

Falton in the Higgs Era Talk given at Planck 2013 Bonn, Germany

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based on work done with F. Botella, M.N. Rebelo, M.N. Rebelo
and earlier work with L. Lavoura, W. Grimus.

Some of the Open Questions in Flavour Physics.

- Why is there "Flavour" ?
i.e. What is the Origin of Family replication
- How to understand the observed spectrum of Fermion Masses and Mixing ?
Why is Leptonic Mixing Large, in contrast to Small Quark Mixing ?
- Why is there Flavour Alignment in the Quark Sector ?

The Question of Alignment

Within the SM, consider a set of Yukawa couplings Y_u, Y_d leading to hierarchical quark masses and small mixing. By small mixing we mean that there is a Weak Basis (WB) where:

$$M_u = \text{diag.} (m_u, m_c, m_t); \quad M_d \approx \text{diagonal}$$

The absolute ordering of quark mass eigenstates has no physical meaning.

But the relative ordering, does have physical meaning. Since in the SM, χ_u, χ_d are entirely independent it is as likely that, in the

basis where $M_u = \text{diag.} (m_u, m_c, m_t)$, one has

$M_d \approx \text{diag} (m_d, m_s, m_b)$ corresponding to Alignment as to have $M_d \approx (m_b, m_d, m_s)$ implying Misalignment Obviously, even assuming "small mixing" the probability of Alignment is only $1/5$.

CP Violation is closely related to the Question of Flavour

An open question :

- Is CP broken explicitly through complex Yukawa couplings like in the SM

or

(T.D. Lee)

- Spontaneously : Lagrangian is invariant under CP but the vacuum violates CP?

At present, there is solid experimental evidence for a complex V_{CKM} .

Does this imply complex Yukawa couplings?

NO!!

There are realistic models of spontaneous CP violation where the vacuum phase generates a complex V_{CKM} .

Can one have geometrical CP violation?

For definiteness, let us consider an extension of the SM where n $SU(2) \times U(1)$ scalar doublets are introduced. In order to include

the possible existence of "family" symmetries of the Lagrangian under which the scalar doublets transform non-trivially, one has to consider the most general CP transformation which leaves invariant the kinetic energy terms of the scalar doublets:

$$CP \phi_i (CP)^\dagger = \sum_{j=1}^n U_{ij} \phi_j^*$$

Let us assume that the vacuum is CP invariant, meaning that:

$$CP |0\rangle = |0\rangle$$

One can then derive, the following relation:

$$\sum_{j=1}^n U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \phi_i | 0 \rangle$$

If the vacuum is such that none of the symmetries allowed by the Lagrangian satisfy the above equation, then this means that the vacuum is not CP invariant and we say that CP is spontaneously broken.

There are various **examples** of multi-Higgs models with "family" symmetries, where in a certain region of **parameter space**, the minimum of the scalar potential corresponds to **fixed values of the vacuum phase**, which do not depend on the specific values of the parameters of the potential.

$$\mathbb{Z}_2 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v ; \langle \phi_2^0 \rangle = v e^{i\pi/2}$$

$$\mathbb{S}_3 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v ; \langle \phi_2^0 \rangle = v e^{i2\pi/3} ; \langle \phi_3^0 \rangle = v e^{i4\pi/3}$$

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It has been shown by G.B. Girard, J.M. Guimard, W. that, contrary to naive expectation all these vacua are CP conserving.

However, an example was found, based on the group $\Delta(27)$ leading to genuine CP and T geometrical violation.

Recently, interesting other examples have been found:

I. M. Varzielas, D. E. Costa, P. Lezer Phys. Lett (2012)

I. P. Ivanov, L. Lavagna, Eur. Phys. (2013)

Realistic model?

Generation of the Baryon Asymmetry of the Universe (BAU)

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The ingredients to dynamically generate

BAU from an initial state with zero

B. A. , were formulated by Sakharov (1967)

- (i) Baryon number violation
- (ii) C and CP Violation
- (iii) Departure from Thermal equilibrium

All these ingredients exist in the SM, but it has been established that in the SM, one cannot generate the observed BAU:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}$$

$n_B, n_{\bar{B}}, n_\gamma$ number densities of baryons, anti baryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small

$$\frac{\text{Im}[H_u, H_d]^3}{T_{\text{ew}}^{12}} \approx 10^{-20}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \leq 70 \text{ GeV}$$

The various manifestations of

CP violation:

- CP violation in the quark sector
- CP violation in the lepton sector
- CP violation needed to generate **BAU**

A Common Origin?

- What about the Strong CP problem?

We need to suppress Flavor-

Changing - neutral - currents (FCNC)
 led to the following two Dogmas:

- No Z -mediated FCNC at tree level

- No Scalar - mediated FCNC at tree level

{ Glashow, Weinberg
 Paschos

Can one violate these two dogmas in reasonable extensions of the SM? Yes!

"Reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

• In the gauge sector, the Dogma can be violated through the introduction of a $Q = 1/3$ and/or $Q = 2/3$ vector-like quark.

Naturally small violations of 3×3 unitarity of V_{CKM}

Z-mediated, Naturally suppressed FCNC at tree level

Yukawa interactions in the *General* *Two Higgs doublet Model*

$$-\mathcal{L}_Y = \bar{Q}_L^0 \Gamma_1 \phi_1 d_R^0 + \bar{Q}_L^0 \Gamma_2 \phi_2 d_R^0 + \bar{Q}_L^0 \Delta_1 \tilde{\phi}_1 u_R^0 + \bar{Q}_L^0 \Delta_2 \tilde{\phi}_2 u_R^0 + \text{h.c.}$$

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}} (\nu_1 \Gamma_1 + \nu_2 e^{i\alpha} \Gamma_2) ; M_u = \frac{1}{\sqrt{2}} (\nu_1 \Delta_1 + \nu_2 e^{-i\alpha} \Delta_2)$$

Diagonalized by:

$$(u_d)_L^\dagger M_d (u_d)_R = D_d = \text{diag}(m_d, m_s, m_b)$$

$$(u_u)_L^\dagger M_u u_{uR} = D_u = \text{diag}(m_u, m_c, m_t)$$

Expanding around the vacuum:

$$\langle \Phi_j^0 \rangle = e^{i\alpha_j} \frac{1}{\sqrt{2}} (\nu_j + \rho_j + i\eta_j) \quad j = 1, 2$$

It is useful to introduce new fields:

$$\begin{bmatrix} H^0 \\ \rho \end{bmatrix} = O \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}; \quad \begin{bmatrix} G^0 \\ I \end{bmatrix} = O \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}; \quad \begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = O \begin{bmatrix} \phi_1^+ \\ \phi_2^0 \end{bmatrix}$$

$$O = \frac{1}{v} \begin{bmatrix} \nu_1 & \nu_2 \\ \nu_2 & -\nu_1 \end{bmatrix}; \quad v = \sqrt{\nu_1^2 + \nu_2^2} \approx 246 \text{ GeV}$$

$H^0 \rightarrow$ has couplings to quarks proportional to mass matrices
 $G^0 \rightarrow$ neutral pseudo Goldstone boson
 $H^\pm \rightarrow$ charged pseudo-Goldstone bosons

Neutral and charged Higgs interactions in the quark sector

$$\begin{aligned}
 -\mathcal{L}_Y = & \bar{d}_L^0 \frac{1}{v} (M_d H^0 + N_d^0 R + i N_d^0 I) d_R^0 + \\
 & + \bar{u}_L^0 \frac{1}{v} (M_u H^0 + N_u^0 R + i N_u^0 I) u_R^0 + \\
 & + \frac{\sqrt{2}}{v} H^+ (\bar{u}_L^0 N_d^0 d_R^0 - \bar{u}_R^0 N_u^0 d_L^0) + \text{h.c.}
 \end{aligned}$$

$$N_d^0 = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2) ; \quad N_u^0 = \frac{1}{\sqrt{2}} (v_2^{-\Delta_1} v_1 e^{-i\alpha} \Delta_2)$$

In the quark mass eigenstate basis N_d, N_u are not flavour diagonal.

Physical neutral Higgs are combinations of H^0, R, I

Yukawa Couplings in terms of quark mass eigenstates:

$$\begin{aligned}
 \mathcal{L}_Y = & \frac{\sqrt{2} H^+}{v} \bar{u} [-V N_d \delta_R + N_u^+ V \delta_L] d + \text{h.c.} - \\
 & - \frac{H^0}{v} \left[\bar{u} D_u u + \bar{d} D_d d \right] - \\
 & - \frac{R}{v} \left[\bar{u} (N_u \delta_R + N_u^+ \delta_L) u + \bar{d} (N_d \delta_R + N_d^+ \delta_L) d \right] \\
 & + \frac{I}{v} \left[\bar{u} (N_u \delta_R - N_u^+ \delta_L) u - \bar{d} (N_d \delta_R - N_d^+ \delta_L) d \right] \\
 \delta_L = & \frac{1}{2} (1 - \gamma_5) \quad ; \quad \delta_R = \frac{1}{2} (1 + \gamma_5) \quad ; \quad V \text{ is the}
 \end{aligned}$$

CKM matrix

Flavour changing neutral currents are controlled

by :

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For generic two Higgs doublet models
 N_u, N_d are non-diagonal, arbitrary
 matrices.



too large Higgs mediated Flavour changing
 neutral currents, unless a suppression mechanism
 is introduced

G.C.B, Grimus, W and Lavoura, L (BGL) have shown that it is possible to find a symmetry which when imposed on a 2-Higgs doublet extension of the SM, leads to a structure of the Yukawa couplings such that there are FCNC at tree level, with strength completely controlled by V_{CKM} , i.e., N_d , N_u only depend on V_{CKM} and on $\frac{V_1}{V_2}$.

Possible choice of the symmetry S :

$$Q_{L3}^0 \rightarrow \exp(i\alpha) Q_{L3}^0 ; \quad U_{R3}^0 \rightarrow \exp(i2\alpha) U_{R3}^0$$

$$\phi_2 \rightarrow \exp(i\alpha) \phi_2, \quad \text{where } \alpha \neq 0, \pi$$

all other fields transform trivially under S .
This leads to the Yukawa couplings:

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

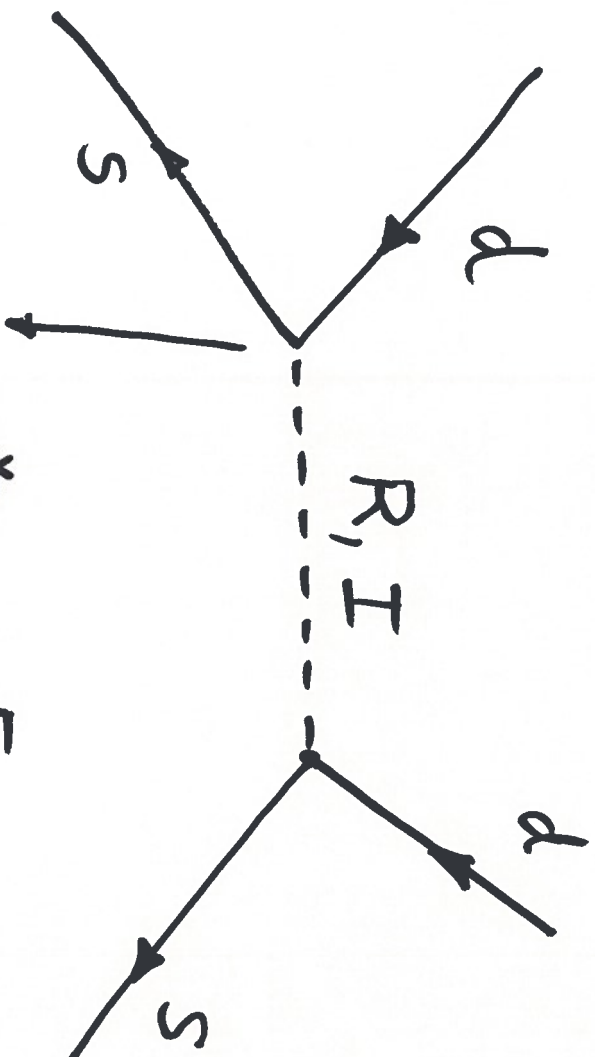
These Yukawa structures lead to :

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) [V_{CKM}]_j (D_d)_{ji}$$

$$(N_u) = -\frac{v_1}{v_2} \text{diag.} (0, 0, m_t) + \frac{v_2}{v_1} \text{diag.} (m_u, m_c, 0)$$

In this particular **BGL** model there are **FCNC** only in the down sector, but there is a **strong, natural suppression** of the most dangerous processes

$K^0 - \bar{K}^0$ mixing



$$V_{td} V_{ts}^* \sim \lambda^5$$

λ^{10} suppression !!

There are 6 different BGL models
in the quark sector

An important Question:

Are there any other models, based on abelian symmetries, which lead to FCCNC at tree level, completely controlled by V_{CKM} , without further parameters?

Intuitive Answer: Yes

Correct Answer: No!

P. Ferreira and J. Silva (Phys. Rev. D 83 (2011))

have classified all possible implementations of an **Abelian symmetry** in 2-Higgs doublet models, imposing the request of having non-vanishing quark masses and not block diagonal VCKM.

Then conclusion : **BGL models are unique**

Open question : Non-abelian symmetries?

Other Approaches to Multi-Higgs Models

- Minimal Flavour Violation

G. D'Ambrosio, G. F. Giudice, G. Isidori, A. Strumia

A. Buras, P. Gambino, M. Gorbahn, S. Jäger, (2002)

L. Silvestrini (2001)

A. Dery, A. Efrati, G. Hiller, Y. Hochberg, Y. Nir

(2013)

- Alignment

A. Pich, P. Tugon (2009)

In the quark mass eigenstate basis flavour mixing can be parametrised by various unitary matrices which arise from the misalignment in flavour space between pairs of various Hermitian flavour matrices. This is entirely analogous to what one encounters in the SM

✓ CKM $\rightarrow$ arises from the misalignment between $H_u \equiv M_u M_u^\dagger$ and $H_d \equiv M_d M_d^\dagger$

Weak-basis invariants are a useful to study $\widetilde{\text{Flavour}}$.

Example :

$$\text{tr} [H_u, H_d] = \delta i (m_c^2 - m_u^2) (m_t^2 - m_c^2) (m_t^2 - m_u^2) \times$$

$$\times (m_s^2 - m_d^2) (m_b^2 - m_s^2) (m_b^2 - m_d^2) \times$$

$\rightarrow$ J. Bernabeu, G.C.B., M. Gronau

$$\propto \det [H_u, H_d]$$

for 3 generations

(C. Jarlskog)

$\text{Im } Q$

$Q \rightarrow$ invariant quartet of Y_{CKM}

One may also study invariants under Higgs basis transformations

G.C.B., M.N. Rebelo, Silva-Marcos ;

H. Haber, F. Gunion
S. Davidson, H. Haber

We have seen that under a WB transformation:

$$d_L^0 = W_L d_L^{0'} ; \quad d_R^0 = W_R^u d_R^{0'}$$

$$u_L^0 = W_L u_L^{0'} ; \quad u_R^0 = W_R^u u_R^{0'}$$

Under a WB transformation the flavour matrices transform as:

$$M_d \rightarrow M_d' = W_L^\dagger M_d W_R^d ; \quad M_u \rightarrow M_u' = W_L^\dagger M_u W_R^u$$

$$N_d^0 \rightarrow N_d^{0'} = W_L^\dagger N_d^0 W_R^d ; \quad N_u^0 \rightarrow N_u^{0'} = W_L^\dagger N_u^0 W_R^u$$

In view of the freedom of choice of WB, it is useful to express the physical content of M_d, M_u, N_d^0, N_u^0 in terms of WB invariants.

Examples of I invariants:

$$I_1 \equiv \text{tr} (M_d N_d^{\circ \dagger}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33}$$

where N_d is N_d° in the basis where the down quark mass matrix is diagonal. This invariant is not sensitive to Higgs mediated FCNC but $\text{Im } I_1$ is specially important, since it probes the phase of $(N_d)_j$ which contribute to the electric dipole moment of down quarks.

Analogous comments apply to $\text{tr} (M_u N_u^{\circ \dagger})$

Consider now a WB invariant which is sensitive to the off-diagonal elements of N_d :

$$I_2 \equiv \text{tr} [M_d N_d^{\circ\dagger}, M_d M_d^\dagger]^2 = -2 m_d m_s (m_s^2 - m_d^2) (N_d^* /_{12} N_d^* /_{21} - 2 m_d m_b (m_b^2 - m_d^2) (N_d^* /_{13} N_d^* /_{31} - 2 m_s m_b (m_b^2 - m_s^2) (N_d /_{23} N_d^* /_{32})$$

CP-odd WB invariant:

$$I_2^{CP} \equiv \text{tr} [H_u, H_{N_d^0}]^3 = 6 i \Delta_a \Delta_{N_d} \text{Im } Q_2$$

$(m_b^2 - m_c^2) \dots$
 $\rightarrow$ analogous

Q_2 is a rephasing invariant quartet of V_2

$$V_2 \equiv U_u^\dagger (U_{N_d})_L$$

These invariants can be very useful to study the CP properties (as well as FCNC) in specific models. For example in BGL models:

$$\text{Im tr}(M_d N_d^{\dagger}) = 0$$

In BGL models the lowest order invariant sensitive to CP violation is:

$$I_9^{\text{CP}} \equiv \text{tr}[M_d N_d^{\dagger} M_d M_u^{\dagger} M_u M_d^{\dagger}]$$

This may be relevant for Baryogenesis.

See M.N. Rebelo talk

Conclusion

• We are entering the **Higgs Era** in **Particle Physics**.

• A crucial question is: Will Higgs particle(s) play an important rôle in Flavour?

• **My guess:** *Yes, it will not be just Vanilla Higgs.*

• It may take some time until Higgs Flavour be tested experimentally. We have to be patient and remember **Neutrinos**

• We can safely violate the two Flavour Dogmas.

A Minimal Model

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- Consider an *extension* of the SM, where the following new fields are introduced:
- A vectorial quark D^0 , with both D_L^0 and D_R^0 are $SU(2)_L$ singlets with charge $Q = -1/3$ (or $Q = 2/3$)
 - 3 right-handed neutrinos ν_R^i
 - A neutral complex singlet S

- Since we want to have Spontaneous CP violation, we impose CP invariance at the Lagrangian level: All couplings real.
- Introduce a Z_4 symmetry on the Lagrangian, under which:

$$\psi_L^0 \rightarrow i \psi_L^0; \quad e_{Rj}^0 \rightarrow i e_{Rj}^0; \quad \nu_{Rj}^0 \rightarrow i \nu_{Rj}^0$$

$$D^0 \rightarrow -D^0; \quad S \rightarrow -S$$

The Z_4 symmetry is crucial to obtain a solution of the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence.

$$V_{\text{phase}} = \left[\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^\dagger \phi \right] (S^2 + S^{*2}) + \lambda_3 (S^4 + S^{*4})$$

There is a range of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi \rangle = \frac{v}{\sqrt{2}} ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)_L \times U(1) \times SU(3)_c \times \mathbb{Z}_4$
 invariant Yukawa couplings in the quark
 sector :

$$\mathcal{L}_Y = -(\bar{u}^0 \bar{d}^0)_L [g_{ij} d_R^0 + h_{ij} \tilde{\phi} u_R^0] - \bar{M} (\bar{D}_L^0 D_R^0) \\
- (f_i S + f'_i S^*) \bar{D}^0 d_{Ri}^0 + h.c.$$

Quark mass matrix for down-type quarks :

$$(\bar{d}_{1L}^0 \bar{d}_{2L}^0 \bar{d}_{3L}^0 \bar{D}_L^0) \begin{matrix} 3 \times 3, \text{rad} \\ m_d \end{matrix} \begin{matrix} 0 \\ \vdots \\ \vdots \\ \vdots \end{matrix} \begin{matrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{matrix} \begin{matrix} d_{1R}^0 \\ d_{2R}^0 \\ d_{3R}^0 \\ D_R^0 \end{matrix}$$

"zeros" due to $\mathbb{Z}_4$ symmetry

$$M_j = f_j V e^{i\theta} + f'_j V e^{-i\theta}$$

$$N \equiv \begin{pmatrix} m_d & \vdots & 0 \\ \hline \bar{M}_1, \bar{M}_2, \bar{M}_3, & \bar{M} & \end{pmatrix};$$

$$U_L^\dagger (N N^\dagger) U_L = \text{diag.} (d^2, D^2)$$

$$U = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$$

; One can easily derive: $\xrightarrow{\text{Complex}}$

$$K^{-1} \left[m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{(M M^\dagger + \bar{M} \bar{M}^\dagger)} \right] K = d^2$$

A remarkable feature of the Model:

The phase θ arising from $\langle S \rangle$, generates a non-trivial CKM phase, provided $|M_j|$ and $\bar{M}$ are of the same order of magnitude (This is "natural")

$$K^{-1} m_{\text{eff}} m_{\text{eff}}^{\dagger} K = \text{diag.} (m_d^2, m_s^2, m_b^2)$$

$$m_{\text{eff}} m_{\text{eff}}^{\dagger} = m_d m_d^{\dagger} - \frac{m_d M^{\dagger} M m_d^{\dagger}}{M M^{\dagger} + \bar{M}^2}$$

$$M_j = (f_j \cdot V e^{i\theta} + f_j' \cdot V e^{-i\theta})$$

Naturally small
deviations of 3×3 unitarity

Naturally Small
Flavour-Changing
Neutral Currents

For definiteness, consider the case of one isosinglet $Q = -1/3$
 3×3 VCKM quark

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u} \bar{e} \bar{\tau})_L \gamma^\mu \begin{bmatrix} K \\ R \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} W_\mu^+$$

$$\mathcal{L}_Z = -\frac{g}{2 \cos \theta_W} \left\{ (\bar{u} \bar{e} \bar{\tau})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L - [\bar{d} \bar{s} \bar{b} \bar{D}] \begin{bmatrix} K^+ & K^+ & R^+ \\ R^+ & K^+ & R^+ \end{bmatrix} \gamma^\mu \begin{bmatrix} d \\ s \\ b \end{bmatrix} \right. \\ \left. - \sin^2 \theta_W J_{em}^\mu \right\} Z_\mu$$

Why deviations of 3×3 unitarity are *naturally small*:

$$U_L^\dagger \mathcal{M} \mathcal{M}^\dagger U_L = \text{diag.} (m_d^2, m_s^2, m_b^2, M_D^2)$$

$$U_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix}; \quad K^\dagger K + S^\dagger S = 1$$

$$\text{but } S \approx -\frac{M m_d^\dagger K}{\bar{M}^2} \rightarrow O(m/M);$$

$K^\dagger K = 1 - O(m^2/M^2)$. Note that there is nothing strange about violations of 3×3 unitarity.

The PMNS matrix is not unitary in the framework of seesaw mechanism, type 1.

Some Comments :

• Nothing "Strange" in having deviations of 3×3 unitarity of V_{CKM} . The PMNS matrix in the leptonic sector, in the context of type-I seesaw, is not 3×3 unitary.

• Vector-like quarks provide the simplest model with Spontaneous CP violation, with a complex V_{CKM} , in agreement with experiment

• Provide a framework to have a Common Origin of all CP violations.

• Potential Solution of Strong CP Problem without Axions