
C.U. UNAM
México, D.F.
Junio 2007

Trabajo presentado en la XXI Reunión Anual de la División de Partículas y Campos de la Sociedad Mexicana de Física

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Los vértices neutros VVV y VVVV en el modelo estándar no conmutativo

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M.C. Fernando Procopio Garcia

Asesor:
Dr. J. Jesús Toscano Chávez.

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Modelo estándar no conmutativo (MENC)

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$$V = \gamma, Z.$$

INTRODUCCIÓN

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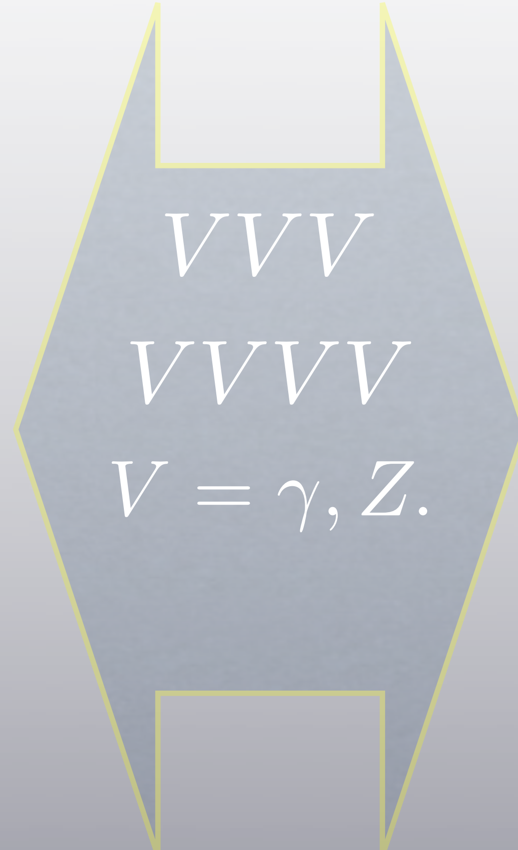

Modelo estándar no conmutativo (MENC)

INTRODUCCIÓN

Modelo estándar(ME)

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Fluctuación cuántica

INTRODUCCIÓN

Modelo estándar(ME)

$$VVV$$

$$VVVV$$

$$V = \gamma, Z.$$

$$v = 246\ GeV$$

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Fluctuación cuántica

Acción clásica

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$$VVV$$
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Acción clásica

Simetría de Bose.	Simetría de Bose.
Invariancia de Lorentz	Violación de la simetría de Lorentz.

INTRODUCCIÓN

Modelo estándar(ME)

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$v = 246\ GeV$

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Modelo estándar(ME)

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$$V = \gamma, Z.$$

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Teorías de norma sobre espacios no conmutativos

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Existencia de un límite conmutativo

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Equivalencia de norma

Teorías de norma sobre espacios no conmutativos

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Estudio perturbativo de la teoría

- Coordenada covariante
- Localidad
- Equivalencia de norma
- Condiciones de consistencia

Modelo Estándar no conmutativo

Modelo Estándar No Conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Modelo Estándar No Conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{A}_x$$

Modelo Estándar No Conmutativo

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$$\mathcal{A}_x \left\{ \right.$$

Modelo estándar no conmutativo

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{A}_x \left\{ \hat{x}^i \right.$$

Modelo Estándar No Conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$A_x \begin{cases} \hat{x}^i \\ R \end{cases}$$

Modelo estándar no conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{A}_x \begin{cases} \hat{x}^i \\ R \end{cases}$$

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Modelo Estándar no conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{A}_x \begin{cases} \hat{x}^i \\ R \end{cases} \qquad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Violación de la simetría de Lorentz

Modelo estándar no conmutativo

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{A}_x \left\{ \begin{array}{l} \hat{x}^i \\ R \end{array} \right. \qquad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Violación de la simetría de Lorentz

↓

Modelo Estándar no conmutativo

$SU(3)_C \times SU(2)_L \times U(1)_Y$

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Violación de la simetría de Lorentz

↓

Nuevos eventos fenomenológicos

Modelo Estándar No Conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{A}_x \begin{cases} \hat{x}^i \\ R \end{cases} \qquad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Violación de la simetría de Lorentz

↓

Nuevos eventos fenomenológicos

$$f \star g = fg + \frac{1}{2} i\theta^{ij} \partial_i f \partial_j g + \mathcal{O}(\theta^2)$$

Modelo Estándar no conmutativo

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{A}_x \begin{cases} \hat{x}^i \\ R \end{cases} \qquad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Violación de la simetría de Lorentz

↓

Nuevos eventos fenomenológicos

$$f \star g = fg + \frac{1}{2} i\theta^{ij} \partial_i f \partial_j g + \mathcal{O}(\theta^2)$$

$$\hat{A}(A + \delta_\lambda A) = \hat{A}(A) + \hat{\delta}_{\hat{\lambda}} \hat{A}(A)$$

$$\int Trf \star g = \int Trg \star f$$

$$\int Tr f \star g = \int Tr g \star f$$

$$S_{norma} = -\frac{1}{2}\int d^4x Tr \frac{1}{G^2}\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}, \;\; \frac{1}{g_I^2} = Tr \frac{1}{G^2} T_I^a T_I^a$$

$$\int Tr f \star g = \int Tr g \star f$$

$$S_{norma} = -\frac{1}{2}\int d^4x Tr \frac{1}{G^2}\hat{F}_{\mu\nu}\star\hat{F}^{\mu\nu}, \quad \frac{1}{g_I^2} = Tr\frac{1}{G^2}T_I^a T_I^a$$

$$\begin{aligned}
\int d^4x Tr \frac{1}{G^2}\hat{F}_{\mu\nu}\star\hat{F}^{\mu\nu} \quad = \quad & -\frac{1}{2}\int d^4x Tr\frac{1}{G^2}F_{\mu\nu}F^{\mu\nu} \\
& +\theta^{\mu\nu}\int d^4x Tr\frac{1}{G^2}[(-F_{\rho\mu}F_{\tau\nu}F^{\rho\tau} + \frac{1}{4}F_{\mu\nu}F_{\rho\tau}F^{\rho\tau})] \\
& +\theta^{\mu\nu}\theta^{\kappa\lambda}\int d^4x Tr\frac{1}{G^2}[(-\frac{1}{16}F_{\mu\nu}F_{\kappa\lambda}F_{\rho\tau}F^{\rho\tau} \\
& +\frac{i}{8}(D_\kappa F_{\rho\tau})(D_\lambda F^{\rho\tau}) + \frac{1}{16}(D_\mu D_\kappa F_{\rho\tau})(D_\nu D_\lambda F^{\rho\tau}) \\
& -\frac{i}{4}(D_\mu F_{\rho\kappa})(D_\nu F_{\tau\lambda})F^{\rho\tau} - \frac{1}{4}F_{\mu\rho}F_{\nu\tau}F^\rho_\kappa F^\tau_\lambda \\
& -\frac{1}{4}F_{\mu\rho}F_{\nu\tau}F^\tau_\kappa F^\rho_\lambda + \frac{1}{4}F_{\mu\nu}F^\rho_\kappa F_{\lambda\tau}F^{\rho\tau} + \frac{1}{4}F_{\kappa\rho}F_{\lambda\tau}F_{\mu\nu}F^{\rho\tau} \\
& -\frac{1}{4}(F_{\mu\kappa}F_{\nu\rho}F_{\lambda\tau} + 2F_{\nu\rho}F_{\mu\kappa}F_{\lambda\tau} + F_{\lambda\tau}F_{\nu\rho}F_{\mu\kappa})F^{\rho\tau})] \\
& +O(\theta^3)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{\gamma\gamma\gamma} &= \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}(A_{\mu\nu}A_{\rho\tau}-4A_{\mu\rho}A_{\nu\tau}),\\
K_{\gamma\gamma\gamma} &= \frac{1}{2}gg'(\kappa_1+3\kappa_2);\\
\mathcal{L}_{Z\gamma\gamma} &= \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\tau}[2Z^{\mu\nu}(2A_{\mu\rho}A_{\nu\tau}-A_{\mu\nu}A_{\rho\tau})+8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau}-Z_{\rho\tau}A_{\mu\nu}A^{\mu\nu})],\\
K_{Z\gamma\gamma} &= \frac{1}{2}[g'^2\kappa_1+(g'^2-2g^2)\kappa_2];\\
\mathcal{L}_{ZZ\gamma} &= \mathcal{L}_{Z\gamma\gamma}(A\leftrightarrow Z),\\
K_{ZZ\gamma} &= \frac{-1}{2gg'}[g'^4\kappa_1+g^2(g^2-2g'^2)\kappa_2];\\
\mathcal{L}_{ZZZ} &= \mathcal{L}_{\gamma\gamma\gamma}(A\rightarrow Z),\\
K_{ZZZ} &= \frac{-1}{2g^2}[g'^4\kappa_1+3g^4\kappa_2].
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\gamma\gamma\gamma} &= \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}(A_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}),\\
K_{\gamma\gamma\gamma} &= \frac{1}{2}gg'(\kappa_1 + 3\kappa_2);\\
\mathcal{L}_{Z\gamma\gamma} &= \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\tau}[2Z^{\mu\nu}(2A_{\mu\rho}A_{\nu\tau} - A_{\mu\nu}A_{\rho\tau}) + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - Z_{\rho\tau}A_{\mu\nu}A^{\mu\nu})],\\
K_{Z\gamma\gamma} &= \frac{1}{2}[g'^2\kappa_1 + (g'^2 - 2g^2)\kappa_2];\\
\mathcal{L}_{ZZ\gamma} &= \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z),\\
K_{ZZ\gamma} &= \frac{-1}{2gg'}[g'^4\kappa_1 + g^2(g^2 - 2g'^2)\kappa_2];\\
\mathcal{L}_{ZZZ} &= \mathcal{L}_{\gamma\gamma\gamma}(A \to Z),\\
K_{ZZZ} &= \frac{-1}{2g^2}[g'^4\kappa_1 + 3g^4\kappa_2].
\end{aligned}
$$

$$
\begin{aligned}
\kappa_1 &= -\frac{1}{g_1^2} - \frac{1}{4g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{1}{36g_5^2} + \frac{1}{4g_6^2},\\
\kappa_2 &= -\frac{1}{4g_2^2} + \frac{1}{4g_5^2} + \frac{1}{4g_6^2}.
\end{aligned}
$$

$$\mathcal{L}_{\gamma\gamma\gamma\gamma} = \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{g'^4 g^4(\kappa_4 + 6\kappa_5)}{(\sqrt{g^2+g'^2})^4}]((-\frac{1}{16}A_{\kappa\lambda}A_{\mu\nu}A_{\rho\tau} + \frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}A_{\mu\nu} - A_{\lambda\tau}A_{\mu\kappa}A_{\nu\rho})A^{\rho\tau})$$

$$\mathcal{L}_{\gamma\gamma\gamma\gamma} = \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}]((-\frac{1}{16}A_{\kappa\lambda}A_{\mu\nu}A_{\rho\tau}+\frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}A_{\mu\nu}$$
$$-A_{\lambda\tau}A_{\mu\kappa}A_{\nu\rho})A^{\rho\tau})$$

$$\mathcal{L}_{Z\gamma\gamma\gamma} = \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{(g'^5 g^3\kappa_4+(3g'^5g^3-3g'^3g^5)\kappa_5)}{(\sqrt{g^2+g'^2})^4}](Z_{\mu\nu}(\frac{1}{16}A_{\rho\tau}A_{\rho\tau}A_{\kappa\lambda}$$
$$-\frac{1}{2}A_{\rho\tau}A_{\kappa\rho}A_{\lambda\tau})-Z_{\lambda\tau}(\frac{1}{2}A_{\rho\tau}A_{\kappa\rho}A_{\mu\nu}-A_{\rho\tau}A_{\mu\kappa}A_{\nu\rho})$$
$$+Z_{\mu\kappa}A_{\rho\tau}A_{\lambda\tau}A_{\nu\rho}+Z_{\nu\rho}A_{\rho\tau}A_{\lambda\tau}A_{\mu\kappa}+\frac{1}{16}Z_{\kappa\lambda}A_{\mu\nu}A_{\rho\tau}A^{\rho\tau}$$
$$+Z_{\rho\tau}(\frac{1}{8}A_{\rho\tau}A_{\kappa\lambda}A_{\mu\nu}-\frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}A_{\mu\nu}+A_{\lambda\tau}A_{\mu\kappa}A_{\nu\rho})).$$

$$
\begin{aligned}
\mathcal{L}_{\gamma\gamma\gamma\gamma} &= \theta^{\mu\nu}\theta^{\kappa\lambda}\Big[\frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\Big]((-\frac{1}{16}A_{\kappa\lambda}A_{\mu\nu}A_{\rho\tau}+\frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}A_{\mu\nu}\\
&\quad -A_{\lambda\tau}A_{\mu\kappa}A_{\nu\rho})A^{\rho\tau})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{Z\gamma\gamma\gamma} &= \theta^{\mu\nu}\theta^{\kappa\lambda}\Big[\frac{(g'^5 g^3\kappa_4+(3g'^5g^3-3g'^3g^5)\kappa_5)}{(\sqrt{g^2+g'^2})^4}\Big](Z_{\mu\nu}(\frac{1}{16}A_{\rho\tau}A_{\rho\tau}A_{\kappa\lambda}\\
&\quad -\frac{1}{2}A_{\rho\tau}A_{\kappa\rho}A_{\lambda\tau})-Z_{\lambda\tau}(\frac{1}{2}A_{\rho\tau}A_{\kappa\rho}A_{\mu\nu}-A_{\rho\tau}A_{\mu\kappa}A_{\nu\rho})\\
&\quad +Z_{\mu\kappa}A_{\rho\tau}A_{\lambda\tau}A_{\nu\rho}+Z_{\nu\rho}A_{\rho\tau}A_{\lambda\tau}A_{\mu\kappa}+\frac{1}{16}Z_{\kappa\lambda}A_{\mu\nu}A_{\rho\tau}A^{\rho\tau}\\
&\quad +Z_{\rho\tau}(\frac{1}{8}A_{\rho\tau}A_{\kappa\lambda}A_{\mu\nu}-\frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}A_{\mu\nu}+A_{\lambda\tau}A_{\mu\kappa}A_{\nu\rho})).
\end{aligned}
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\begin{aligned}
\mathcal{L}_{ZZ\gamma\gamma} &= \theta^{\mu\nu}\theta^{\kappa\lambda}\Big[\frac{g'^6 g^2\kappa_4+(g'^6g^2+g'^2g^6-4g'^4g^4)\kappa_5}{(\sqrt{g^2+g'^2})^4}\Big]((-\frac{1}{16}Z_{\kappa\lambda}Z_{\mu\nu}A_{\rho\tau}\\
&\quad +\frac{1}{2}A_{\mu\nu}Z_{\kappa\rho}Z_{\lambda\tau}-A_{\nu\rho}Z_{\lambda\tau}Z_{\mu\kappa}+A_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu}\\
&\quad -A_{\mu\kappa}Z_{\lambda\tau}Z_{\nu\rho}-A_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho}\\
&\quad -\frac{1}{8}A_{\mu\nu}Z_{\kappa\lambda}Z_{\rho\tau}-\frac{1}{8}A_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau})A^{\rho\tau}\\
&\quad -(\frac{1}{16}A_{\kappa\lambda}A_{\mu\nu}Z_{\rho\tau}-A_{\lambda\tau}A_{\mu\kappa}Z_{\nu\rho}+\frac{1}{2}A_{\kappa\rho}A_{\lambda\tau}Z_{\mu\nu}\\
&\quad -A_{\lambda\tau}A_{\nu\rho}Z_{\mu\kappa}-A_{\mu\kappa}A_{\nu\rho}Z_{\lambda\tau}+A_{\kappa\rho}A_{\mu\nu}Z_{\lambda\tau})Z^{\rho\tau}.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{ZZZ\gamma} \quad = \quad & \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{g'^7 g\kappa_4 + (3g'^3 g^5 - 3g'^5 g^3)\kappa_5}{(\sqrt{g^2+g'^2})^4}](A_{\mu\nu}(\frac{1}{16}Z_{\rho\tau}Z_{\rho\tau}Z_{\kappa\lambda} \\
& -\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\lambda\tau}) - A_{\lambda\tau}(\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\mu\nu} - Z_{\rho\tau}Z_{\mu\kappa}Z_{\nu\rho}) \\
& +A_{\mu\kappa}Z_{\rho\tau}Z_{\lambda\tau}Z_{\nu\rho} + A_{\nu\rho}Z_{\rho\tau}Z_{\lambda\tau}Z_{\mu\kappa} + \frac{1}{16}A_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau}Z^{\rho\tau} \\
& +A_{\rho\tau}(\frac{1}{8}Z_{\rho\tau}Z_{\kappa\lambda}Z_{\mu\nu} - \frac{1}{2}Z_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu} + Z_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho})).
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\mathcal{L}_{ZZZ\gamma} \quad = \quad & \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{g'^7 g\kappa_4 + (3g'^3 g^5 - 3g'^5 g^3)\kappa_5}{(\sqrt{g^2+g'^2})^4}](A_{\mu\nu}(\frac{1}{16}Z_{\rho\tau}Z_{\rho\tau}Z_{\kappa\lambda} \\
& -\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\lambda\tau}) - A_{\lambda\tau}(\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\mu\nu} - Z_{\rho\tau}Z_{\mu\kappa}Z_{\nu\rho}) \\
& +A_{\mu\kappa}Z_{\rho\tau}Z_{\lambda\tau}Z_{\nu\rho} + A_{\nu\rho}Z_{\rho\tau}Z_{\lambda\tau}Z_{\mu\kappa} + \frac{1}{16}A_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau}Z^{\rho\tau} \\
& +A_{\rho\tau}(\frac{1}{8}Z_{\rho\tau}Z_{\kappa\lambda}Z_{\mu\nu} - \frac{1}{2}Z_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu} + Z_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho})).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{ZZZZ} \quad = \quad & \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{(g'^8\kappa_4 + g'^4 g^4 6\kappa_5)}{(\sqrt{g^2+g'^2})^4}]((-\frac{1}{16}Z_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau} + \frac{1}{2}Z_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu} \\
& -Z_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho})Z^{\rho\tau}).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{ZZZ\gamma} &= \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{g'^7 g\kappa_4 + (3g'^3 g^5 - 3g'^5 g^3)\kappa_5}{(\sqrt{g^2+g'^2})^4}](A_{\mu\nu}(\frac{1}{16}Z_{\rho\tau}Z_{\rho\tau}Z_{\kappa\lambda} \\
&\quad -\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\lambda\tau}) - A_{\lambda\tau}(\frac{1}{2}Z_{\rho\tau}Z_{\kappa\rho}Z_{\mu\nu} - Z_{\rho\tau}Z_{\mu\kappa}Z_{\nu\rho}) \\
&\quad +A_{\mu\kappa}Z_{\rho\tau}Z_{\lambda\tau}Z_{\nu\rho} + A_{\nu\rho}Z_{\rho\tau}Z_{\lambda\tau}Z_{\mu\kappa} + \frac{1}{16}A_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau}Z^{\rho\tau} \\
&\quad +A_{\rho\tau}(\frac{1}{8}Z_{\rho\tau}Z_{\kappa\lambda}Z_{\mu\nu} - \frac{1}{2}Z_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu} + Z_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho})).
\end{aligned}
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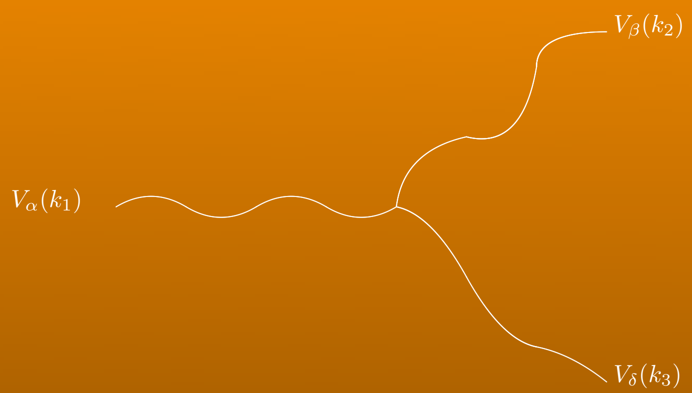
$$
\begin{aligned}
\mathcal{L}_{ZZZZ} &= \theta^{\mu\nu}\theta^{\kappa\lambda}[\frac{(g'^8\kappa_4 + g'^4 g^4 6\kappa_5)}{(\sqrt{g^2+g'^2})^4}]((-\frac{1}{16}Z_{\kappa\lambda}Z_{\mu\nu}Z_{\rho\tau} + \frac{1}{2}Z_{\kappa\rho}Z_{\lambda\tau}Z_{\mu\nu} \\
&\quad -Z_{\lambda\tau}Z_{\mu\kappa}Z_{\nu\rho})Z^{\rho\tau}).
\end{aligned}
$$

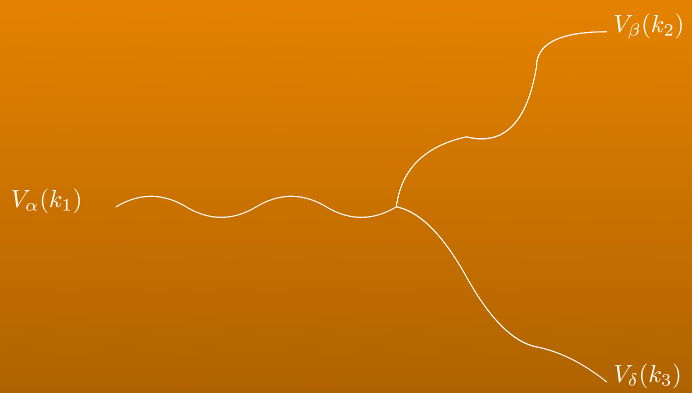
$$
\begin{aligned}
\kappa_4 &= \frac{1}{g_1^2} + \frac{1}{8g_2^2} + \frac{16}{27g_3^2} + \frac{1}{27g_4^2} + \frac{1}{216g_5^2} + \frac{1}{8g_6^2}, \\
\kappa_5 &= \frac{1}{2}(\frac{1}{4g_2^2} + \frac{1}{12g_5^2} + \frac{1}{4g_6^2}).
\end{aligned}
$$

Reglas de Feynman

Reglas de Feynman

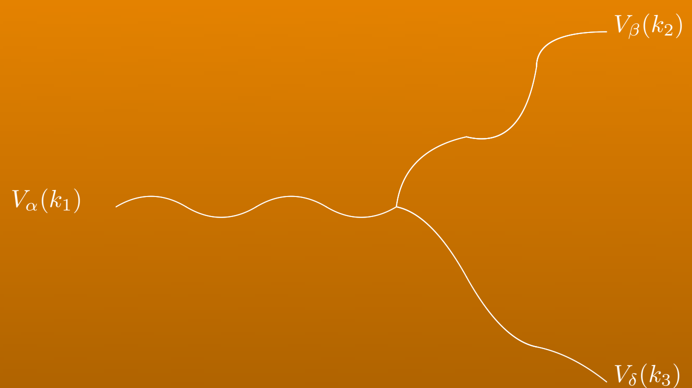


Reglas de Feynman



$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=\Gamma_{\beta\alpha\delta}(k_2,k_1,k_3)=\Gamma_{\delta\beta\alpha}(k_3,k_2,k_1)=\cdots\,,$$

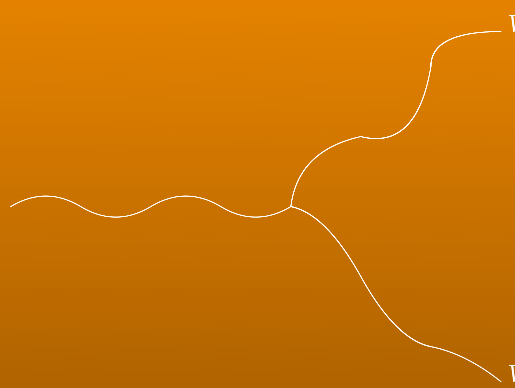
Reglas de Feynman



$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=\Gamma_{\beta\alpha\delta}(k_2,k_1,k_3)=\Gamma_{\delta\beta\alpha}(k_3,k_2,k_1)=\cdots\,,$$

$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=C_{VVV}\theta^{\rho\tau}[R_{\rho\tau\alpha\beta\delta}+T_{\rho\tau\alpha\beta\delta}],$$

Reglas de Feynman

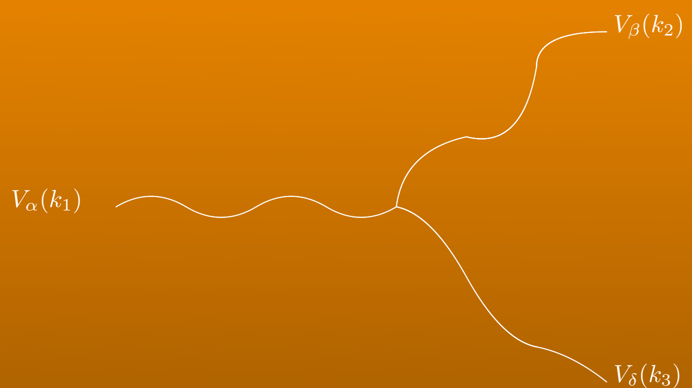


$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=\Gamma_{\beta\alpha\delta}(k_2,k_1,k_3)=\Gamma_{\delta\beta\alpha}(k_3,k_2,k_1)=\cdots\,,$$

$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=C_{VVV}\theta^{\rho\tau}[R_{\rho\tau\alpha\beta\delta}+T_{\rho\tau\alpha\beta\delta}],$$

$$\begin{aligned}
C_{\gamma\gamma\gamma} &= e\ sen\,\theta_W K_{\gamma\gamma\gamma},\\
C_{Z\gamma\gamma} &= -3e\ sen\,\theta_W K_{Z\gamma\gamma},\\
C_{ZZ\gamma} &= -3e\ sen\,\theta_W K_{ZZ\gamma},\\
C_{ZZZ} &= e\ sen\,\theta_W K_{ZZZ}.
\end{aligned}$$

Reglas de Feynman



$$\Gamma_{\alpha\beta\delta}(k_1,k_2,k_3)=\Gamma_{\beta\alpha\delta}(k_2,k_1,k_3)=\Gamma_{\delta\beta\alpha}(k_3,k_2,k_1)=\cdots\ ,$$

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C_{\gamma\gamma\gamma} &= e\ sen\,\theta_W K_{\gamma\gamma\gamma},\\
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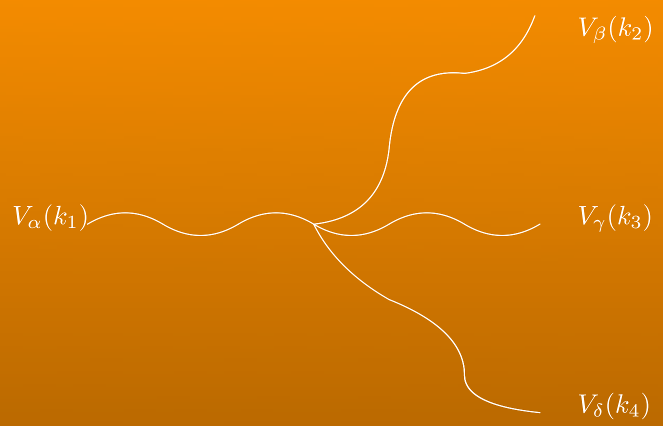
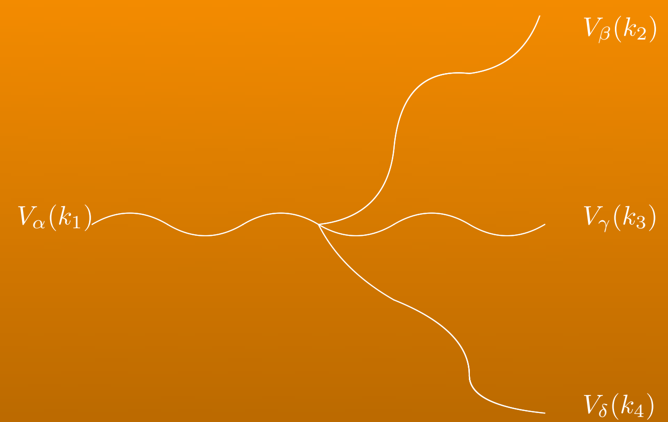
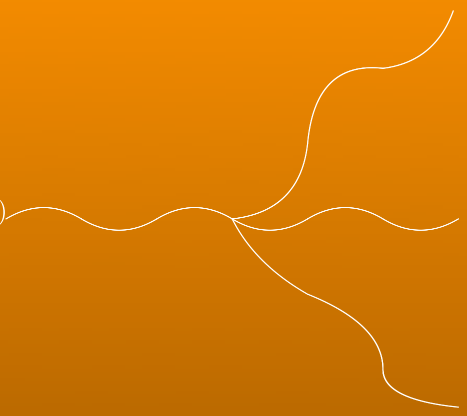
C_{ZZ\gamma} &= -3e\ sen\,\theta_W K_{ZZ\gamma},\\
C_{ZZZ} &= e\ sen\,\theta_W K_{ZZZ}.
\end{aligned}$$

$$\begin{aligned}
k_1^\alpha\Gamma_{\alpha\beta\delta} &= 0,\\
k_2^\beta\Gamma_{\alpha\beta\delta} &= 0,\\
k_3^\delta\Gamma_{\alpha\beta\delta} &= 0,\\
k_1^\alpha k_2^\beta k_3^\delta\Gamma_{\alpha\beta\delta} &= 0.
\end{aligned}$$


$V_\alpha(k_1)$
$V_\beta(k_2)$
$V_\gamma(k_3)$
$V_\delta(k_4)$

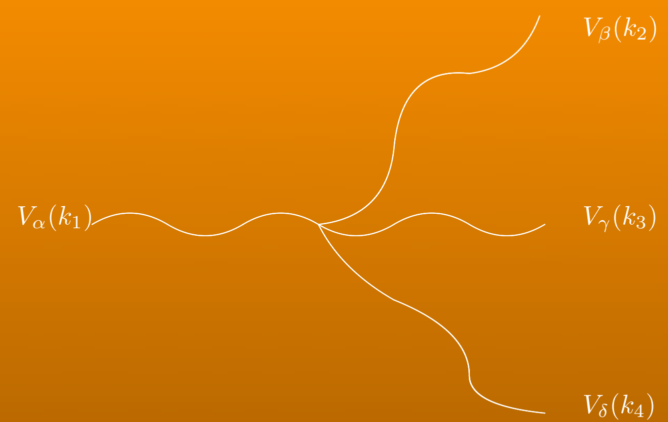


$$\Gamma^{AAAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\;\Gamma^1_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$



$$\Gamma^{AAAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\;\Gamma^1_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$

$$\Gamma^{ZAAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{(g'^5 g^3\kappa_4+(3g'^5g^3-3g'^3g^5)\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\Gamma^2_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$



$$\Gamma^{AAAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\;\Gamma^1_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$

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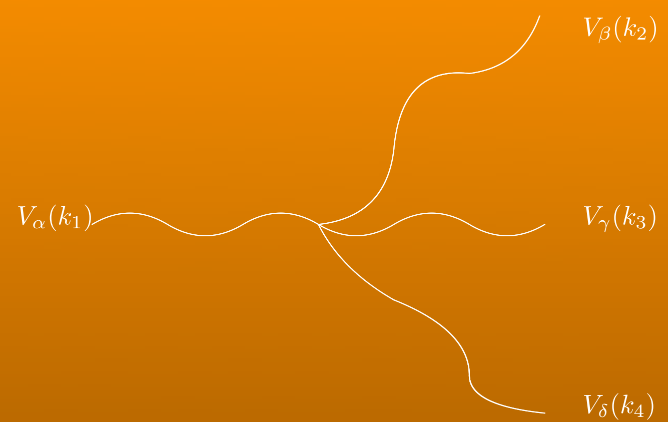
$$\Gamma^{ZZAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{g'^6 g^2\kappa_4+(g'^6g^2+g'^2g^6-4g'^4g^4)\kappa_5}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\Gamma^3_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$



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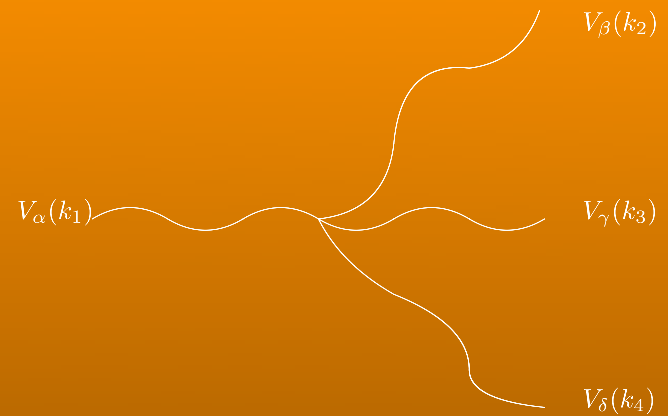

$$\Gamma^{AAAA}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{g'^4 g^4(\kappa_4+6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\ \Gamma^1_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$

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$$\Gamma^{ZZZZ}_{\alpha\beta\gamma\delta}(k_1,k_2,k_3,k_4) = \frac{(g'^8\kappa_4 + g'^4 g^4 6\kappa_5)}{(\sqrt{g^2+g'^2})^4}\theta^{\mu\nu}\theta^{\kappa\lambda}\Gamma^5_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda}.$$

Conclusiones

Conclusiones

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Conclusiones

- Se realizó el desarrollo de la acción del MENC hasta segundo orden en θ con el fin de derivar los vértices cuárticos VVVV. Se obtuvieron las lagrangianas para todos los vértices cuárticos ZZZZ, ZZZɣ, ZZɣɣ, Zɣɣɣ y ɣɣɣɣ. Se calcularon las reglas de Feynman para todos los vértices cuárticos, incluyendo también las correspondientes a los vértices trilineales ZZZ, ZZɣ, Zɣɣ y ɣɣɣ, las cuales no han sido dadas en la literatura.

Perspectivas

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Decaimiento raro $Z \to \gamma\gamma\gamma$

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$Br(Z \to \gamma\gamma\gamma) < 10^{-10}$

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Colisionadores fotónicos

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Colisionadores fotónicos

Electrodinámica no conmutativa

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Electrodinámica no conmutativa

Z

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Electrodinámica no conmutativa

~~Z~~ $\gamma\gamma\gamma\gamma$

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Electrodinámica no conmutativa

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Dispersión de luz por luz: $\gamma\gamma \to \gamma\gamma$

Colisionadores fotónicos

Electrodinámica no conmutativa

~~Z~~ ~~$\gamma\gamma\gamma\gamma$~~

Estudio de la dispersión $\gamma\gamma \to \gamma Z$

Estudio de la dispersión $\gamma\gamma \to ZZ$

Perspectivas

Decaimiento raro $Z \rightarrow \gamma\gamma\gamma$

$Br(Z \rightarrow \gamma\gamma\gamma) < 10^{-10}$

90 %

Dispersión de luz por luz: $\gamma\gamma \rightarrow \gamma\gamma$

Colisionadores fotónicos

Electrodinámica no conmutativa

Z $\gamma\gamma\gamma$

Estudio de la dispersión $\gamma\gamma \rightarrow \gamma Z$

$MENC$

Estudio de la dispersión $\gamma\gamma \rightarrow ZZ$


Introducción
MENC
¿Preguntas?
C&P
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	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e_R^{(i)}$	1	1	-1	-1	0
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
$u_R^{(i)}$	3	1	$\frac{2}{3}$	$\frac{2}{3}$	0
$d_R^{(i)}$	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$	0
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$
W^+, W^-, Z	1	3	0	$(\pm 1, 0)$	$(\pm 1, 0)$
A	1	1	0	0	0
G^b	8	1	0	0	0

$$\begin{aligned}
\frac{1}{g_1^2} + \frac{1}{2g_2^2} + \frac{4}{3g_3^2} + \frac{1}{3g_4^2} + \frac{1}{6g_5^2} + \frac{1}{2g_6^2} &= \frac{1}{2g'^2}, \\
\frac{1}{g_2^2} + \frac{3}{g_5^2} + \frac{1}{g_6^2} &= \frac{1}{g^2}, \\
\frac{1}{g_3^2} + \frac{1}{g_4^2} + \frac{2}{g_5^2} &= \frac{1}{g_S^2}.
\end{aligned}$$

$$
\begin{aligned}
\theta^{\rho\tau} R_{\rho\tau\alpha\beta\delta} \quad = \quad & (k_3^\beta k_2^\delta - g^{\beta\delta} k_2 \cdot k_3)\theta^{\alpha\phi} k_1^\phi \\
& +(k_3^\alpha k_1^\delta - g^{\alpha\delta} k_1 \cdot k_3)\theta^{\beta\phi} k_2^\phi \\
& +(k_2^\alpha k_1^\beta - g^{\alpha\beta} k_1 \cdot k_2)\theta^{\delta\phi} k_3^\phi
\end{aligned}
$$

$$
\begin{aligned}
\theta^{\rho\tau} T_{\rho\tau\alpha\beta\delta} \quad = \quad & (g^{\alpha\delta} k_3^\beta - g^{\beta\delta} k_3^\alpha)\theta^{\xi\phi} k_1^\xi k_2^\phi \\
& +(g^{\alpha\beta} k_2^\delta - g^{\beta\delta} k_2^\alpha)\theta^{\xi\phi} k_1^\xi k_3^\phi \\
& +(g^{\alpha\beta} k_1^\delta - g^{\alpha\delta} k_1^\beta)\theta^{\xi\phi} k_2^\xi k_3^\phi \\
& +k_1 \cdot k_2(-\theta^{\beta\delta} k_3^\alpha - \theta^{\alpha\delta} k_3^\beta + g^{\alpha\delta}\theta^{\beta\phi} k_3^\phi + g^{\beta\delta}\theta^{\alpha\phi} k_3^\phi) \\
& +k_1 \cdot k_3(-\theta^{\alpha\beta} k_2^\delta + \theta^{\beta\delta} k_2^\alpha + g^{\beta\delta}\theta^{\alpha\phi} k_2^\phi + g^{\alpha\beta}\theta^{\delta\phi} k_2^\phi) \\
& +k_2 \cdot k_3(\theta^{\alpha\beta} k_1^\delta + \theta^{\alpha\delta} k_1^\beta + g^{\alpha\beta}\theta^{\delta\phi} k_1^\phi + g^{\alpha\delta}\theta^{\beta\phi} k_1^\phi) \\
& -\theta^{\alpha\phi} k_2^\phi k_3^\beta k_1^\delta - \theta^{\beta\phi} k_3^\phi k_2^\alpha k_1^\delta - \theta^{\delta\phi} k_1^\phi k_2^\alpha k_3^\beta \\
& -\theta^{\beta\phi} k_1^\phi k_3^\alpha k_2^\delta - \theta^{\delta\phi} k_2^\phi k_3^\alpha k_1^\beta - \theta^{\alpha\phi} k_3^\phi k_1^\beta k_2^\delta .
\end{aligned}
$$

$$
\begin{aligned}
\theta^{\mu\nu}\theta^{\kappa\lambda}\Gamma^{1}_{\alpha\beta\gamma\delta\mu\nu\kappa\lambda} \quad = \quad & (k_2^\delta k_3\cdot k4 - k_3^\delta k_2\cdot k4)\theta^{\alpha\xi}\theta^{\beta\gamma}k_{1\xi} - \\
& -(k_4^\alpha k_1\cdot k_3 - k_3^\alpha k_1\cdot k_4)\theta^{\delta\xi}\theta^{\beta\gamma}k_{2\xi} + \\
& +(k_3^\alpha k_1^\delta - g^{\alpha\delta}k_1\cdot k_3)\theta^{\eta\xi}\theta^{\beta\gamma}k_{2\eta}k_{4\xi} + (k_1^\gamma k_2\cdot k_3 - k_2^\gamma k_1\cdot k_3)\theta^{\alpha\eta}\theta^{\beta\delta}k_{4\eta} \\
& +(k_3^\delta k_2\cdot k_4 - k_2^\delta k_3\cdot k4)\theta^{\alpha\gamma}\theta^{\beta\eta}k_{1\eta} + (k_4^\gamma k_2^\delta - g^{\gamma\delta}k_2\cdot k_4)\theta^{\alpha\eta}\theta^{\beta\xi}k_{3\eta}k_{1\xi} \\
& +\frac{1}{2}(k_4^\gamma k_3^\delta - g^{\gamma\delta}k_3\cdot k_4)\theta^{\alpha\eta}\theta^{\beta\xi}k_{1\eta}k_{2\xi} + (g^{\gamma\delta}k_2\cdot k_4 - k_4^\gamma k_2^\delta)\theta^{\alpha\eta}\theta^{\beta\xi}k_{1\eta}k_{3\xi} \\
& +(k_2^\gamma k_1\cdot k_3 - k_1^\gamma k_2\cdot k_3)\theta^{\alpha\delta}\theta^{\beta\eta}k_{4\eta} + (k_4^\beta k_1\cdot k_2 - k_1^\beta k_2\cdot k_4)\theta^{\alpha\eta}\theta^{\gamma\delta}k_{3\eta} \\
& +(k_3^\alpha k_1\cdot k_4 - k_4^\alpha k_1\cdot k_3)\theta^{\beta\eta}\theta^{\gamma\delta}k_{2\eta} - (k_3^\delta k_2\cdot k_4 - k_2^\delta k_3\cdot k_4)\theta^{\alpha\beta}\theta^{\gamma\eta}k_{1\eta} \\
& +(k_4^\beta k_3^\delta - g^{\beta\delta}k_3\cdot k_4)\theta^{\alpha\eta}\theta^{\gamma\xi}k_{2\eta}k_{1\xi} + (g^{\beta\delta}k_3\cdot k_4 - k_4^\beta k_3^\delta)\theta^{\alpha\eta}\theta^{\gamma\xi}k_{1\eta}k_{2\xi} \\
& +(k_4^\alpha k_1\cdot k_3 - k_3^\alpha k_1\cdot k_4)\theta^{\beta\delta}\theta^{\gamma\eta}k_{2\eta} + (k_3^\alpha k_2^\delta - g^{\alpha\delta}k_1\cdot k_3)\theta^{\beta\eta}\theta^{\gamma\xi}k_{4\eta}k_{2\xi} \\
& +(k_1^\beta k_2\cdot k_4 - k_4^\beta k_1\cdot k_2)\theta^{\alpha\delta}\theta^{\gamma\eta}k_{3\eta} + (g^{\beta\delta}k_1\cdot k_2 - k_1^\beta k_2^\delta)\theta^{\alpha\eta}\theta^{\gamma\xi}k_{4\eta}k_{3\xi} \\
& +\frac{1}{2}(k_4^\alpha k_1^\delta - g^{\alpha\delta}k_1\cdot k_4)\theta^{\beta\eta}\theta^{\gamma\xi}k_{2\eta}k_3\xi + (k_1^\beta k_2^\delta - g^{\beta\delta}k_1\cdot k_2)\theta^{\alpha\eta}\theta^{\gamma\xi}k_{3\eta}k_{4\xi} \\
& +(g^{\alpha\delta}k_1\cdot k_3 - k_3^\alpha k_1^\delta)\theta^{\beta\eta}\theta^{\gamma\xi}k_{2\eta}k_{4\xi} + (k_3^\alpha k_2^\gamma - g^{\alpha\gamma}k_2\cdot k_3)\theta^{\beta\eta}\theta^{\delta\xi}k_{4\eta}k_{1\xi} \\
& +(g^{\alpha\beta}k_2\cdot k_4 - k_2^\alpha k_4^\beta)\theta^{\gamma\eta}\theta^{\delta\xi}k_{3\eta}k_{1\xi} + (k_3^\beta k_1^\gamma - g^{\beta\gamma k_1\cdot k_3})\theta^{\alpha\eta}\theta^{\delta\xi}k_{4\eta}k_{2\xi} \\
& +(k_4^\alpha k_1^\gamma - g^{\alpha\gamma}k_1\cdot k_4)\theta^{\beta\eta}\theta^{\delta\xi}k_{3\eta}k_{2\xi} + (k_4^\beta k_1\cdot k_2 - k_1^\beta k_2\cdot k_4)\theta^{\alpha\gamma}\theta^{\delta\eta}k_{3\eta} \\
& +(g^{\alpha\gamma}k_1\cdot k_4 - k_4^\alpha k_1^\gamma)\theta^{\beta\eta}\theta^{\delta\xi}k_{2\eta}k_{3\xi} + (k_2^\alpha k_4^\beta - g^{\alpha\beta}k_2\cdot k_4)\theta^{\gamma\eta}\theta^{\delta\xi}k_{1\eta}k_{3\xi} \\
& +(k_1^\gamma k_2\cdot k_3 - k_2^\gamma k_1\cdot k_3)\theta^{\alpha\beta}\theta^{\delta\eta}k_{4\eta} + \frac{1}{2}(k_3^\beta k_2^\gamma - g^{\beta\gamma}k_2\cdot k_3)\theta^{\alpha\eta}\theta^{\delta\xi}k_{1\eta}k_{4\xi} \\
& +(g^{\beta\gamma}k_1\cdot k_3 - k_3^\beta k_1^\gamma)\theta^{\alpha\eta}\theta^{\delta\xi}k_{2\eta}k_{4\xi} + (g^{\alpha\gamma}k_2\cdot k_3 - k_3^\alpha k_2^\gamma)\theta^{\beta\eta}\theta^{\delta\xi}k_{1\eta}k_{4\xi} \\
& +\frac{1}{2}(k_2^\alpha k_1^\beta - g^{\alpha\beta}k_1\cdot k_2)\theta^{\gamma\eta}\theta^{\delta\xi}k_{3\eta}k_{4\xi} + (k_4^\beta k_3^\delta - g^{\beta\delta}k_3\cdot k_4)\theta^{\alpha\gamma}\theta^{\eta\xi}k_{1\eta}k_{2\xi} \\
& +(g^{\beta\delta}k_4^\gamma - k_4^\beta g^{\gamma\delta})\theta^{\alpha\eta}\theta^{\xi\zeta}k_{3\eta}k_{1\xi}k_{2\zeta} + (g^{\alpha\gamma}k_3^\beta - k_3^\alpha g^{\beta\gamma})\theta^{\delta\eta}\theta^{\xi\zeta}k_{4\eta}k_{1\xi}k_{2\zeta} \\
& +(k_4^\gamma k_2^\delta - g^{\gamma\delta}k_2\cdot k_4)\theta^{\alpha\beta}\theta^{\eta\xi}k_{1\eta}k_{3\xi} + (k_4^\beta g^{\gamma\delta} - g^{\beta\delta}k_4^\gamma)\theta^{\alpha\eta}\theta^{\xi\zeta}k_{2\eta}k_{1\xi}k_{3\zeta} \\
& -(k_2^\alpha k_4^\beta - g^{\alpha\beta}k_2\cdot k_4)\theta^{\gamma\delta}\theta^{\eta\xi}k_{1\eta}k_{3\xi} + (k_2^\alpha g^{\beta\delta} - g^{\alpha\beta}k_2^\delta)\theta^{\gamma\eta}\theta^{\xi\zeta}k_{4\eta}k_{1\xi}k_{3\zeta} \\
& +(k_3^\alpha k_3^\gamma - g^{\alpha\gamma}k_2\cdot k_3)\theta^{\beta\delta}\theta^{\eta\xi}k_{1\eta}k_{4\xi} + (g^{\alpha\beta}k_2^\delta - k_2^\alpha g^{\beta\delta})\theta^{\gamma\eta}\theta^{\xi\zeta}k_{3\eta}k_{1\xi}k_{4\zeta} \\
& +(k_3^\alpha g^{\beta\gamma} - g^{\alpha\gamma}k_3^\beta)\theta^{\delta\eta}\theta^{\xi\zeta}k_{2\eta}k_{1\xi}k_{4\zeta} + (g^{\beta\delta}k_4^\gamma - k_4^\beta g^{\gamma\delta})\theta^{\alpha\eta}\theta^{\xi\zeta}k_{1\eta}k_{2\xi}k_{3\zeta} \\
& +(k_4^\alpha k_1^\gamma - g^{\alpha\gamma}k_1\cdot k_4)\theta^{\beta\delta}\theta^{\eta\xi}k_{2\eta}k_{3\xi} + (g^{\alpha\gamma}k_1^\delta - g^{\alpha\delta}k_1^\gamma)\theta^{\beta\eta}\theta^{\xi\zeta}k_{4\eta}k_{2\xi}k_{2\zeta} \\
& +(k_3^\beta k_1^\gamma - g^{\beta\gamma}k_1\cdot k_3)\theta^{\alpha\delta}\theta^{\eta\xi}k_{2\eta}k_{4\xi} + (g^{\alpha\delta}k_1^\gamma - g^{\alpha\gamma}k_1^\delta)\theta^{\beta\eta}\theta^{\xi\zeta}k_{3\eta}k_{2\xi}k_{4\zeta} \\
& -(k_3^\alpha g^{\beta\gamma} - g^{\alpha\gamma}k_3^\beta)\theta^{\delta\eta}\theta^{\xi\zeta}k_{1\eta}k_{2\xi}k_{4\zeta} - (k_1^\beta k_2^\delta - g^{\beta\delta}k_1\cdot k_2)\theta^{\alpha\gamma}\theta^{\eta\xi}k_{3\eta}k_{4\xi} \\
& +(g^{\alpha\gamma}k_1^\delta - g^{\alpha\delta}k_1^\gamma)\theta^{\beta\eta}\theta^{\xi\zeta}k_{2\eta}k_{3\xi}k_{4\zeta} + (k_2^\alpha g^{\beta\delta} - g^{\alpha\beta}k_2^\delta)\theta^{\gamma\eta}\theta^{\xi\zeta}k_{1\eta}k_{3\xi}k_{4\zeta}.
\end{aligned}
$$