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-8	(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-1	(d)	$\Psi_{[A]R}$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
A=1	(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\tfrac{1}{2},\tfrac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
	(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
	(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\tfrac{1}{2}\,\tfrac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
	(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
					$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
	(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-\,+\,+\,+};-\,+\,+\,+)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
	(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
	(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
	(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
	(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
	(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
	(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
	(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1\,0\,0\,0};0\,0\,\tfrac{-1}{2}\,\tfrac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
					$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

		$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
A=2	(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
A=6	(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\frac{3}{4}\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-8	(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-1	(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
A=1	(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
	(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
	(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-10^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
	(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
					$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
	(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
	(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
	(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
	(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
	(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
	(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
	(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
	(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
					$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

2 from here

		$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
A=2	(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
A=6	(b)	$2\,\Psi^{[A]}$	T_3	$\left(\underline{\tfrac{3}{4}\,\tfrac{3}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}};\tfrac{1}{4}\right)\,(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-8	(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)\,(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
A=-1	(d)	$\Psi_{[A]R}$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
A=1	(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\tfrac{1}{2},\tfrac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
	(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)\,(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
	(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\tfrac{1}{2}\,\tfrac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
	(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
					$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
	(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
	(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
	(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
	(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
	(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
	(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
	(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
	(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\,\tfrac{-1}{2}\,\tfrac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
					$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

2 from here

1 from here

A=2

A=6

A=-8

A=-1

A=1

	$\mathcal{P}\times$(rep.)	[illegible]ect[illegible]	We[illegible]	V^k	$Q_{[illegible]}$	[illegible]$_2$	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-\,-\,-\,-+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	2 [illegible]$^{[A]}$	T_3	$\left(\underline{\text{[illegible]}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T[illegible]	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-++\,+};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus\,((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus\,((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1\,0\,0\,0};0\,0\,\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

2 from here

1 from here

A=2
A=6
A=-8
A=-1
A=1

	$\mathcal{P}\times$(rep.)	Sect[illegible]	W[illegible]	V^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{- - [illegible] - + + +}; +)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi^{[A]}$ [illegible]	T_3	$\left(\underline{[illegible]\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}}; \frac{1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T[illegible]	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}}; \frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}; \frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1, 0^6}; 0)\,(0^6, \frac{1}{2}, \frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R} + \Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}}; \frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})' \oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R} + \Phi_R^{[A]}\right)$	T_6	$(\underline{-1 0^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})' \oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R} + \Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}; \frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})' \oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{- + + +}; - + + +)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7; \frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}}; \frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12}; \frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7; \frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}}; 0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7; \frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}}; \frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R} + \Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus ((\frac{1}{12})^7; \frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}}; \frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12}; \frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R} + \Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7; \frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}}; 0\,\frac{1}{3}\,\frac{1}{3}\,0\right)' \oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R} + \Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus ((\frac{1}{12})^7; \frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}}; \frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R} + \Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1 0 0 0}; 0 0\,\frac{-1}{2}\,\frac{-1}{2})' \oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

JEK, JHEP 1506 (2015) 114 [1503.03104]

t quark and missing partner mechanism

For Yukawa couplings, we use just the effective field theory approach. There may be other supp factors which are assumed to be O(1).

t quark Yukawa coupling is, from $\Psi^{[AB]}$: T3,

$$T_3^{\mathbf{21}} T_3^{\overline{\mathbf{7}}} T_{6,\mathrm{BEH}}^{\overline{\mathbf{7}}} \quad (t \text{ mass}).$$

θ'_2

13

9

4

For Yukawa couplings, we use just the effective field theory approach. There may be other supp factors which are assumed to be O(1).

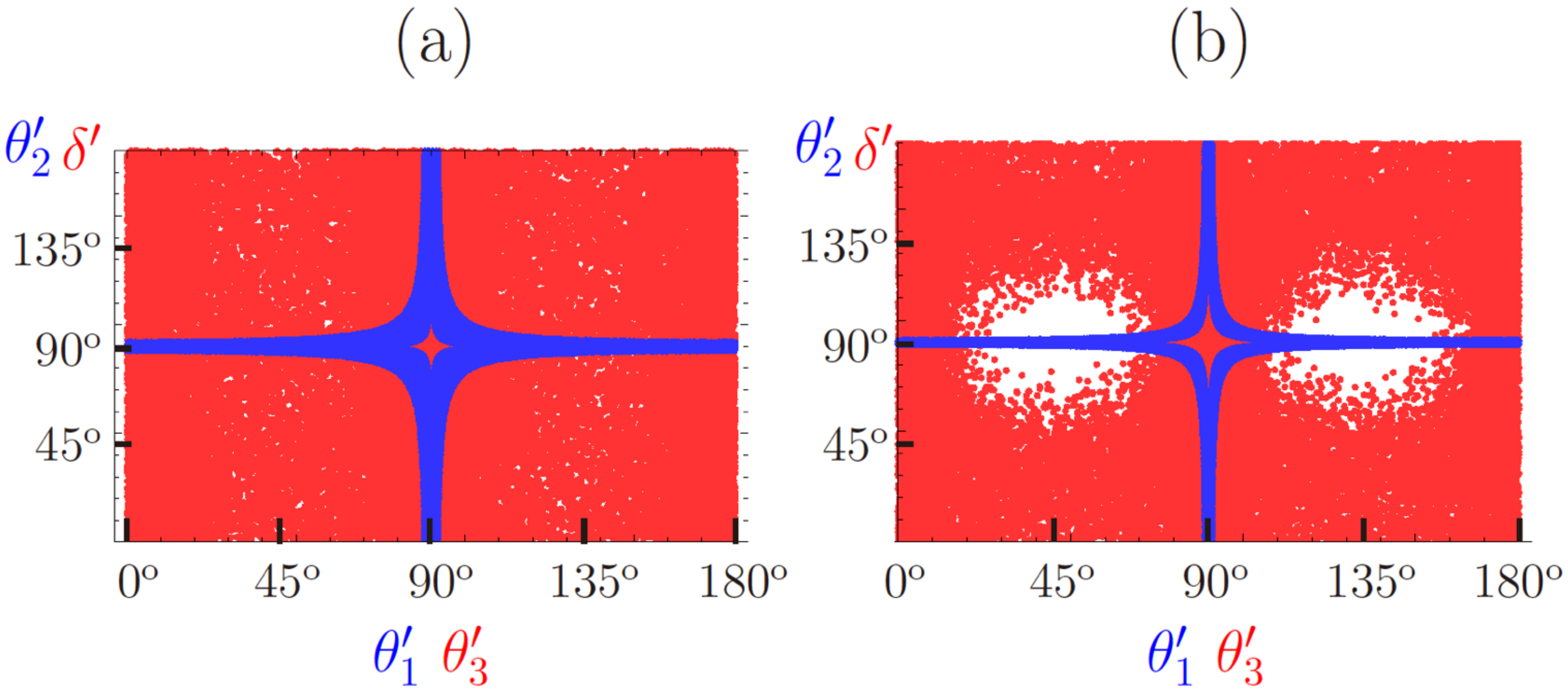
t quark Yukawa coupling is, from $\Psi^{[AB]}$: T3,

$$T_3^{\mathbf{21}} T_3^{\overline{\mathbf{7}}} T_{6,\mathrm{BEH}}^{\overline{\mathbf{7}}} \ (t \text{ mass}).$$

On the other hand, b quark Yukawa coupling is

$$\sim \frac{1}{M_s} T_3^{\mathbf{21}} T_3^{\mathbf{21}} T_{3,\mathrm{BEH}}^{\mathbf{21}} T_{3,\mathrm{BEH}}^{\mathbf{7}}.$$

θ_2'
13
9
4



JEK + D. Y. Mo + M-S. Seo, arXiv:1506.08984

Since our theory is a GUT, we must realize the doublet-triplet splitting. Some examples are

1) Kawamura's 5D SU(5) GUT with Z2 fixed points.
2) Dimopoulos-Georgi fine-tuned SU(5)

$$W = M_1\, \mathbf{5}^T_{\rm u,BEH}\mathbf{5}_{\rm d,BEH} + \mathbf{5}^T_{\rm u,BEH} \begin{pmatrix} v_c & 0 & 0 & 0 & 0 \\ 0 & v_c & 0 & 0 & 0 \\ 0 & 0 & v_c & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2}v_c & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2}v_c \end{pmatrix} \mathbf{5}_{\rm d,BEH}$$

$$\text{with adjoint BEH boson,}\ \ M_1 = \frac{3}{2}v_c.$$

Since our theory is a GUT, we must realize the doublet-triplet splitting. Some examples are

1) Kawamura's 5D SU(5) GUT with Z2 fixed points.
2) Dimopoulos-Georgi fine-tuned SU(5)

$$W = M_1\, \mathbf{5}^T_{\rm u,BEH} \mathbf{5}_{\rm d,BEH} + \mathbf{5}^T_{\rm u,BEH} \begin{pmatrix} v_c & 0 & 0 & 0 & 0 \\ 0 & v_c & 0 & 0 & 0 \\ 0 & 0 & v_c & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2}v_c & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2}v_c \end{pmatrix} \mathbf{5}_{\rm d,BEH}$$

with adjoint BEH boson, $M_1 = \frac{3}{2} v_c$.

Being GUT, we need to answer on the doublet-triplet splitting problem.

$$\frac{1}{M_s}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[EF]}\Phi_{[G]},\ \text{and/or}$$

$$\frac{1}{M_s^2}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[E]}\langle\Phi'_{[F]}\rangle\langle\Phi''_{[G]}\rangle,$$

$\Phi_{[AB]} = \Phi_{[45]}$ of Eq. (61) are essential

$$\frac{1}{M_s}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[EF]}\Phi_{[G]},\ \text{and/or}$$

$$\frac{1}{M_s^2}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[E]}\langle\Phi'_{[F]}\rangle\langle\Phi''_{[G]}\rangle,$$

$\Phi_{[AB]} = \Phi_{[45]}$ of Eq. (61) are essential

This kind was already noted in 1980 in the SU(7) model (JEK). The SU(7) UGUTF is the almost unique possibility for family unification.

4. Is $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$?

4. Is $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$?

JEK + S. Nam, arXiv:1506.08491
JEK + D. Y. Mo + M-S. Seo, arXiv:1506.08984

$$\begin{pmatrix} & \vline & u & \vline & \\ d^c & \vline & & \vline & N^0 \\ & \vline & d & \vline & \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L , \quad e_L^+$$

In our UGUTF with anti-SU(5) subgroup, the CKM and PMNS matrices use W couplings.

$$\begin{pmatrix} & \vdots & u & \vdots & \\ d^c & \vdots & & \vdots & N^0 \\ & \vdots & d & \vdots & \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L , \quad e_L^+$$

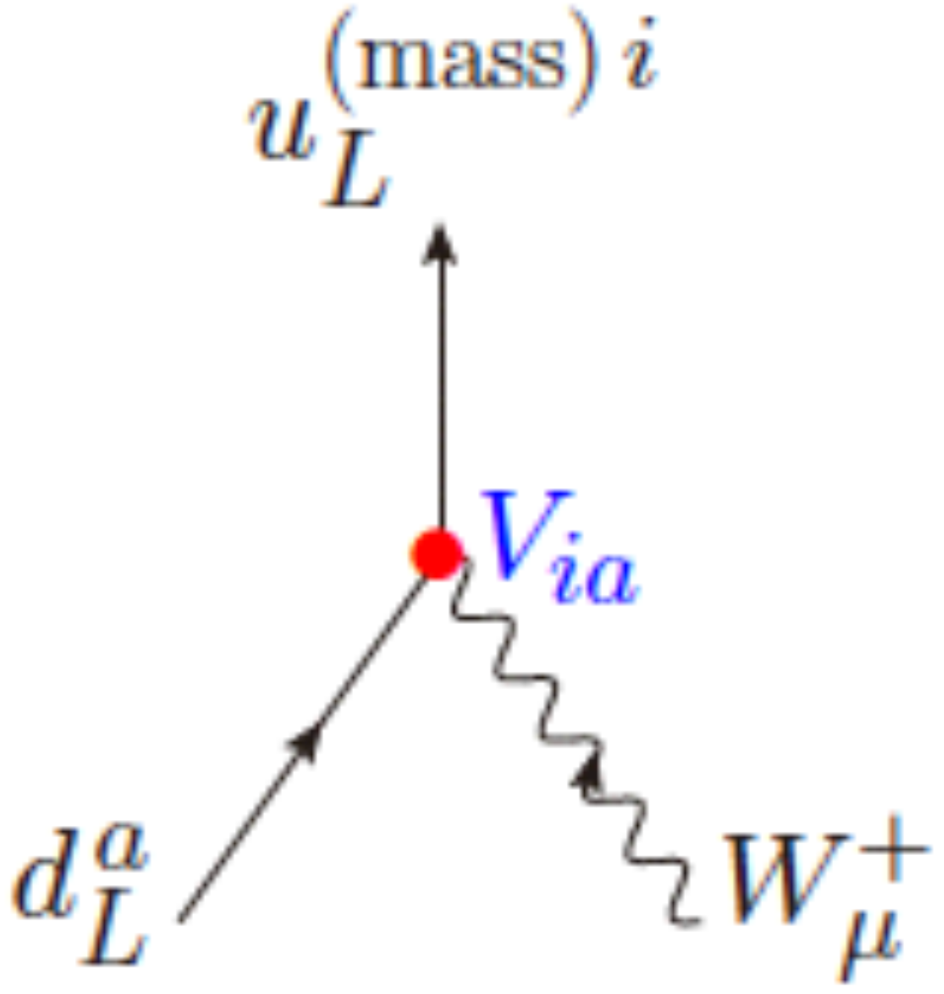
In our UGUTF with anti-SU(5) subgroup, the CKM and PMNS matrices use W couplings.

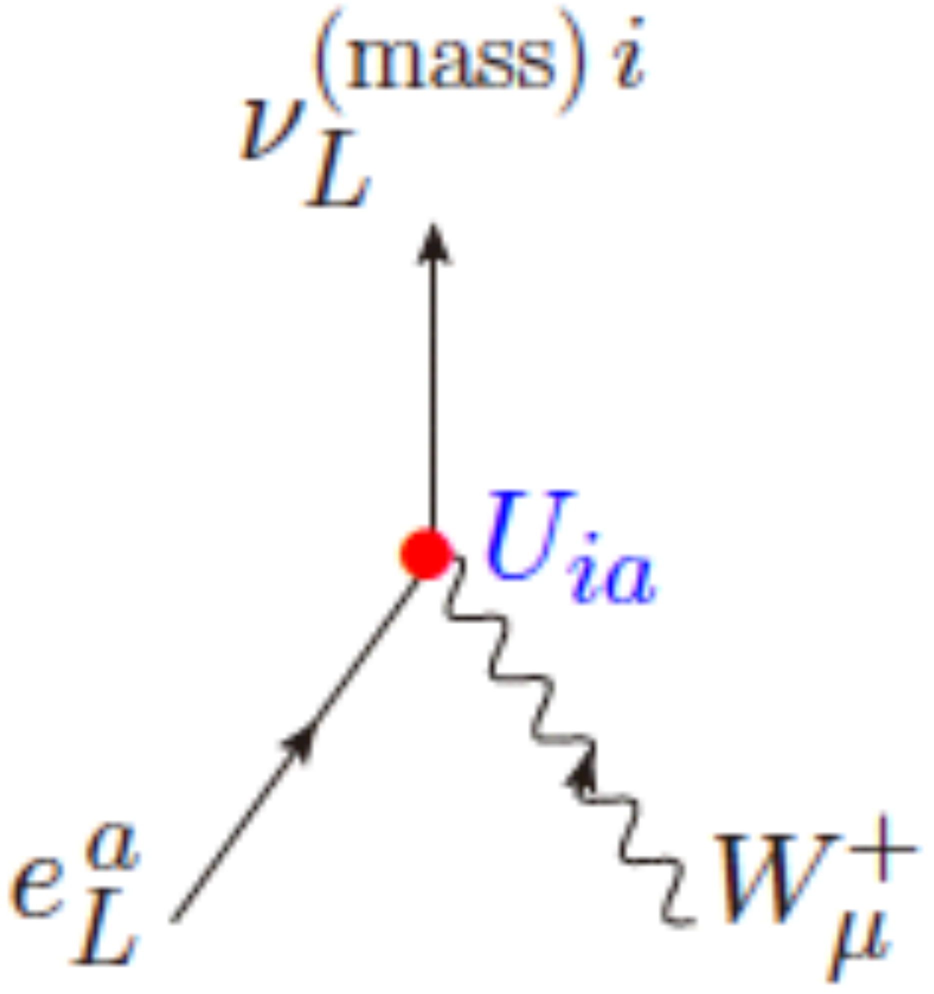
$$\begin{pmatrix} d^c & \begin{array}{c} u \\ \\ d \end{array} & N^0 \end{pmatrix}_L, \quad \begin{pmatrix} u^c \\ \hline \nu_e \\ e \end{pmatrix}_L, \quad e_L^+$$

In our UGUTF with anti-SU(5) subgroup, the CKM and PMNS matrices use W couplings.

$$\begin{pmatrix} d^c & \begin{array}{|c|} u \\ \\ d \end{array} & N^0 \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ \hline \nu_e \\ e \end{pmatrix}_L , \quad e^+_L$$

These W couplings define the CKM and PMNS matrices.





The CKM and PMNS matrices arise when diagonalizing quark and mass matrices. Quarks and leptons are related here: 5-bar

$$\begin{pmatrix} d^c & \begin{array}{c|c|c} & u & \\ & & \\ & d & \end{array} & N^0 \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ \hline \nu_e \\ e \end{pmatrix}_L , \quad e_L^+$$

The CKM and PMNS matrices arise when diagonalizing quark and mass matrices. Quarks and leptons are related here: 5-bar

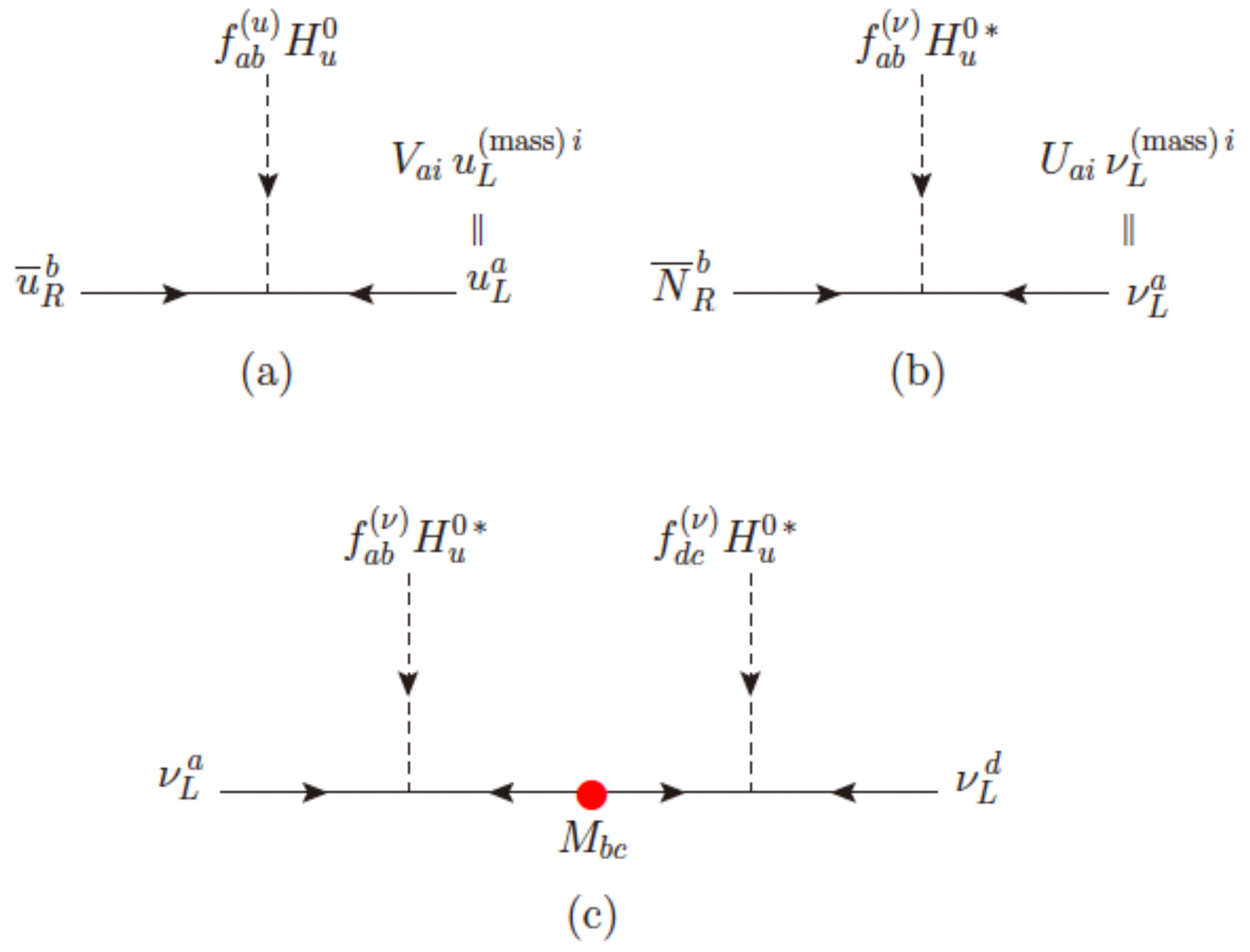
$$\begin{pmatrix} d^c & \begin{array}{|c|} u \\ \\ d \end{array} & N^0 \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L , \quad e_L^+$$

The CKM and PMNS matrices arise when diagonalizing quark and mass matrices. Quarks and leptons are related here: 5-bar

$$\begin{pmatrix} d^c & \Big| \begin{matrix} u \\ \\ d \end{matrix} \Big| & N^0 \end{pmatrix}_L, \quad \begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L, \quad e_L^+$$

The CKM and PMNS matrices arise when diagonalizing quark and mass matrices. Quarks and leptons are related here: 5-bar

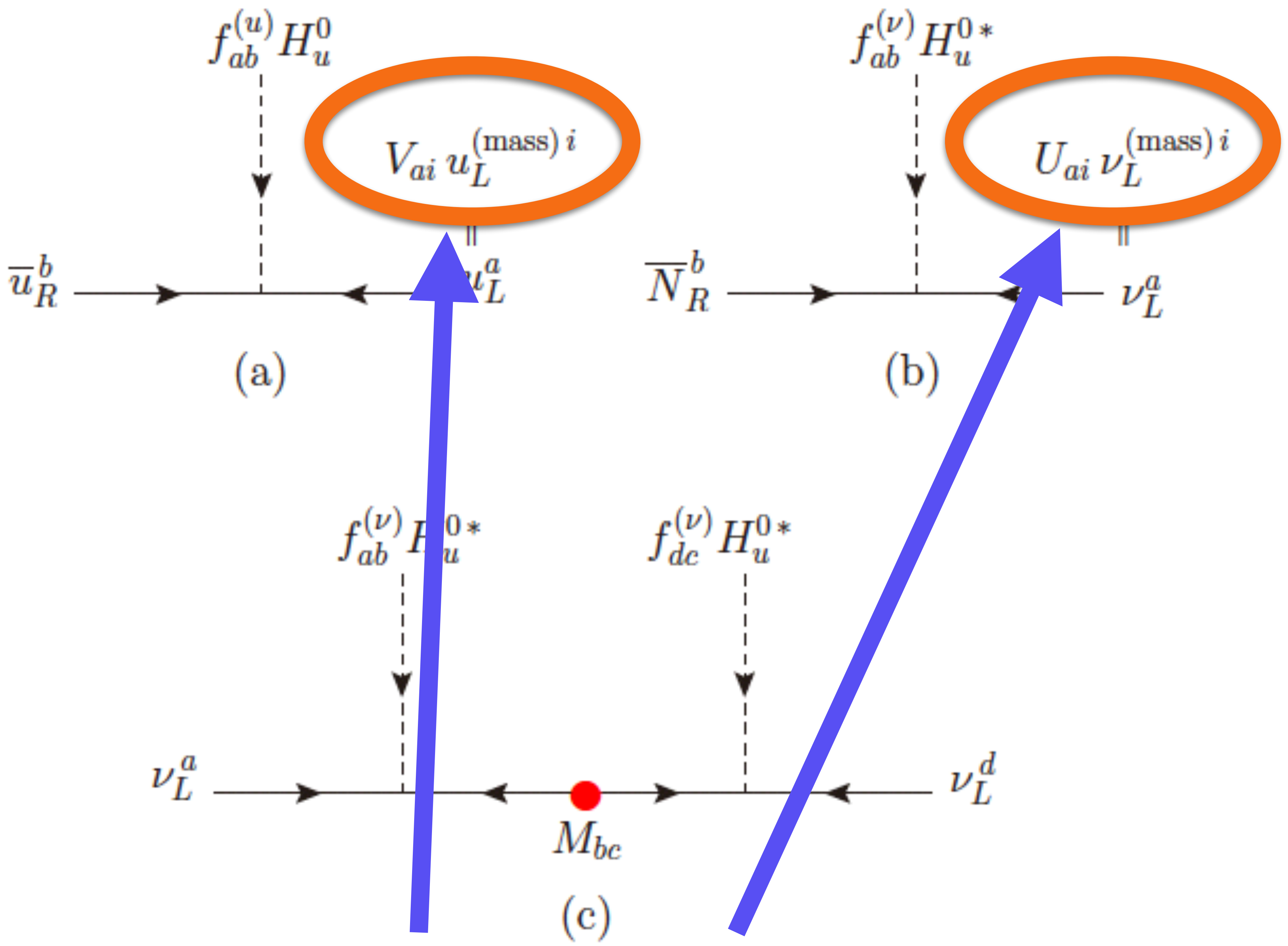
$$\begin{pmatrix} d^c & \begin{vmatrix} u \\ d \end{vmatrix} & N^0 \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ \hline \nu_e \\ e \end{pmatrix}_L , \quad e^+_L$$



Using the bases where e and d masses are diagonalized, only neutrino and u-quark masses are important.

The CKM and PMNS matrices arise when diagonalizing quark and mass matrices. Quarks and leptons are related here: 5-bar

$$\begin{pmatrix} d^c & \begin{matrix} u \\ d \end{matrix} & N^0 \end{pmatrix}_L , \quad \begin{pmatrix} u^c \\ \nu_e \\ e \end{pmatrix}_L , \quad e_L^+$$



Using the bases where e and d masses are diagonalized, only neutrino and u-quark masses are important.

Thus, CKM and PMNS matrices are related

$$V^{\rm KS}_{\rm CKM} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta_{\rm CKM}} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta_{\rm CKM}} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta_{\rm CKM}} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\rm CKM}} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\rm CKM}} \end{pmatrix}$$

$s_i = \sin\theta_i$ for $i = 1, 2, 3$ $J = c_1 c_2 c_3 s_1^2 s_2 s_3 \sin(\mathrm{delta}_{\rm CKM})$

$$V_{\rm CKM}^{\rm KS} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -c_2s_1 & e^{-i\delta_{\rm CKM}}s_2s_3 + c_1c_2c_3 & -e^{-i\delta_{\rm CKM}}s_2c_3 + c_1c_2s_3 \\ -e^{i\delta_{\rm CKM}}s_1s_2 & -c_2s_3 + c_1s_2c_3e^{i\delta_{\rm CKM}} & c_2c_3 + c_1s_2s_3e^{i\delta_{\rm CKM}} \end{pmatrix}$$

$s_i = \sin\theta_i$ for $i = 1,2,3$ $\quad J = c_1c_2c_3s_1^2s_2s_3 \sin(\text{delta}_{\rm CKM})$

$$V_{\rm PMNS}^{\rm KS} = \begin{pmatrix} C_1 & S_1C_3 & S_1S_3 \\ -C_2S_1 & e^{-i\delta_{\rm PMNS}}S_2S_3 + C_1C_2C_3 & -e^{-i\delta_{\rm PMNS}}S_2C_3 + C_1C_2S_3 \\ -e^{i\delta_{\rm PMNS}}S_1S_2 & -C_2S_3 + C_1S_2C_3e^{i\delta_{\rm PMNS}} & C_2C_3 + C_1S_2S_3e^{i\delta_{\rm PMNS}} \end{pmatrix}$$

$S_i = \sin\Theta_i$ for $i = 1,2,3$ $\quad J = C_1C_2C_3S_1^2S_2S_3 \sin(\text{delta}_{\rm PMNS})$

$$V^{\rm KS}_{\rm CKM} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -c_2s_1 & e^{-i\delta_{\rm CKM}}s_2s_3 + c_1c_2c_3 & -e^{-i\delta_{\rm CKM}}s_2c_3 + c_1c_2s_3 \\ -e^{i\delta_{\rm CKM}}s_1s_2 & -c_2s_3 + c_1s_2c_3e^{i\delta_{\rm CKM}} & c_2c_3 + c_1s_2s_3e^{i\delta_{\rm CKM}} \end{pmatrix}$$

$s_i = \sin\theta_i$ for $i = 1,2,3$ $J = c_1c_2c_3s_1^2s_2s_3 \sin(\mathrm{delta}_{\rm CKM})$

$$V^{\rm KS}_{\rm PMNS} = \begin{pmatrix} C_1 & S_1C_3 & S_1S_3 \\ -C_2S_1 & e^{-i\delta_{\rm PMNS}}S_2S_3 + C_1C_2C_3 & -e^{-i\delta_{\rm PMNS}}S_2C_3 + C_1C_2S_3 \\ -e^{i\delta_{\rm PMNS}}S_1S_2 & -C_2S_3 + C_1S_2C_3e^{i\delta_{\rm PMNS}} & C_2C_3 + C_1S_2S_3e^{i\delta_{\rm PMNS}} \end{pmatrix}$$

$S_i = \sin\Theta_i$ for $i = 1,2,3$ $J = C_1C_2C_3S_1^2S_2S_3 \sin(\mathrm{delta}_{\rm PMNS})$

Even though si is not equal to Si, |deltaCKM| and |deltaPMNS| can be equal. We may satisfy the following in this program

$$\delta_{\rm PMNS} \simeq \pm\delta_{\rm CKM}$$

if CP violation is spontaneous a la Froggatt-Nielsen by ONE complex vev of a SM singlet X. [JEK-Nam, 1506.08491]

Comments on Jarlskog determinant [JEK-Mo-Seo]

Comments on Jarlskog determinant [JEK-Mo-Seo]

- There are three possibilities for $\delta_{\rm CKM}$: α, β, γ of PDG book.

Comments on Jarlskog determinant [JEK-Mo-Seo]

- There are three possibilities for $\delta_{\rm CKM}$: α , β , γ of PDG book.

(i) Make Det=1 as in KS. Make the real part of (22) element is very large as in many parametrizations.

Comments on Jarlskog determinant [JEK-Mo-Seo]

- There are three possibilities for $\delta_{\rm CKM}$: α, β, γ of PDG book.

- If 1st row = real, or 1st column = real
  $\delta_{\rm CKM}$ is α
  Kobayashi-Maskawa parametrization,
  Kim-Seo parametrization

- (i) If both 1st row = real, and 1st column = real, then
  $\delta_{\rm CKM} = \gamma$: Chau-Keung-Maiani parametrization

Comments on Jarlskog determinant [JEK-Mo-Seo]

- There are three possibilities for $\delta_{\rm CKM}$: α, β, γ of PDG book.

- If 1st row = real, or 1st column = real
  $\delta_{\rm CKM}$ is α
  Kobayashi-Maskawa parametrization,
  Kim-Seo parametrization

- (i) If both 1st row = real, and 1st column = real, then
  $\delta_{\rm CKM} = \gamma$: Chau-Keung-Maiani parametrization

  - The identity $\delta_{\rm CKM}$ = Imaginary part of V(31)*V(22)*V(13)* was very useful.

6. Conclusion

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}{}^{*}$ $V_{22}{}^{*}$ $V_{13}{}^{*}$ in the KS form.

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}{}^{*}$ $V_{22}{}^{*}$ $V_{13}{}^{*}$ in the KS form.
2. δ_{CKM} is maximal.

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}^* V_{22}^* V_{13}^*$ in the KS form.
2. $\delta_{\rm CKM}$ is maximal.
3. GUT family unification in SU(7)xU(1) from Z(12-I).

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}^* V_{22}^* V_{13}^*$ in the KS form.
2. $\delta_{\rm CKM}$ is maximal.
3. GUT family unification in SU(7)xU(1) from Z(12-I).
4. Doublet-Triplet splitting is possible in anti-SU(N).

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}^* V_{22}^* V_{13}^*$ in the KS form.
2. $\delta_{\rm CKM}$ is maximal.
3. GUT family unification in SU(7)xU(1) from Z(12-I).
4. Doublet-Triplet splitting is possible in anti-SU(N).
5. $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$ possibility with spontaneous CP violation (cf:T2K on $\text{delta}_{\rm PMNS} = -90^{\rm o}$)

6. Conclusion

1. Jarlskog det. is J=Im V_{31}^{*} V_{22}^{*} V_{13}^{*} in the KS form.
2. $\delta_{\rm CKM}$ is maximal.
3. GUT family unification in SU(7)xU(1) from Z(12-I).
4. Doublet-Triplet splitting is possible in anti-SU(N).
5. $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$ possibility with spontaneous CP violation (cf:T2K on $\text{delta}_{\rm PMNS} = -90^{\circ}$)
6. Our model is free from gravity worries: from string.