String theory, holography and condensed matter systems

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String theory

- What is string theory ?
- Theory of quantum strings.

- Originally proposed as a theory for QCD.
- Later shown to contain gravity in 10 (26) dimension.

$$R_{\mu\nu} + O(l_s) = 0 \tag{1}$$

- Does it contain other forces and particles ? It does.
- Can we reproduce some thing like Standard model ? TOE ?
- Possible, the “problem” of landscape.

Holography

- Large N $SU(N)$ gauge theory and t'Hooft exapansion.

$$L = \int \mathrm{T}r F^a_{\mu\nu} F^a_{\mu\nu} \tag{2}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_{YM} [A_\mu, A_\nu] \tag{3}$$

$$A_\mu = A^a_\mu \tau^a \tag{4}$$

  τ^a's are traceless hermitian matrices. There are $N^2 - 1$ matrice. For QCD ($N = 3$) they may be identify as gluons.
- t'Hooft expansion in $\lambda = g^2_{YM} N$
- $f_0(\lambda) + \frac{1}{N^2} f_1(\lambda) + ...$
- $\frac{1}{N^2} \propto g_s$

Gauge/Gravity duality

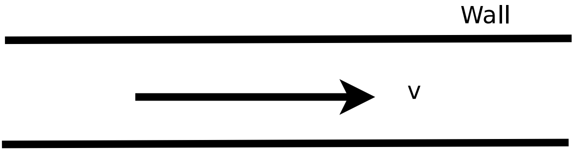
- String theory in $AdS^5 \times S^5$ is equivalent to $\mathcal{N} = 4SU(N)$ gauge theory.
- $ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + r^2(dx^2 + dy^2)$. A funnel like space.
- $l_s \propto \lambda^{-\frac{1}{4}}$
- Various variation is known now.

Superfluidity

- What is Bose condensation ?
  Cool down a bosonic system and a macroscopic fraction of the particles will go to the ground state.
- What is superfluidity ? The condensate shows strange properties like zero viscosity flow.

Landau's criterion

- Fluid moving through a capilary.



- Ground state is moving with velocity v.
- The ground state may loose energy with an excitation of momentum $-p$,
- The energy of the quasiparticle in the capillary frame,

$$E'(p) = E(p) - p.v \tag{5}$$

$$\tag{6}$$

- $min(E'(p)) < 0$ or $min(E(p) < p.v$.

For free particles $E(p) = p^2/2m$, so we always need interaction for superfluidity.

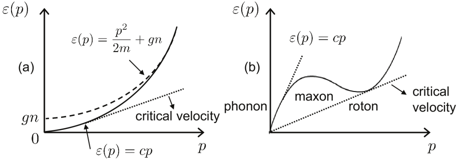


Figure 5.1: (a) The excitation spectrum of a weakly interacting Bose gas, for which the critical velocity is equal to the sound velocity, $v_c = c$. (b) The excitation spectrum of a strongly interacting Bose liquid, for which the critical velocity is smaller than the sound

- First order transition at low temperature.
- Second order near superfluid transition temperature.

EYMH model I

Based on JHEP 1010 (2010) 006 Daniel Arean,PB, Chethan Krishnan
Also, Phys.Rev. D79 (2009) 045010, PB, Anindya Mukherjee, Hsein-Hang Shieh
Phys.Rev. D79 (2009) 066002, Kovtun, Son, Herzog

We start with minimal ingredients,

- Holographic back ground at finite temperature, we have a black hole.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2) \quad (7)$$

$$f(r) = r^2\left(1 - \frac{1}{r^{d-1}}\right) \quad (8)$$

- A complex scalar field ϕ and a gauge field A_μ.

- Holography : global symmetry in boundary $\Rightarrow$ Local symmetry in bulk.
- Boundary value of gauge field A_t $\Rightarrow$ Chemical potential for boundary particle number.
- Boundary value of A_x is superfluid velocity.
- $A_t(\infty) = \mu$ and $A_x(\infty) = S$.

- The action is given by,

$$S = \frac{1}{q^2}\int d^{d+1}x\sqrt{-g}\left(-\frac{1}{4}F^{ab}F_{ab} - V(|\Psi|) - |\nabla\Psi - i\mathcal{A}\Psi|^2\right) \tag{9}$$

- EOM's

$$\psi'' + \left(\frac{f'}{f} + \frac{d-1}{r}\right)\psi' + \left(\frac{A_t^2}{f^2} - \frac{A_x^2}{r^2 f}\right)\psi - \frac{m^2}{f}\psi = 0 \tag{10}$$

$$A_t'' + \frac{d-1}{r}A_t' + \frac{\psi^2}{f^2}A_t = 0$$

$$A_x'' + (\frac{f'}{f} + \frac{d-3}{r})A_x' + \frac{\psi^2}{f^2}A_x = 0$$

- $\psi = 0, A_t = \mu(1 - \frac{1}{r^{d-3}}), A_x = S$ is the normal phase.
- This phase becomes unstable as we increase μ. A zero mode of ψ forms at $\mu = \mu_c$.

- The scalar EOM is (assuming $V(\psi) = m^2\psi^2$)

$$\psi'' + \left(\frac{f'}{f} + \frac{d-1}{r}\right)\psi' + \left(\frac{A_t^2}{f^2} - \frac{A_x^2}{r^2 f}\right)\psi - \frac{m^2}{f}\psi = 0\,. \quad (11)$$

- After applying the following change of variables:

$$\psi = \frac{\tilde{\psi}}{r^{\frac{d-1}{2}}}\,, \qquad \frac{dr}{dy} = \frac{1}{f}\,, \quad (12)$$

the scalar EoM (**??**) takes the form of a Schrödinger equation:

$$\frac{d^2}{dy^2}\tilde{\psi} - \tilde{V}_{\text{eff}}(y)\tilde{\psi} = 0 \quad (13)$$

Notice that $y \to \infty$ as $r \to 1$ and $y \to 0$ as $r \to \infty$.

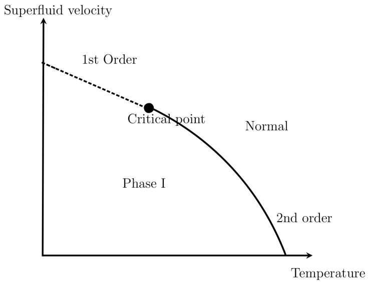
- The potential V_{eff}, written in terms of r, reads

$$V_{\text{eff}}(r) = -f^2\left(-\frac{(d-1)(d-3)}{4r^2} - \frac{(d-1)f'}{2rf} + \frac{A_t^2}{f^2} - \frac{A_x^2}{r^2 f} - \frac{m^2}{f}\right).$$

- The phase with $\psi = 0, A_t = \mu(1 - \frac{1}{r^2}), A_x = S_x$ is always there.

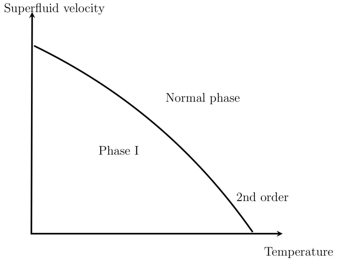
$$V_{\text{eff}}(r) = -\mu^2\Big(\big(1 - \frac{1}{r^2}\big)^2 - \tilde{S}^2\frac{f}{r^2}\Big) \quad (15)$$
$$-f^2\Big(- \frac{(d-1)(d-3)}{4r^2} - \frac{(d-1)f'}{2rf} - \frac{m^2}{f}\Big),$$

- This phase becomes unstable as we increase μ. A zero mode of ψ forms at $\mu = \mu_c$.
- The questions are:
  1. Which phase dominates?
  2. What is the associated phase transition?



(a) For $d = 3$ and $d = 4$ with lower values of m^2. The undotted curve is the line of second order transition which ends in a special/critical point. The dotted line is for first order transition.

ıperfluid velocity



(c) Phase diagram for $d = 4$ with large values of m^2. Only a second order transition is possible between superconducting and normal phase.

- For $S_x = 0$ this question may be answered without getting into too much detail. With a chemical potential fixed to μ, the solution of A_t in the new phase can be written as

$$A_t = \mu(1 - \frac{1}{r^{d-2}}) + \delta A_t \tag{16}$$

where $\delta A_t \to 0$ at $r \to 1$ and $r \to \infty$. Then, from eq. (**??**) we get,

$$\begin{aligned}
\frac{\Omega_{new}}{T^d V} &= -\frac{\mu^2(d-2)^2}{2} + (d-2)\int dr\partial_r(\delta A_t) - \int r^{d-1}\frac{(\delta A_t)'^2}{2}dr = \\
&= -\frac{\mu^2(d-2)^2}{2} + (d-2)\delta A_t\Big|_0^\infty - \int r^{d-1}\frac{(\delta A_t)'^2}{2}dr = \\
&= -\frac{\mu^2(d-2)^2}{2} - \int r^{d-1}\frac{(\delta A_t)'^2}{2}dr\,. \qquad (17)
\end{aligned}$$

Hence

$$\delta\Omega = \Omega_{new} - \Omega_{normal} = -(T^d V)\int r^{d-1}\frac{(\delta A_t'^2)}{2}dr < 0\,. \qquad (18)$$

Therefore if a phase with non-trivial ψ exists it will always have a lower free energy than the normal phase.

Double Scaling limit

- How to solve the scalar equation of motion. Only partial results to solve scalar equation of motion. None at finite $\tilde{S}$.
- We define double scaling limit where $\tilde{S} \to 1$ and $\mu \to \infty$.
- In this limit the potential is negative only in a very small neighbourhood near the boundary $y = 0$ (or $r = \infty$),

$$V_{\rm eff}(r) = -\mu^2\Big(\big(1 - \frac{1}{r}\big)^2 - \tilde{S}^2 \frac{f}{r^2}\Big) \qquad (19)$$
$$-f^2\Big(- \frac{(d-1)(d-3)}{4r^2} - \frac{(d-1)f'}{2rf} - \frac{m^2}{f}\Big)\,,$$

- We define the following double scaling limit:

$$\tilde{S} \longrightarrow 1\,, \qquad \mu \longrightarrow \infty\,,$$
$$\tilde{\mu} \equiv \mu^{2-2\alpha}(1 - \tilde{S}^2) \ \ \text{kept fixed and} \quad \alpha = \frac{2}{d}\,. \quad (20)$$

- Where we have taken into account that at leading order in y, $f(y) \sim \frac{1}{y^2}$, $A_t = \mu(1 - y^{d-2})$, and $A_x = S_x$. In this limit equation (**??**) reduces to

Zero temperature limit

- Extremal solutions at zero temperature,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2) \quad (21)$$

$$f(r) = r^2\left(1 - \frac{1+Q^2}{r^4} + \frac{Q^2}{r^3}\right) \quad (22)$$

$$T \propto \sqrt{(}3 - Q^2) \quad (23)$$

$AdS^4 \to AdS^2$.

- What is the zero temperature limit of holographic superfluid/supercondcutor ?
- We may include gravity back reaction.
- How these solutions look at zero temperature ? New extremal solutions. $AdS^4 \to AdS^4$. Zero sized horizon.

- Frequency response of such back grounds ? For example conductivity ?
- What is the energy gap(mass gap) at zero temperature ? δ_2
- It seems that $\delta_2 = 0$ for holographic supercondcutors. **Horowitz and Roberts**.
- For p-wave supercondcutor $\delta_2 \neq 0$. **Phys.Lett. B689 (2010) 45-50 Pallab Basu, Jianyang He, Anindya Mukherjee, Hsien-Hang Shieh.**

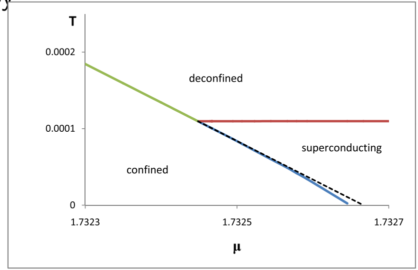
- $\Re(\sigma(0)) \propto \exp(-\frac{\delta_1}{T})$
- $\delta_1 \neq \delta_2$
- for p-wave superconductors $\frac{\delta_1}{\delta_2} \approx 0.699$ (analytic) *arXiv* : 1101.0215 **Pallab Basu**

People have studied various things like,

- Frequency response.
- Velocity of second sound.
- Fluid dynamics.
- Josephson junction.
- Frequency dependence of conductivity.
- Fermionic propagator.
- Competing order.

QCD application I

- Flavor superconductivity. $U(N_f)$ global symmetry in the boundary. Hign isospin chemical potentail. Meson condensation may happen.
- D-Brane construction. We have a nonabelian model like $p - wave$ supercondcutor.
- Color supercondcutor, $\psi\psi$ condensation. Can we model that in string theroy?



Non-fermi liquid from holography I

- What is a fermi liquid?
- Near the fermi surface. $\omega \propto k$
- Liu et al, considered fermion's in an extremal black hole background.

$$ds^2 = \tag{24}$$

- They got non-fermi liquid like behaviour $\omega \propto k^\alpha$.
- We studied the same problem using dyonic black hole.
- We got Landau level and De Haas-Van Alphen like oscillations. **Phys.Rev. D82 (2010) 044036** Pallab Basu, JianYang He, Anindya Mukherjee, Hsien-Hang Shieh.

Current research I

- What is a quench ?. Some parameter(s) of a theory is changing with time.
- A simple scalar system in $0+1$ dim.
- For a time dependent $J(t)$ we have,

$$m^2\phi + \phi^3 + J = 0 \tag{25}$$

Denote the solution of this static equation by $\phi_0(J, m)$.

- We then introduce a $J(t)$ which is slowly varying with time. The idea is to start with adiabatic initial conditions at some early enough time and study the time evolution of the order parameter. To study the adiabatic expansion we write,

$$\phi = \phi_0(J(t), m) + \epsilon\phi_1(t) + \cdots \tag{26}$$

Current research II

- To lowest order in ϵ, the equation governing ϕ_1 is,

$$\phi_1(m^2 + 3\phi_0^2(t)) = \dot{\phi}_0(t) = \dot{J}(t)\frac{\partial \phi_0}{\partial J} \tag{27}$$

- The adiabatic expansion is good for a finite m, ϕ_0 for a sufficiently slowly varying $J(t)$. The adiabatic expansion fails when

$$\phi_1 \sim \phi_0 \tag{28}$$

$$\Rightarrow \quad \frac{1}{m^2 + 3\phi_0^2}\dot{\phi}_0 \sim \phi_0 \tag{29}$$

For a sufficiently slowly varying $J(t)$, this is possible if both m^2 and ϕ_0 are small. i.e. when we are close to the critical point $m^2 = J = 0$.

Current research III

- What happens if the system goes through a phase transition point? Approaching the critical point along the direction $m^2 = 0$ we have $\phi_0(t) \approx (-J(t))^{\frac{1}{3}}$.
- The condition for breakdown of adiabaticity ,

$$\dot{J} \sim J^{\frac{5}{3}}. \tag{30}$$

- If the quench is linear near the critical point, that is $J(t) \sim J_0 vt$ for small J, we get

$$vt^{\frac{5}{2}} \sim 1 \tag{31}$$

  as the condition for breakdown of adiabaticity.
- When adiabaticity breaks down, the system enters a scaling region. We have a scaling solution,

$$\phi(t) = v^{\frac{1}{5}} \tilde{\phi}(v^{\frac{2}{5}} t). \tag{32}$$

Current research IV

- This scaling solution leads to an estimate of the magnitude of fluctuations $(\delta\phi(0))$,i.e. the departure of ϕ from the equilibrium value at $t = 0$.
- Outside the critical region $\delta\phi(0) \sim \frac{v}{m^2}$, whereas in the critical region $\delta\phi(0) \sim v^{\frac{1}{5}}$.
- We also have the "zero crossing time" $(t_\star)$ defined by $\phi(t_\star) = 0$. We have $t_\star \sim v^{-\frac{2}{5}}$ at the critical point. Which diverges in the $v \to 0$ limit. Out of criticality $t_\star$ actually approaches a constant in the $v \to 0$ limit.

Holographic setup

- $$\mathcal{L} = \frac{1}{2\kappa^2\lambda}\sqrt{-g}[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}(\phi^2 + m^2)^2 - \frac{m^4}{4}] \quad (33)$$
- $$\phi = \phi_0 r^{-\Delta_-} + J(t) r^{-\Delta_-} \quad (34)$$
- Needs a system with second order phase transition.
- There is one! Scalar field in extremal black hole back ground with
$$-\frac{9}{4} < m^2 < -\frac{3}{2} \quad (35)$$
- Make the black hole a little non-extremal.
- Near the phase transition point what happens?
- Only zero mode is important.
- The radial direction is non-important. We get back similar results to that of $0+1$ dim theory.

Future direction

- What happens at zero temperature ? BKT like transition.
- Non-integrability and Chaotic motion : in generic the dynamics of a scalar fields in AdS (attractors, Langevin equations: non-equilibrium stat mechs).
- More understand of applicability of holography in various setup.