High precision at the LHC: The Loop-Tree duality

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Outline

- High precision measurements: Higgs discovery
- Theoretical tools: Phenomenology
- Loop-Tree Duality: Ideas & developments

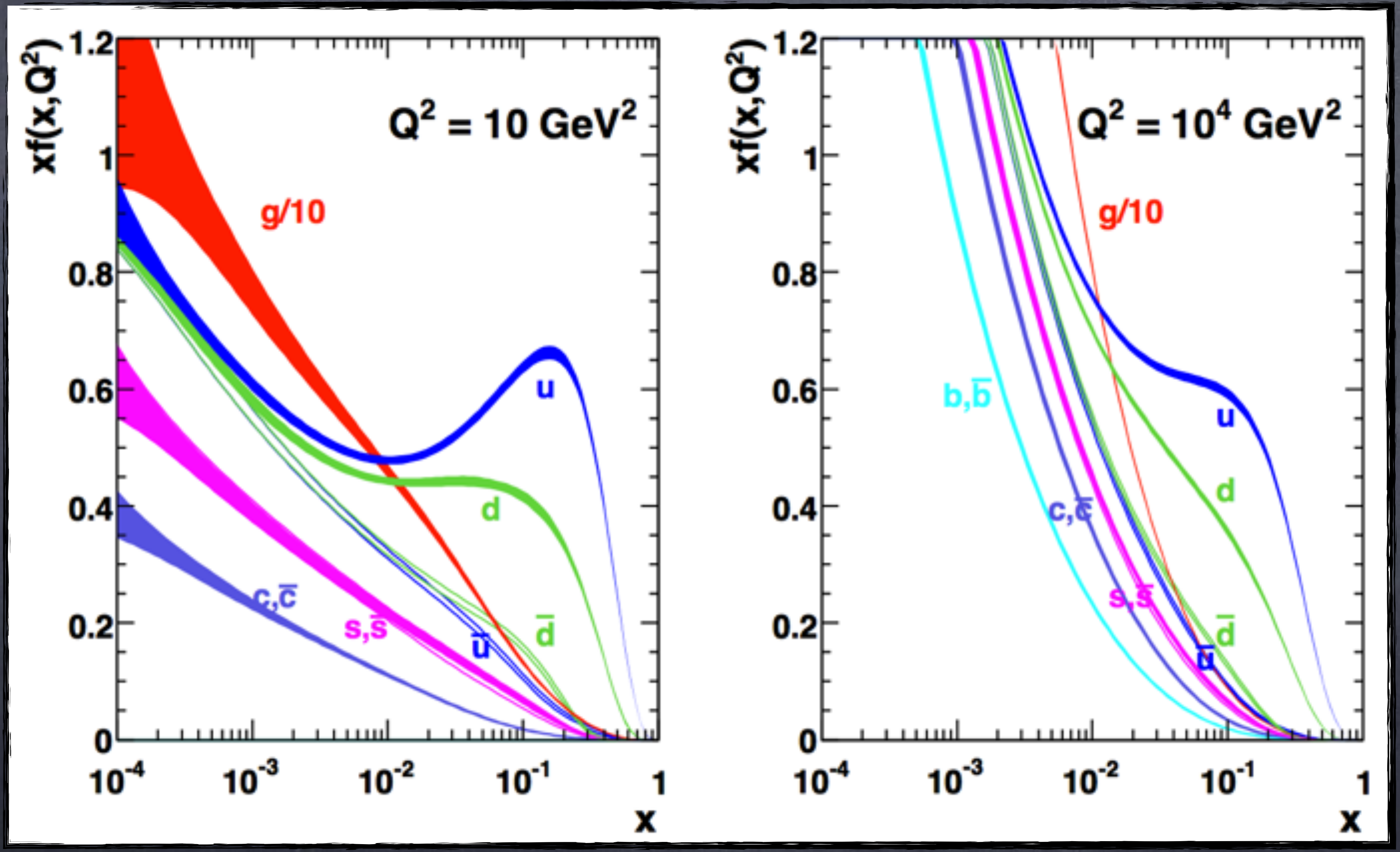
Higgs production @LHC

- Higgs discovery has been achieved with a high precision due to theoretical and experimental advantages
- Both groups have done such a hard work that in July 4th 2012, the 5sigma discovery was announced at CERN
- How was the path of the Higgs discovery ?

Long ago...

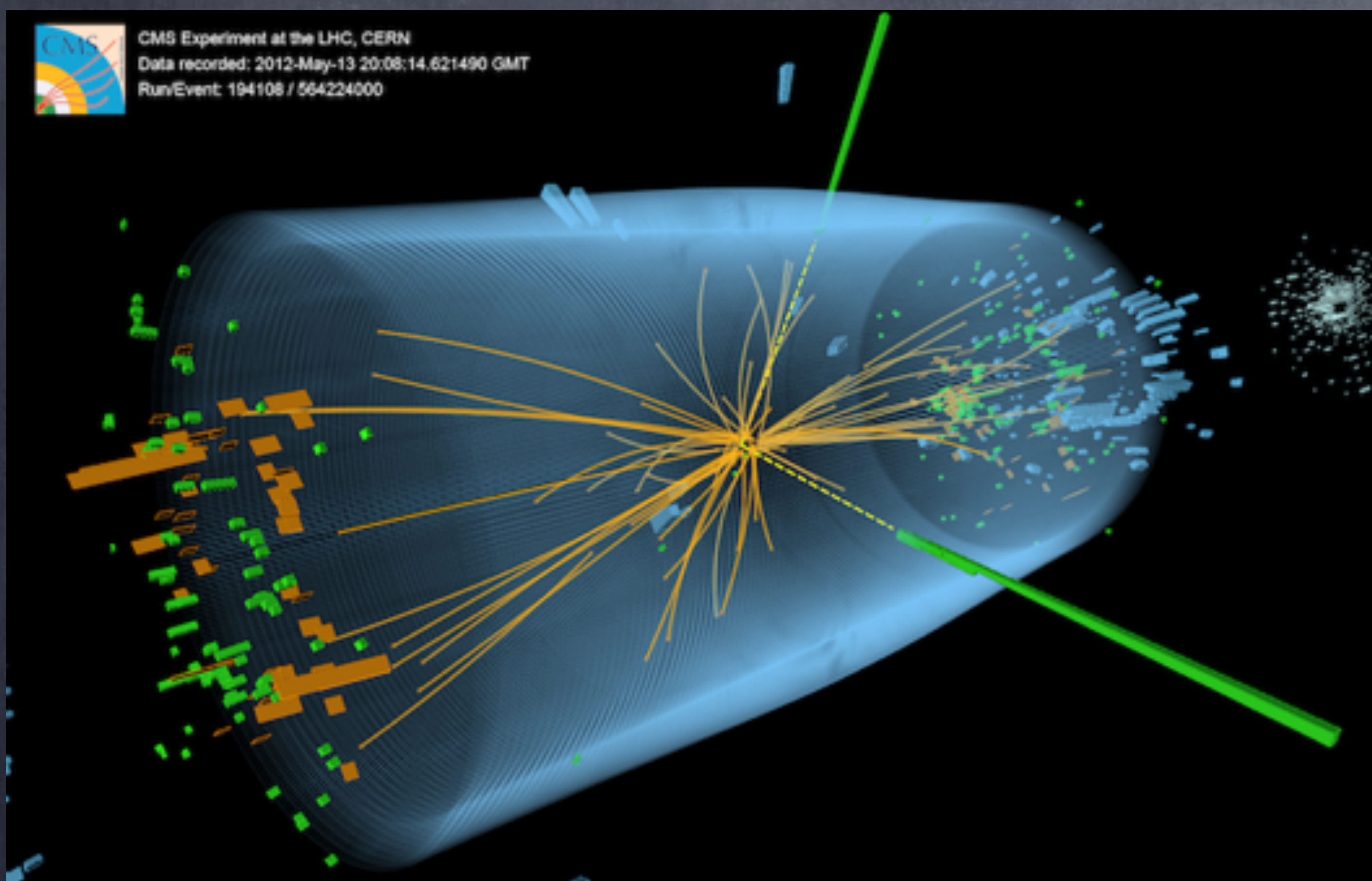
- Colliders have been searching for all fundamental particles since long ago, and the Higgs particle (the only scalar in the SM) was the most difficult one to find.
- The first issue is that the production of the Higgs at colliders is dominated by gluon densities.

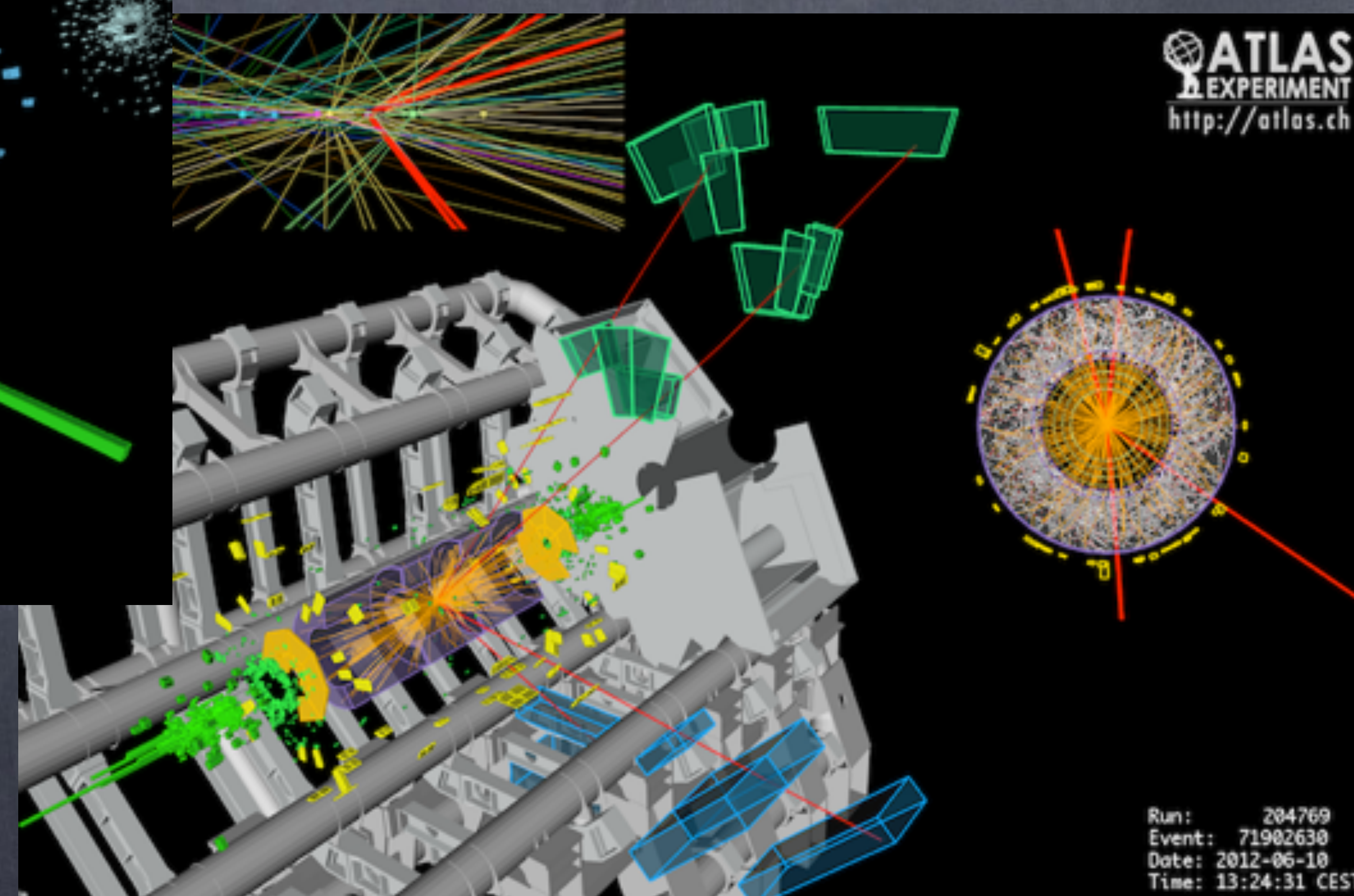
Parton Distribution Functions



- Protons are made of quarks and gluons

- Then, fundamental particles are not detected in the LHC detectors, only the products of its decay



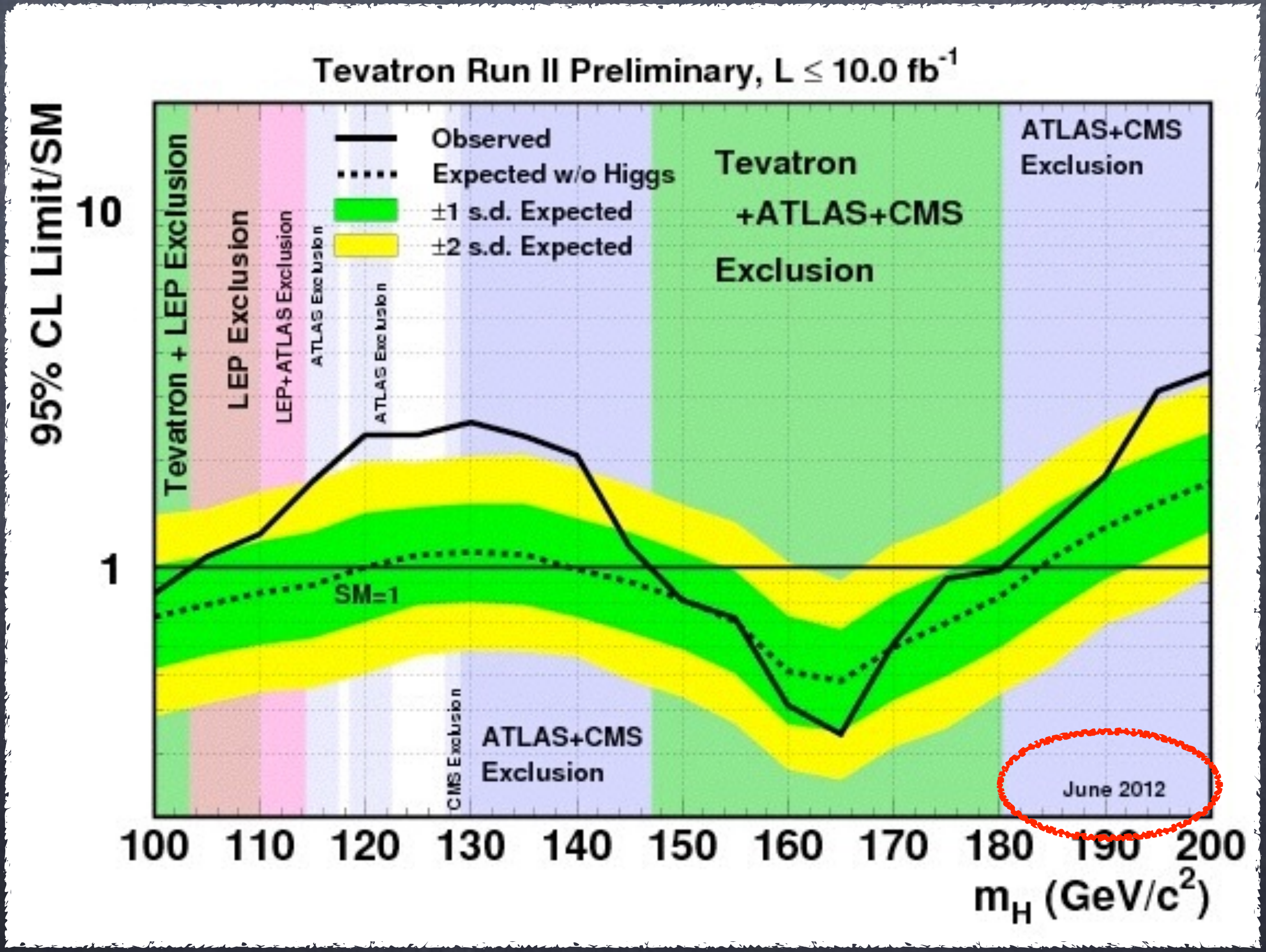


- How does the Higgs decay?

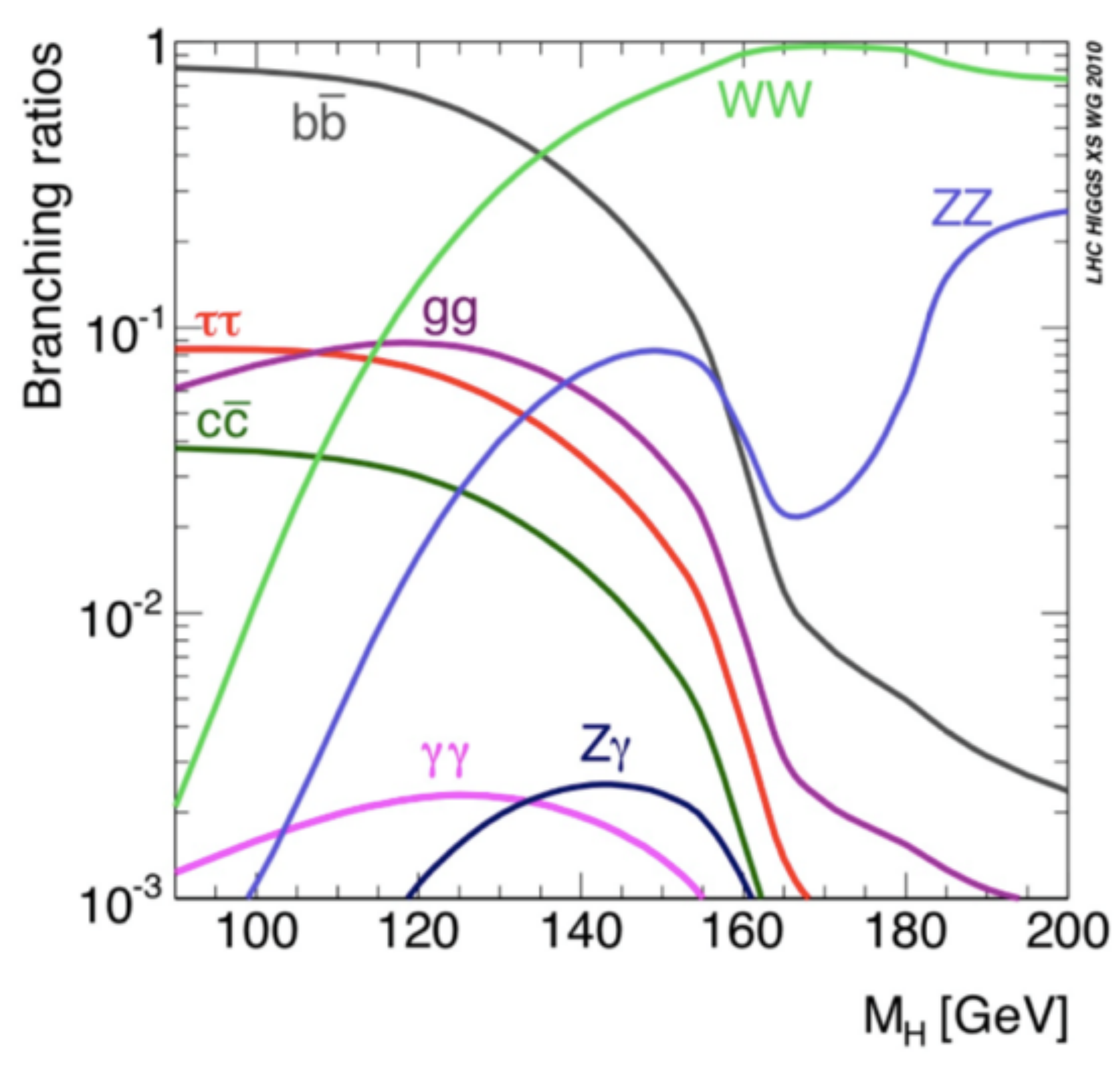
Higgs discovery

- Higgs has so many different channels to decay, once it is produced
- One has to be able to separate all contributions from each individual channel, i.e. distinguish all signatures in the most accurate way
- A Monte Carlo simulation is needed in order to provide the distributions which are the smoking gun of the Higgs particle

- Tevatron and the LHC discarded the Higgs in a very wide range except around 122 - 128 GeV
- The only region available was the most complicated one, where all decay channels have to be controlled with very high precision



Decay modes of Higgs

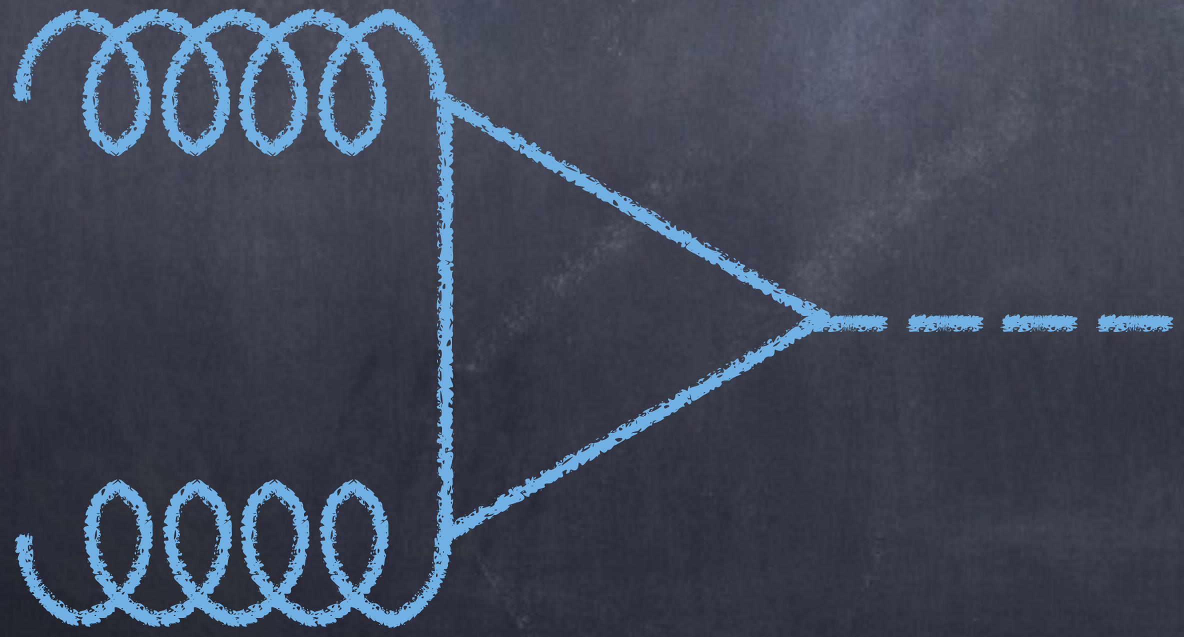


One needs to analyse all channels in order to confirm that the particle is the one predicted by the SM or to claim new physics

Everything have to be extremely precise in the measurement (experimentally) and in the prediction (theoretically)

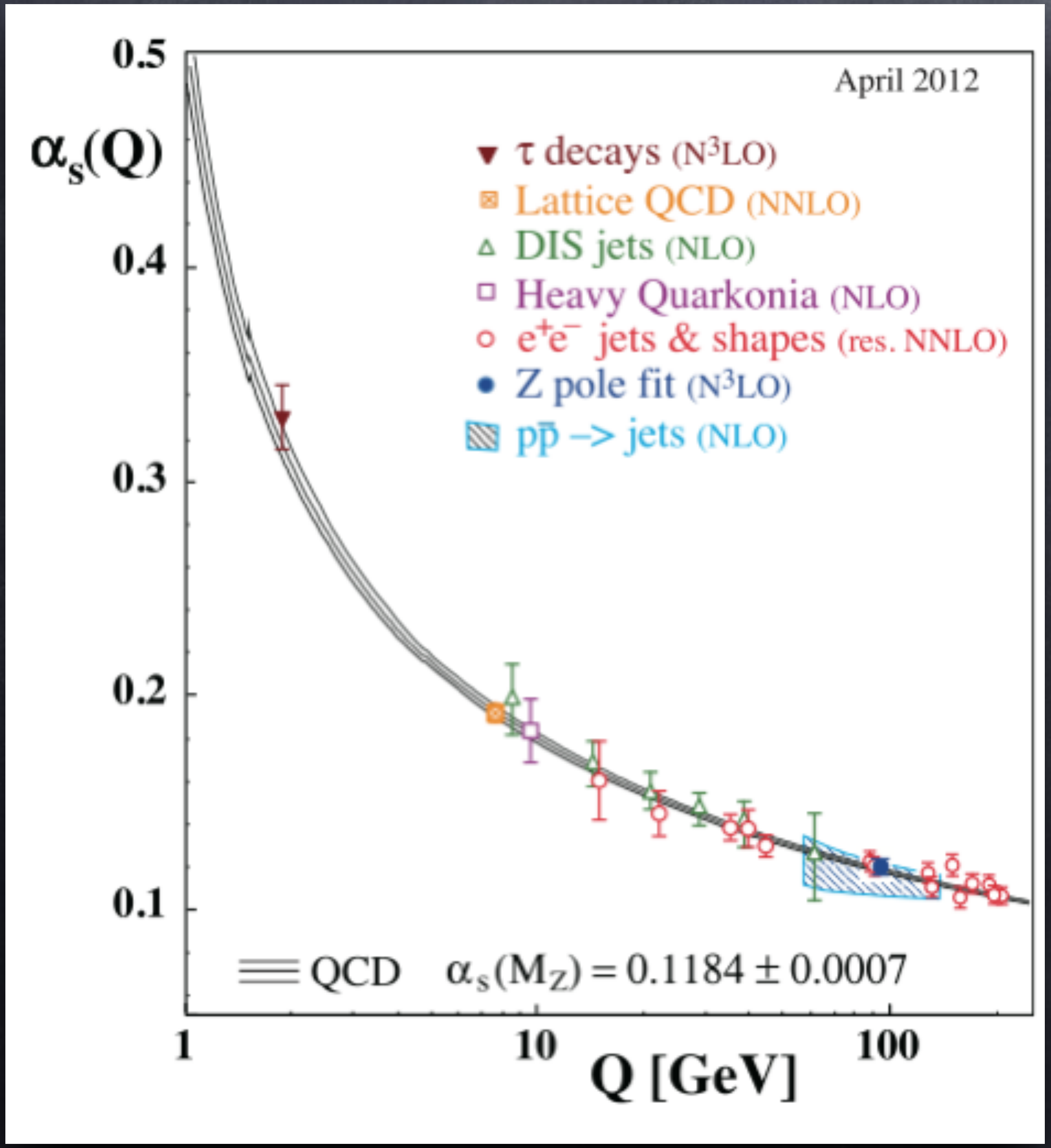
Why so complicated?

- Let's consider only Higgs production
- The lowest order in pQCD involves a triangle diagram (so called, gluon fusion):



Divergences appear due to all possible configurations of the loop momentum

- But LO is not reliable due to running of the strong coupling at LHC energies, so, it is crucial to know the NLO or NNLO theoretical results

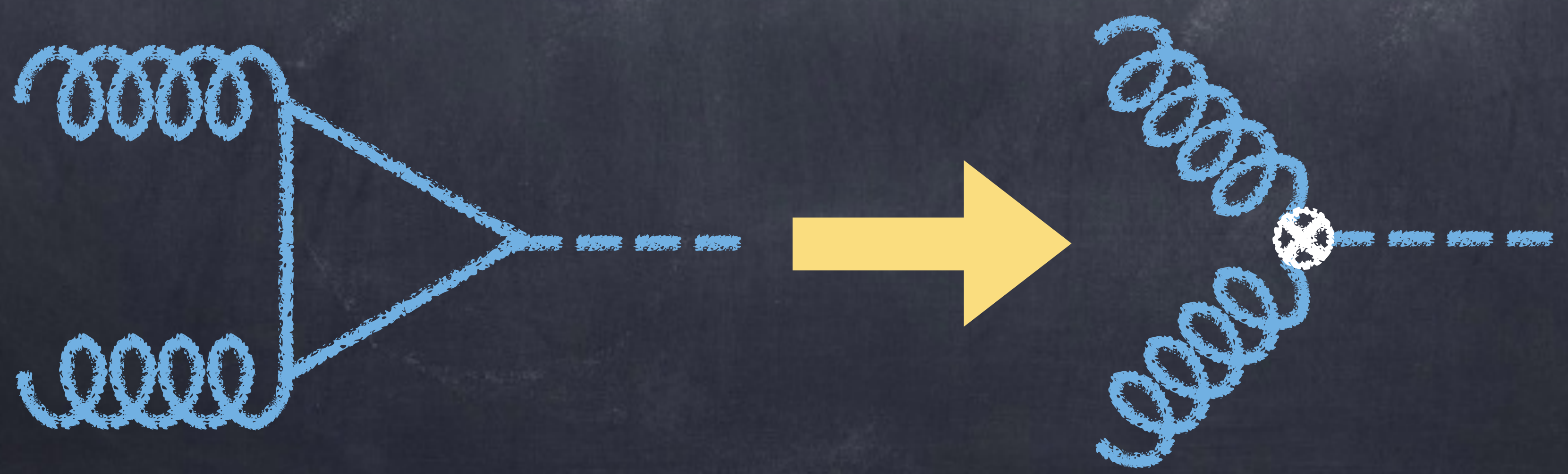


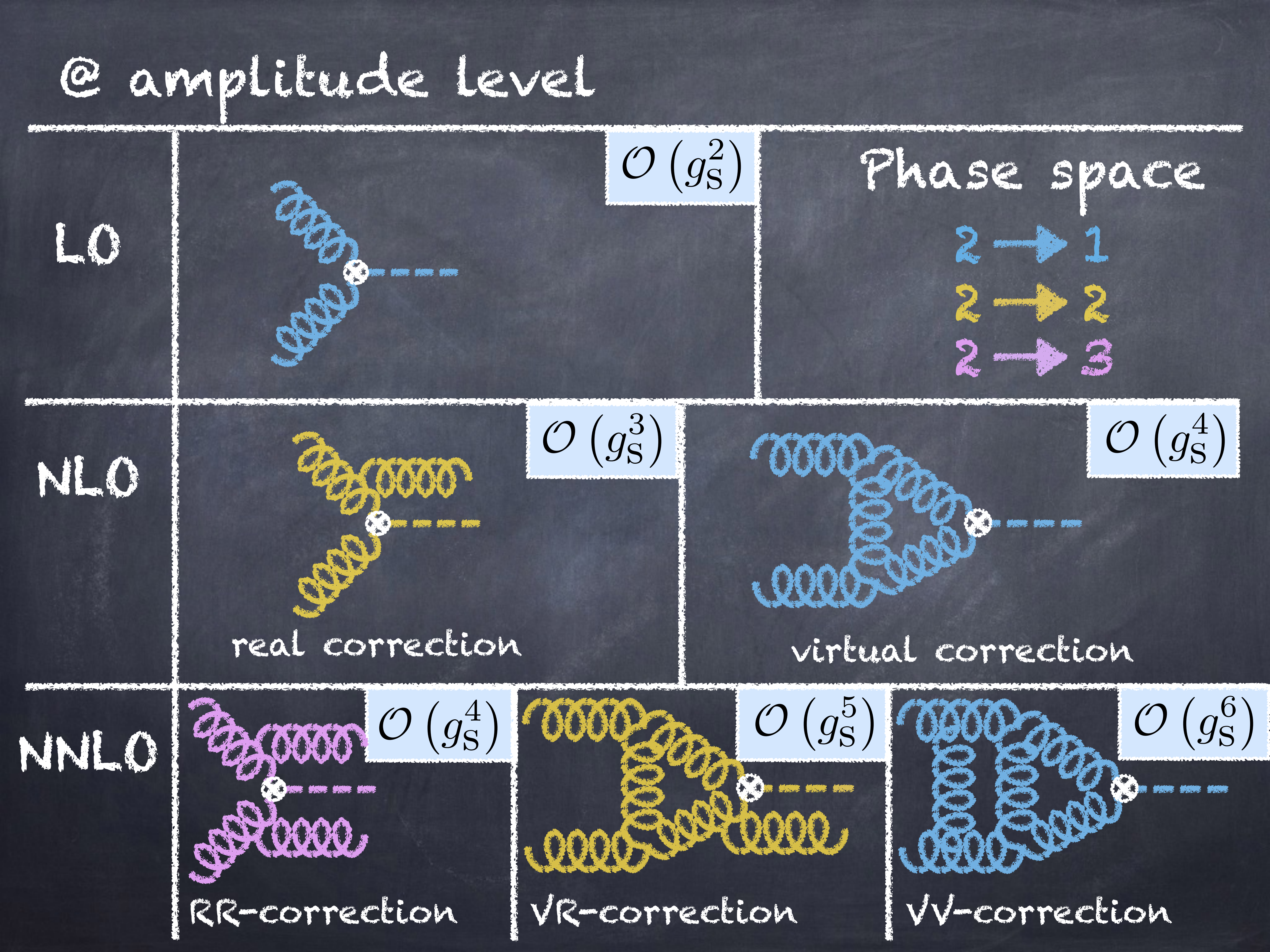
- It means that NLO or NNLO is basically the order of magnitude which dominates the order of magnitude of the cross section

- But, what does it means phenomenologically NLO or NNLO?
- The cross section in pQCD can be expressed at certain accuracy as,

$$\sigma = \sigma^{(0)} + \alpha_S(\mu)\sigma^{(1)} + \alpha_S^2(\mu)\sigma^{(2)} + \cdots$$

- Where the sum is finite after the complete sum
- As an example, let's work in the EFT framework, i.e.





@ amplitude level
LO
$\mathcal{O}\left(g_S^2\right)$
Phase space
2 → 1
2 → 2
2 → 3
NLO
$\mathcal{O}\left(g_S^3\right)$
real correction
$\mathcal{O}\left(g_S^4\right)$
virtual correction
NNLO
$\mathcal{O}\left(g_S^4\right)$
RR-correction
$\mathcal{O}\left(g_S^5\right)$
VR-correction
$\mathcal{O}\left(g_S^6\right)$
VV-correction

- The total cross section will be then,

$$\sigma = \sigma^{(LO)} + \alpha_S(\mu)\sigma^{(NLO)} + \alpha_S^2(\mu)\sigma^{(NNLO)} + \cdots$$

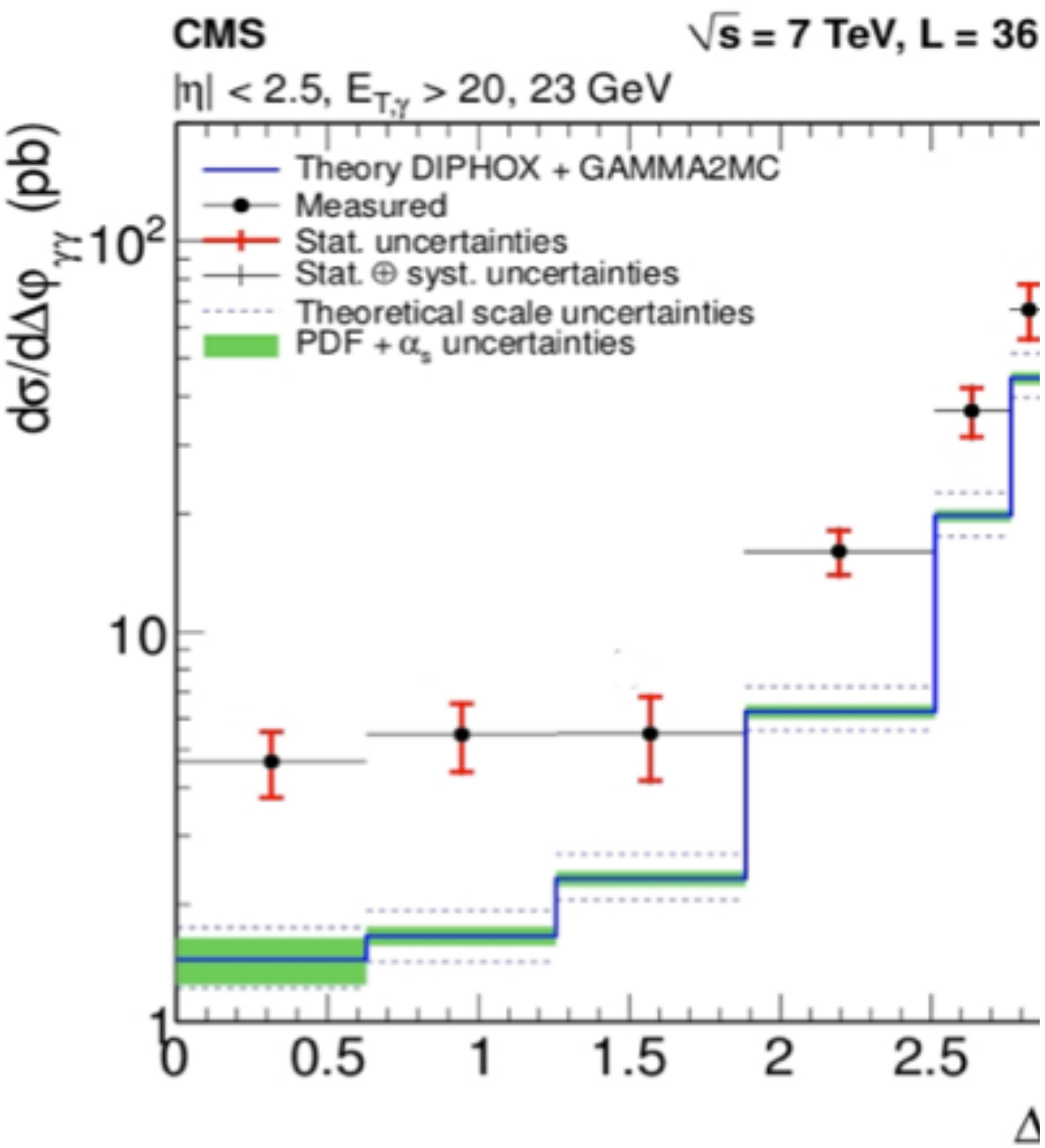
- where,

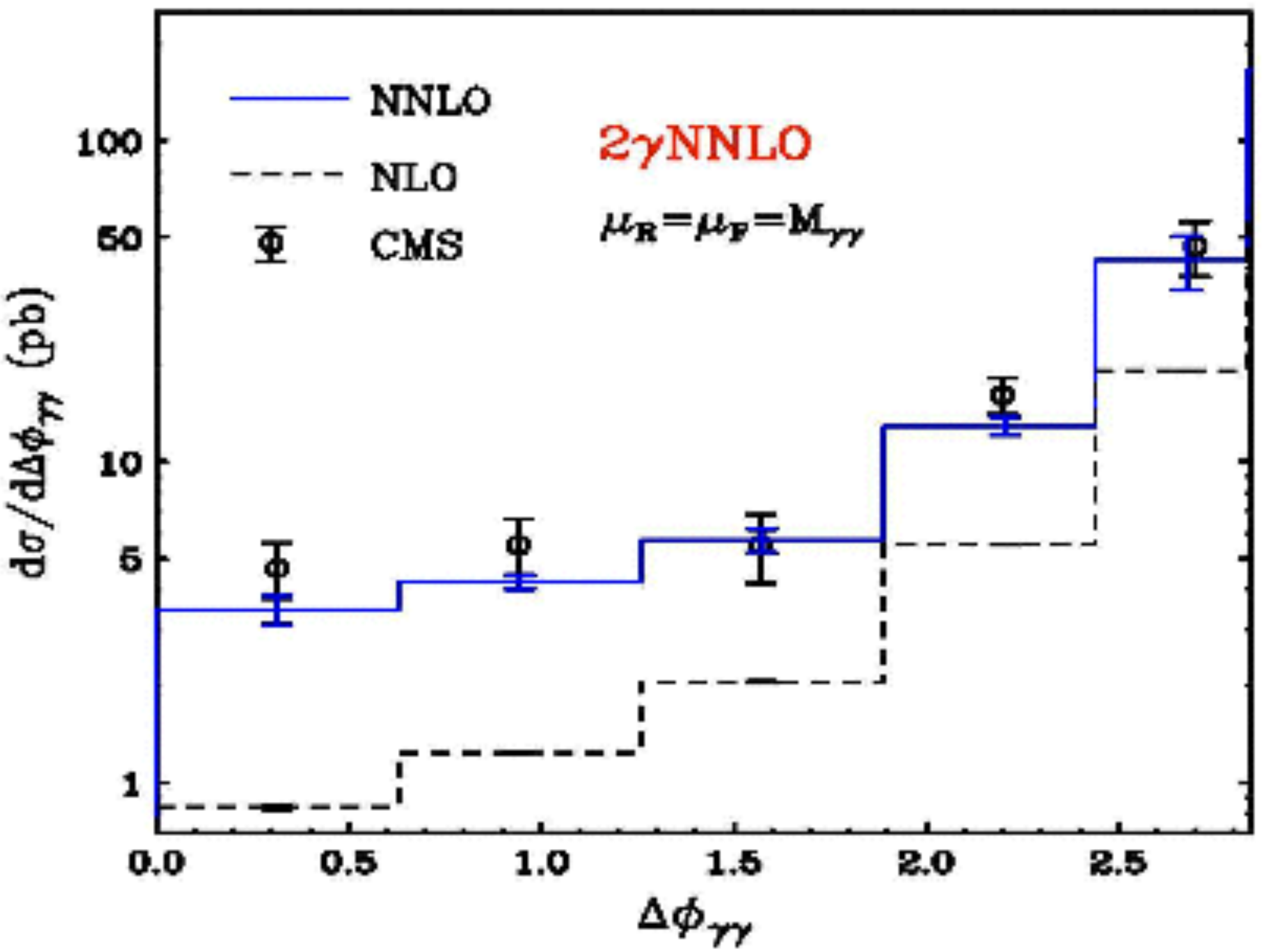
$$\sigma^{(NLO)} = \int_{\Omega} d\sigma^V + \int_{\Omega+1} d\sigma^R$$

$$\sigma^{(NNLO)} = \int_{\Omega} d\sigma^{VV} + \int_{\Omega+1} d\sigma^{RV} + \int_{\Omega+2} d\sigma^{RR}$$

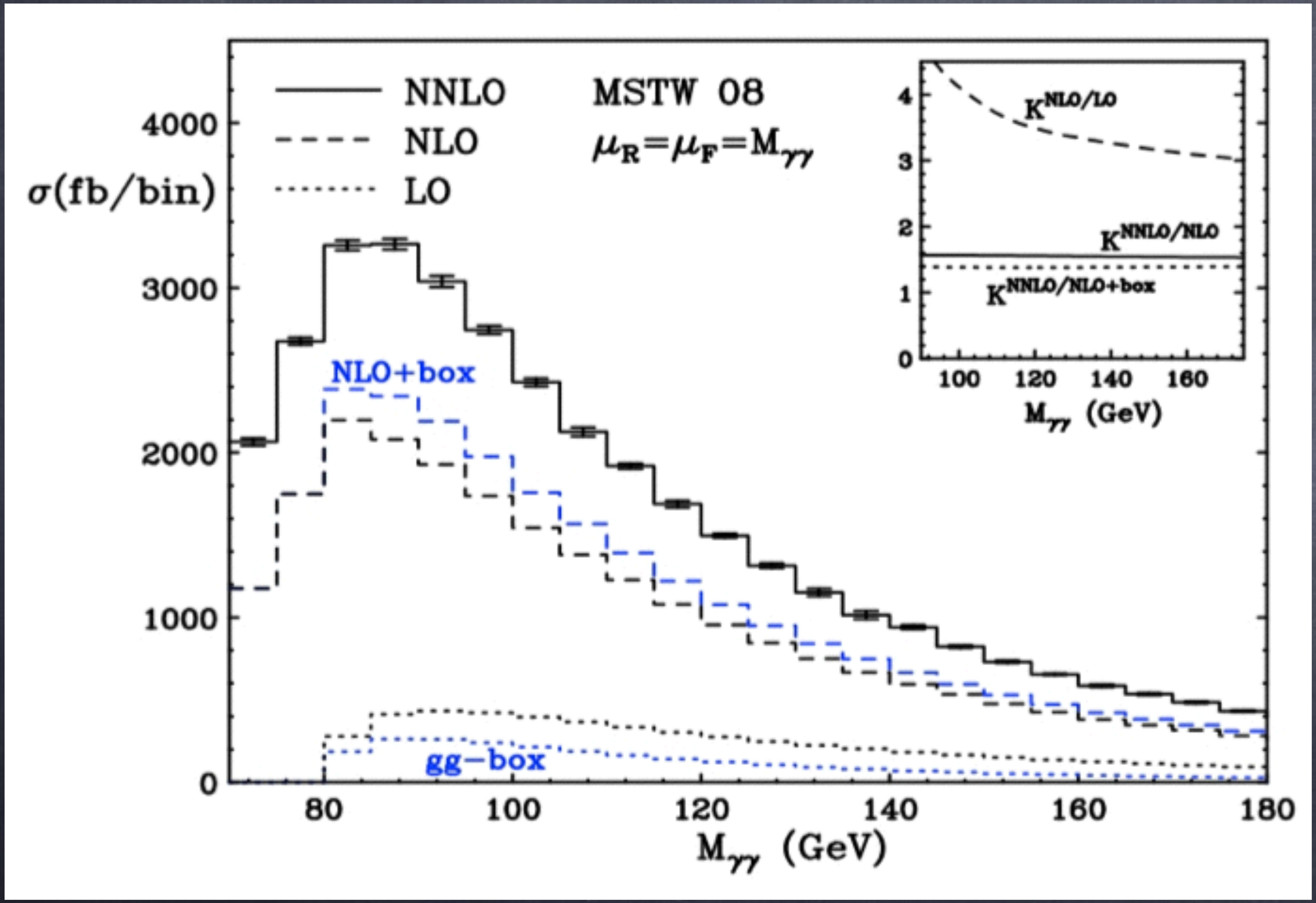
- But, divergences appears everywhere

- Moreover, the complete understanding of physics has to achieved in all senses
- LO, NLO and NNLO could sometimes explain the misunderstanding of some distributions





- NNLO calculations are important for the LHC now and for the next run



virtual corrections could have an enormous impact on the physics@LHC

High precision implies new Feynman diagrams to be considered

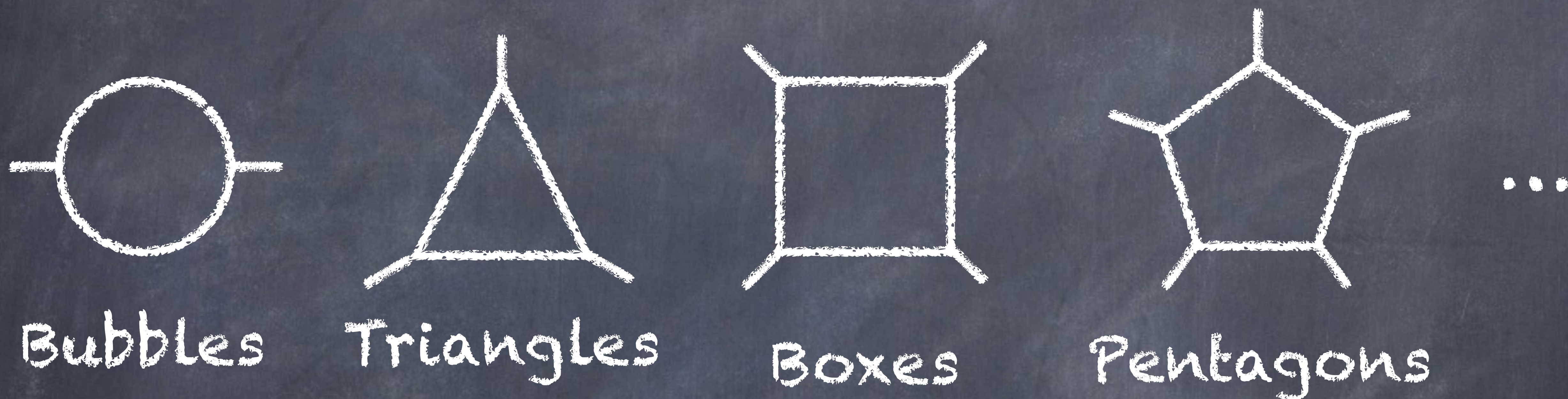
Remarks

- Several studies are being performed in order to control properly the divergences.
- Collider phenomenology requires Monte Carlo simulations which provides not only the total cross section accurately, BUT the distributions of some observables.
- Distributions are made taking into account the experimental cuts, and also the experimental configurations.

Virtual corrections

- The main problem is to control the divergencies which appears in the computation of the cross section
- Experimentally, divergences never appear, thus, theoretically they cannot exist in any computation
- In this talk, we focus on a method which implies tree level amplitudes with loop level ones
- Loop level amplitudes can be classified by its topologies

Topologies@1-Loop



- In all of them, there is a loop momentum running 'freely'
- All configurations are possible for it, from zero to infinity, always respecting the momentum energy conservation, BUT still, it is free
- At the end, all these diagrams represents an integral over all these possible configurations.

- 1-loop integrals have been computed using conventional dimensional regularisation (CDR), for massless case and for certain massive cases

- For instance, in the most general case, it is known that in CDR that:

$$= \int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell^2 - m_1^2 + i\epsilon)((\ell + q_1)^2 - m_2^2 + i\epsilon)}$$

$$= \int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell^2 - m_1^2 + i\epsilon)((\ell + q_1)^2 - m_2^2 + i\epsilon)((\ell + q_2)^2 - m_3^2 + i\epsilon)}$$

- The solution for the massless case is

$$= \left(\frac{\mu^2}{-s-i\varepsilon}\right)^{\epsilon}\left\{\frac{1}{\epsilon}+2\right\}+\mathcal{O}(\epsilon)$$

$$= \frac{1}{p_2^2-p_3^2}\left\{\frac{1}{\epsilon}\ln\left(\frac{-p_3^2}{-p_2^2}\right)+\frac{1}{2}\left[\ln^2\left(\frac{-p_2^2}{\mu^2}\right)\right]-\left[\ln^2\left(\frac{-p_3^2}{\mu^2}\right)\right]\right\}+\mathcal{O}(\epsilon)$$

- More accurate measurements require multi-loops and multi-legs; i.e. we need a better way to compute Feynman diagrams

- How do we compute easily at more than one loop ?
- Two loops

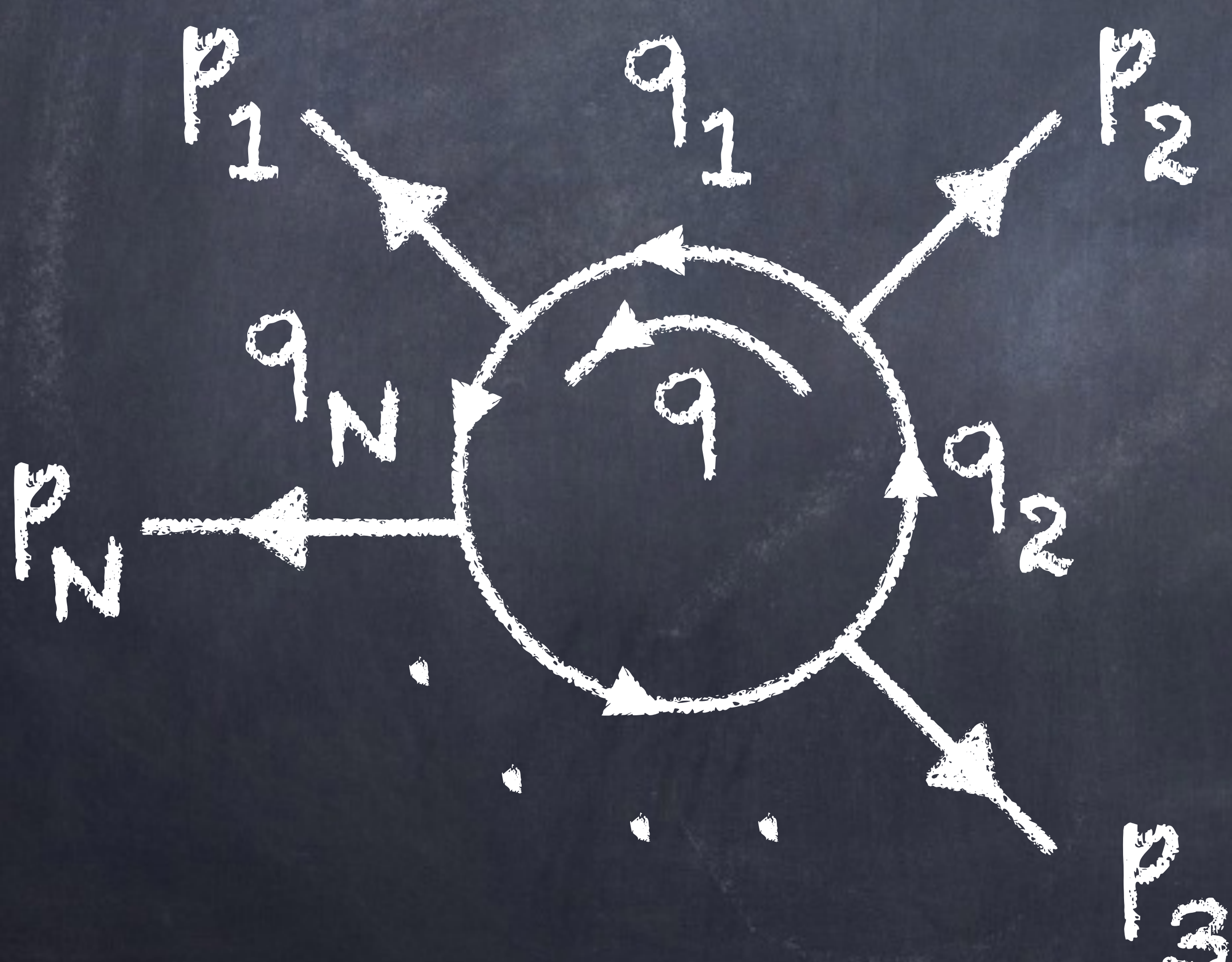
- Three loops

- ...

The loop-tree duality

Idea

- The purpose is to have a prescription to any virtual correction
- Let's start from the most general 1-loop diagram with massless particles in the loop



$$= -i \int \frac{d^D q}{(2\pi)^D} \prod_{i=1}^{N} \frac{1}{q_i^2 + i0}$$

$$= \int_q \prod_{i=1}^{N} G_F(q_i)$$

Duality @ 1-loop

Recalling
$$L^{(1)}(p_1, \cdots, p_N) = \int_{\ell_1} \prod_{i=1}^{N} G_F(q_i)$$

where
$$\int_{\ell_1} \cdots = -i \int \frac{d^D \ell_i}{(2\pi)^D} \cdots$$

and the Feynman propagator

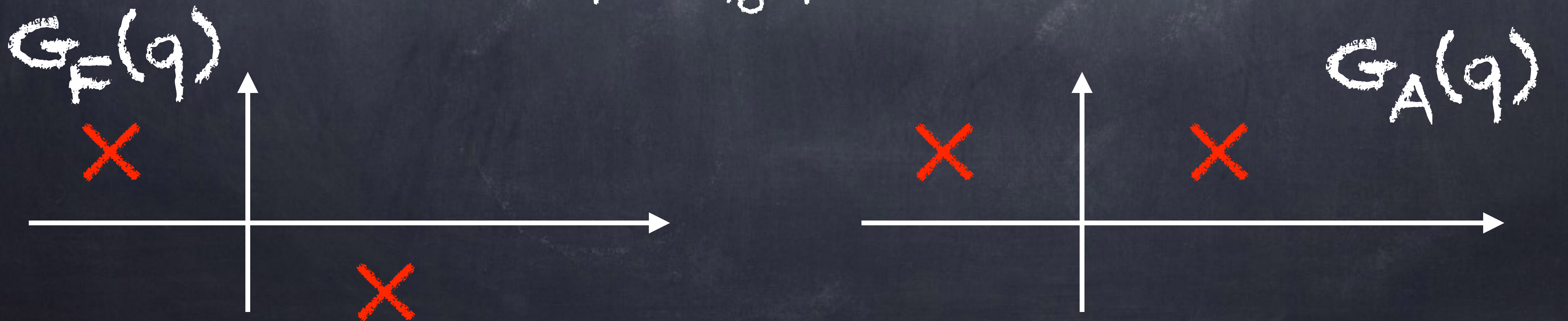
$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

and
$$q_i = \ell_1 + \sum_{k=1}^{i} p_k$$

- The duality relation is achieved when the system is integrated over the advanced propagator
- In this system, all poles are located in one half of the plane and the Cauchy Residues Theorem holds
- The advanced propagator is defined as,

$$G_A(q) \equiv \frac{1}{q^2 - i0q_0}$$

- Poles in the complex q_0-plane are located in,



- Using the identity

$$\frac{1}{x \pm i0} = PV\left(\frac{1}{x}\right) \mp i\pi\delta(x)$$

- we can relate the Feynman and the Advanced propagators by

$$G_A(q) = G_F(q) + \tilde{\delta}(q)$$

- And then

$$L_A^{(N)}(p_1, p_2, \cdots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G_A(q_i)$$

$$= \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G_A(q_i) = -2\pi i \int_{\mathbf{q}} \sum \text{Res}_{\{\text{Im } q_0 < 0\}} \left[\prod_{i=1}^{N} G_A(q_i)\right] = 0$$

- Then,

$$L^{(1)}(p_1, \cdots, p_N) = -\sum \int_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G_D(q_i; q_j)$$

- where the dual propagator is,

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$

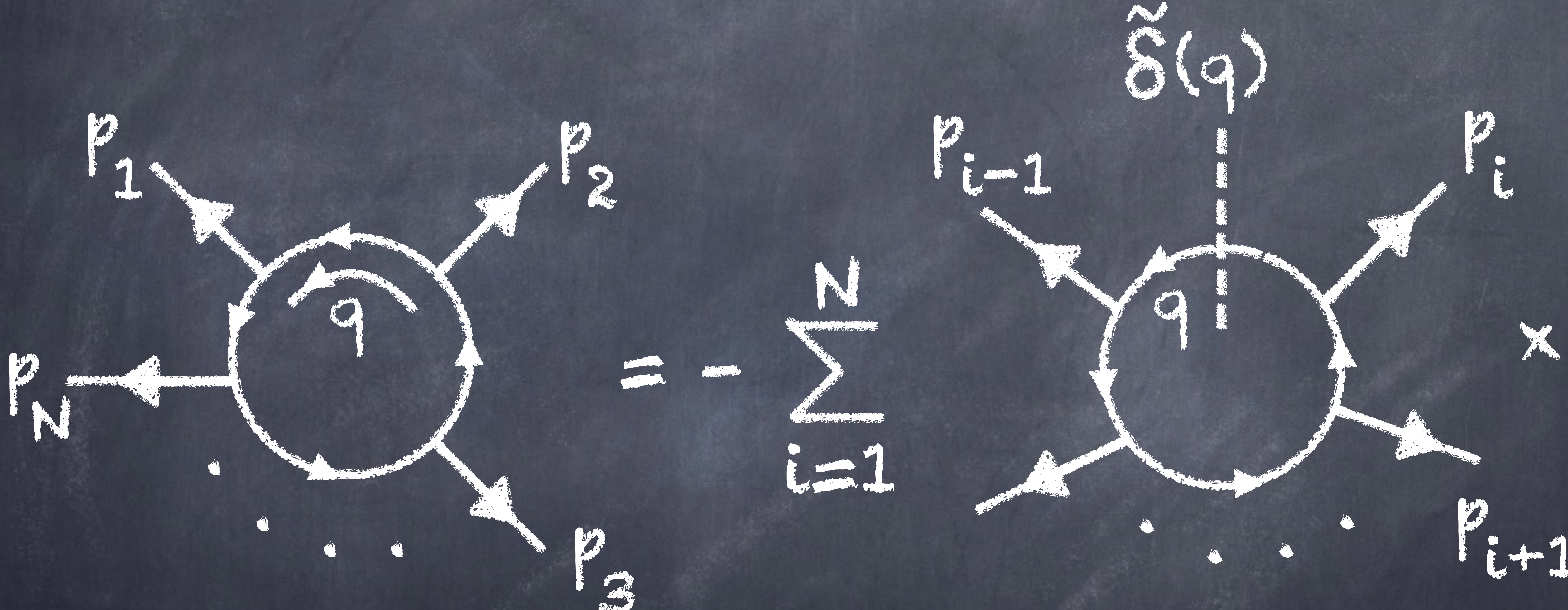
defining

$$\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$$

and a future-like vector,

$$\eta_\mu = (\eta_0, \eta), \quad \eta_0 \geq 0, \eta^2 = \eta_\mu \eta^\mu \geq 0$$

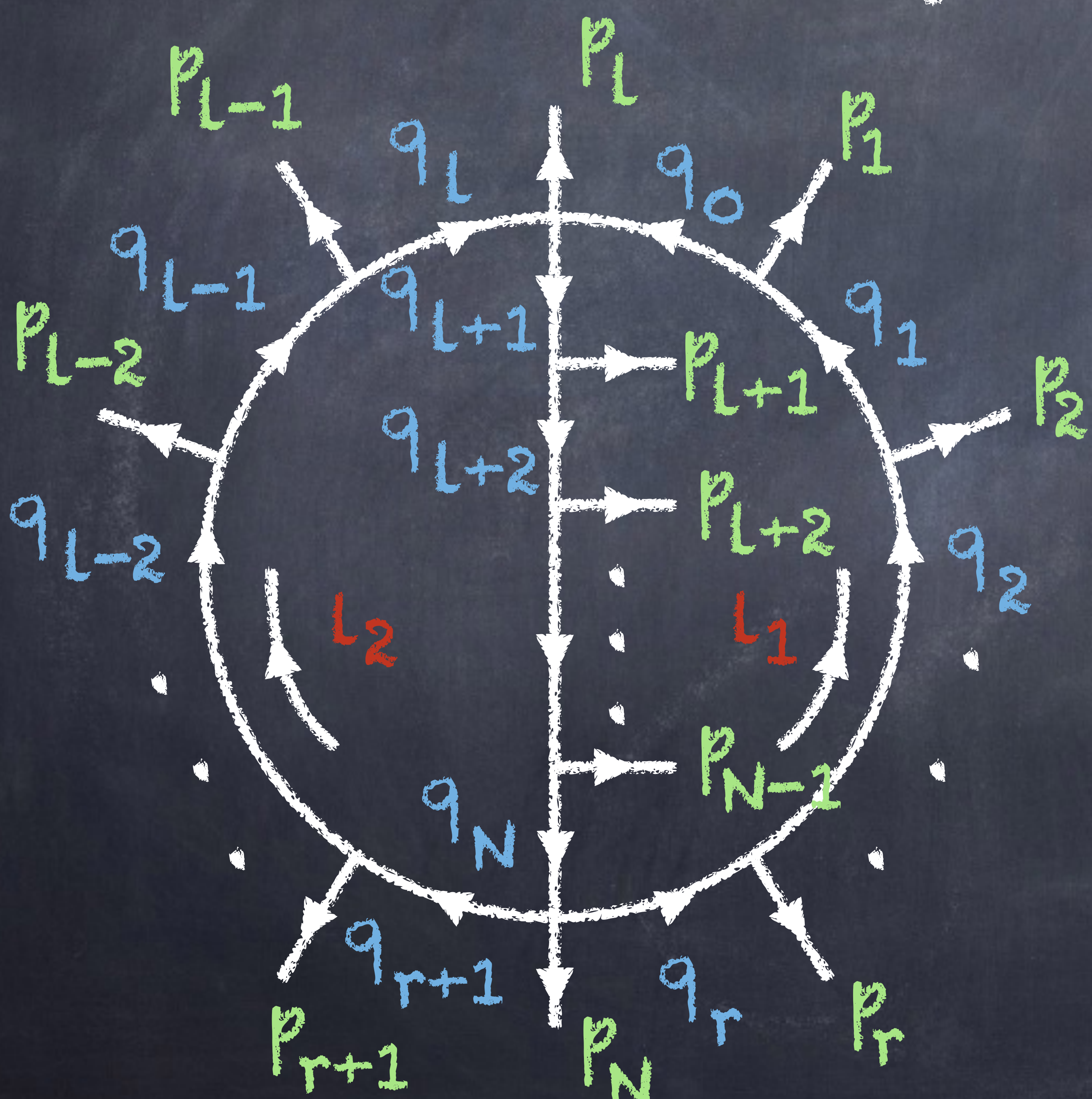
- Diagramatically, it means,



$$\times \frac{1}{(q+p_i)^2+in.p}$$

Duality @ 2-loop

- The most general Feynman diagram has the form



- There is a mixing between the loop momentum l_1 and l_2
- Integrals over loop momentum makes the visualisation harder

- Momenta of internal lines are denoted by

$$q_i = \begin{cases} \ell_1 + p_{1,i} & , i \in \alpha_1 \\ \ell_2 + p_{i,l-1} & , i \in \alpha_2 \\ \ell_1 + \ell_2 + p_{i,l-1} & , i \in \alpha_3 \end{cases}$$

- and the three parameters,

$$\alpha_1 \equiv \{0, 1, \ldots, r\}, \quad \alpha_2 \equiv \{r+1, r+2, \cdots, l\},$$
$$\alpha_3 \equiv \{l+1, l+2, \cdots, N\}.$$

- Now, the integral has the form

$$L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3)$$

- Applying the duality to the first loop, i.e. cutting α_1 and α_3

$$L^{(2)}(p_1, p_2, \ldots, p_N) = -\int_{\ell_1}\int_{\ell_2} G_D(\alpha_1 \cup \alpha_3) G_F(\alpha_2)$$

- Using the relation

$$G_D(\alpha_1 \cup \alpha_2) = G_D(\alpha_1)G_D(\alpha_2) + G_D(\alpha_1)G_F(\alpha_2) + G_F(\alpha_1)G_D(\alpha_2)$$

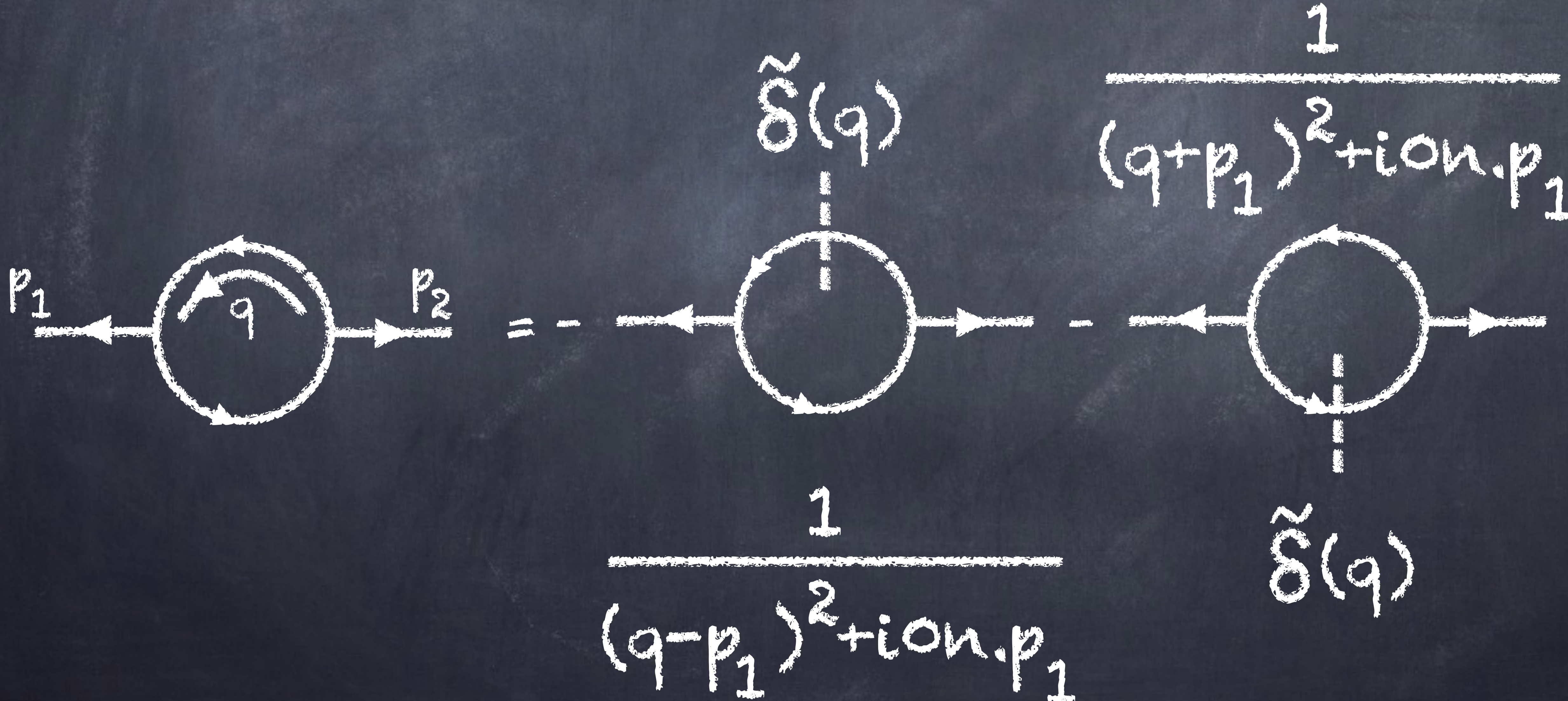
one ends, after some rearrangement

$$L^{(2)}(p_1, p_2, \ldots, p_N)$$
$$= \int_{\ell_1}\int_{\ell_2} \{-G_D(\alpha_1)G_F(\alpha_2)G_D(\alpha_3) + G_D(\alpha_1)G_D(\alpha_2 \cup \alpha_3) + G_D(\alpha_3)G_D(-\alpha_1 \cup \alpha_2)\}$$

Dual representation of two-loop scalar integral as a function of double cut integrals only

The simplest case

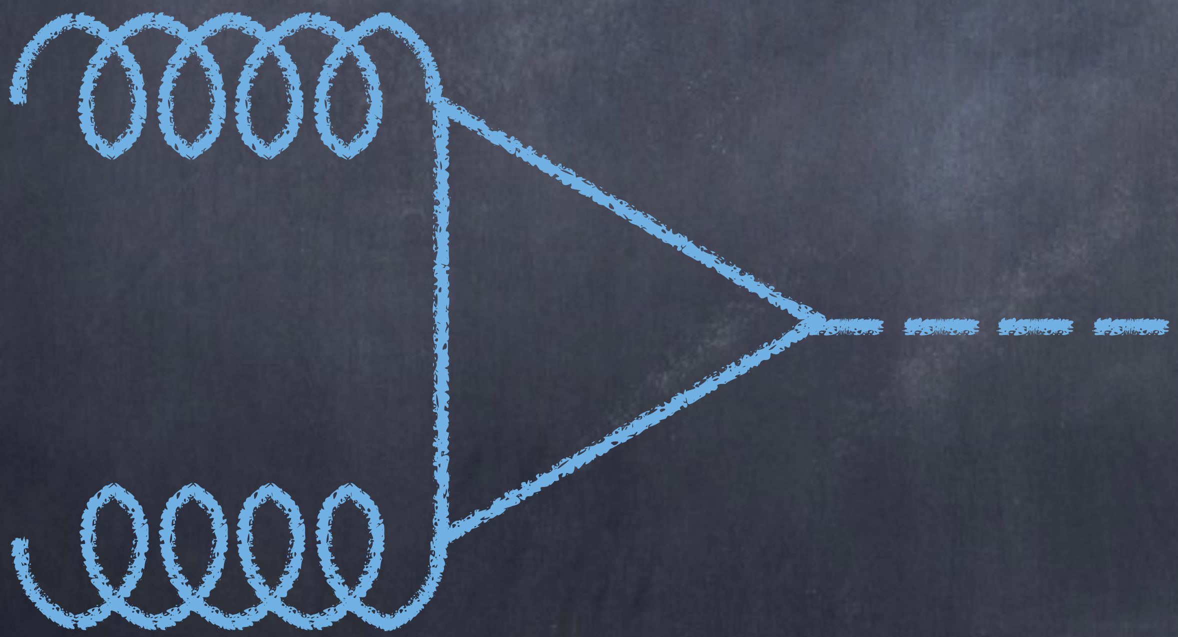
- The bubble is the simples one loop calculation in which it is possible to test the duality



Are we done ?

- The method has been used so far in order to check singularities
- One would like to have a recursion method in order to go to whatever precision needed
- The computation is not completed due to dual propagator still needs one integration to be performed
- The remaining integral has the advantage that is along the light-cone coordinates

- Along this path, UV singularities are cancelled while the IR has to be cancelled with the real emission
- However, it has been implemented in scalar loops
- Back to physics:



The simple GGF is not a scalar integral but a tensor one

It is possible to use the old IBP beyond 1 loop, even if the phase space is reduced ?

Conclusions

- In this talk, it has been presented the basic ideas of the loop-tree duality method
- The final goal is to provide a method in order to go to higher and higher in precision
- Virtual corrections can be computed in the loop-tree duality, in principle to all orders
- Difficulties appear when the method is extended to tensor integrals
- Tensor integrals are being studied in the most simple but relevant channel for the Higgs discovery, the GGF

Thanks ...