

# Predictions for the Leptonic Dirac CP Violation Phase



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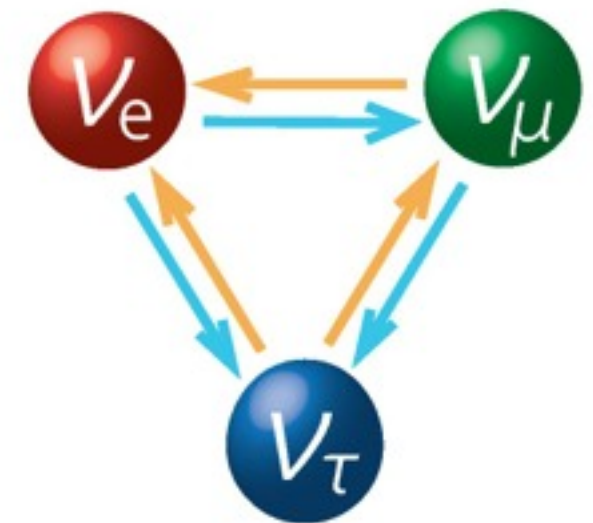
# Neutrino Mixing

# Neutrino Mixing

All the compelling data on neutrino oscillations, coming from **atmospheric** (K2K, MINOS, T2K, OPERA...), **solar** (Kamiokande, SK, SNO, Borexino, KamLAND), **reactor** (Daya Bay, RENO, Double Chooz) and **accelerator** (T2K, MINOS) neutrino experiments are compatible with the existence of three light neutrinos.

$$|\nu_l\rangle = \sum_{j=1}^3 U_{lj}^* |\nu_j\rangle, \quad \nu_{lL}(x) = \sum_{j=1}^3 U_{lj} \nu_{jL}(x),$$

$$l = e, \mu, \tau \quad m_j \lesssim 1 \text{ eV}$$



$U$  is the **P**ontecorvo, **M**aki, **N**akagawa, and **S**akata neutrino mixing matrix

Pontecorvo 1957, 1958, 1967  
Maki, Nakagawa, Sakata 1962

# Standard Parametrisation

The unitary PMNS mixing matrix  $U$  contains

$$n(n-1)/2 = 3 \text{ angles}$$

$$(n-1)(n-2)/2 = 1 \text{ phases for Dirac Neutrinos}$$

$$n(n-1)/2 = 3 \text{ phases Majorana Neutrinos}$$

S.M. Bilenky, J. Hosek, S.T. Petcov 1980

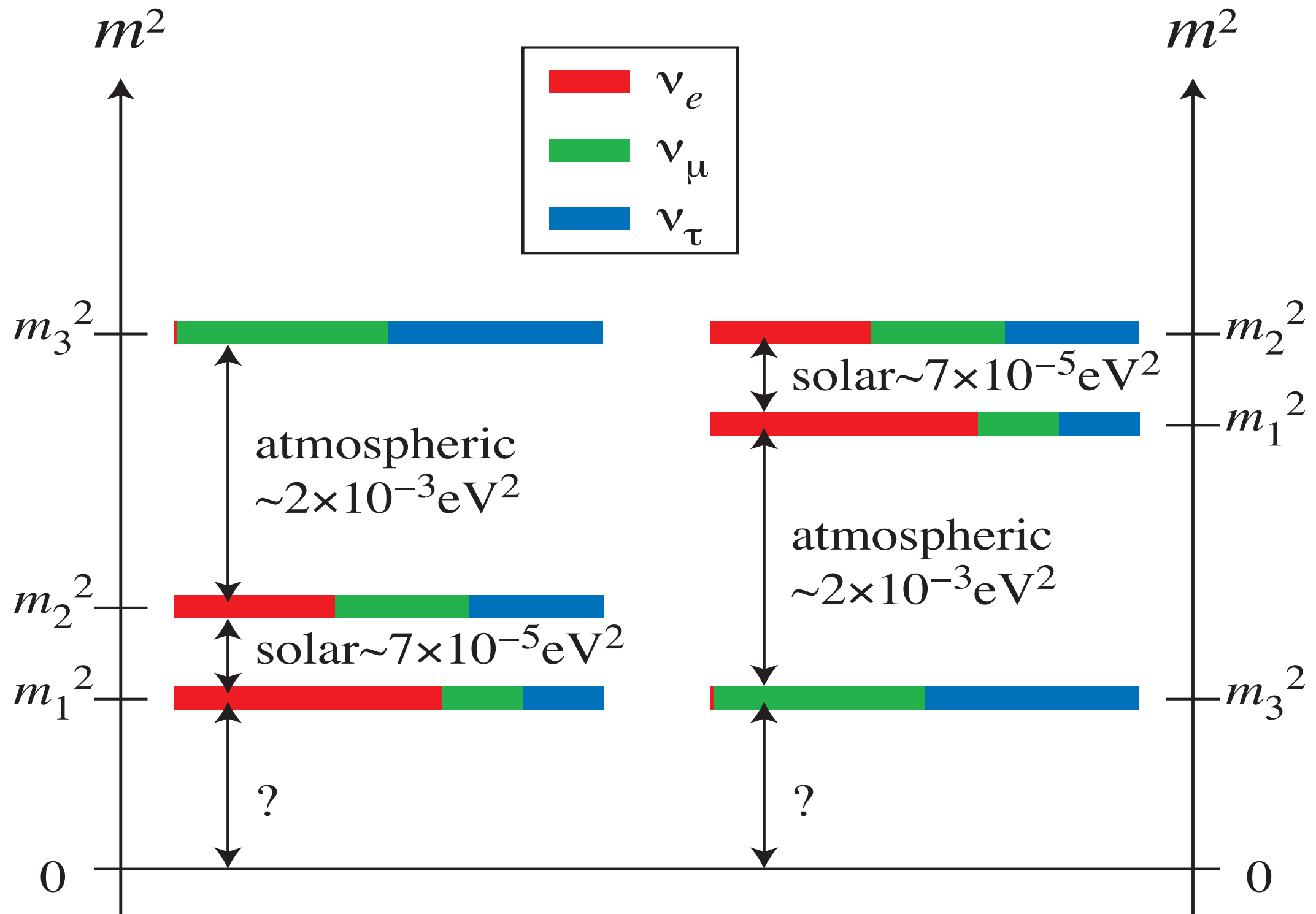
$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor + Dirac}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}_{\text{Majorana}}$$

With current values for NO (IO) neutrino mass spectrum

$$\begin{aligned} (\sin^2 \theta_{12})_{\text{BF}} &= 0.308, & 0.259 \leq \sin^2 \theta_{12} \leq 0.359 \\ (\sin^2 \theta_{23})_{\text{BF}} &= 0.437 \text{ (0.455)}, & 0.374 \text{ (0.380)} \leq \sin^2 \theta_{23} \leq 0.626 \text{ (0.641)} \\ (\sin^2 \theta_{13})_{\text{BF}} &= 0.0234 \text{ (0.0240)}, & 0.0176 \text{ (0.0178)} \leq \sin^2 \theta_{13} \leq 0.0295 \text{ (0.0298)} \end{aligned}$$

F. Capozzi et al. (Bari group) 2014

# Neutrino Mixing in Colors



S.F. King and C. Luhn, 2013

# Current vs Future Sensitivity

The precision on the mixing angles from current global fit of neutrino oscillation data, from future single experiment

Parameter	Current	Future
$\sin^2 \theta_{12}$	6%	0.7% (JUNO)
$\sin^2 \theta_{23}$	10%	5% (T2K, NOvA, T2HK)
$\sin^2 \theta_{13}$	9%	3% (Daya Bay)

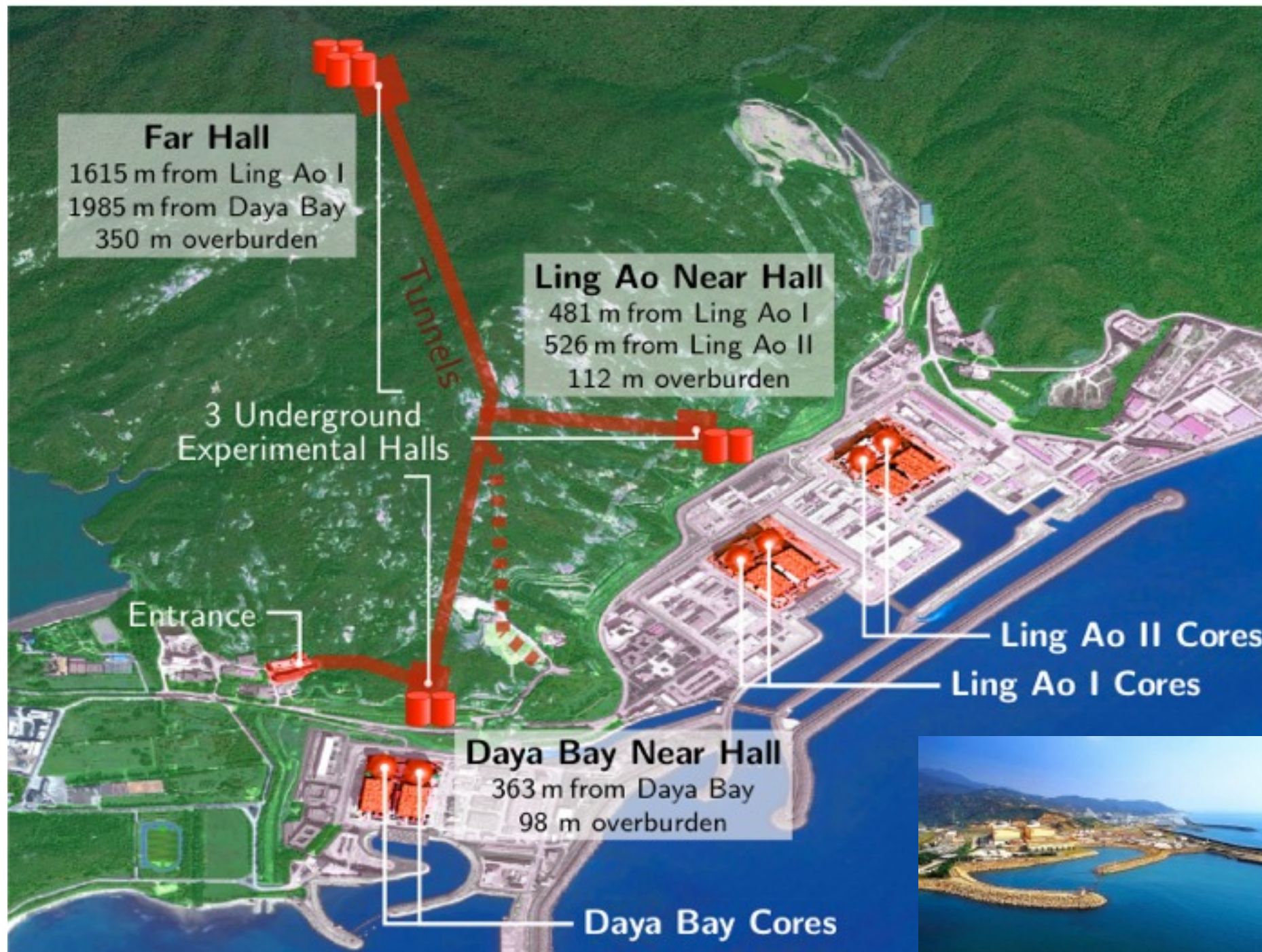
Y. Wang 2013, P. Coloma et al. 2014,  
C. Zhang et al. 2015, A. de Gouvea et al. 2013

The recent "hints for CPV" come from Daya Bay + T2K

T2K Collaboration 2015



# Daya Bay Experiment



Daya Bay: 6 **reactors** (D1, D2, L1-L4) and 8 **detectors** (ADs)



# Daya Bay Experiment

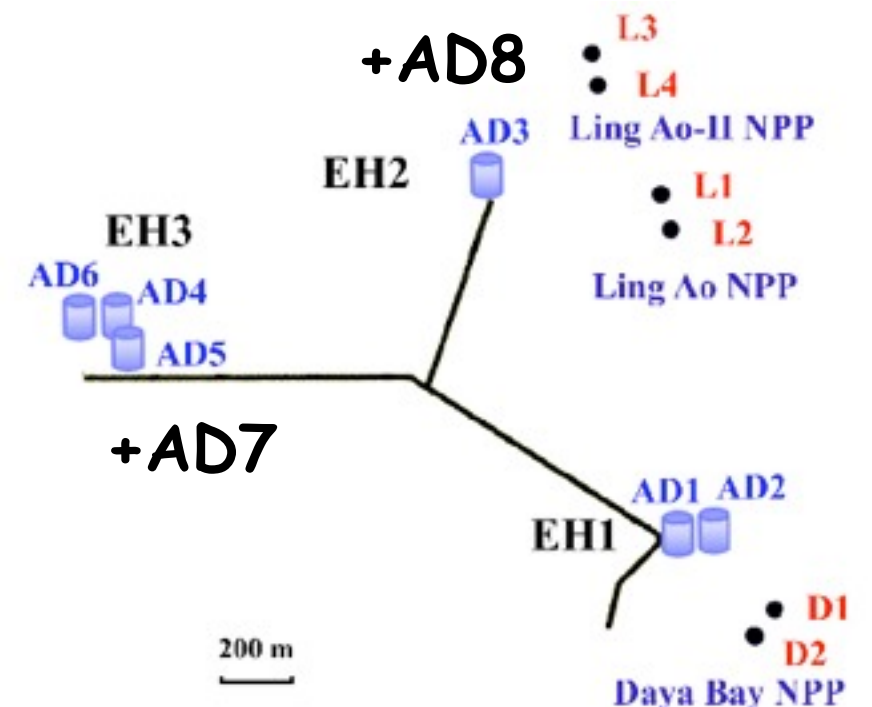
In the one dominant mass-scale approximation  $\Delta m^2_{31}L/(4E) = O(1)$   
 ( $\Delta m^2_{21}L/(4E) \approx 0.03$  for  $L = 1$  km and  $\langle E \rangle \approx 0.003$  GeV):

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left[ \frac{\Delta m^2_{31} L}{4E_\nu} \right]$$

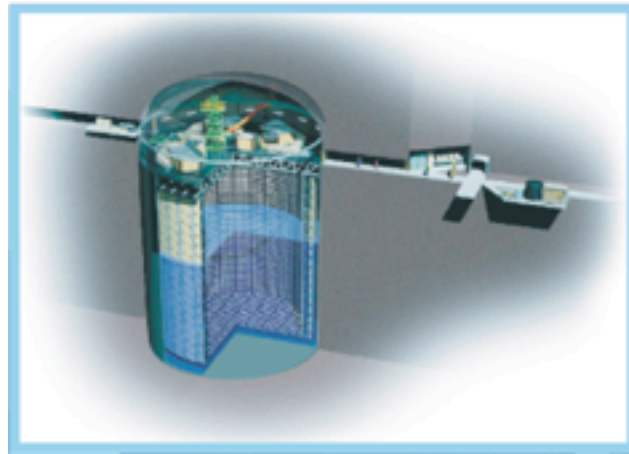
- ◆ no sensitivity to  $\Delta m^2_{21}$  and  $\theta_{21}$
- ◆ no sensitivity to neutrino mass ordering and CPV phase

$\sin^2 2\theta_{13} = 0.090 \pm 10\%$  with 6 ADs (2014)  
 $\sin^2 2\theta_{13} = 0.084 \pm 6\%$  with 8 ADs (2015)

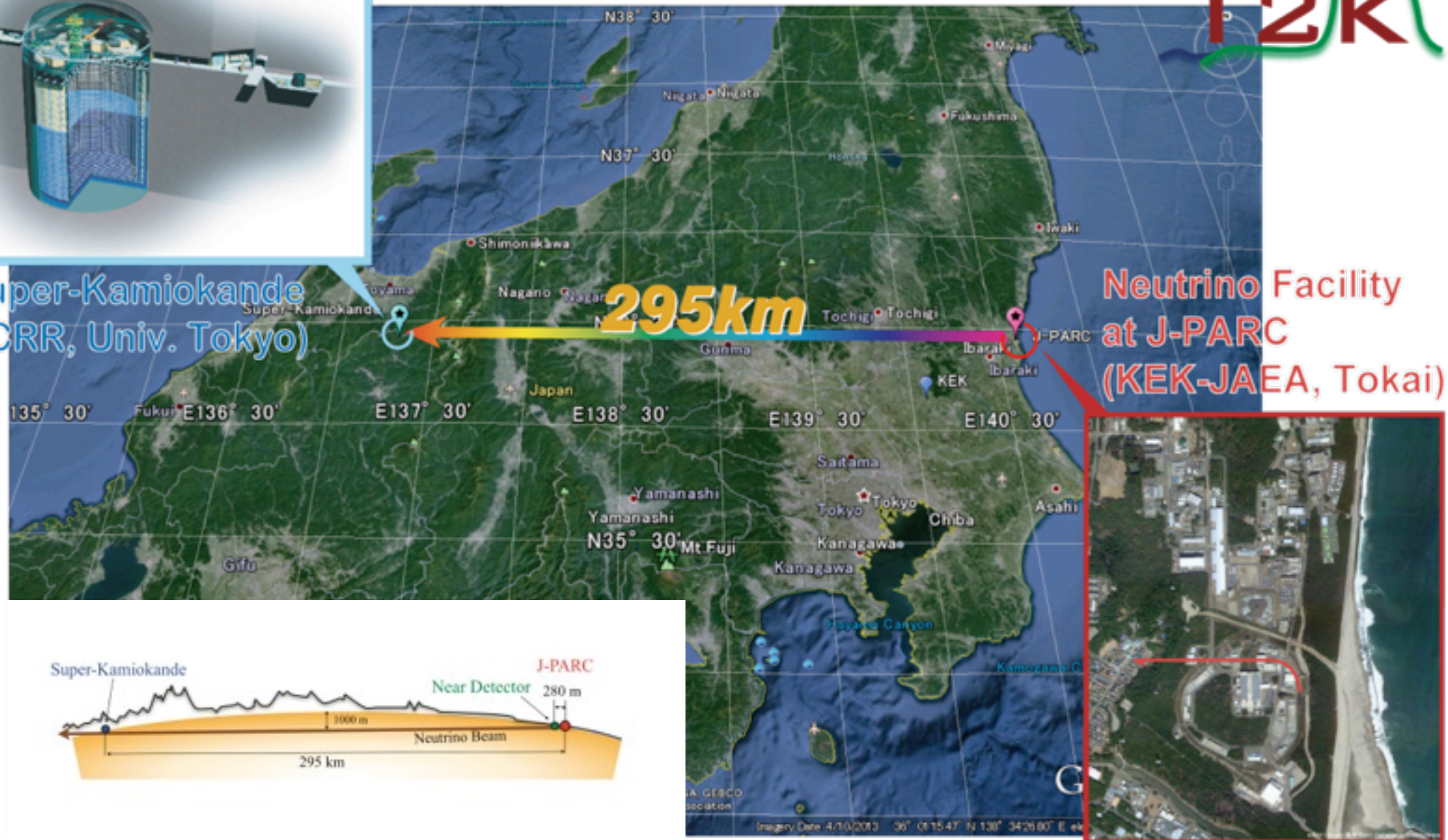
$|\Delta m^2_{ee}| = (2.59 \pm 8\%) \times 10^{-3} \text{ eV}^2$  with 6 ADs (2014)  
 $|\Delta m^2_{ee}| = (2.42 \pm 4.5\%) \times 10^{-3} \text{ eV}^2$  with 8 ADs (2015)



# T2K Experiment



Super-Kamiokande  
(ICRR, Univ. Tokyo)





Leading

$$\sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$

CPV

$$\frac{\sin 2\theta_{12} \sin 2\theta_{23}}{2 \sin \theta_{13}} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \delta$$

$\sim 0.03$

$$\sim \frac{\pi}{4} \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \frac{\sin 2\theta_{12} \sin 2\theta_{23}}{\sin^2 \theta_{23} \sin \theta_{13}} \frac{E_{1st \max}}{E} [leading] \sin \delta$$

$\sim 1.8$  (6.4 from  $1/\sin \theta_{13}$ )

$$\sim 0.27 \times [leading] \times \frac{E_{1st \max}}{E} \times \sin \delta$$

**27%**

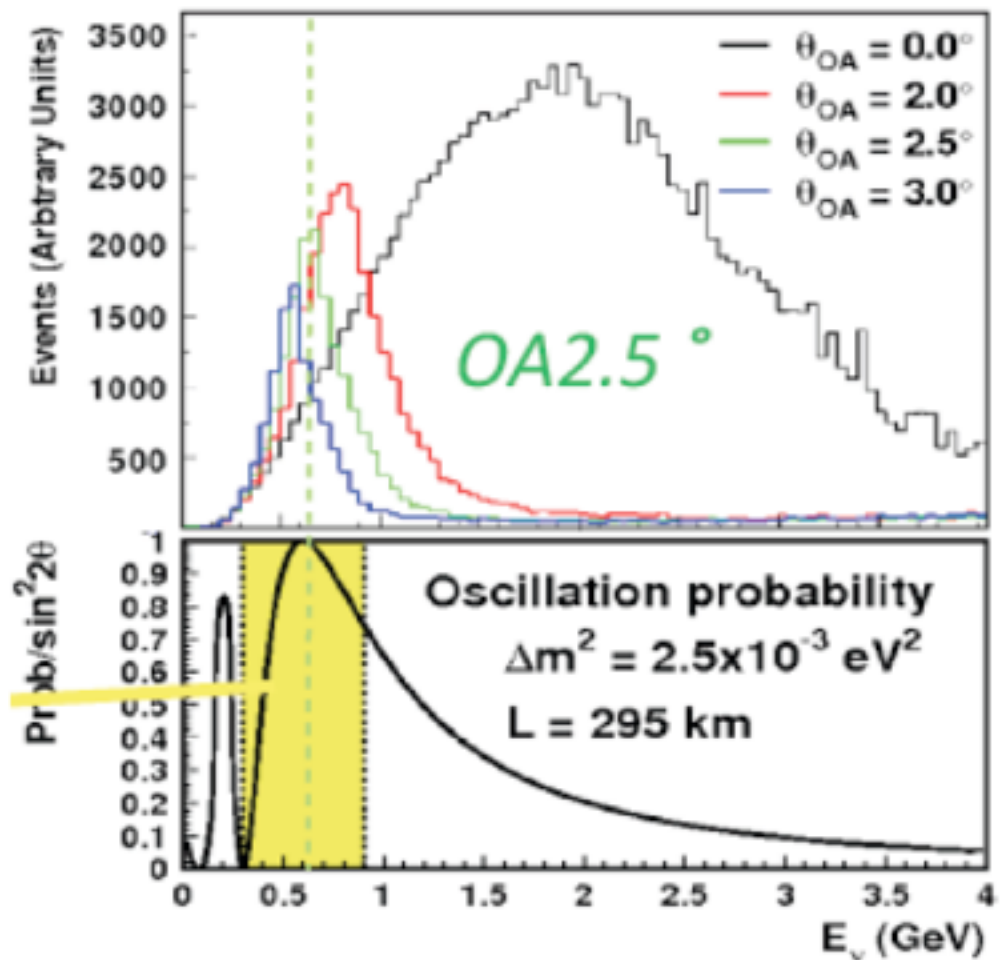
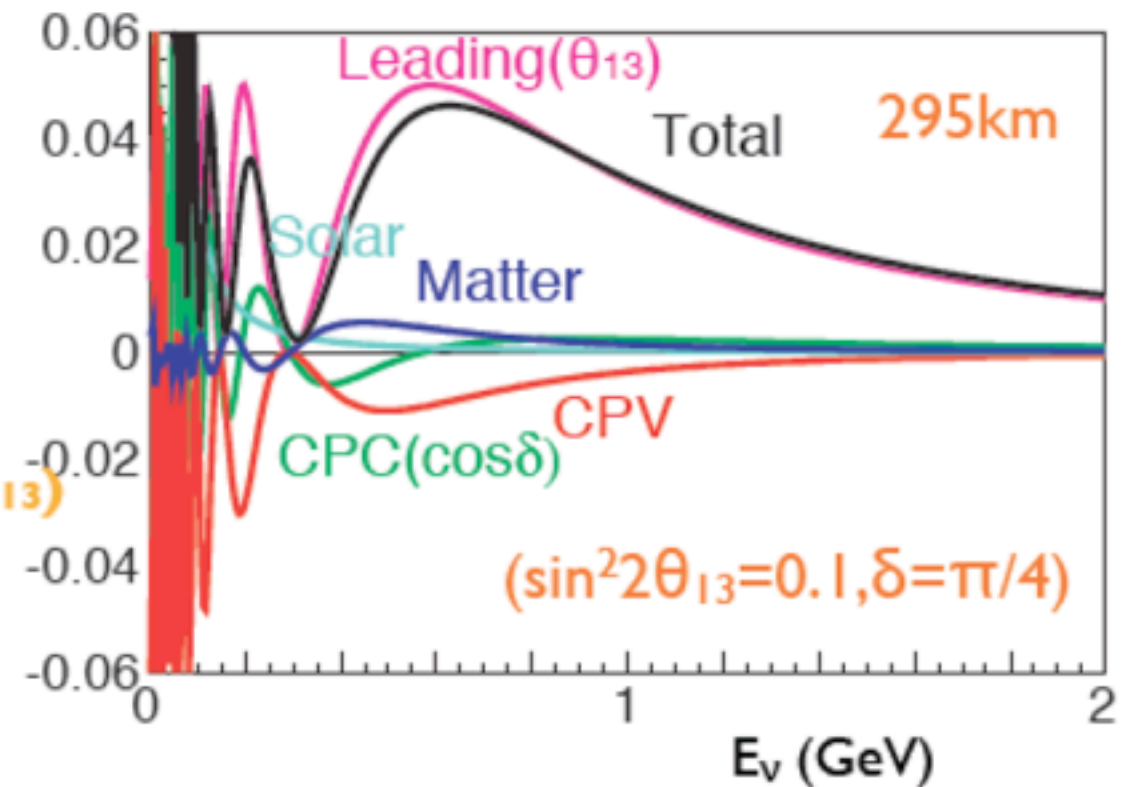
M. Tanimoto's talk (SISSA 2015)

$\nu + \bar{\nu}$  mode reduces the uncertainties in the leading term

T2K appearance channel:

- ◆  $\nu_\mu$  beam from J-PARC to SK
- ◆  $L = 295$  km and  $\langle E \rangle \approx 0.6$  GeV
- ◆  $\Delta m_{31}^2 L / (4E) \approx \pi/2$

6 events (2011)  
28 events (2013)



Up to 2nd order in  $\sin^2 \theta_{13}$  and  $a$ :

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}, \quad a = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$$

$$P_m^{3\nu}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3$$

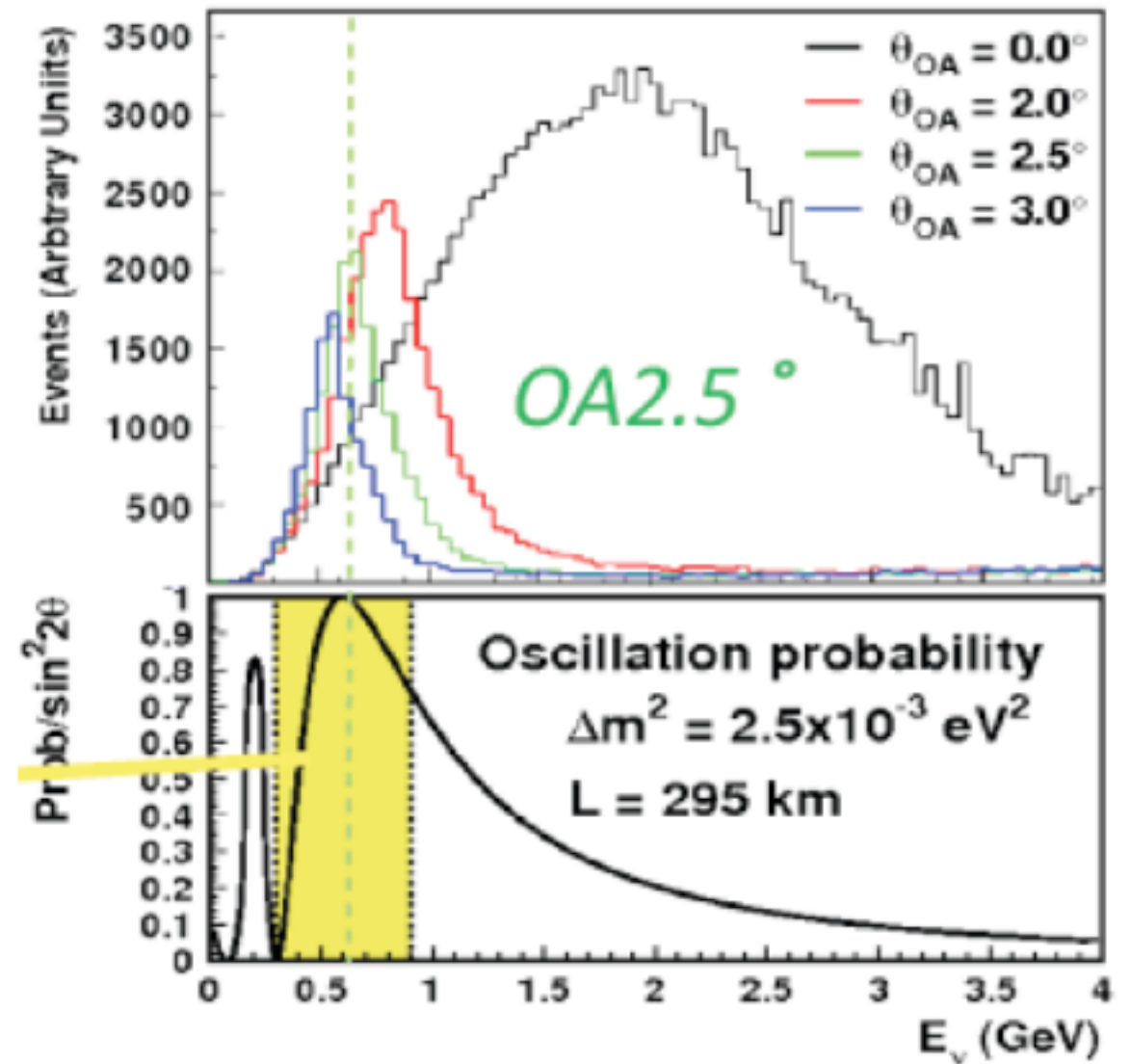
$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta]$$

$$P_{\sin \delta} = -a \frac{\sin \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{A(1-A)} \sin(\Delta) \sin(A\Delta) \sin[(1-A)\Delta]$$

$$P_{\cos \delta} = a \frac{\cos \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{A(1-A)} \cos(\Delta) \sin(A\Delta) \sin[(1-A)\Delta] \approx 0$$

$$P_3 = a^2 \frac{\cos^2 \theta_{23} \sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta)$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e : \delta \rightarrow -\delta, \quad A \rightarrow -A$$



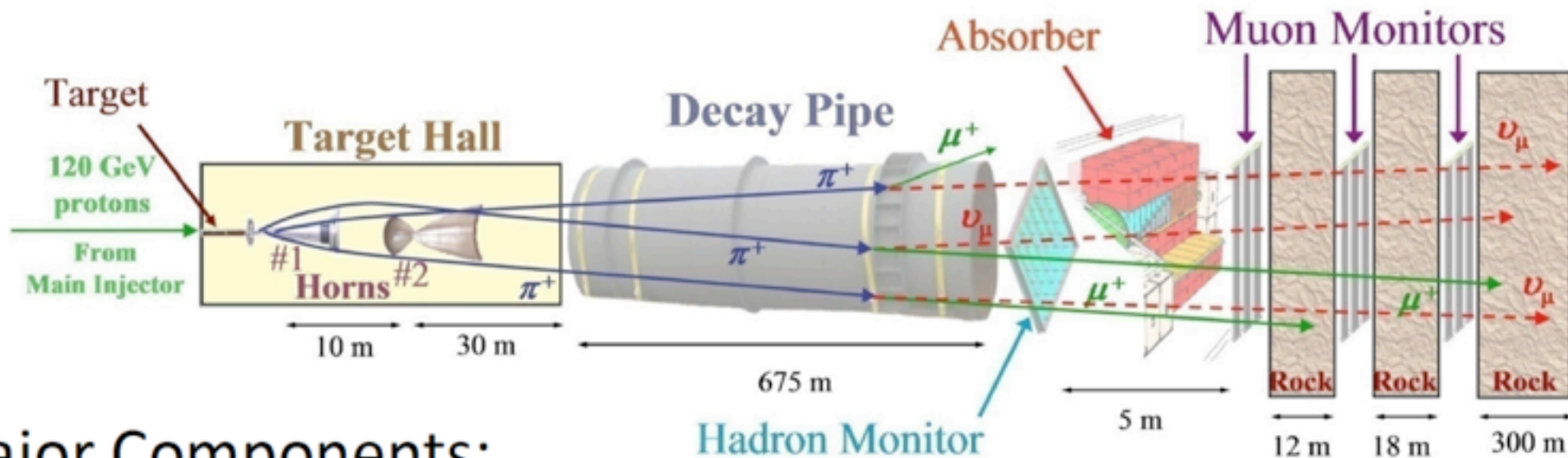
$8J_{CP}$

@1st osc max one cannot distinguish  $\delta$  from  $\pi-\delta$

M. Freund, 2001

K. Nakamura and S.T. Petcov 2014

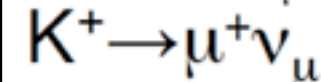
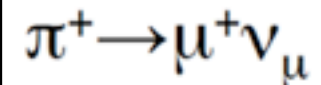
# Example: NuMI beamline at Fermilab



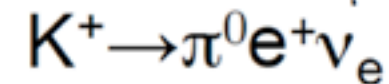
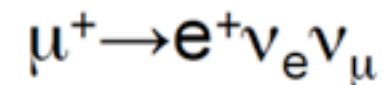
## Major Components:

- Proton Beam
- Pion Production Target
- Focusing System
- Decay Region
- Absorber
- Shielding...

Most  $\nu_\mu$ 's from 2-body decays:



Most  $\nu_e$ 's from 3-body decays:



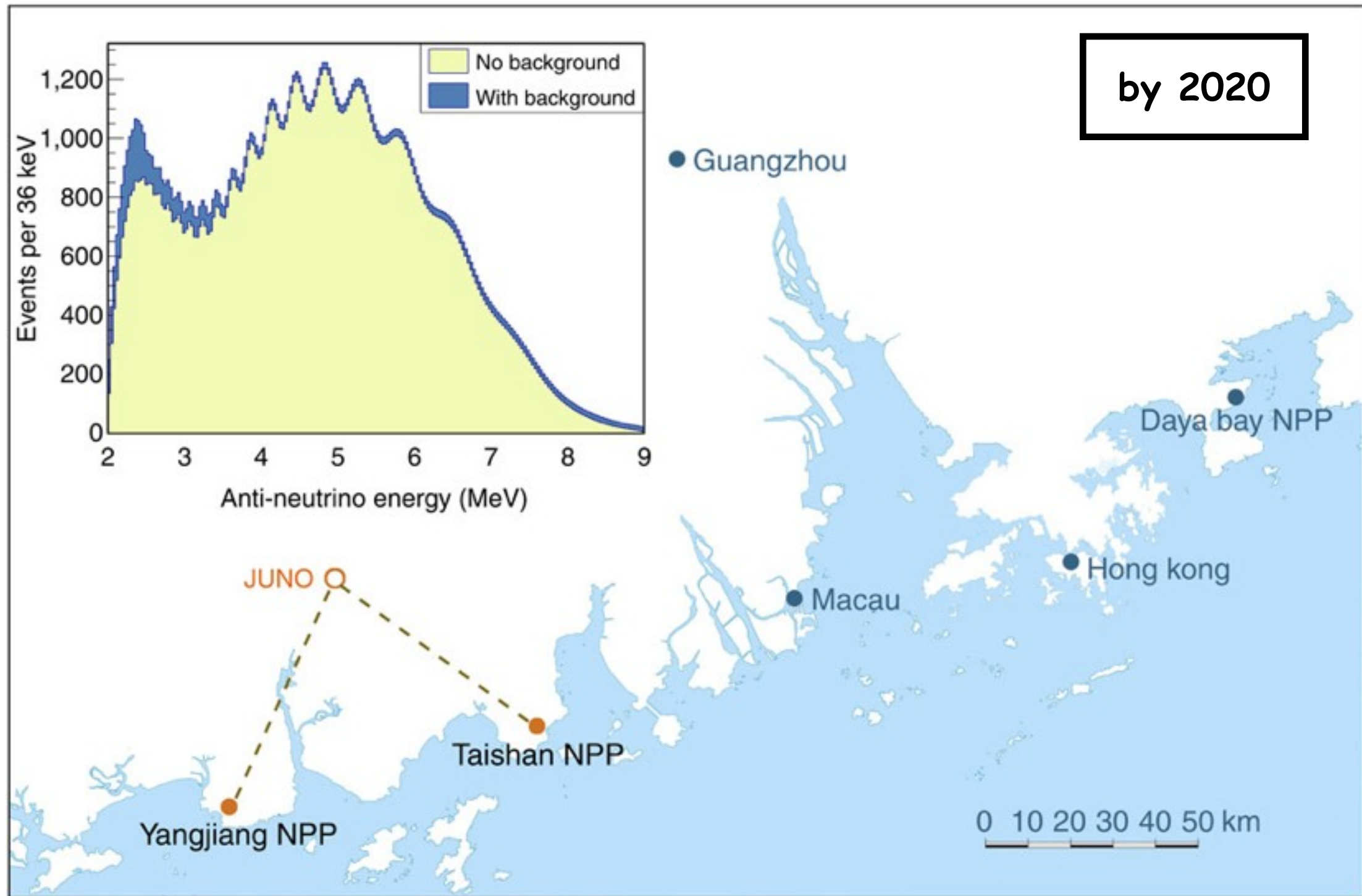
**Two body decay is mono-energetic**

K. McFarland, INSS 2013

D. Harris, Invisibles 2014



# JUNO Experiment



by 2020

M. Piai, S. T. Petcov 2001

P. Vogel et al. 2014

# Dirac Phase: "Hints for CP Violation"

# Dirac Phase $\delta$

**CP Violation (CPV) is a difference between neutrino and antineutrino oscillations**

$$P(\nu_l \rightarrow \nu_k) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_k), \quad l \neq k$$

$$A_{\text{CP}}^{(kl)} \equiv P(\nu_l \rightarrow \nu_k) - P(\bar{\nu}_l \rightarrow \bar{\nu}_k), \quad l \neq k$$

**For neutrino oscillations in vacuum**

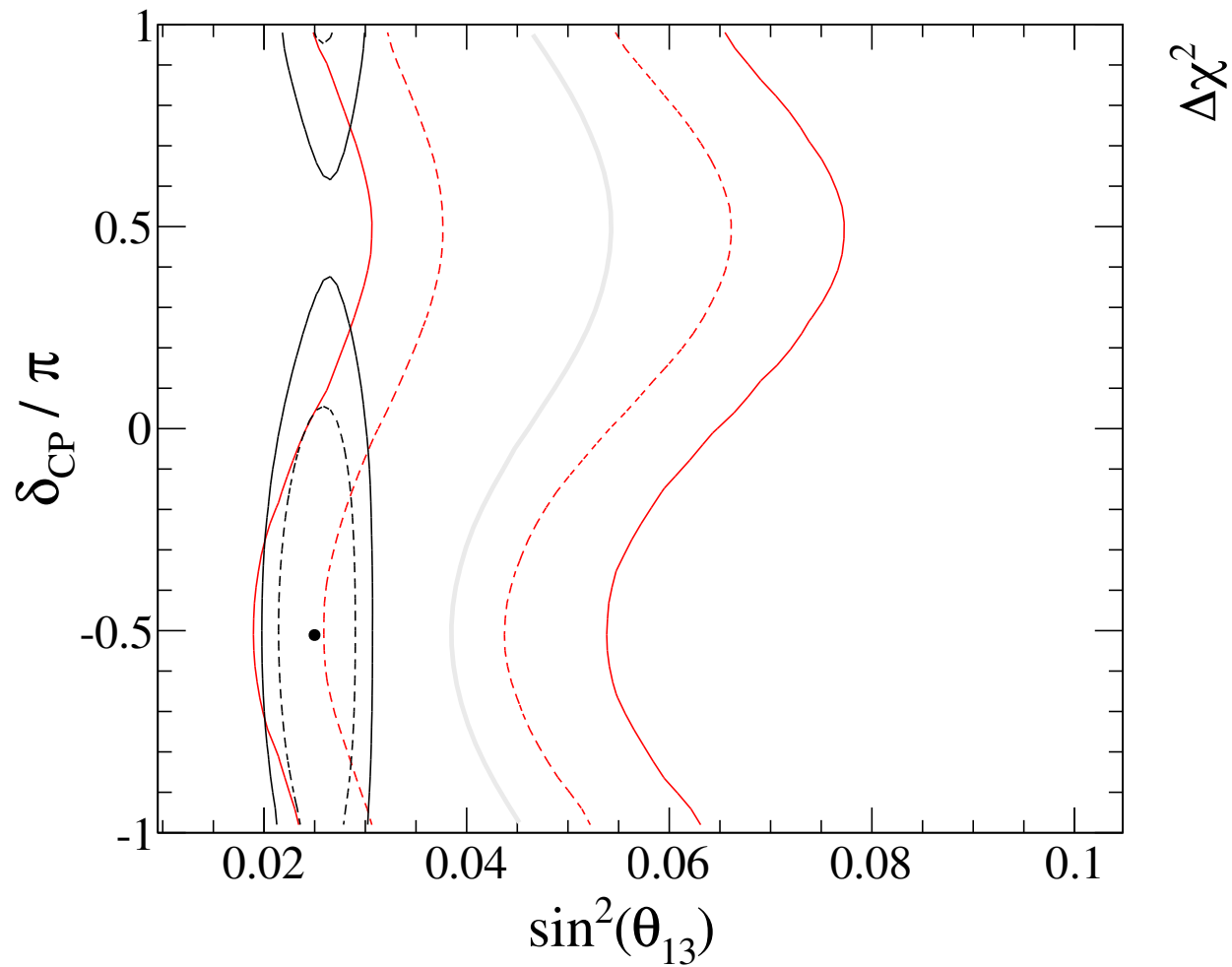
$$A_{\text{CP}}^{(\mu e)} = 4J_{\text{CP}} \left[ \sin \frac{\Delta m_{32}^2 L}{2E} + \sin \frac{\Delta m_{21}^2 L}{2E} + \sin \frac{\Delta m_{13}^2 L}{2E} \right]$$

**For neutrino oscillations in matter**

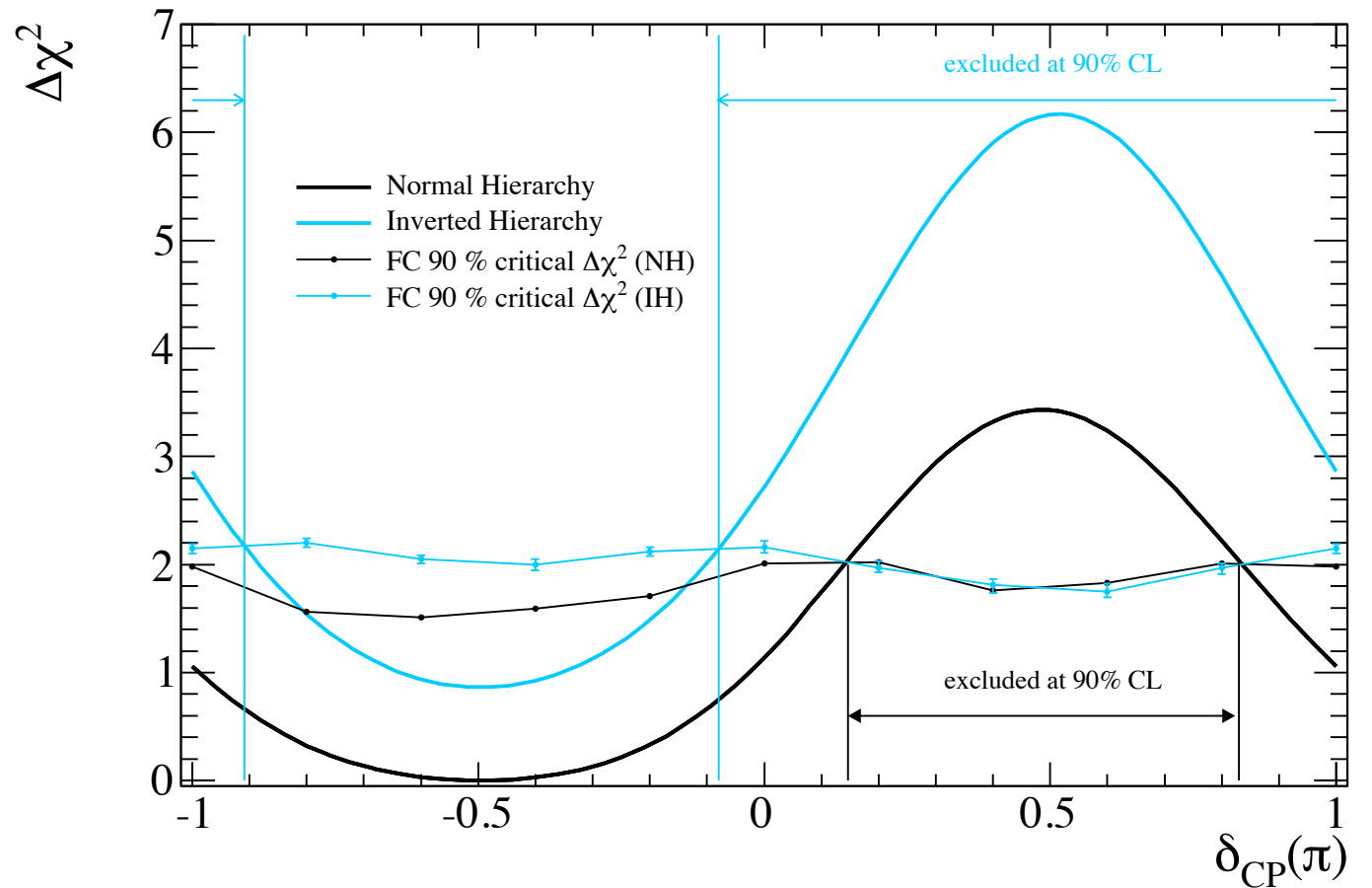
◆ **matter effects violate CP**

P. I. Krastev and S. T. Petcov 1988  
see also K. Nakamura and S. T. Petcov in  
PDG (2014)

# T2K + Daya Bay



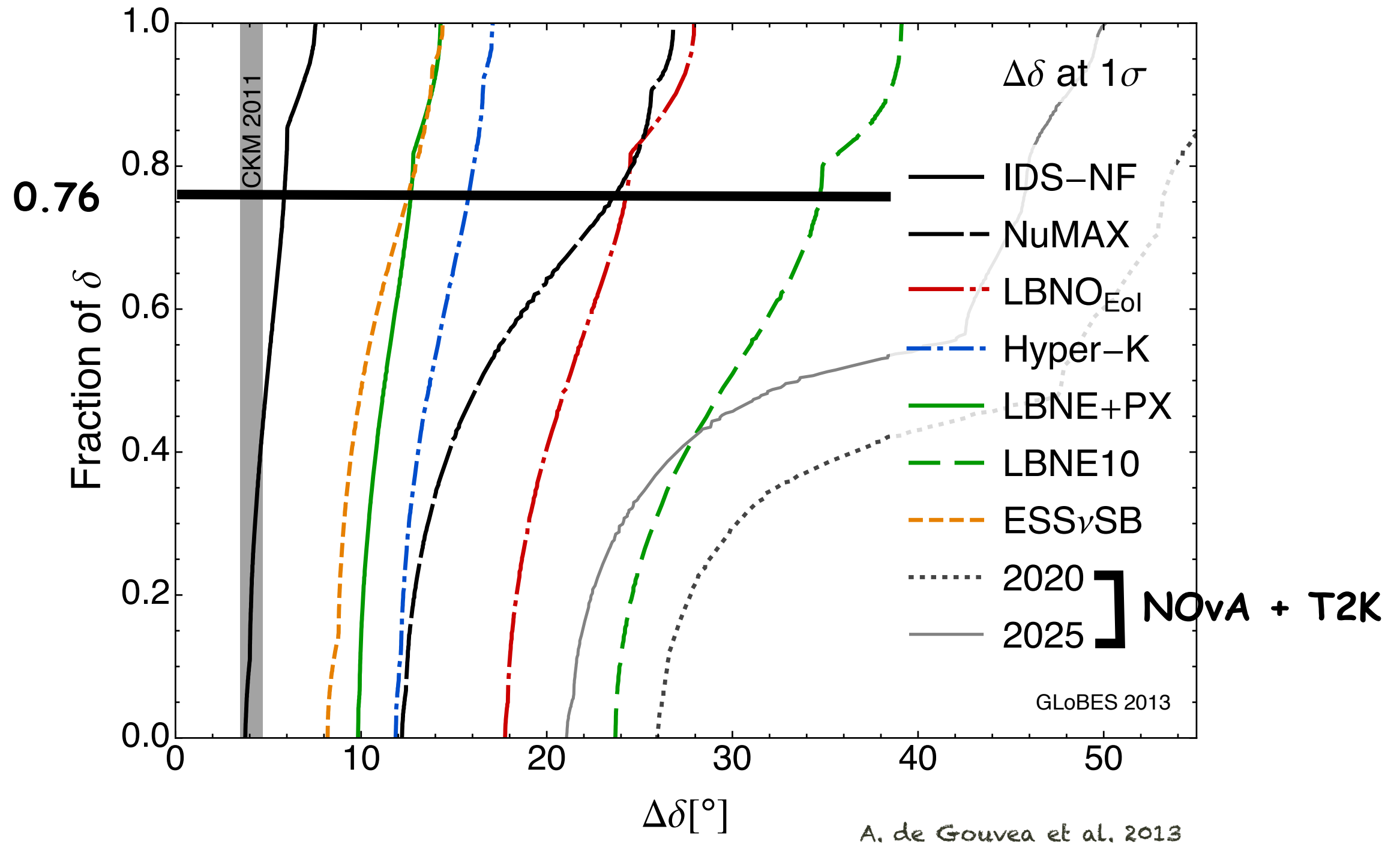
- T2K+Reactor 68% Credible Region
- T2K Only 68% Credible Region
- T2K+Reactor 90% Credible Region
- T2K Only 90% Credible Region
- T2K+Reactor Best Fit Point
- T2K Only Best Fit Line



T2K Collaboration 2015

**“Hints for CP Violation”**

# Expected Precision on $\delta$





# Expected Precision on $\delta$

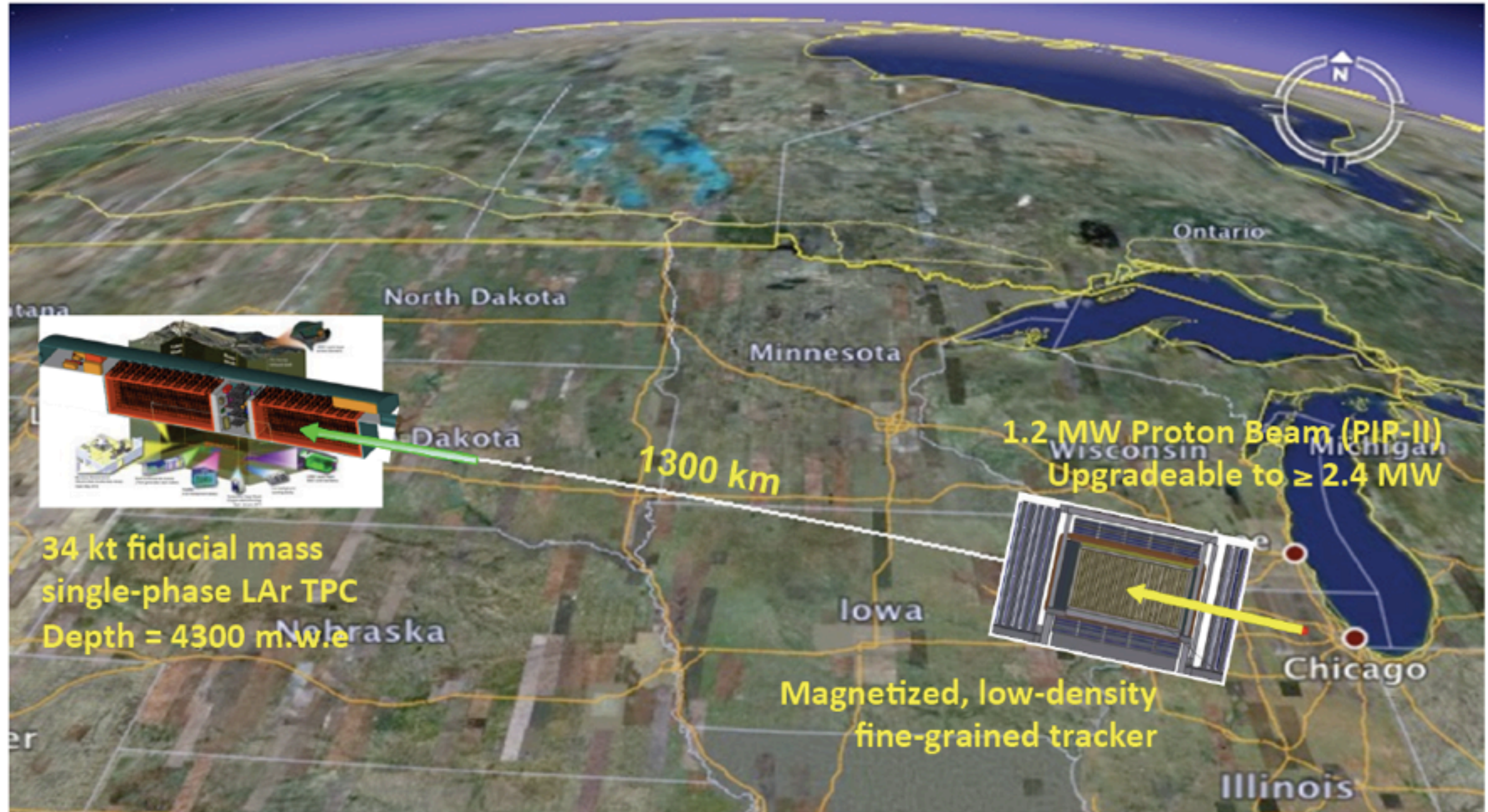
Assuming  $|\cos \delta| < 0.93$ , for the 76% of values of  $\delta$ :

for  $\Delta(\cos \delta) = 0.1$  (0.08) the error on  $\delta$   
does not exceed  $\Delta(\cos \delta)/(1 - 0.93^2)^{1/2}$

$$\Delta\delta \leq \Delta(\cos \delta)/(1 - 0.93^2)^{1/2} = 16^\circ (12^\circ) \text{ T2HK (ESSvSB)}$$

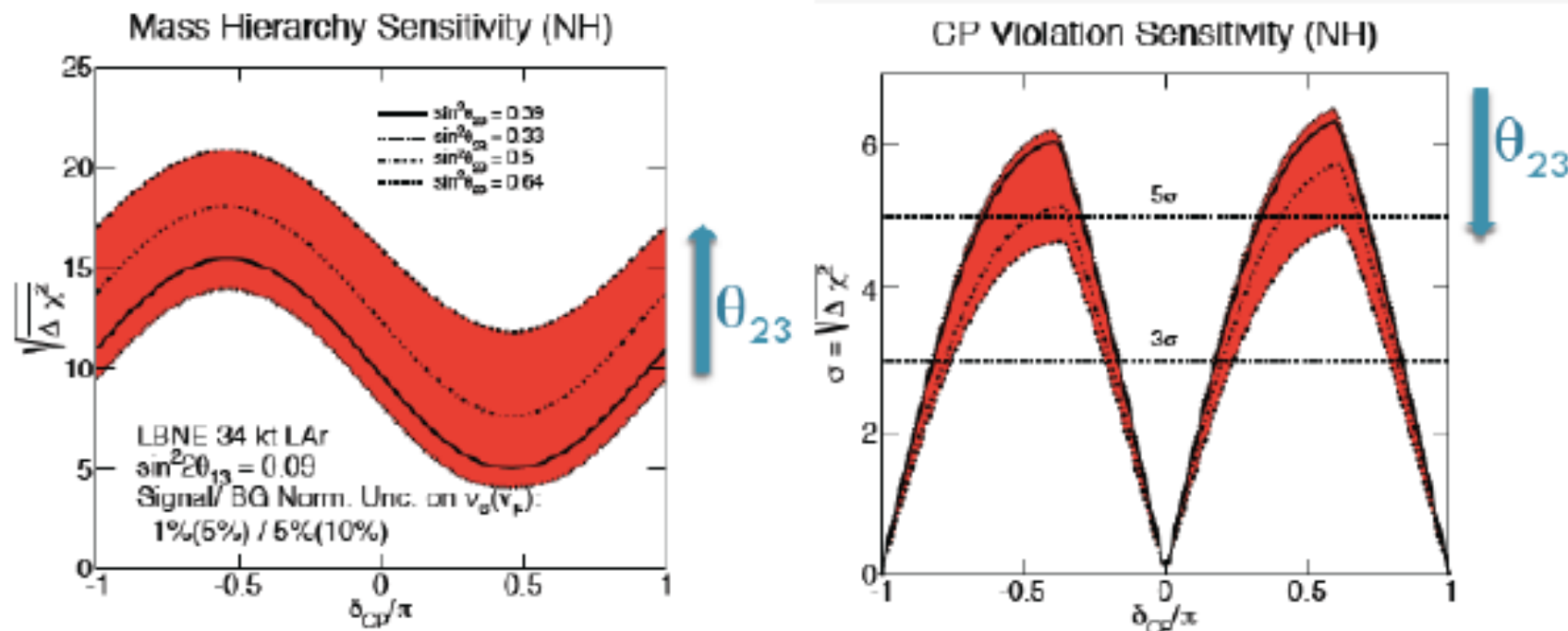
# Experimental Challenges (e.g. DUNE, $\mu$ DAR)

# DUNE (formerly LBNE)



B. C. Choudhary's talk  
(Queen Mary 2014)

# DUNE (formerly LBNE)



**Width of the band indicates variation within the 2013 allowed range for  $\theta_{23}$ .**  
**Exposure  $\sim 245\text{kTon} \sim 34\text{ kt} \times 1.2\text{ MW} \times (3\nu + 3\bar{\nu})$  years**

LBNE Collaboration 2013

B. C. Choudhary's talk  
 (Queen Mary 2014)

**Fiducial Mass Far Detector  $\times$  Beam Power  $\times$  Years Running**

**Wide band neutrino beam,**

**$L \approx 1300\text{ km}$ ,  $E \approx [2,4]\text{ GeV}$ :  $\Delta \approx [0.3,0.6]\pi$**



# $\mu$ DAR Proposal

## Limitations of a Long-Baseline Accelerator Approach

off-axis

T2K, NOvA

This off-axis, accelerator approach, comparing  $\nu_e$  and  $\bar{\nu}_e$  appearance at the oscillation maximum, has two disadvantages:

- 1) High energy proton accelerators produce  $\nu$  more efficiently than  $\bar{\nu}$ .

This is because they use  $\nu_\mu$  from  $\pi^+ \rightarrow \mu^+ + \nu_\mu$

As a result the  $\bar{\nu}$  mode occupies most of the beam time and still dominates the uncertainty

- 2) Even with a perfect measurement, only  $\sin(\delta)$  is determined: Can't distinguish  $\delta$  from  $\pi - \delta$ .

Jarah Evslin's talk (WIN 2015)



# $\mu$ DAR Proposal

## $\mu^+$ Decay at Rest

**Solution:** Measure  $\bar{\nu}$  oscillations using  $\mu^+$  decay at rest (DAR)

How does it work?

- 1) A high intensity 400 MeV-2 GeV proton beam hits a fixed target
- 2) The target produces pions which stop.  
The  $\pi^-$  are absorbed in the target while the  $\pi^+$  decay at rest  
$$\pi^+ \rightarrow \mu^+ + \nu_\mu.$$
- 3) The  $\mu^+$  then stop and also decay at rest  
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu.$$
- 4) The  $\bar{\nu}_\mu$  travel isotropically in all directions, oscillating as they go
- 5) A detector measures the  $\bar{\nu}_e$  arising from the oscillations  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ .

# Theoretical Framework

# Neutrino Mixing suggests

Tri-bimaximal Mixing (TBM, e.g., from  $A_4$ )  $(s_{12})^2 \approx 1/3$ ,  $(s_{23})^2 \approx 1/2$ ,  $(s_{13})^2 \approx 0$

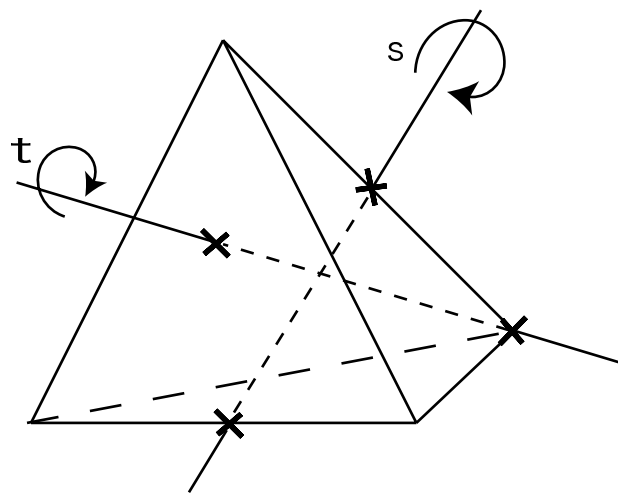
Bi-bimaximal Mixing (BM, e.g., from  $S_4$ )  $(s_{12})^2 \approx 1/2$ ,  $(s_{23})^2 \approx 1/2$ ,  $(s_{13})^2 \approx 0$

Golden Ratio A (GRA, e.g., from  $A_5$ )  $(s_{12})^2 \approx 1/(2+r)$ ,  $(s_{23})^2 \approx 1/2$ ,  $(s_{13})^2 \approx 0$

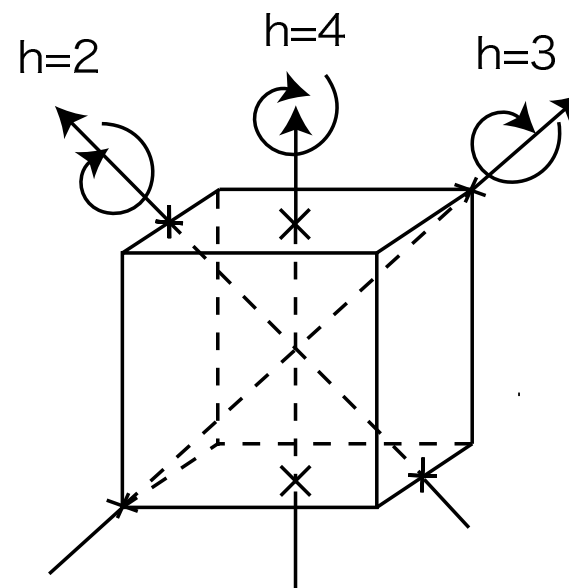
(GRB:  $D_{10}$  HG:  $D_{12}$ )

( $r$  is the golden ratio)

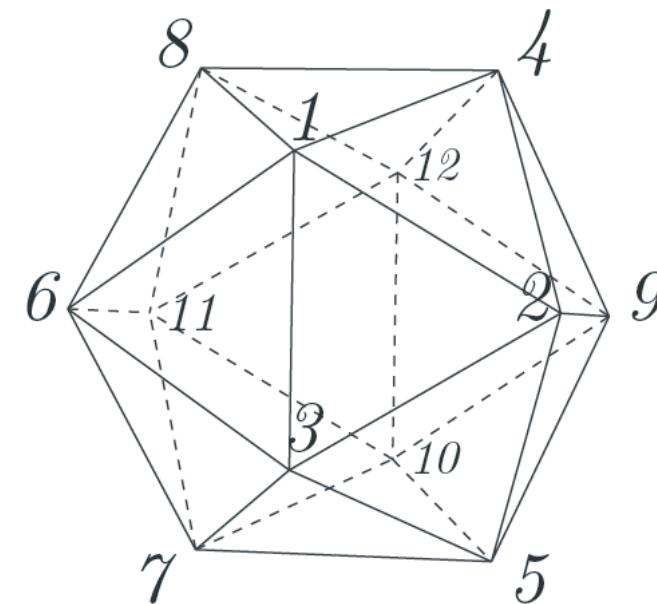
Is there a symmetry that nature is using in order to generate the lepton mixing?



Tetrahedron:  $A_4$



Cube:  $S_4$



Icosahedron:  $A_5$

Figures from M. Tanimoto et al. 2010

# Is there a broken symmetry?

The largest possible symmetry of the exact Neutrino Majorana mass matrix is a  $Z_2 \times Z_2$  symmetry

If this residual  $Z_2 \times Z_2$  symmetry arises from a largest group  $G_f$ , given

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$$

$U_\nu$  is fixed by the residual  $Z_2 \times Z_2$  symmetry (e.g. subgroup of  $G_f$ )

# Charged Lepton Corrections to $U_\nu$

$$U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$$

P. Frampton, S. T. Petcov,  
W. Rodejohann 2003

**1** rotation from charged leptons and **2** from neutrinos: **(1,2)**

$$U = R_{ij}(\theta_{ij}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

**2** rotations from charged leptons and **2** from neutrinos: **(2,2)**

$$U = R_{ij}(\theta_{ij}^e) R_{kl}(\theta_{kl}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

**1** rotation from charged leptons and **3** from neutrinos: **(1,3)**

$$U = R_{ij}(\theta_{ij}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

 **"Exact" and testable Sum Rules for  $\cos \delta$**



# Sum Rules

# "Exact" Sum Rules: (1,2)

$$U = R_{12}(\theta_{12}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

---

$$\cos \delta = \frac{(\cos 2\theta_{13} - \cos 2\theta_{23}^\nu)^{\frac{1}{2}}}{\sqrt{2} \sin 2\theta_{12} \sin \theta_{13} |\cos \theta_{23}^\nu|} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) \frac{2 \sin^2 \theta_{23}^\nu - (3 + \cos 2\theta_{23}^\nu) \sin^2 \theta_{13}}{\cos 2\theta_{13} - \cos 2\theta_{23}^\nu} \right]$$

$$U = R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

---

$$\cos \delta = -\frac{(\cos 2\theta_{13} + \cos 2\theta_{23}^\nu)^{\frac{1}{2}}}{\sqrt{2} \sin 2\theta_{12} \sin \theta_{13} |\sin \theta_{23}^\nu|} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) \frac{2 \cos^2 \theta_{23}^\nu - (3 - \cos 2\theta_{23}^\nu) \sin^2 \theta_{13}}{\cos 2\theta_{13} + \cos 2\theta_{23}^\nu} \right]$$

diff in global minus sign and  $\theta_{23}^\nu$

I. G., S. T. Petcov, A. V. Titov 2015

## "Exact" Sum Rules: (2,2)

$$U = R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

---

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}) \right]$$

S. T. Petcov 2015

see also D. Marzocca, S. T. Petcov,  
A. Romanino, M. C. Sevilla 2013

$$U = R_{13}(\theta_{13}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

---

$$\cos \delta = -\frac{\cot \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \tan^2 \theta_{23} \sin^2 \theta_{13}) \right]$$

**diff in global minus sign and  $\theta_{23}$**

I. G., S. T. Petcov, A. V. Titov 2015

# "Exact" Sum Rules: (1,3)

$$U = R_{12}(\theta_{12}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$


---

$$\cos \delta = \frac{1}{\sin 2\theta_{12} \sin \theta_{13} |\cos \theta_{13}^\nu \cos \theta_{23}^\nu| (\cos^2 \theta_{13} - \cos^2 \theta_{13}^\nu \cos^2 \theta_{23}^\nu)^{\frac{1}{2}}} \left[ (\cos^2 \theta_{13} - \cos^2 \theta_{13}^\nu \cos^2 \theta_{23}^\nu) \sin^2 \theta_{12} + \cos^2 \theta_{12} \sin^2 \theta_{13} \cos^2 \theta_{13}^\nu \cos^2 \theta_{23}^\nu - \cos^2 \theta_{13} (\cos \theta_{12}^\nu \sin \theta_{13}^\nu \cos \theta_{23}^\nu - \sin \theta_{12}^\nu \sin \theta_{23}^\nu)^2 \right]$$

$$U = R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$


---

$$\cos \delta = -\frac{1}{\sin 2\theta_{12} \sin \theta_{13} |\cos \theta_{13}^\nu \sin \theta_{23}^\nu| (\cos^2 \theta_{13} - \cos^2 \theta_{13}^\nu \sin^2 \theta_{23}^\nu)^{\frac{1}{2}}} \left[ (\cos^2 \theta_{13} - \cos^2 \theta_{13}^\nu \sin^2 \theta_{23}^\nu) \sin^2 \theta_{12} + \cos^2 \theta_{12} \sin^2 \theta_{13} \cos^2 \theta_{13}^\nu \sin^2 \theta_{23}^\nu - \cos^2 \theta_{13} (\cos \theta_{12}^\nu \sin \theta_{13}^\nu \sin \theta_{23}^\nu + \sin \theta_{12}^\nu \cos \theta_{23}^\nu)^2 \right]$$

diff in global minus sign and  $\theta_{23}^\nu$

I. G., S. T. Petcov, A. V. Titov 2015

# "Exact" Sum Rules for $\sin^2 \Theta_{23}$

Parametrisation of $U$	$\sin^2 \theta_{23}$
$R_{12}(\theta_{12}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	$\frac{\sin^2 \theta_{23}^\nu - \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}$
$R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	$\frac{\sin^2 \theta_{23}^\nu}{1 - \sin^2 \theta_{13}}$
$R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	<b>not fixed</b>
$R_{13}(\theta_{13}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	<b>not fixed</b>
$R_{12}(\theta_{12}^e) R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	<b>not fixed</b>
$R_{12}(\theta_{12}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	$1 - \frac{\cos^2 \theta_{23}^\nu \cos^2 \theta_{13}^\nu}{1 - \sin^2 \theta_{13}}$
$R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$	$\frac{\sin^2 \theta_{23}^\nu \cos^2 \theta_{13}^\nu}{1 - \sin^2 \theta_{13}}$

# Predictions



# Predictions for $\cos \delta$

$\theta_{23}^{\nu} = -\pi/4$  below  $\theta_{12}^{\nu}$  or  $[\theta_{13}^{\nu}, \theta_{12}^{\nu}]$  using the current b.f.v. for  $\theta_{ij}$  for NO

Scheme	TBM	GRA	GRB	HG	BM (LC)
$\theta_{12}^e - (\theta_{23}^{\nu}, \theta_{12}^{\nu})$	-0.114	0.289	-0.200	0.476	—
$\theta_{13}^e - (\theta_{23}^{\nu}, \theta_{12}^{\nu})$	0.114	-0.289	0.200	-0.476	—
$(\theta_{12}^e, \theta_{23}^e) - (\theta_{23}^{\nu}, \theta_{12}^{\nu})$	-0.091	0.275	-0.169	0.445	—
$(\theta_{13}^e, \theta_{23}^e) - (\theta_{23}^{\nu}, \theta_{12}^{\nu})$	0.151	-0.315	0.251	-0.531	—
$''(\theta_{12}^e, \theta_{13}^e) - (\theta_{23}^{\nu}, \theta_{12}^{\nu})''$	-0.122	0.282	-0.208	0.469	—
Scheme	<b>III</b> $[\pi/20, -\pi/4]$	<b>I</b> $[\pi/10, -\pi/4]$	<b>IV</b> $[a, -\pi/4]$	<b>II</b> $[\pi/20, b]$	<b>V</b> $[\pi/20, \pi/6]$
$\theta_{12}^e - (\theta_{23}^{\nu}, \theta_{13}^{\nu}, \theta_{12}^{\nu})$	-0.222	0.760	0.911	-0.775	-0.562
Scheme	<b>III</b> $[\pi/20, c]$	<b>I</b> $[\pi/20, \pi/4]$	<b>IV</b> $[\pi/10, \pi/4]$	<b>II</b> $[a, \pi/4]$	<b>V</b> $[\pi/20, d]$
$\theta_{13}^e - (\theta_{23}^{\nu}, \theta_{13}^{\nu}, \theta_{12}^{\nu})$	-0.866	0.222	-0.760	-0.911	-0.791

with  $\sin a = 1/3$ ,  $\sin b = 1/(2+r)^{1/2}$ ,  $\sin^2 c = 1/3$ ,  $\sin^2 d = (3-r)/4$

e.g. non-zero  $\theta_{13}^{\nu}$  i) TP mixing (**IV**) F. Bazzocchi 2008, ii) R. d. A. Toorop et al. 2011, iii) W. Rodejohann and H. Zhang 2014

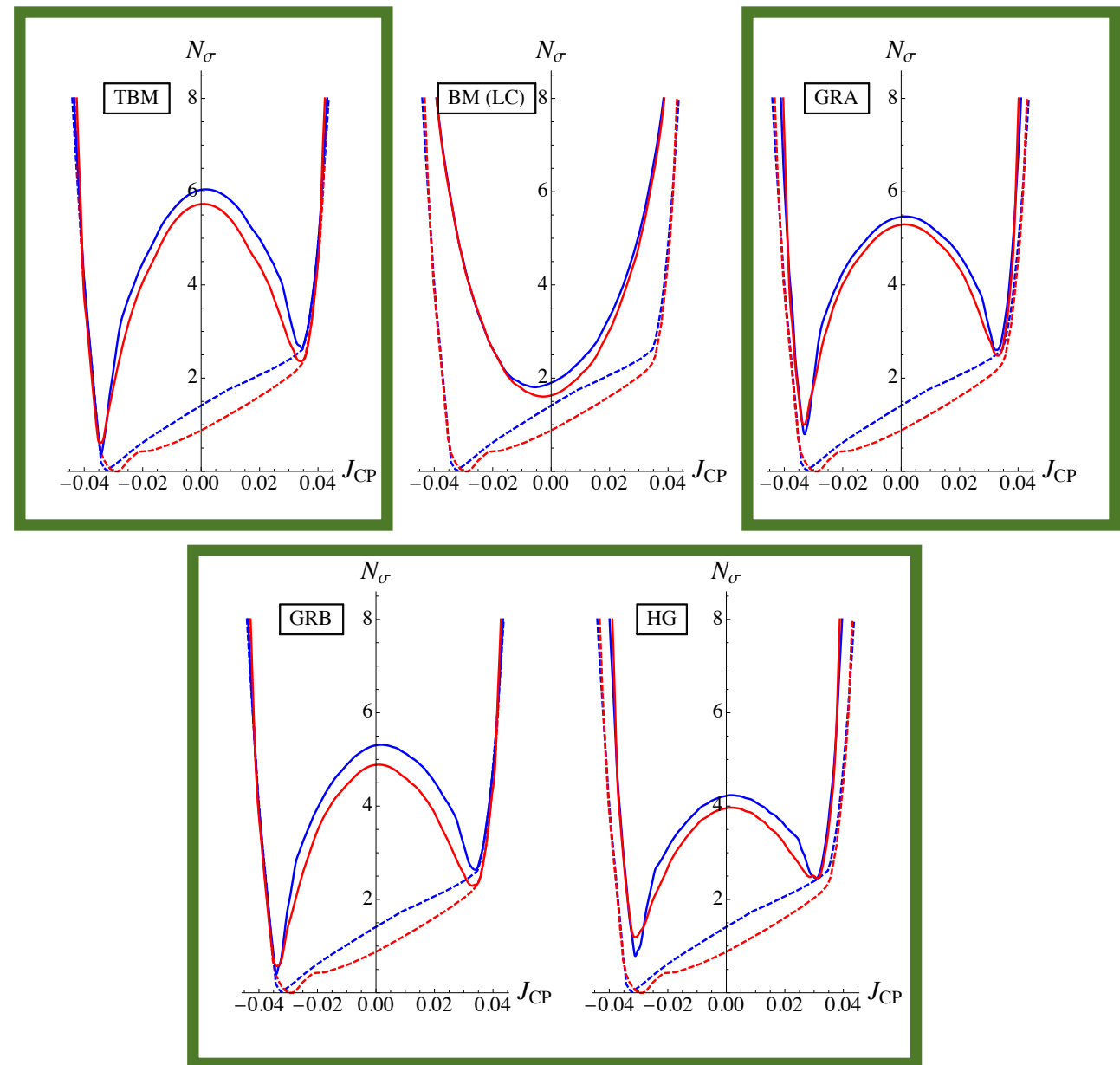
# Predictions for $J_{CP}$ : current sensitivity on $\theta_{ij}$

$J_{CP}$  determines the magnitude of CP-violating effects in neutrino oscillations

$$U = R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

$$N_\sigma \equiv \sqrt{\chi^2}$$

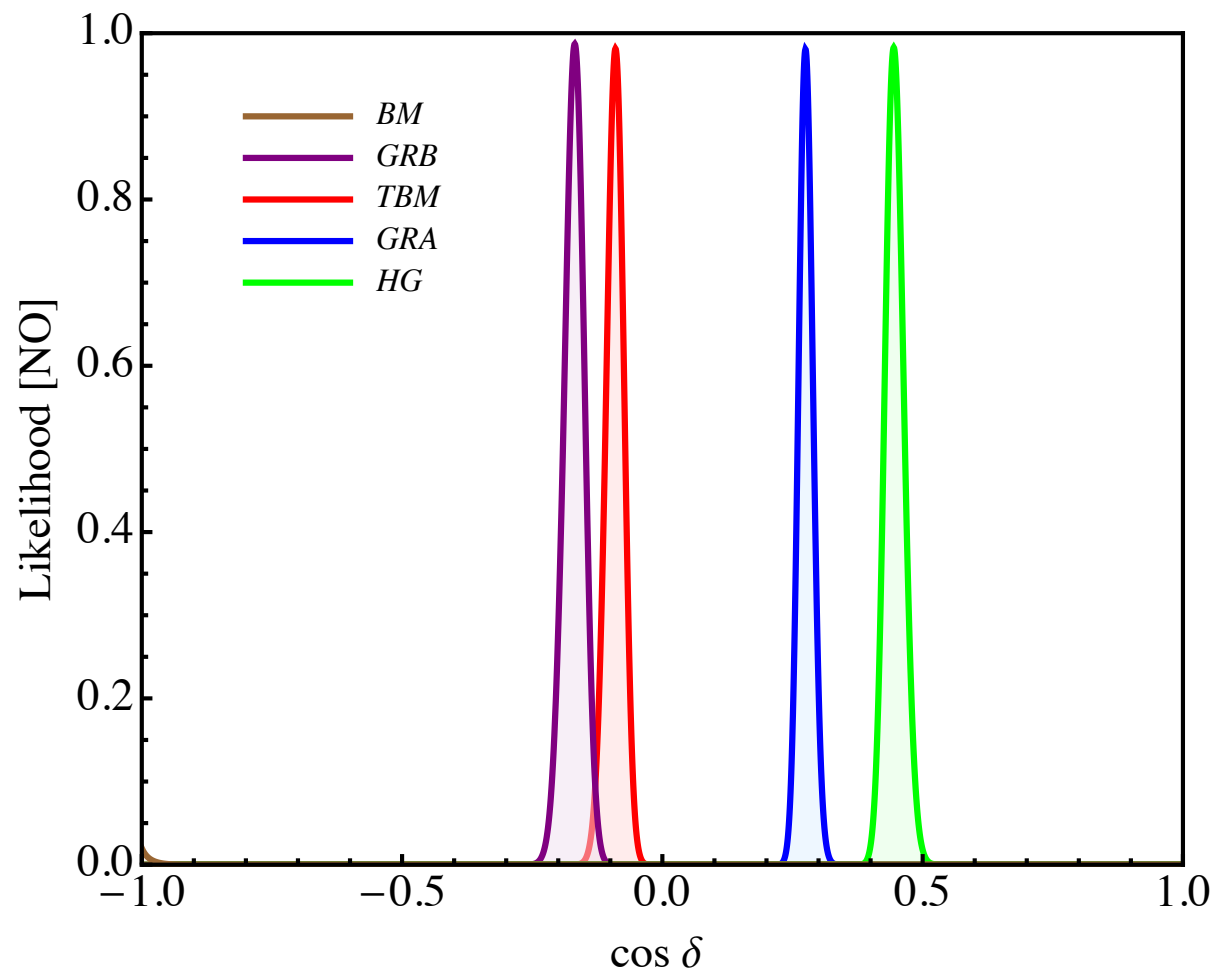
In TBM, GRA, GRB, HG we predict relatively large CPV effects in neutrino oscillations:  $J_{CP} \approx -0.03$   
 $|J_{CP}| > 0.02 @ 3\sigma$



In contrast in BM CPV effects are predicted to be suppressed,  $\delta \approx \pi$

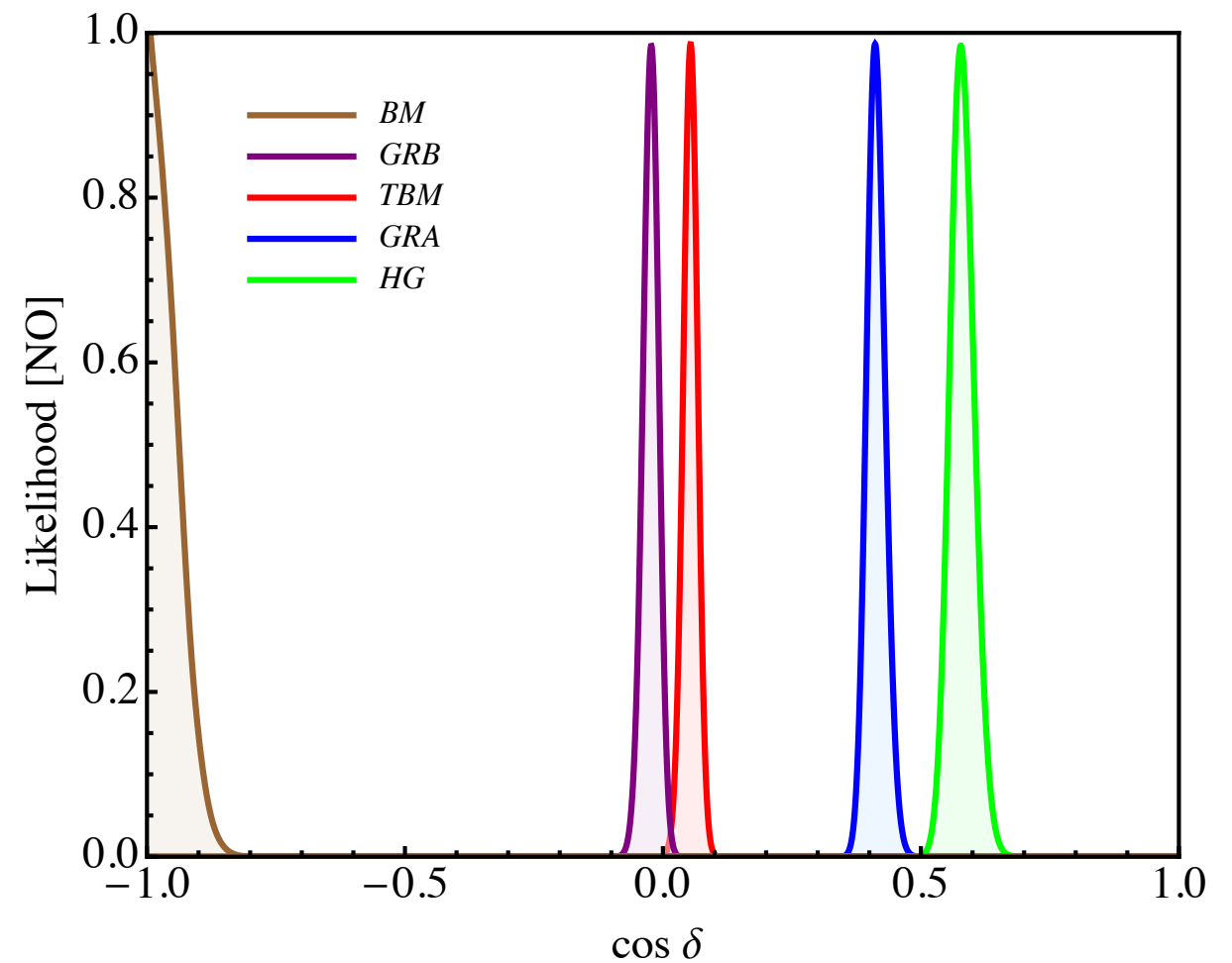
# Predictions for $\cos \delta$ : future sensitivity on $\theta_{ij}$

$$U = R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$



b.f. from F. Capozzi et al. 2014

$$\sin^2 \theta_{12} = 0.308$$



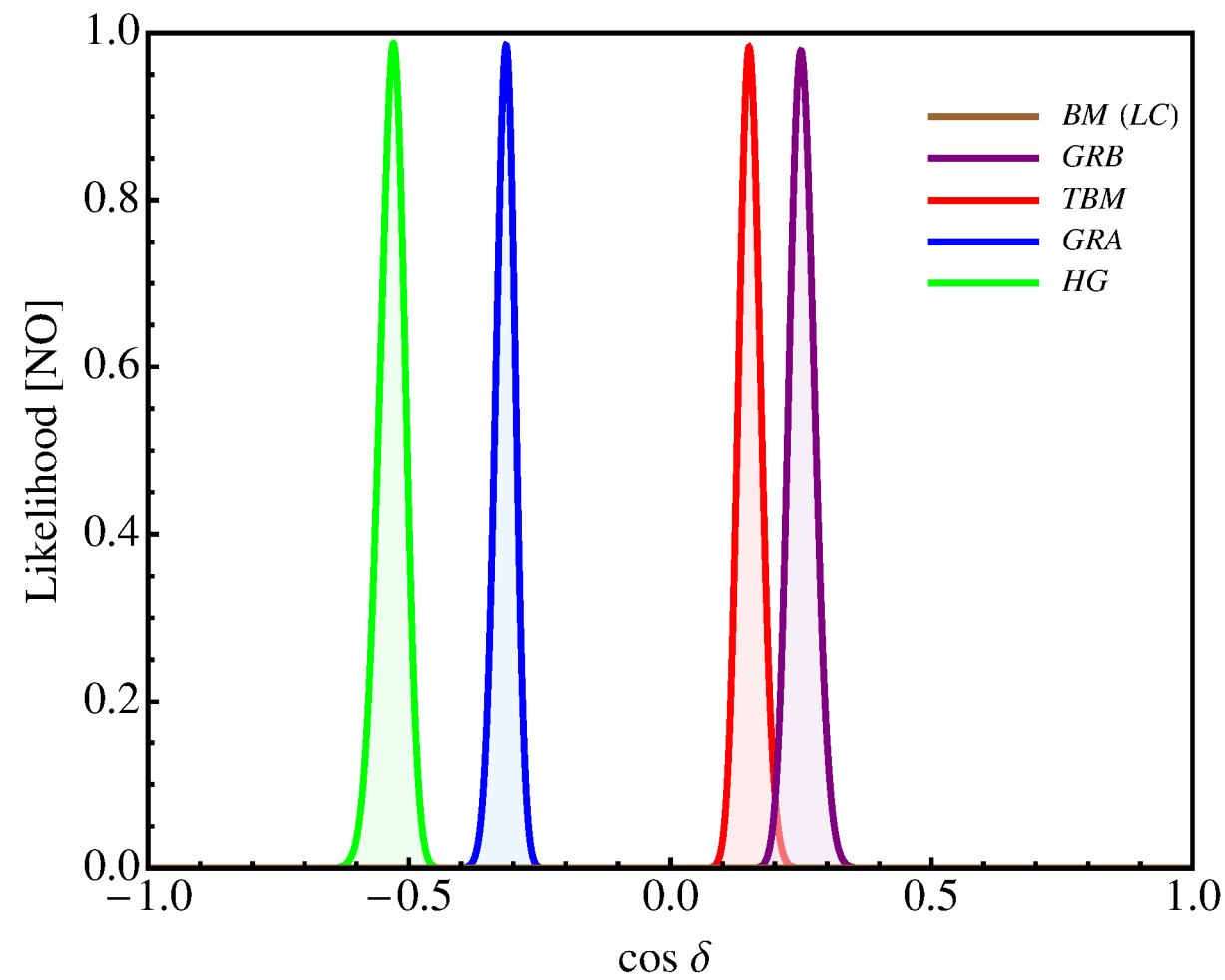
$$\sin^2 \theta_{12} = 0.332$$

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$$L(\cos \delta) \propto \exp\left(-\frac{\chi^2(\cos \delta)}{2}\right)$$

# Predictions for $\cos \delta$ : future sensitivity on $\theta_{ij}$

$$U = R_{13}(\theta_{13}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$



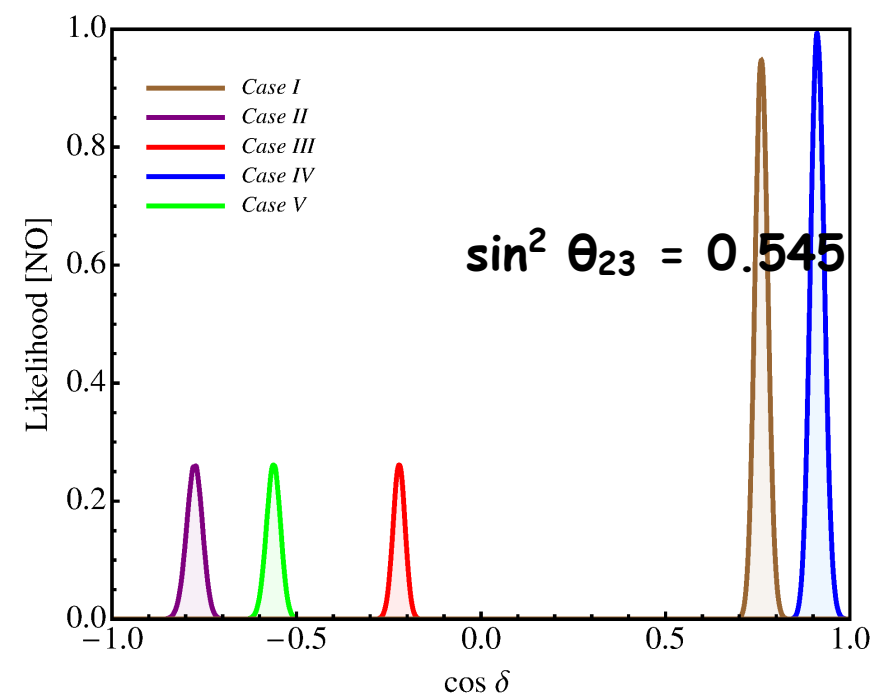
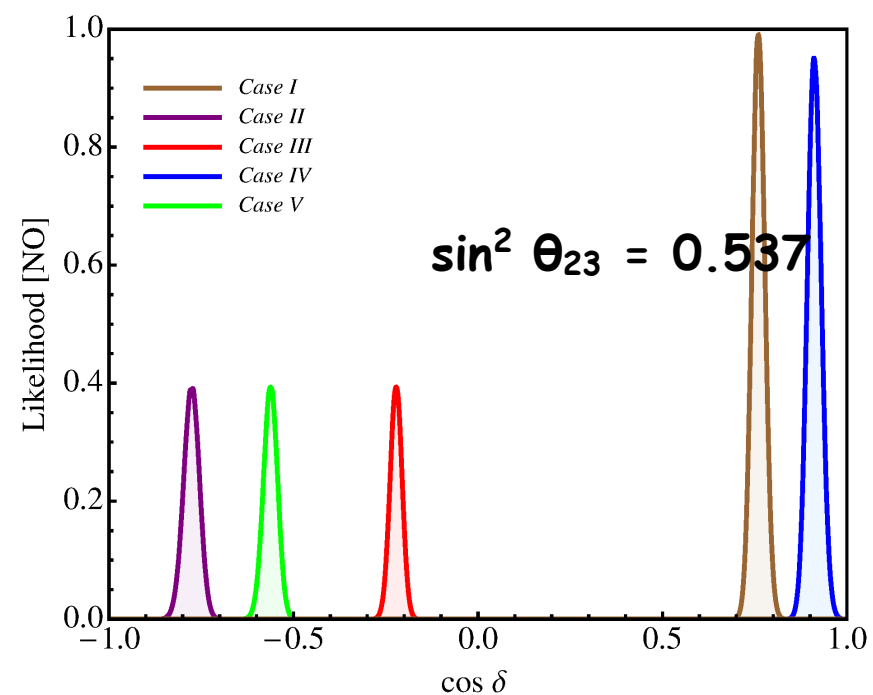
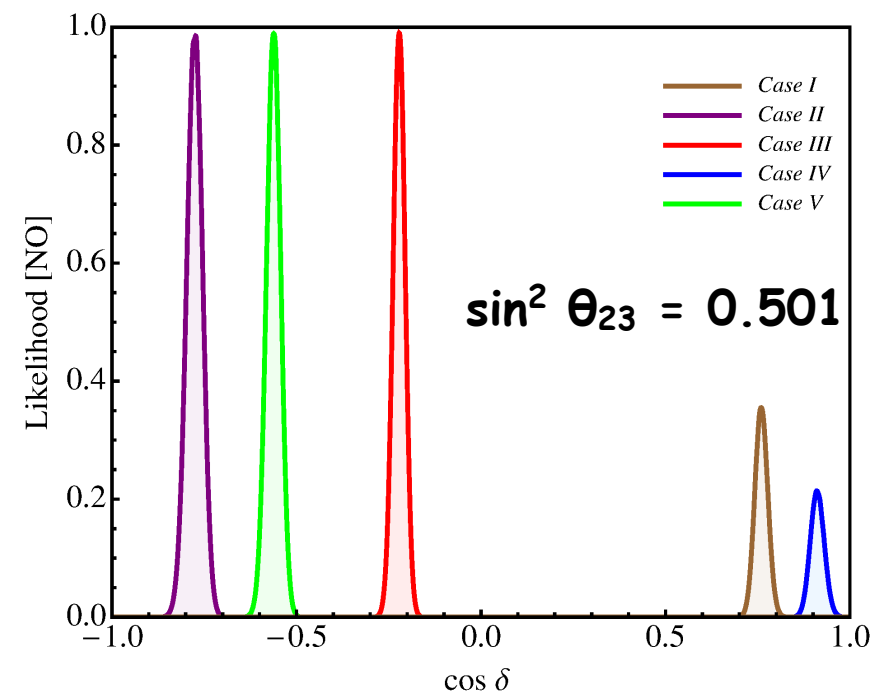
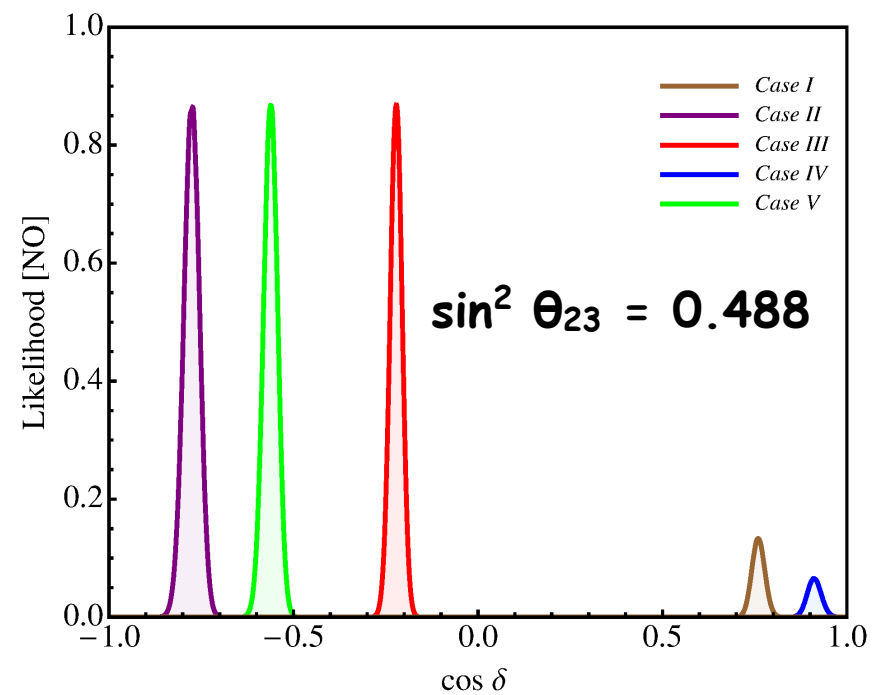
**b.f. from F. Capozzi et al. 2014**

I. G., S. T. Petcov, A. V. Titov 2015

# Predictions for $\cos \delta$ : future sensitivity on $\theta_{ij}$

$$U = R_{12}(\theta_{12}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

$\sin^2 \theta_{23}$  p.b.f.

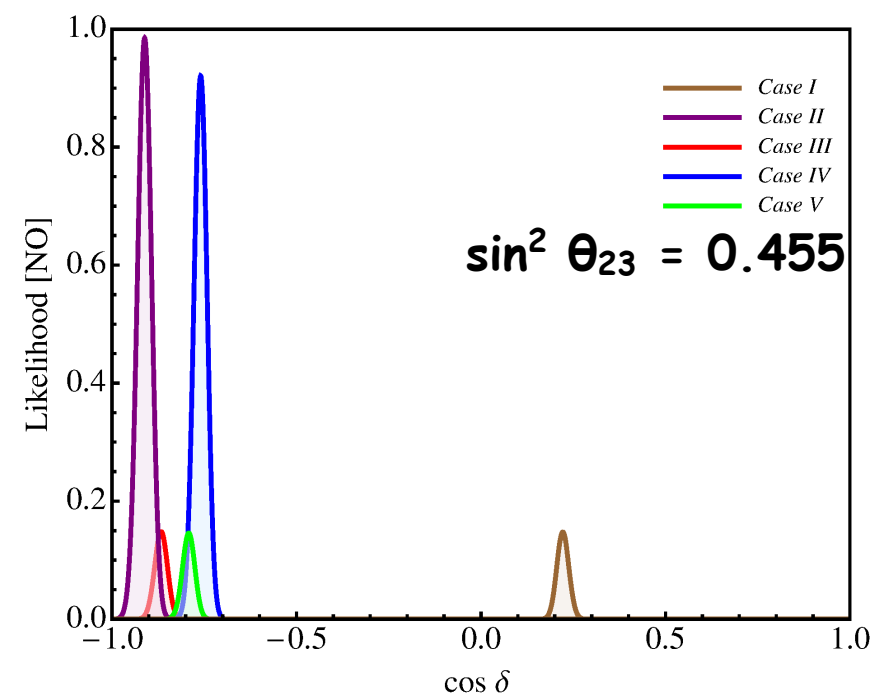
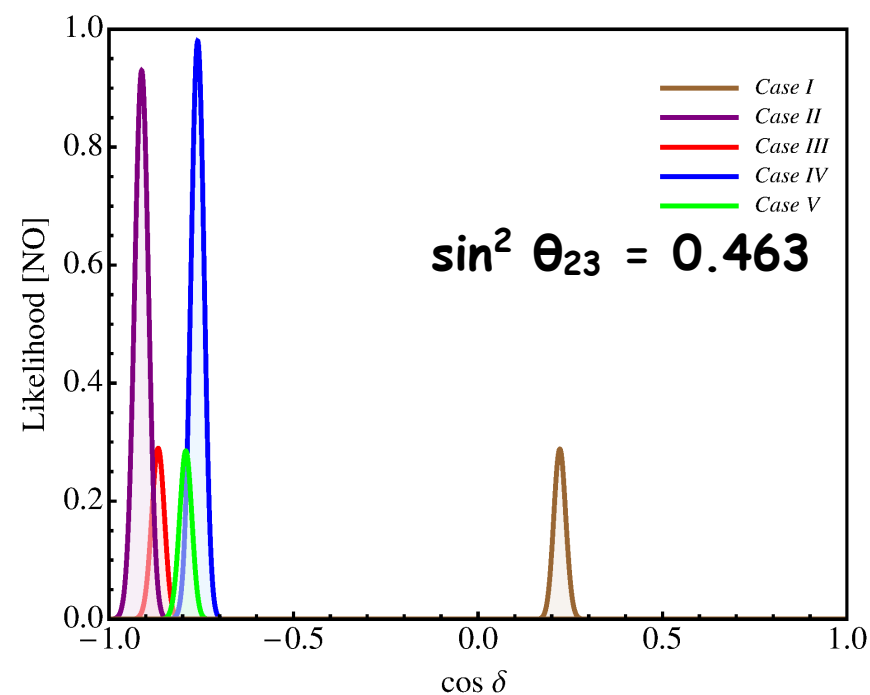
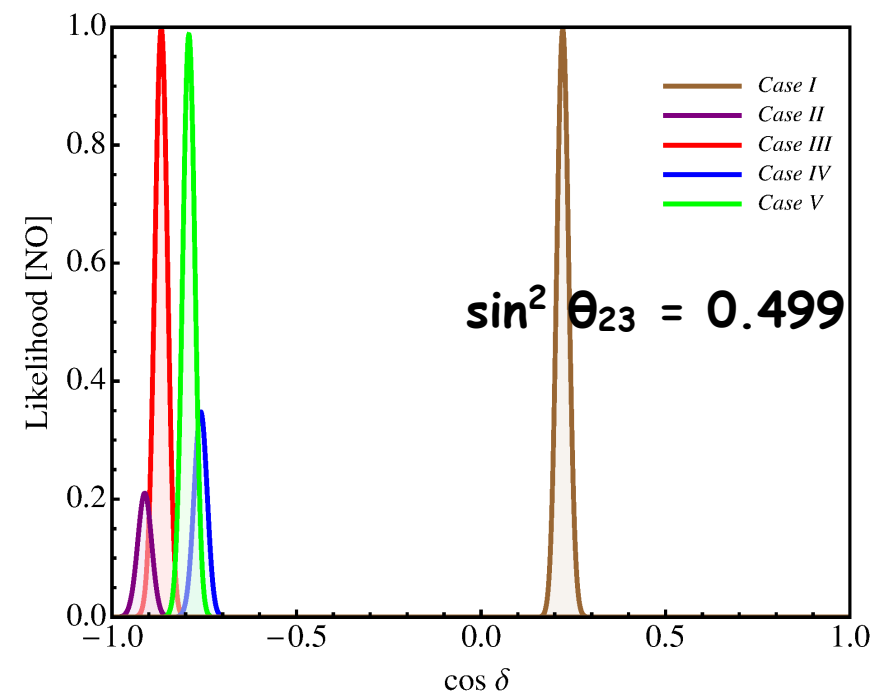
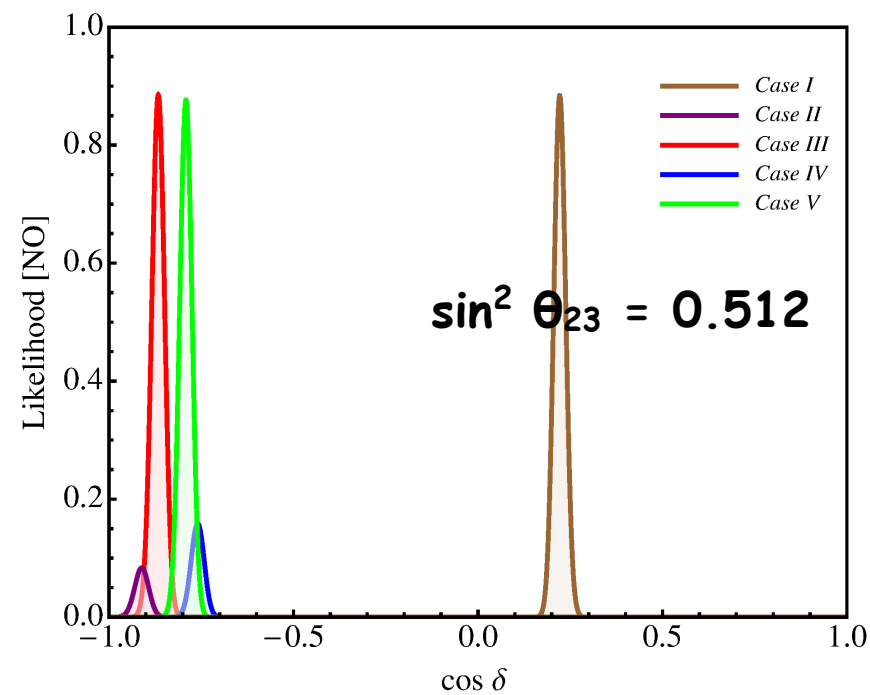


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# Predictions for $\cos \delta$ : future sensitivity on $\theta_{ij}$

$$U = R_{13}(\theta_{13}^e) \Psi R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu) Q_0$$

$\sin^2 \theta_{23}$  p.b.f.



I. G., S. T. Petcov, A. V. Titov 2015

# Summary

“Exact” Sum Rules for  $\cos \delta$ : a systematic analysis

In TBM, GRA, GRB, HG we predict relatively large CPV effects in neutrino oscillations:

$$J_{CP} \approx -0.03 \text{ \& } |J_{CP}| > 0.02 @ 3\sigma$$

In BM CPV effects are predicted to be suppressed, since  $\delta \approx \pi$

One can distinguish between these cases by measuring the value of  $\cos \delta$  (it changes in magnitude and sign)

Needs experimental challenges: DUNE,  $\mu$ DAR, T2HK, ESSvSB...



# Conclusions

A relative precise measurement of  $\cos \delta$  (T2HK, ESS $\nu$ SB, DUNE,  $\mu$ DAR) combined with prospective precision on the neutrino mixing angles (Daya Bay, JUNO, T2K, NO $\nu$ A) can provide unique information about the existence of  
**a new fundamental symmetry in the lepton sector**

**Thank you**

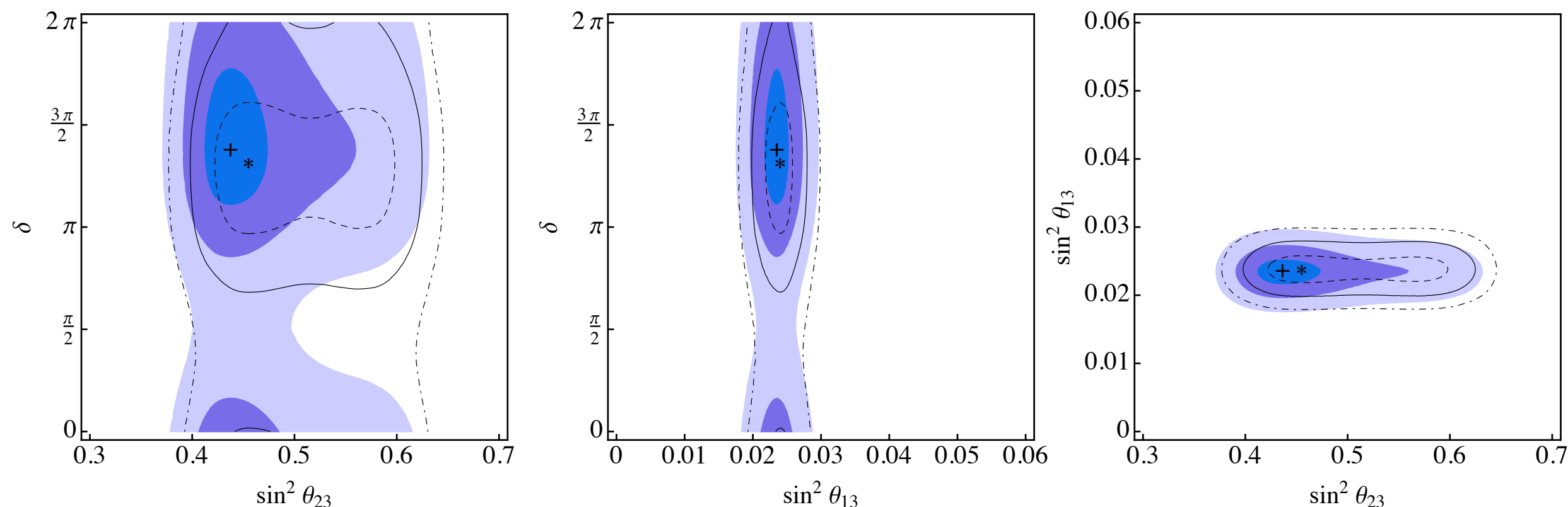
**Details are also in A. V. Titov's talk (WIN 2015)**

# Backup slides

# Statistical Analysis

$$\chi^2(\sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23}, \delta) = \chi_1^2(\sin^2 \theta_{12}) + \chi_2^2(\sin^2 \theta_{13}) + \chi_3^2(\sin^2 \theta_{23}) + \chi_4^2(\delta)$$

from F. Capozzi et al. (2014)



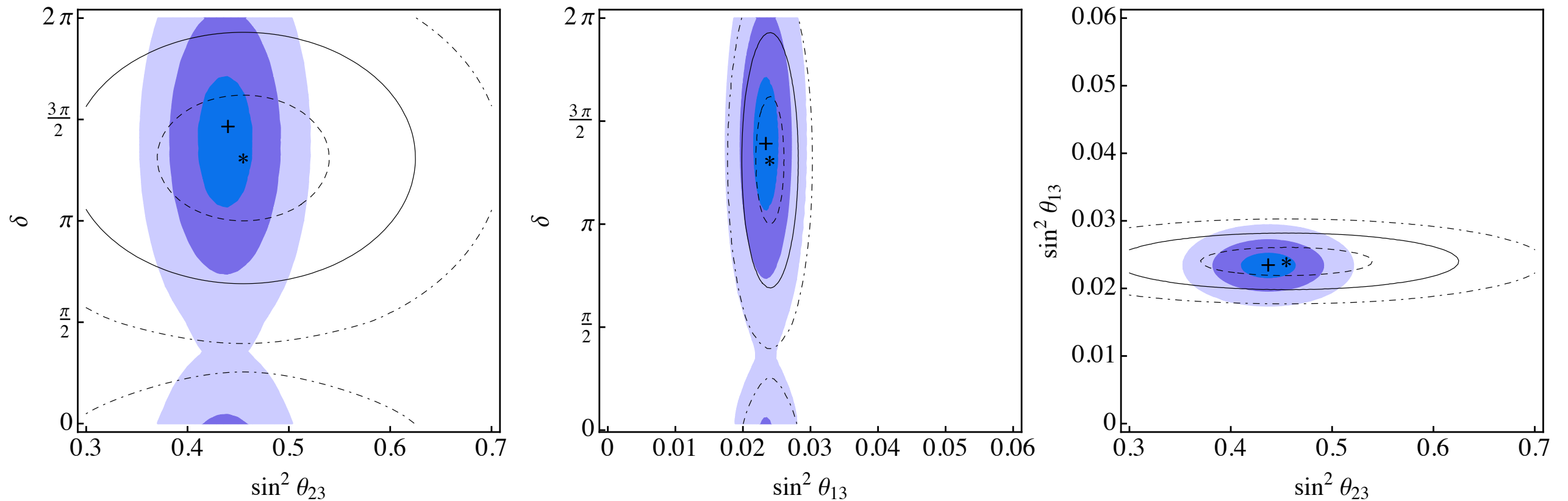
NO: shaded areas  
IO: lines

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Neglecting correlations between the oscillation parameters

# Gaussian Approximation

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$$\chi_G^2 = \sum_i (x_i - \bar{x}_i)^2 / \sigma_{x_i}^2$$

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**with  $x_i = \{\sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23}, \delta\}$  and  $\bar{x}_i, \sigma_{x_i}$  mean and 1 sigma error from F. Capozzi et al. (2014)**  
**the 3 sigma range for the atmospheric angle exceeds [0.3,0.7]**

# Likelihood

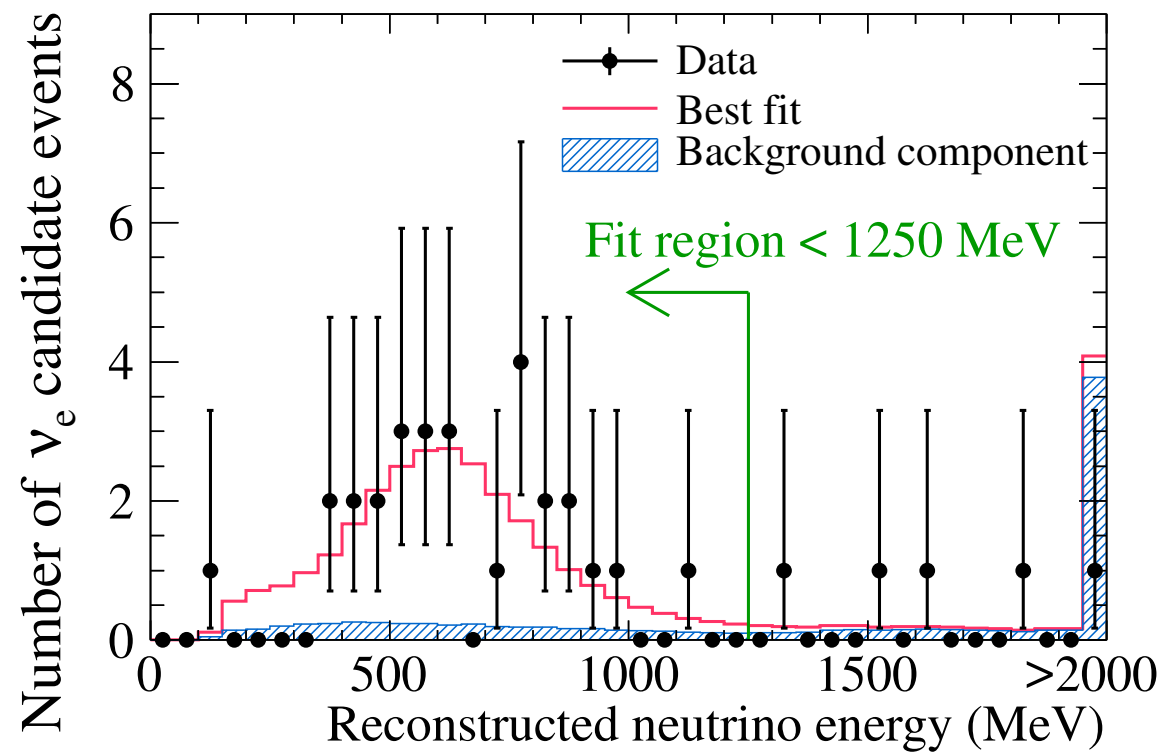
$$L(\cos \delta) \propto \exp \left( -\frac{\chi^2(\cos \delta)}{2} \right)$$

We construct, e.g.,  $\chi^2(\cos \delta)$

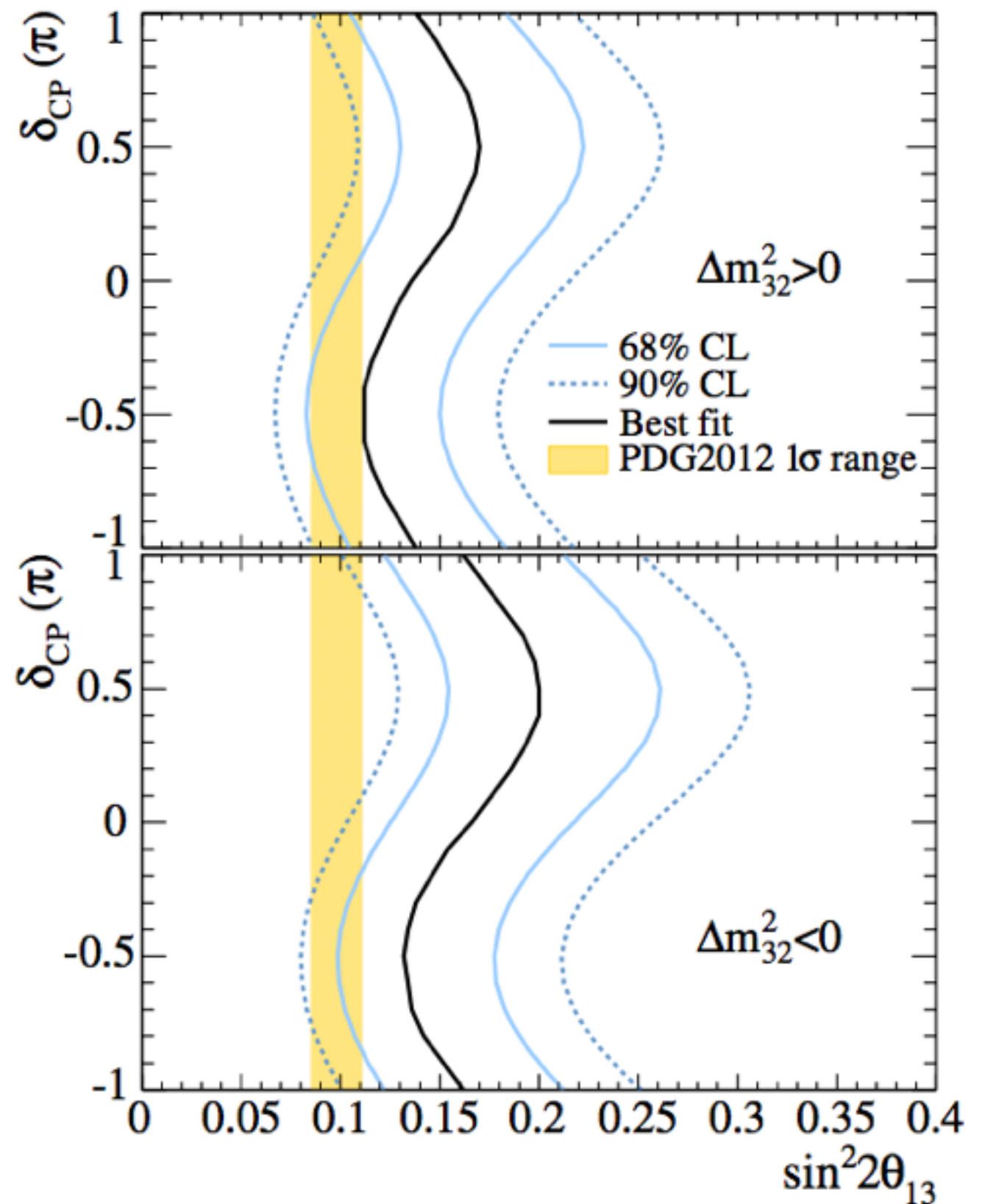
by marginalizing  $\chi^2(\{x_i\})$

over the free parameters for a fixed value of  $\cos \delta$

# T2K results 2014



K. Abe et al. (T2K) 2014

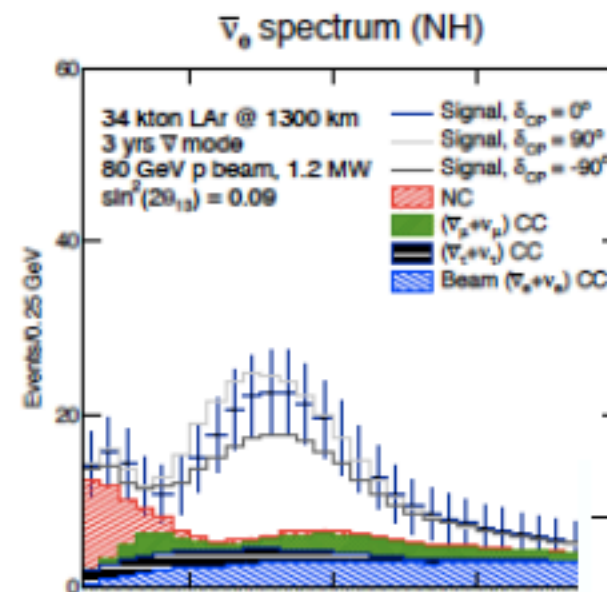
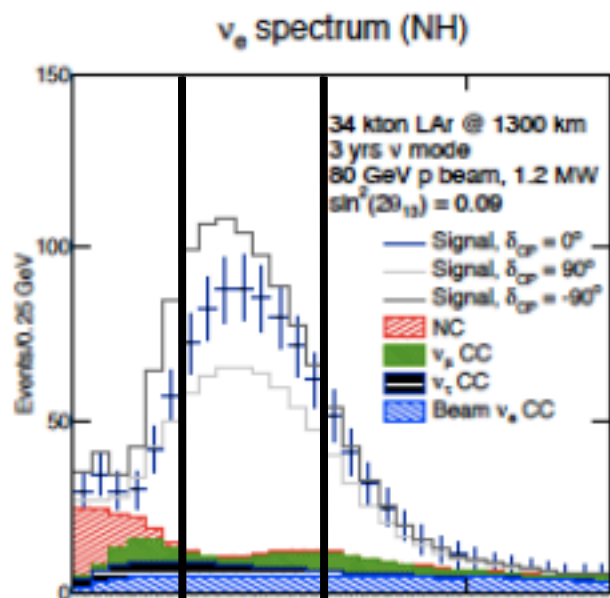




# DUNE (formerly LBNE)

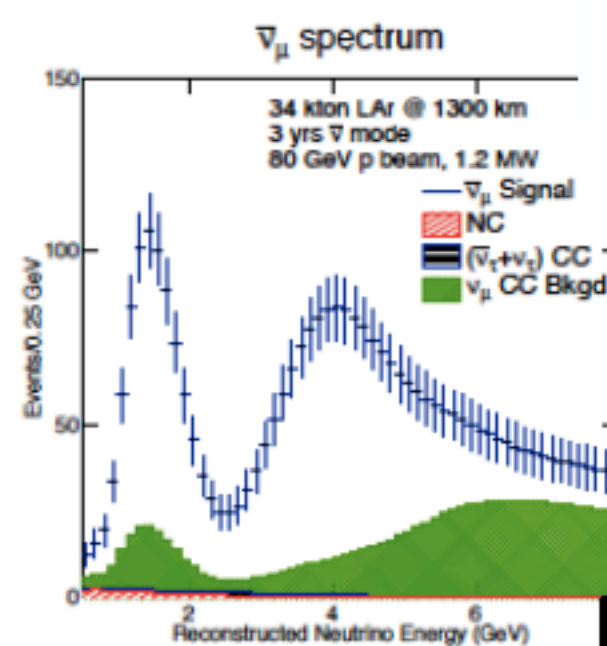
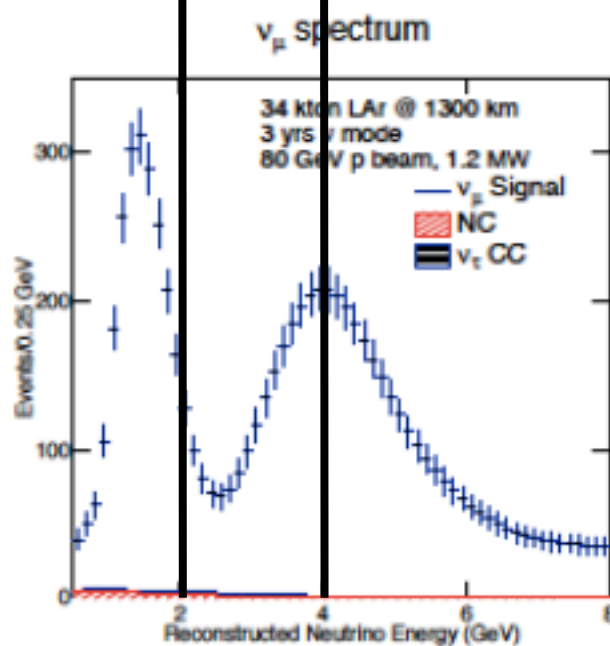


## LBNE Event Rate



**34 kton Lar  
1.2 MW beam  
NH,  $\delta_{CP} = 0$   
Fit parameters  
from 2012**

80 GeV Beam	$\nu$ mode	$\bar{\nu}$ mode
Signal: $\nu_e + \bar{\nu}_e$	777	189
BG: NC	67	39
BG: $\nu_\mu + \bar{\nu}_\mu$ CC	84	39
BG: Beam $\nu_e + \bar{\nu}_e$	147	81
BG: $\nu_\tau + \bar{\nu}_\tau$ CC	49	32



**Full oscillation structure visible in energy spectrum. A combined fit to the spectrum to provide unambiguous parameter sensitivity in one experiment**

**arXiv:1307.7335 – LBNE Document**

2 4 6 8 GeV

B. C. Choudhary's talk  
(Queen Mary 2014)