Jets, flows and Joseph Fourier

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Guy Paić Fest
Puebla, Mexico

Diverse Phenomena – Secret Analogies

Mathematics compares the most diverse phenomena
and
discovers the secret analogies that unite them

Joseph Fourier, 1768-1830

Fourier Transforms and Series

aperiodic function f

$$\mathbf{F(k)} = \int_{-\infty}^{\infty} \mathbf{dx\, f(x) e^{-ikx}} \quad \textit{Fourier transform}$$

bins of width λ

$$= \sum_{n=-\infty}^{\infty} \int_{-\lambda/2}^{\lambda/2} \mathbf{dx'\, f(x' + n\lambda)\, e^{-ik(x'+n\lambda)}}$$

f <u>periodic</u>, with period λ

$$\sum_{m=-\infty}^{\infty} \delta(k - 2\pi m/\lambda)$$

$$= \left[\sum_{n=-\infty}^{\infty} e^{-ikn\lambda} \right] \int_{-\lambda/2}^{\lambda/2} \mathbf{dx\, f(x)\, e^{-ikx}}$$

$k \rightarrow k_m = 2\pi m/\lambda$

$$\mathbf{F_m} = \int_{-\lambda/2}^{\lambda/2} \mathbf{dx\, f(x)\, e^{-i\, 2\pi mx/\lambda}}$$

$x/\lambda \rightarrow \phi/2\pi$

$$\rightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathbf{d\varphi\, f(\varphi)\, e^{-im\varphi}}$$

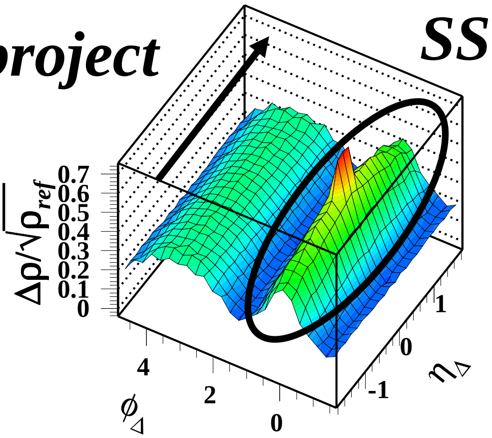
Fourier series coefficients

<u>cylindrical multipoles</u>

m	*multipole*	
0	*monopole*	
1	*dipole*	
2	*quadrupole*	v_2
3	*sextupole*	
4	*octupole*	

“Higher Harmonic Flows”

project *SS 2D peak*



project 2D correlations onto 1D

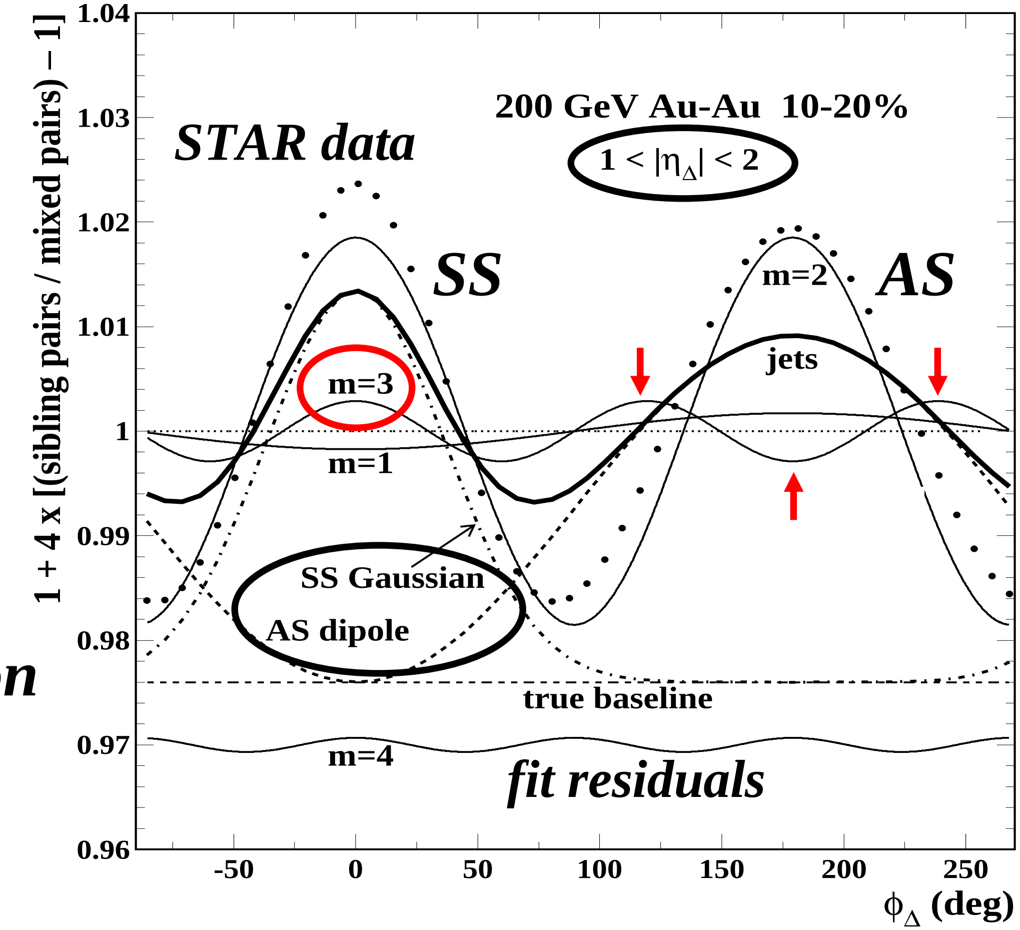
Fourier series fit to data

describes <u>any</u> periodic distribution

thin solid curves

m = 3: sextupole

Alver/Roland **PRC 81, 054905 (2010)**

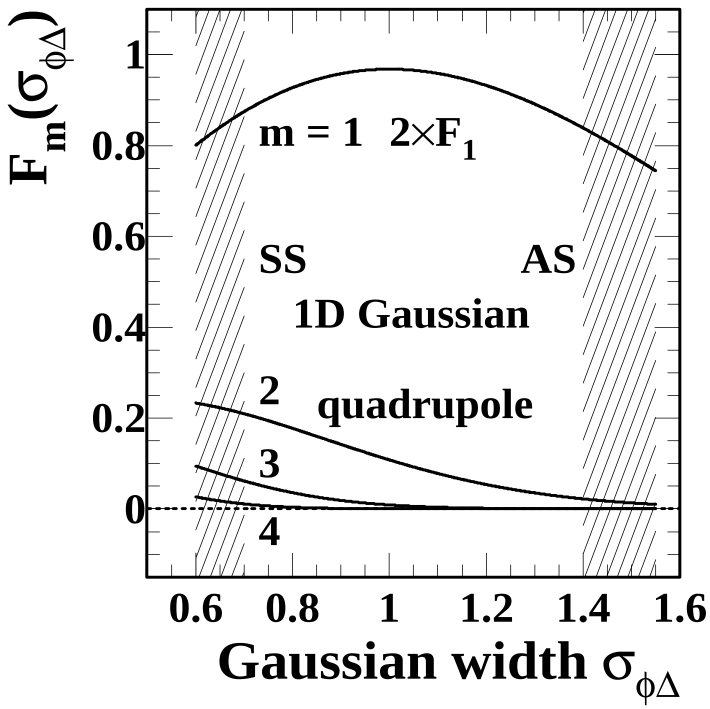


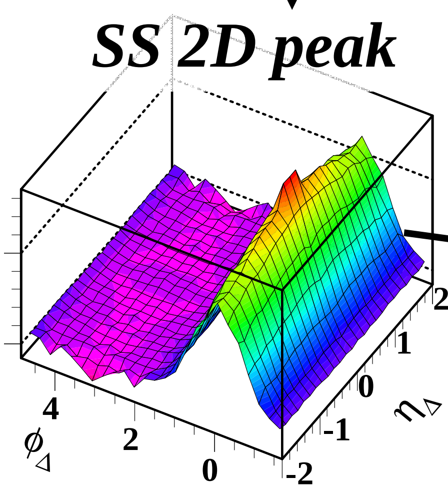
sextupole → “triangular flow” (né Mach Cones)

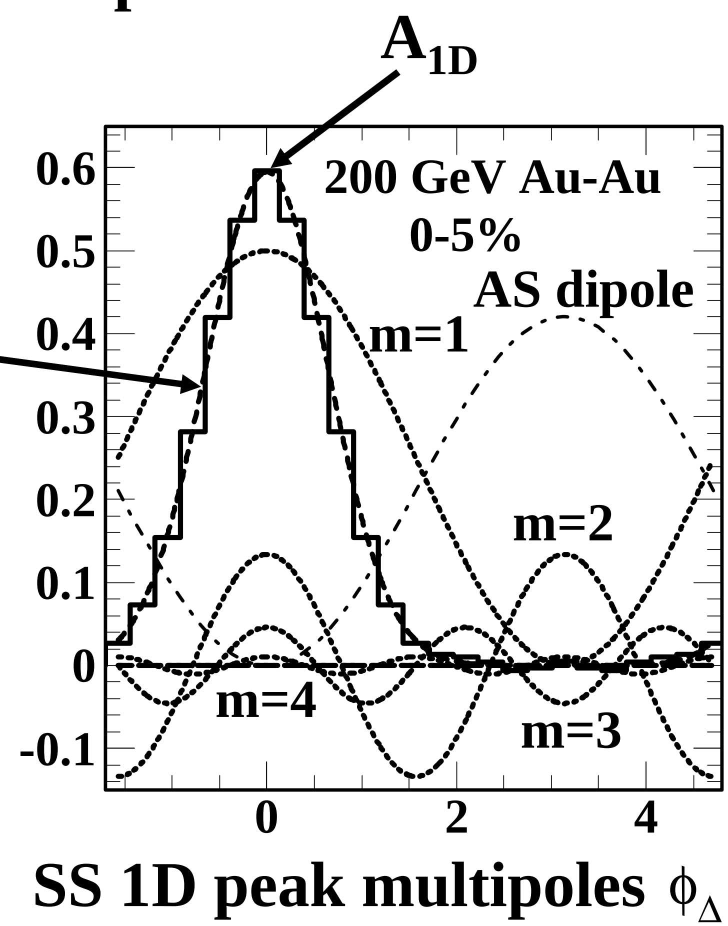
“Higher Harmonics” in 0-5% Au-Au?

data – AS dipole = SS 2D peak

1D Gaussian







$$F_m(\sigma_{\phi_\Delta}) = \sqrt{2/\pi}\,\sigma_{\phi_\Delta}\exp\left\{-m^2\sigma_{\phi_\Delta}^2/2\right\}$$

Fourier coefficients: 1D Gaussian

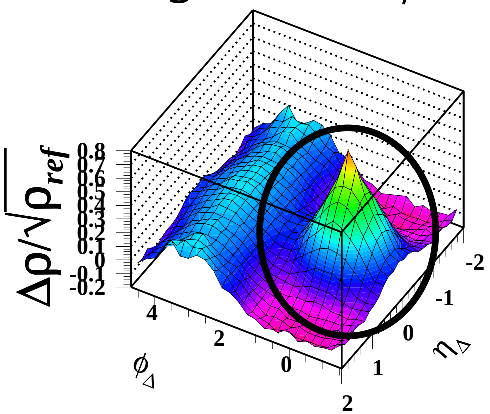
$$A_X\{SS\} \equiv F_m \times A_{1D}/2$$

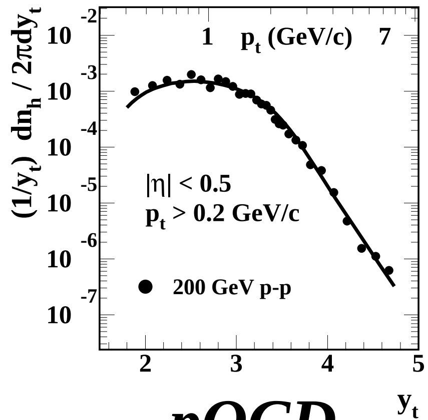
<u>only</u> source of “higher harmonics” is the SS 2D peak

What is the Same-side 2D Peak?

200 GeV p-p **p_t*-integral*** ***200 GeV central Au-Au***

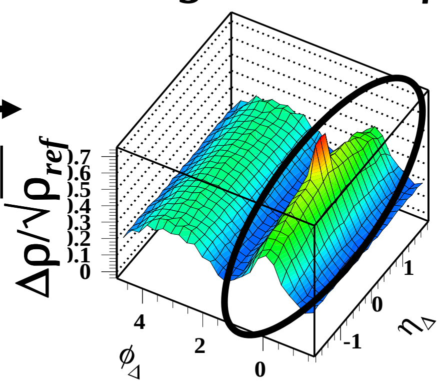
elongation on ϕ

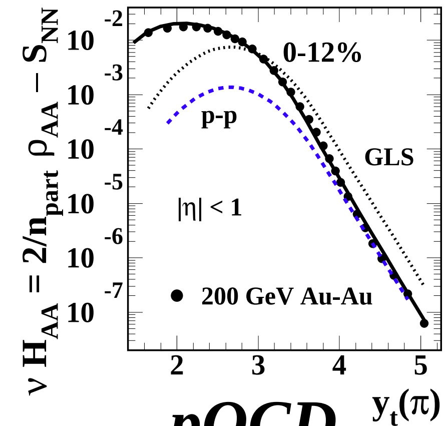




pQCD

elongation on η





pQCD

(mini) jets!

the "Soft Ridge" jets, flows?

persistence of jets is inconvenient for "perfect liquid"

imperative: reinterpret SS 2D peak as flows

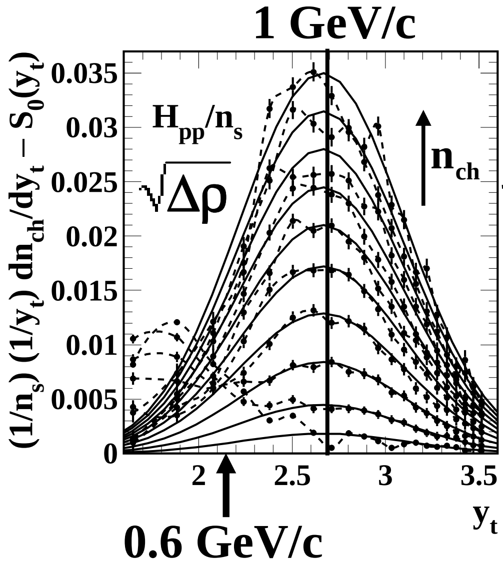
glasma flux tubes, Fourier series → "higher harmonics"

but detailed study confirms a jet mechanism

200 GeV p-p Two-component Model

p_t spectrum hard component

PRD 74, 032006 (2006)

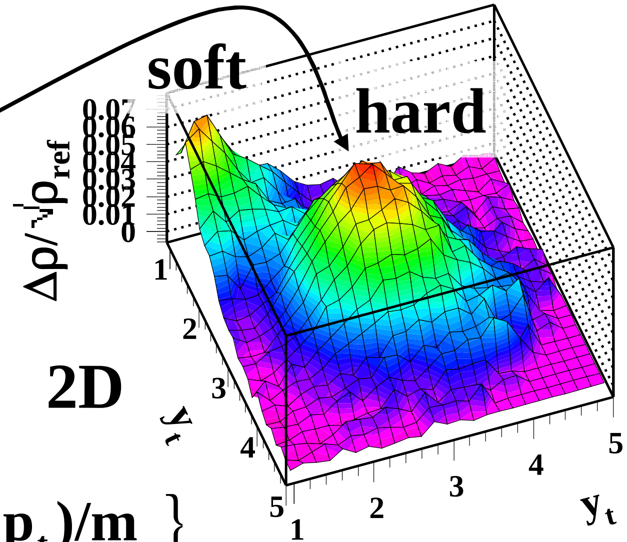


jets $\propto$ $\mathbf{n_{ch}^2}$

1D

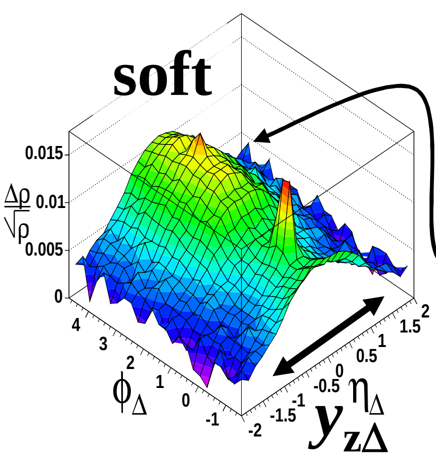
parton fragments

2D



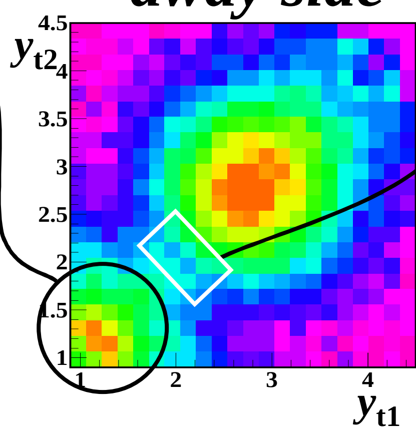
$\mathbf{y_t \equiv \ln\{(m_t + p_t)/m_\pi\}}$

minimum-bias: no trigger condition

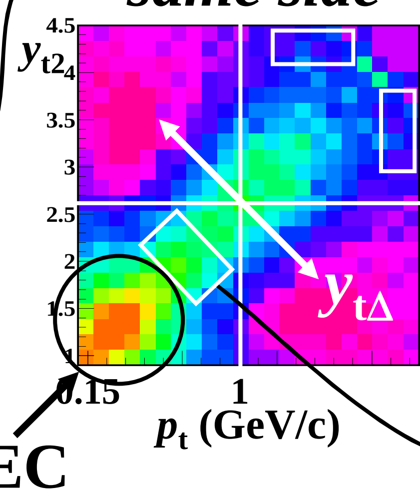


proton fragments

away side

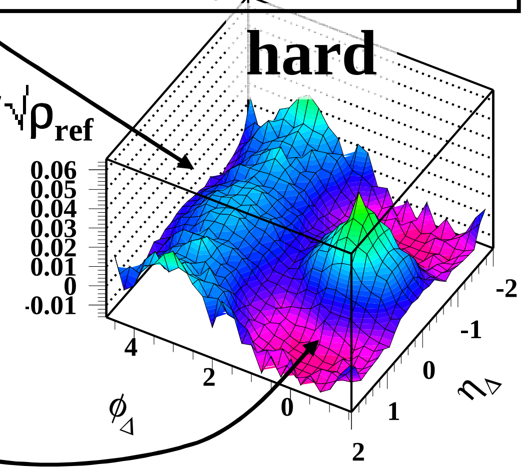


same side



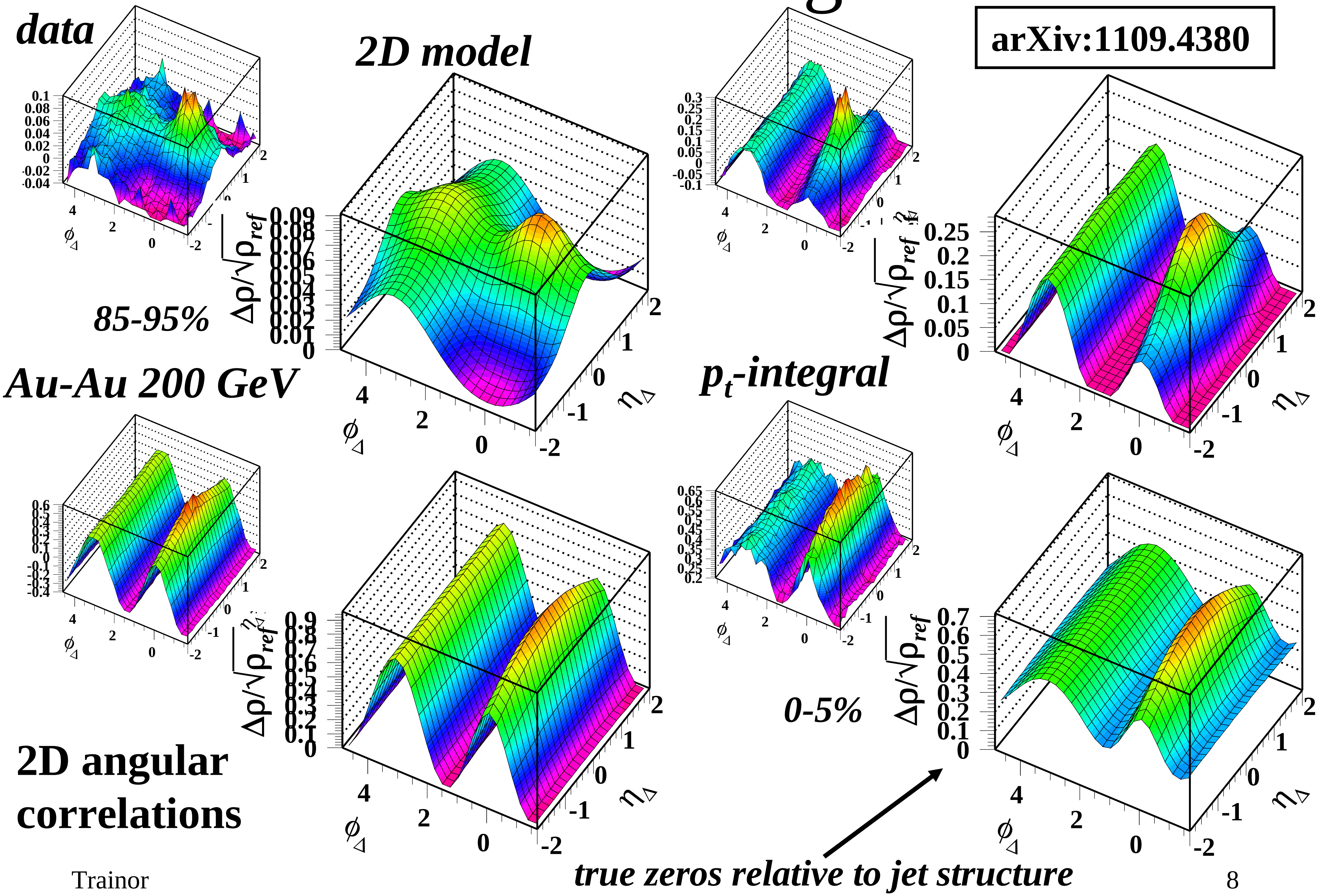
hadron p_t ≈ 0.6 GeV/c

$\Delta\rho/\sqrt{\rho_{ref}}$



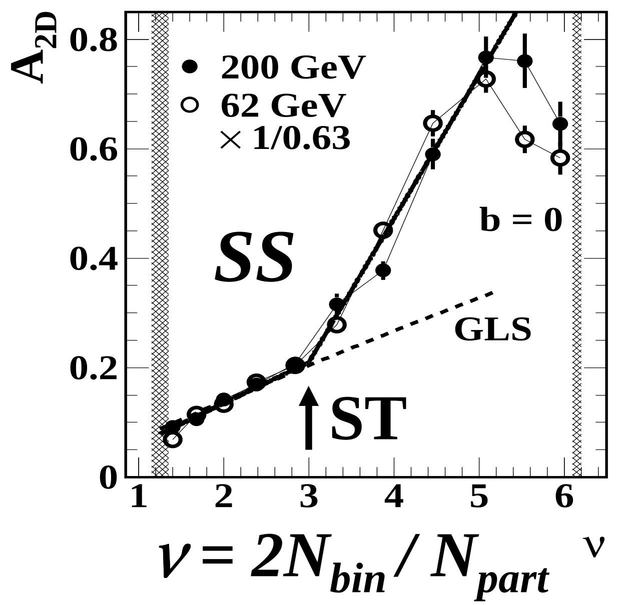
parton fragments

200 GeV Au-Au 2D Angular Structure



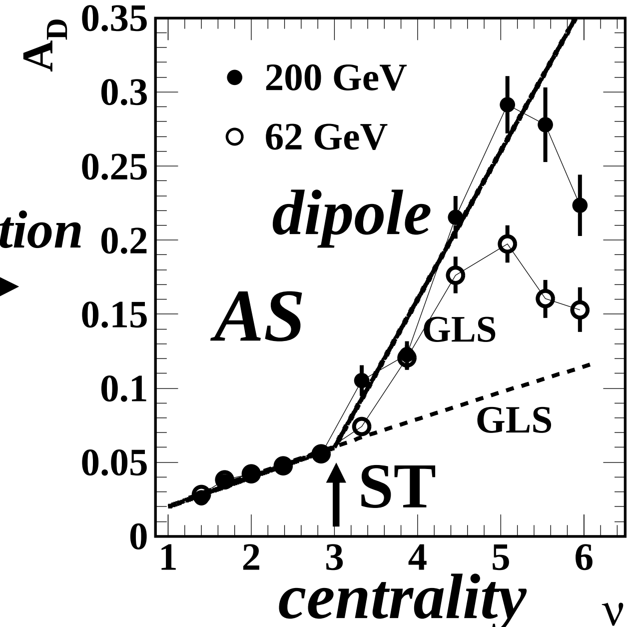
Minimum-bias Jet (minijet) Properties

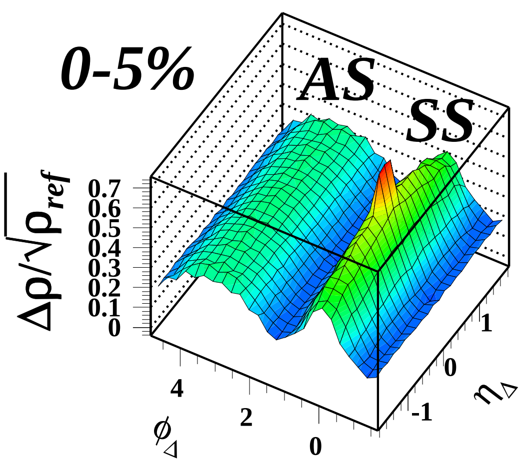
same-side (SS) 2D and away-side (AS) 1D peaks

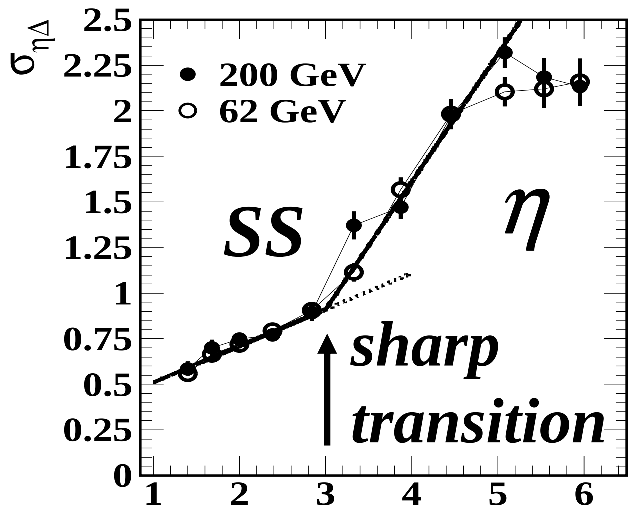


arXiv:1109.4380

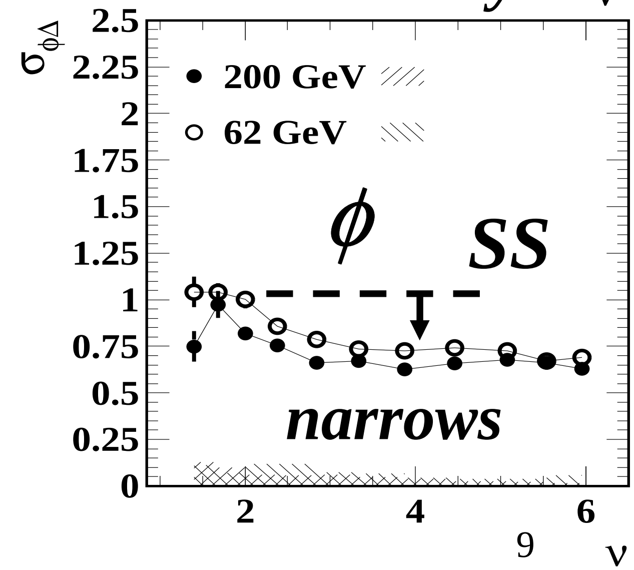
***parton** momentum conservation*







SS 2D peak ≡
single 2D Gaussian
– no “ridge”



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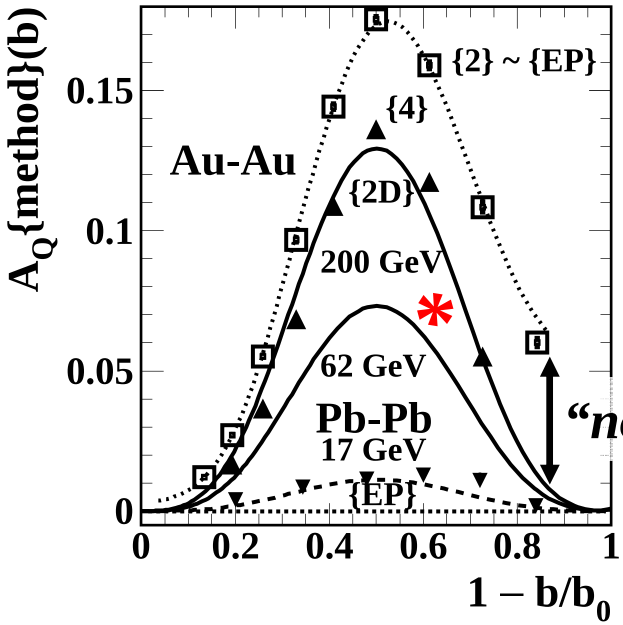
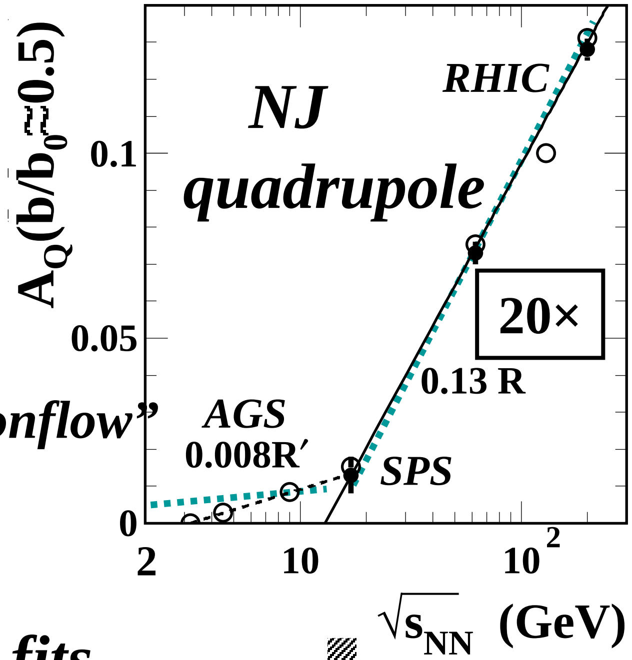
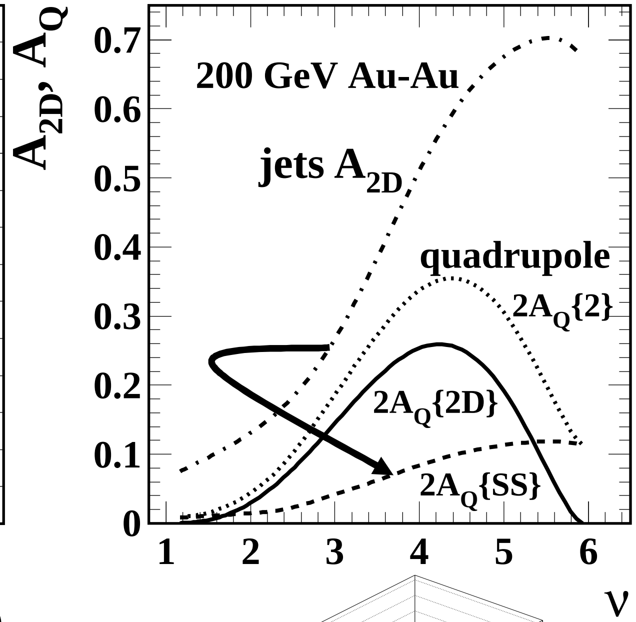
Nonjet Azimuth Quadrupole $v_2\{2D\}$



“nonflow”



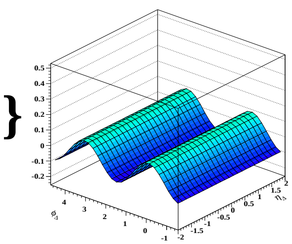


$A_Q\{2D\}$ *from 2D model fits*

arXiv:0907.2686

nucleon coll. → *low-x glue*

$A_Q\{2D\}$



$\rho_0(b) = dn_{ch}/2\pi d\eta$

* $A_Q\{2D\} \equiv \rho_0(b)\, v_2^2\{2D\} = 0.0045\, R\, N_{bin}\, \epsilon_{2,opt}^2$

$A_Q\{EP\} \simeq A_Q\{2\} = A_Q\{2D\} + A_Q\{SS\}$ *SS 2D peak*

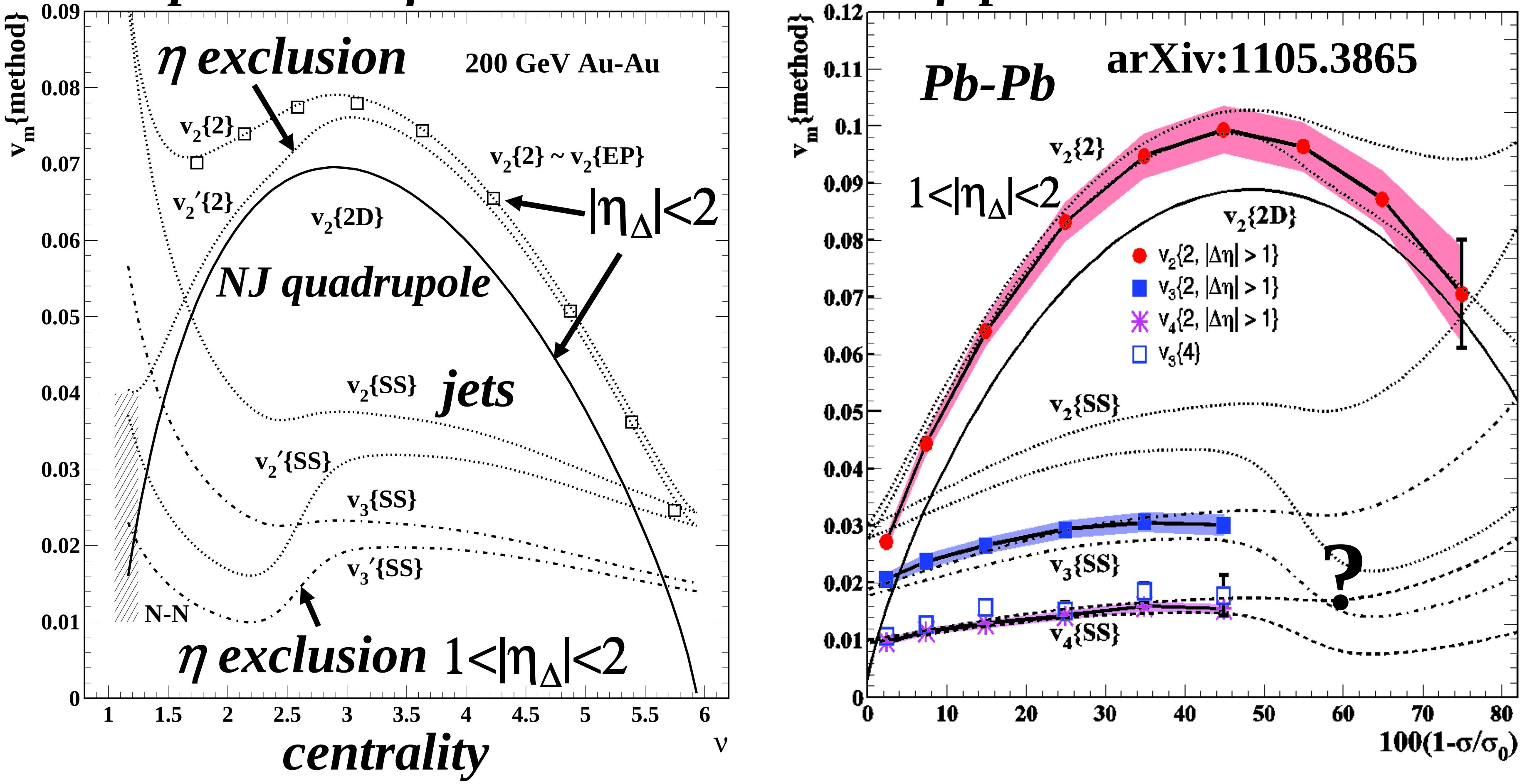
published v2 data *nonjet* *jet-related*

***nonjet** quadrupole distinct from SS, AS jet structure*

"Higher Harmonic Flows" at the LHC?

v_m predictions based on <u>SS 2D peak</u> and nonjet (NJ) quadrupole

prediction for 200 GeV ***left panel × 1.3 → 2.76 TeV***



v_m\{SS\} *at 2.76 TeV similar to 200 GeV*

nonjet quadrupole* v_2\{2D\} *also similar

Converting Jets to Flows

1. ***fit 1D projections on ϕ with a Fourier series***
2. ***interpret each series term as a "harmonic flow"***
3. ***attribute flows to <u>conjectured</u> A-A initial-state geometry***

SS multipoles: $\mathbf{A_X\{SS\} \equiv \rho_0(b) v_m^2\{SS\} = F_m \times A_{1D}/2}$

"global momentum conservation"

m=1 $\mathbf{A_D\{SS\} - A_D\{AS\}}$

difference between <u>two large numbers</u>, jet structure attributed to bulk medium

"elliptic flow" ***nonjet***

m=2 $\mathbf{A_Q\{2D\} + A_Q\{SS\}}$

jet-related SS 2D peak

"triangular flow" **m=3** $\mathbf{A_S\{SS\}}$

- used to be Mach cones

residuals **m=4** $\mathbf{A_O\{SS\}}$

- predicted by SS 2D peak

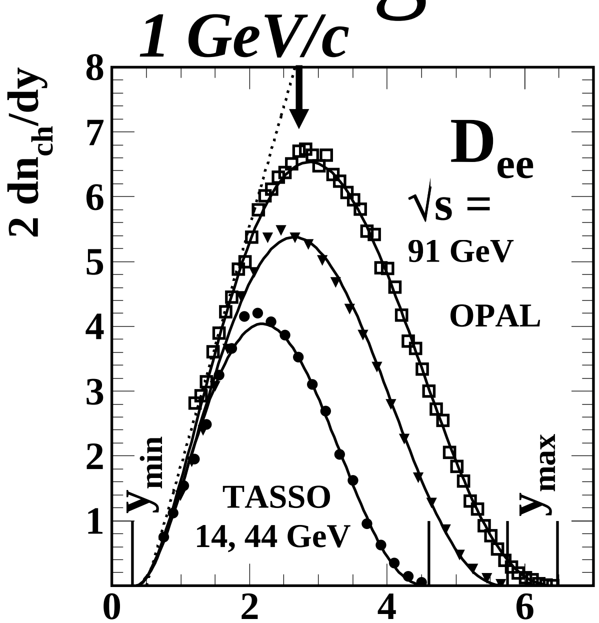
SS 2D jet peak is "fragmented" to become "flows"

SS 2D Peak and Jets

Applying pQCD to spectra and correlations

confirms a jet interpretation for the SS 2D peak

Fragmentation Functions → Jets



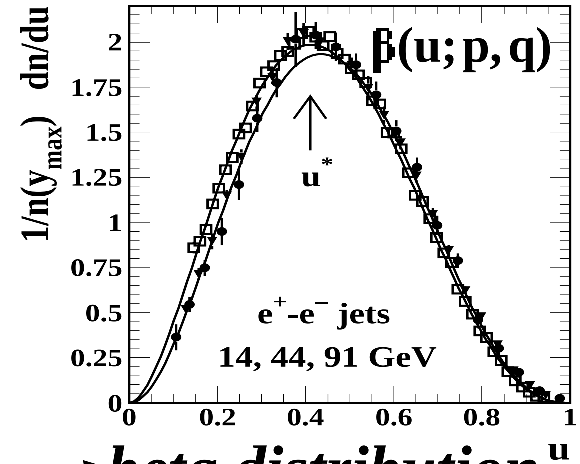
$$y = \ln\{(E+p)/m_\pi\}$$

$$y_{max} = \ln(Q/m_\pi)$$

$$D_{xx}(y, y_{max}) \leftrightarrow D_{xx}(x, Q^2)$$

$$\leftrightarrow 2\,n_{ch}(y_{max})\,\beta(u; p, q)$$

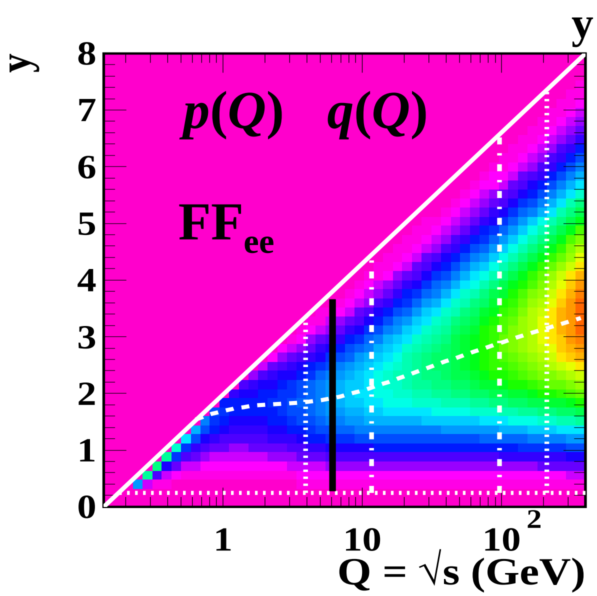
$$u = (y - y_{min})/(y_{max} - y_{min})$$



beta distribution

PRD 74, 034012 (2006)

model parameters p, q



FF data described to statistical limits

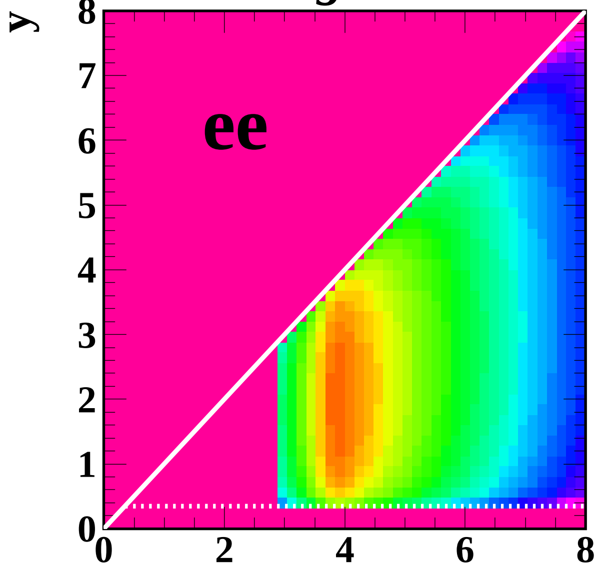
for any jets relevant to RHIC:
<u>more than half</u> of all jet fragments
fall below 1 GeV/c!

all fragmentation [gluon → hadron] is parametrized accurately

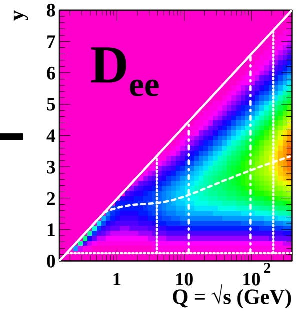
PRC 80, 044901 (2009)

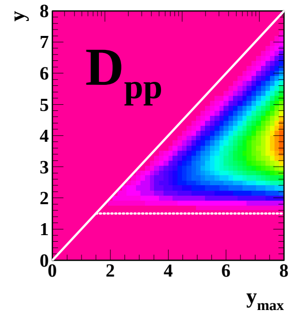
pQCD Folding Integral → FDs

integrand

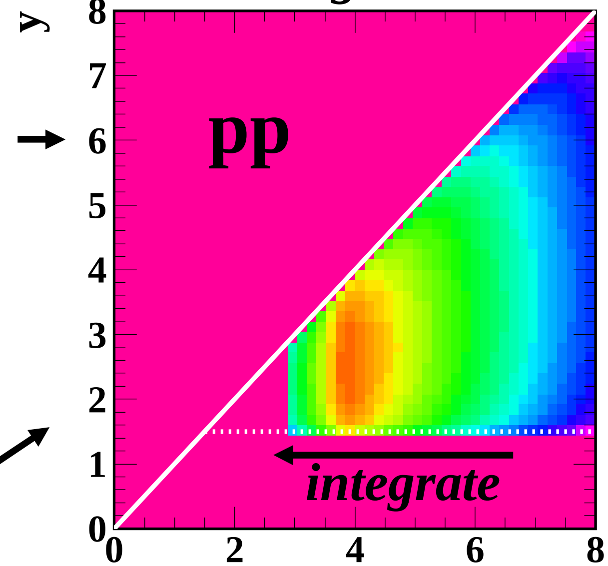


measured FF ensembles





integrand

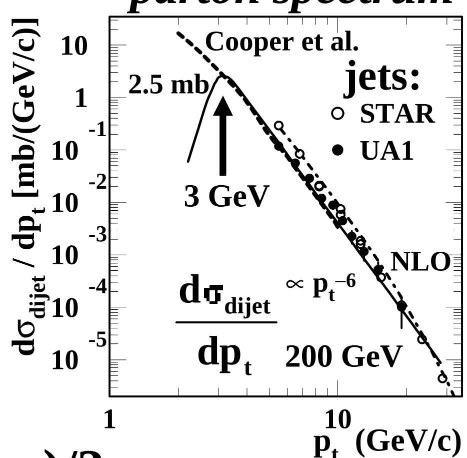


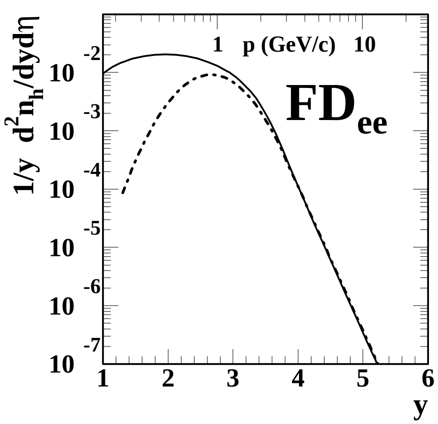
y_{max}

$\ln(2E_{jet}/m_\pi) = y_{max}$

PRC 80, 044901 (2009)

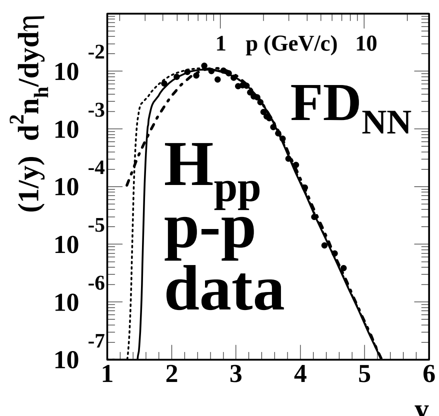
⊗ *parton spectrum* ⊗





$$\frac{d^2n_h}{dy\,d\eta} \approx \frac{\epsilon_j(\Delta\eta)/2}{\sigma_{NSD}\Delta\eta_{4\pi}} \int_0^\infty dy_{max} D_{xx}(y, y_{max}) \frac{d\sigma_{dijet}}{dy_{max}}$$

FD — *FF* — *integrand*



***fragment distributions* FDs ↔**

***spectrum hard components* H**

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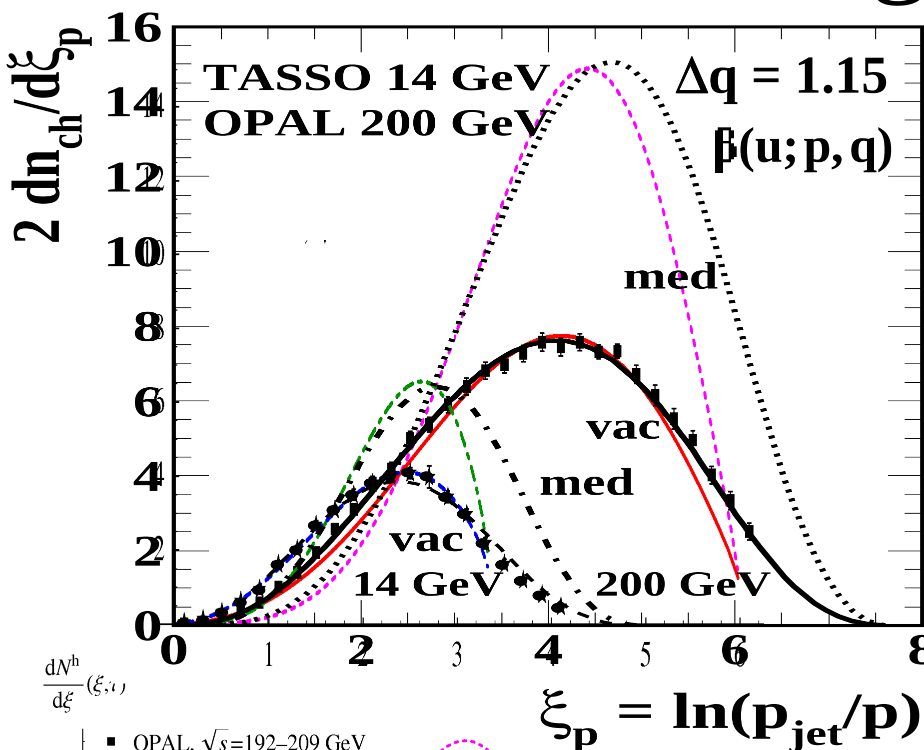
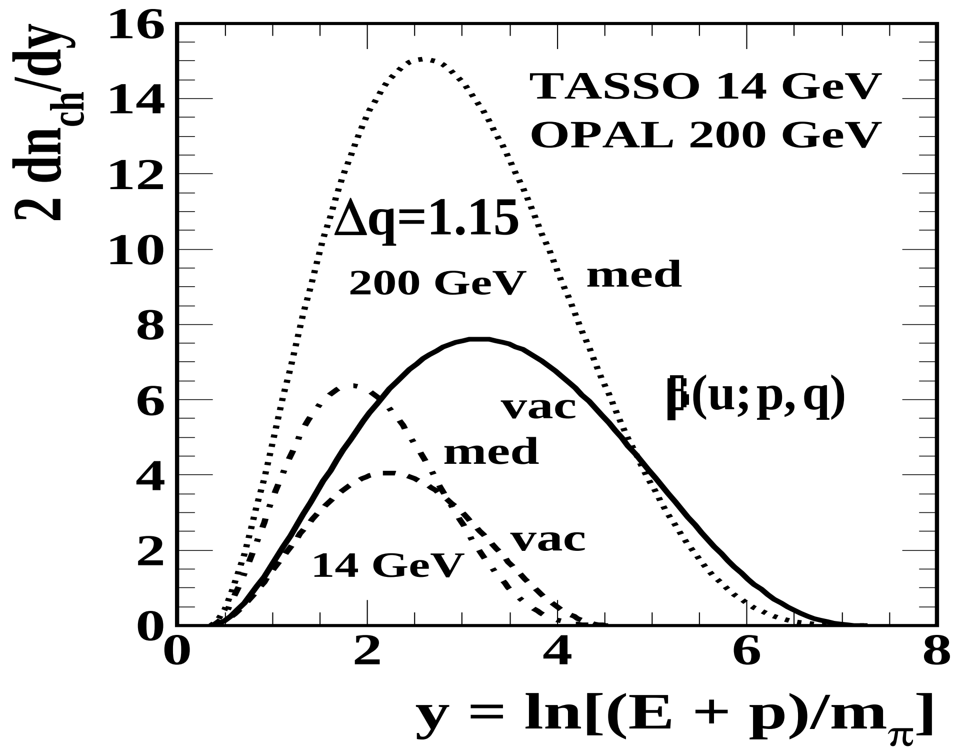
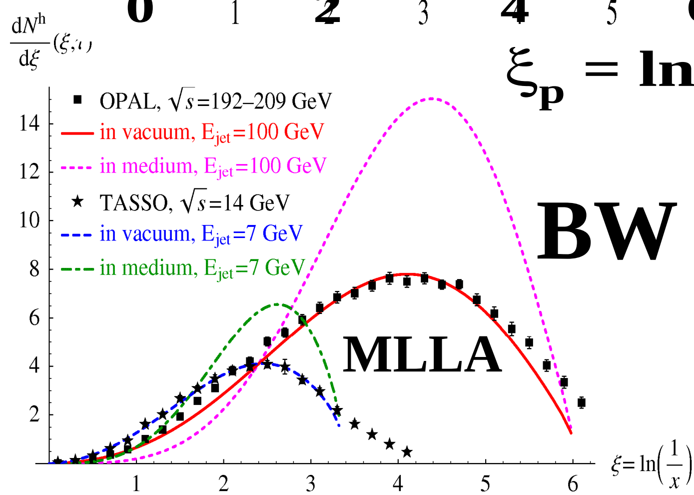
Parton “Energy Loss” in A-A







N. Borghini and U. A. Wiedemann, hep-ph/0506218

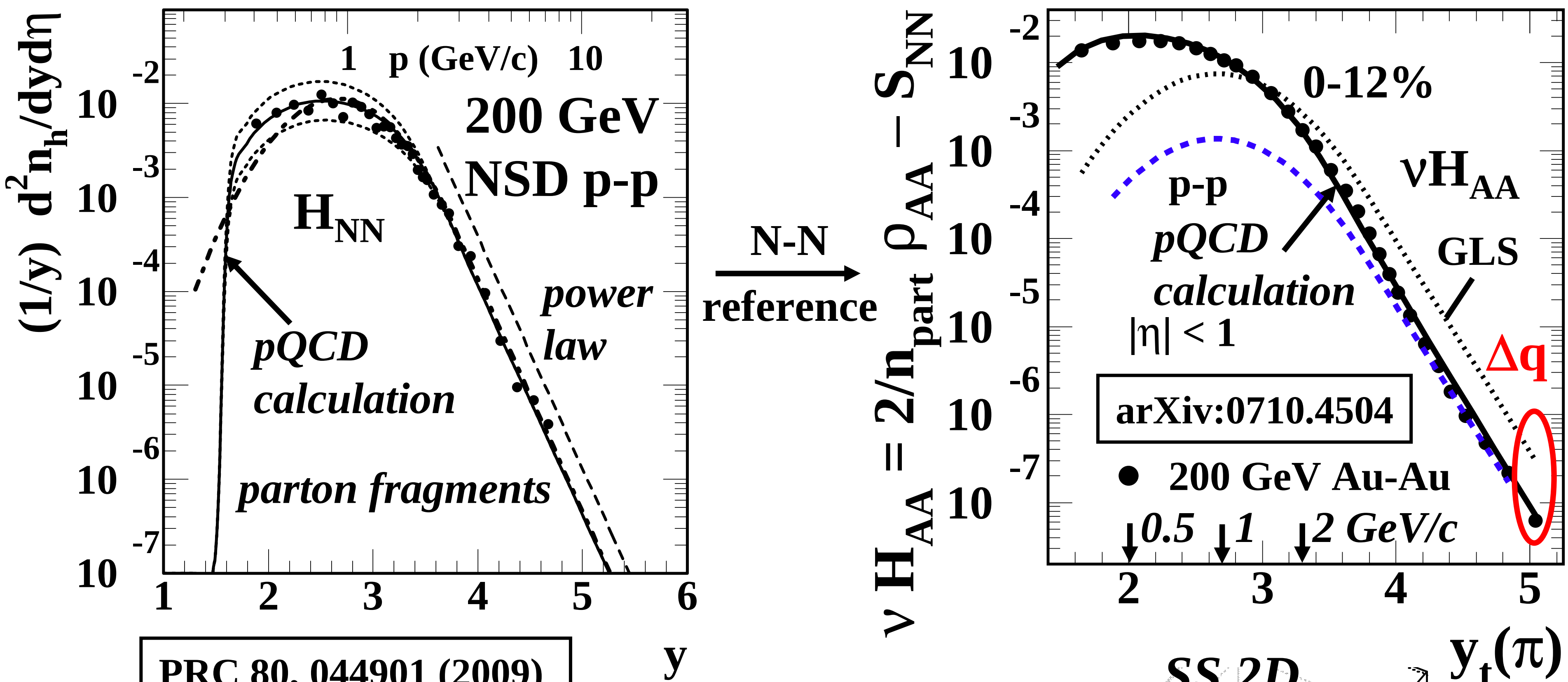
energy conserved by construction

modeled in β(u;p,q) by changing q

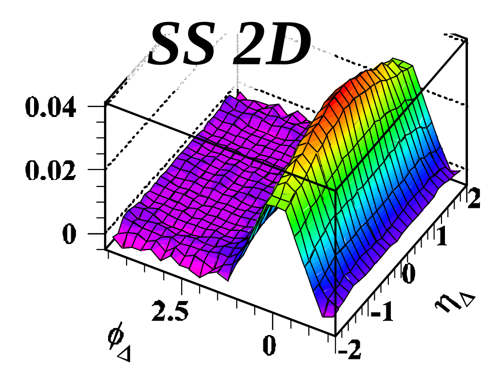
Δq is <u>single</u> “energy-loss” parameter

PRC 80, 044901 (2009)

Hard-component H_{XX} Evolution



PRC 80, 044901 (2009)

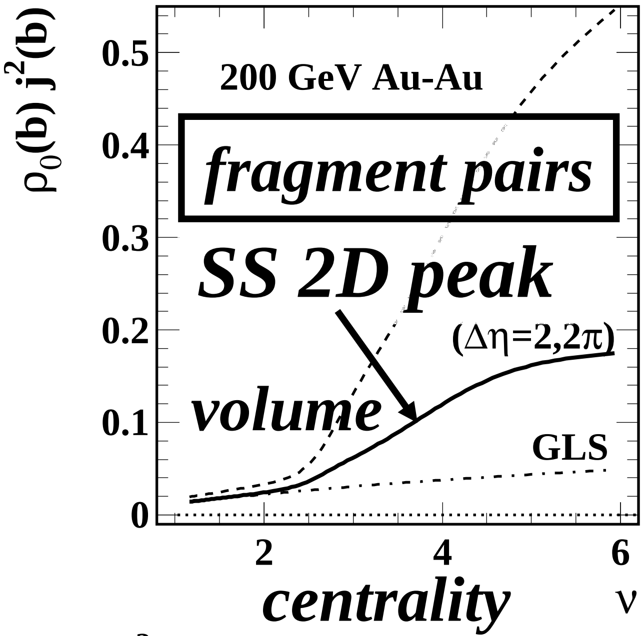


$$\frac{2}{n_{part}}\frac{1}{y_t}\frac{dn_{ch}}{dy_t} = S_{NN}(y_t) + \nu H_{AA}(y_t;\nu)$$

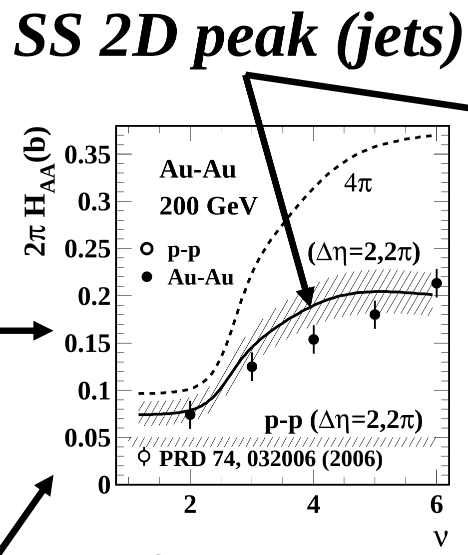
fragmentation evolution

pQCD: describes A-A spectrum hard-component evolution

Jet Correlations and Hadron Production

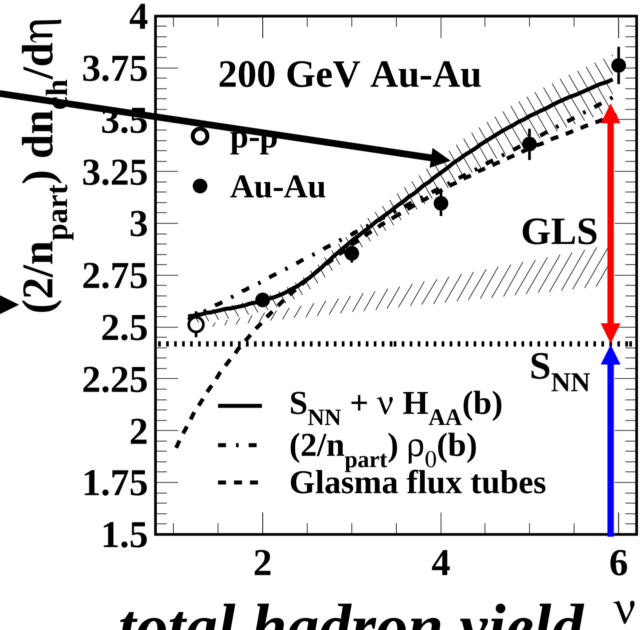


centrality



jet fragment yield
from spectra

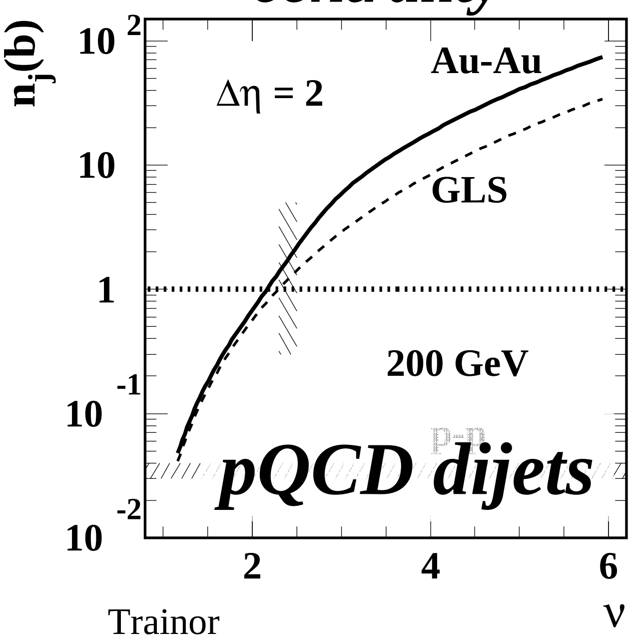
PRC 83, 034903 (2011)



total hadron yield

solid curve: jet correlations
points: spectrum integrals

soft component



quantitative relation:
- *SS 2D peak volume*
- *pQCD jet cross section*
- *spectrum hard-component yields*

1/3 of final state is jet fragments

Summary

- ***2D correlations include a monolithic SS 2D peak***
- ***The SS peak biases all v_m*****{2,EP} data – *“nonflow”***
- ***All “higher harmonics” are part of the SS peak***
- ***The SS peak is quantitatively linked to pQCD (jets)***

Fourier was correct:

A “secret analogy” between
“diverse phenomena” is revealed

“higher harmonic flows” $\rightarrow$ minimum-bias jets