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CP-odd static electromagnetic properties of the W gauge boson and the t quark via the anomalous tbW coupling

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Introducción

- La masa del quark t es del orden de la escala de Fermi.

- El rompimiento espontáneo de la simetría electrodébil puede estar relacionado con la dinámica del quark t

- En un escenario donde el bosón de Higgs no exista

- Los acoplamientos del quark t con los bosones de norma electrodébiles podrían ser muy sensibles a efectos de nueva física

- Una esquema posible es por medio de lagrangianos electrodébiles quirales in which the electroweak symmetry is nonlinearly realized. The resultant Lagrangian is known as the EWCL. In this approach, the Higgs doublet is replaced by a dimensionless matrix field that transforms nonlinearly under the $SU_L(2) \times U_Y(1)$ group [14, 15]:

$$\Sigma = \exp\left(\frac{i\phi^a\sigma^a}{v}\right), \tag{1}$$

where ϕ^a $(a = 1, 2, 3)$ are Goldstone bosons, σ^a are the Pauli matrices, and v is the Fermi scale. Under the $SU_L(2) \times U_Y(1)$ group, Σ transform as:

$$\Sigma' = L\Sigma R^\dagger, \tag{2}$$

where

$$L = \exp\left(\frac{i\alpha^a\sigma^a}{2}\right), \tag{3}$$

$$R = \exp\left(\frac{i\beta\sigma^3}{2}\right), \tag{4}$$

with α^a and β being the parameters of the $SU_L(2)$ and $U_Y(1)$ groups, respectively. From these expressions, it is easy to see that the Goldstone bosons fields transform nonlinearly under the electroweak group. In this scheme, the gauge fields are defined in the following way

$$\hat{W}_\mu = \frac{\sigma^a W^a_\mu}{2i}, \tag{5}$$

$$\hat{B}_\mu = \frac{\sigma^3 B_\mu}{2i}. \tag{6}$$

To define the most general expression for the charged current, it is necessary to introduce some bosonic and fermionic Lorentz structures. We need the following Lorentz tensors:

$$\Sigma_\mu^a = -\frac{i}{2}Tr[\sigma^a \Sigma^\dagger D_\mu \Sigma], \tag{7}$$

$$\Sigma_{\mu\nu}^a = -iTr[\sigma^a \Sigma^\dagger [D_\mu, D_\nu]\Sigma], \tag{8}$$

where

$$D_\mu \Sigma = \partial_\mu \Sigma - g\hat{W}_\mu \Sigma + g' \Sigma \hat{B}_\mu. \tag{9}$$

The charged fields are given by the following relations

$$\Sigma_\mu^\pm = \frac{1}{\sqrt{2}}(\Sigma_\mu^1 \mp i\Sigma_\mu^2), \tag{10}$$

$$\Sigma_{\mu\nu}^\pm = \frac{1}{\sqrt{2}}(\Sigma_{\mu\nu}^1 \mp i\Sigma_{\mu\nu}^2). \tag{11}$$

In the unitary gauge ($\phi^a = 0$), these expressions become $\Sigma_\mu^\pm = g/2\, W_\mu^\pm$ and

In the unitary gauge ($\phi^a = 0$), these expressions become $\Sigma^{\pm}_{\mu} = g/2\, W^{\pm}_{\mu}$ and

$$\Sigma^{\pm}_{\mu\nu} = g[W^{\pm}_{\mu\nu} \pm ie(W^{\pm}_{\mu} A_{\nu} - W^{\pm}_{\nu} A_{\mu}) \pm igc_W(W^{\pm}_{\mu} Z_{\nu} - W^{\pm}_{\nu} Z_{\mu})], \tag{12}$$

where c_W stands for $\cos\theta_W$. We also need the following fermion operators:

$$\Delta_L = \bar{t}P_L b, \quad \Delta^{\mu}_L = \bar{t}\gamma^{\mu} P_L b, \tag{13}$$

$$\Delta_R = \bar{t}P_R b, \quad \Delta^{\mu}_R = \bar{t}\gamma^{\mu} P_R b, \tag{14}$$

$$\Delta^{\mu\nu}_L = \bar{t}\sigma^{\mu\nu} P_L b, \quad \bar{\Delta}^{\mu}_L = i\bar{t}P_L \bar{D}^{\mu} b, \tag{15}$$

$$\Delta^{\mu\nu}_R = \bar{t}\sigma^{\mu\nu} P_R b, \quad \bar{\Delta}^{\mu}_R = i\bar{t}P_R \bar{D}^{\mu} b, \tag{16}$$

where $\bar{D}_{\mu} = \partial_{\mu} - ieQA_{\mu}$ is the electromagnetic covariant derivative, $\sigma_{\mu\nu} = (i/2)[\gamma_{\mu}, \gamma_{\nu}]$, and $P_L(P_R)$ is the left-handed(right-handed) projector.

Using the above expressions, the most general Lagrangian for the charged currents can be written as

$$\begin{aligned}\mathcal{L}^{CC} &= \sqrt{2}a_L\Delta_L^\mu\Sigma_\mu^+ + \sqrt{2}a_R\Delta_R^\mu\Sigma_\mu^+ + \frac{1}{\Lambda}\Big[ib_L\Delta_L\bar{D}^\mu\Sigma_\mu^+ + ib_R\Delta_R\bar{D}^\mu\Sigma_\mu^+ \\ &\quad + c_L\bar{\Delta}_L^\mu\Sigma_\mu^+ + c_R\bar{\Delta}_R^\mu\Sigma_\mu^+ + d_L\Delta_L^{\mu\nu}\Sigma_{\mu\nu}^+ + d_R\Delta_R^{\mu\nu}\Sigma_{\mu\nu}^+\Big] + \text{H.c.},\end{aligned} \tag{17}$$

with Λ being an energy scale. We have introduced the $\sqrt{2}$ factor in order to recover the SM value $(g/\sqrt{2})$ for the left-handed coupling in the appropriate limit.[2] Although this Lagrangian induces the vertices tbW, $tbW\gamma$ and $tbWZ$, we only show the first one:

$$\begin{aligned}\mathcal{L}_{tbW} &= \frac{g}{2}\Big\{\sqrt{2}\bar{t}(a_LP_L + a_RP_R)bW_\mu^+ + \frac{1}{\Lambda}\Big[i\bar{t}(b_LP_L + b_RP_R)b\partial^\mu W_\mu^+ \\ &\quad + i\bar{t}(c_LP_L + c_RP_R)\partial^\mu bW_\mu^+ + 2\bar{t}\sigma^{\mu\nu}(d_LP_L + d_RP_R)bW_{\mu\nu}^+\Big]\Big\} + \text{H.c.}\end{aligned} \tag{18}$$

We will only consider the renormalizable part of this coupling as it is expected to give the dominant contribution to the $WW\gamma$ and $tt\gamma$ vertices. Therefore, all terms proportional to $1/\Lambda$ will be neglected from now on.

A. The W boson EDM and MQM

We now turn to discuss the structure of the $WW\gamma$ vertex and the contribution from the anomalous tbW coupling. The most general on-shell $W_\alpha(p-q)W_\beta(-p-q)A_\mu(2q)$ vertex can be written as

$$\Gamma_{\alpha\beta\mu} = i\,e\Big(g_1\left[2\,p^\mu g^{\alpha\beta} + 4\,(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu})\right] + 2\Delta\kappa(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \frac{4\,\Delta Q}{m_W^2}\,p_\mu q_\alpha q_\beta$$
$$+\ 2\widetilde{\kappa}\,\epsilon_{\alpha\beta\mu\lambda}q^\lambda + \frac{4\,\widetilde{Q}}{m_W^2}\,q_\beta\epsilon_{\alpha\mu\lambda\rho}p^\lambda q^\rho\Big), \tag{19}$$

where all the momenta are incoming. In a renormalizable theory, the form factors $\Delta\kappa$, ΔQ, $\widetilde{\kappa}$, and $\widetilde{Q}$ always arise via radiative corrections. The magnetic (electric) dipole moment μ_W ($\widetilde{\mu}_W$) and the electric (magnetic) quadrupole moment Q_W ($\widetilde{Q}_W$) are given in terms of the electromagnetic form factors as follows

$$\mu_W = \frac{e}{2\,m_W}(2+\Delta\kappa), \tag{20}$$

$$Q_W = -\frac{e}{m_W^2}(1+\Delta\kappa+\Delta Q), \tag{21}$$

$$\widetilde{\mu}_W = \frac{e}{2\,m_W}\widetilde{\kappa}, \tag{22}$$

$$\widetilde{Q}_W = -\frac{e}{m_W^2}(\widetilde{\kappa}+\widetilde{Q}). \tag{23}$$

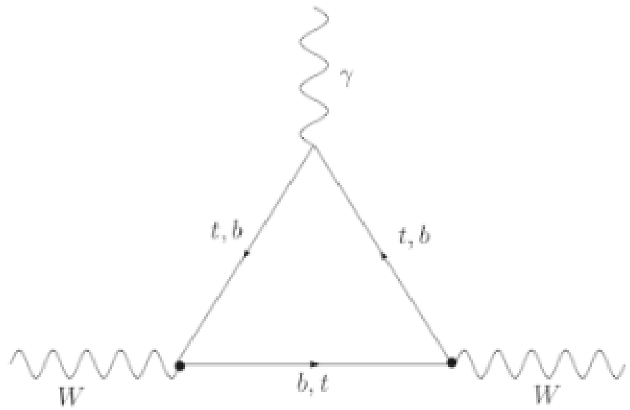

FIG. 1: Feynman diagrams contributing to the on-shell $WW\gamma$ vertex. The dot denotes an anomalous tbW vertex.

The contribution of the tbW coupling arises from the triangle diagrams shown in Fig. 1. We will concentrate only on the CP-odd contribution, although there are also contributions to the CP-even form factors. As already mentioned, at this order of perturbation theory, the renormalizable part of the tbW coupling can only contribute to one CP-odd form factor, namely, $\widetilde{\kappa}$, which reads

$$\widetilde{\kappa} = \frac{\alpha}{8 s_W^2 \pi^2} \left(Q_t F(x_t, x_b) + Q_b F(x_b, x_t) \right) Im \left(a_L a_R^\dagger \right), \tag{24}$$

where $x_a = m_a / m_W$ and

$$F(x,y) = 4\, x\, y \left(\log\left(\frac{x}{y}\right) + \left(1 - x^2 + y^2\right) \frac{f(x,y)}{2\chi(x,y)} \right), \tag{25}$$

with

$$f(x,y) = \log\left(\frac{1 - (x^2 + y^2) - \delta(x,y)}{1 - (x^2 + y^2) + \delta(x,y)} \right), \tag{26}$$

and $\chi^2(x,y) = (x^2 + y^2 - 1)^2 - 4x^2 y^2$. This result is free of ultraviolet divergences, which is a consequence of the fact that only the renormalizable part of the tbW vertex has been considered.

B. The top quark EDM

The most general $tt\gamma$ vertex function is given by

$$\Gamma_\mu = i\,\bar{t}(p_1)\left(\gamma^\mu F_t + i\sigma^{\mu\nu} q_\nu \left(a_t + \gamma^5 d_t\right)\right) t(p_2), \tag{27}$$

where F_t is the electric charge of the top quark, a_t is its MDM, and d_t is its EDM. At tree level $Ft = 2/3e$, whereas a_t and d_t arise via radiative corrections.

The anomalous tbW coupling of Eq. (18) induces an EDM for the top quark through the Feynman diagrams shown in Fig. 2, where all the particles are taken on-shell. After some calculation via the unitary gauge, one can extract the coefficient of the $i\gamma^5\sigma_{\mu\nu}q^\nu$ term from the $\bar{t}t\gamma$ vertex function. Here q_μ is the photon four momentum. This leads to

$$d_t = \frac{N_c\,\alpha}{32\pi}\,\frac{m_b}{m_W}\,\frac{e}{m_W}\,\left(Q_W F_W(x_b, x_W) + Q_b\, F_b(x_b, x_W)\right)\, Im\left(a_L a_R^*\right), \tag{28}$$

with $x_i = m_i/m_t$, $N_c = 3$, $Q_b = -1/3$, and $Q_W = -1$. The F_W and F_b functions stand for the contribution of the Feynman diagram where the photon emerges from the W boson and the b quark line, respectively. They are given by

$$F_W(x_b, x_W) = \left(x_b^2 - 4x_W^2 - 1\right) f_1(x_b, x_W) - \left(x_b^4 + 4x_W^4 - 5x_b^2 x_W^2 - 3x_W^2 - 2x_b^2 + 1\right) f_2(x_b, x_W), \tag{29}$$

$$F_b(x_b, x_W) = \left(x_b^2 - 4x_W^2 - 1\right) f_1(x_W, x_b) + \left(x_b^4 + 4x_W^4 - 5x_b^2 x_W^2 - 3x_W^2 - 2x_b^2 + 1\right) f_2(x_W, x_b), \tag{30}$$

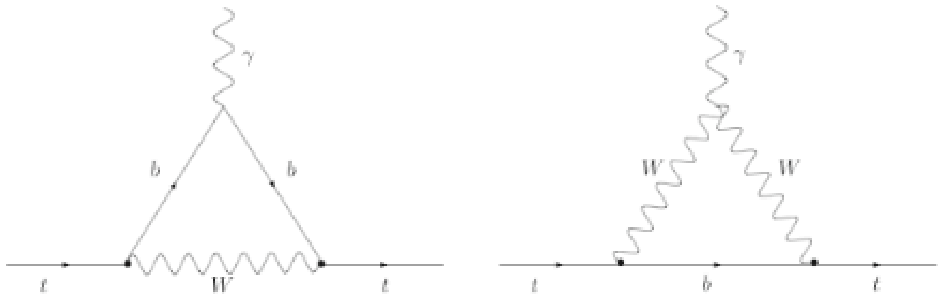

FIG. 2: Feynman diagrams contributing to the on–shell $\bar{t}t\gamma$ vertex.

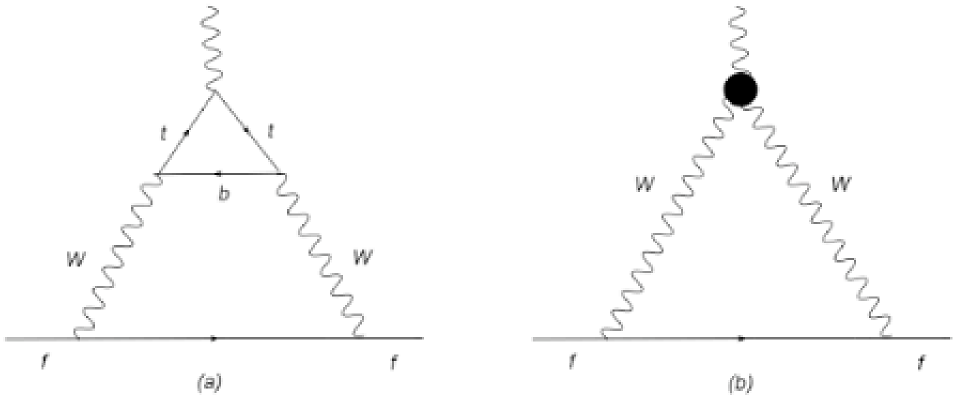

FIG. 3: Feynman diagrams contributing to the EDM of a fermion via the W EDM when (a)the fermions circulating in the $WW\gamma$ loop have a mass of the same order of magnitude than the W boson mass and (b)the internal fermions are much heavier than the W boson, in which case an effective $WW\gamma$ vertex can be used.

with

$$f_1(x,y) = 2 + \left(1 - y^2 + x^2\right)\log\left(\frac{y}{x}\right) + \sqrt{(1-x^2-y^2)^2 - 4x^2y^2}\,\mathrm{sech}^{-1}\left(\frac{2xy}{x^2+y^2-1}\right), \tag{31}$$

$$f_2(x,y) = -\log\left(\frac{y}{x}\right) - \frac{1+x^2-y^2}{\sqrt{(1-x^2-y^2)^2-4x^2y^2}}\,\mathrm{sech}^{-1}\left(\frac{2xy}{x^2+y^2-1}\right).$$

A. Bounds on the anomalous tbW coupling

It is customary to parametrize the left- and right-handed parameters of Eq. (18) in the following way

$$a_L = 1 + \kappa_L e^{i\phi_L}, \quad (32)$$
$$a_R = \kappa_R e^{i\phi_R}, \quad (33)$$

with $\kappa_{L,R}$ and $\phi_{L,R}$ real parameters. It follows that

$$Im(a_L a_R^\dagger) = -\kappa_R \sin\phi_R + \kappa_L \kappa_R \sin(\phi_L - \phi_R). \quad (34)$$

In the above expressions, the SM left-handed coupling was explicitly introduced along with a deviation characterized by the κ_L and ϕ_L parameters. In order to make predictions, we need to assume some values for these parameters. For this purpose, we will consider the bounds reported in the literature, such as the ones obtained in Ref. [17] from B decay processes:

$$\kappa_L \sin\phi_L < 3 \times 10^{-2}, \quad (35)$$
$$\kappa_R \sin\phi_R < 10^{-3}. \quad (36)$$

There are also limits on the right-handed parameters derived from the CLEO Collaboration data on the $b \to s\gamma$ decay [18]:

$$\kappa_R \cos\phi_R < 4 \times 10^{-3}, \quad (37)$$
$$\kappa_R \sin\phi_R < 10^{-3}. \quad (38)$$

In addition, current data on CP-conserving process allows κ_L to be as large as 0.2 [19, 20, 23]. As far as the κ_R parameter is concerned, it seems to be more suppressed than the corresponding left-handed one, as suggested by Eq. (37) and also from the result obtained in Ref. [24], where it was found that $-5 \times 10^{-2} < \kappa_R < 10^{-2}$.

In the following, we will estimate the EDM of the W boson and the top quark using the following values: $\kappa_L \sin\phi_L < 3 \times 10^{-2}$, $\kappa_R \sin\phi_R < 10^{-3}$, and $\kappa_R < 4 \times 10^{-3}$.

V. THE EDM OF THE W BOSON

In the context of renormalizable theories with the simultaneous presence of left- and right-handed fermion currents, the electric dipole and magnetic quadrupole moments are proportional at the one-loop level:

$$\widetilde{Q}_W = -\Big(\frac{2}{m_W}\Big)\widetilde{\mu}_W. \tag{39}$$

This means that $\widetilde{Q}_W$ is suppressed with respect to $\widetilde{\mu}_W$ by a factor of the order of 10^{-16}, provided that units of e and cm are used. However, this hierarchy might not hold at higher orders. Using the known values for the SM parameters, the electric dipole and magnetic quadrupole moments of the W boson can be written as

$$\widetilde{\mu}_W = -4\times 10^{-19} Im(a_L a_R^\dagger) \quad \mathrm{e}\cdot\mathrm{cm}, \tag{40}$$

$$\widetilde{Q}_W = 1.98\times 10^{-34} Im(a_L a_R^\dagger) \quad \mathrm{e}\cdot\mathrm{cm}^2. \tag{41}$$

The constraints of Eqs. (35) and (37) pose two scenarios of interest, for which we get an estimate for the W electric dipole and magnetic quadrupole moments:

- SM-like a_L and complex a_R:

$$\widetilde{\mu}_W = 4\times 10^{-19}\kappa_R \sin\phi_R \ \mathrm{e}\cdot\mathrm{cm} < 4\times 10^{-22} \ \mathrm{e}\cdot\mathrm{cm}, \tag{42}$$

$$\widetilde{Q}_W = -1.98\times 10^{-34}\kappa_R \sin\phi_R \ \mathrm{e}\cdot\mathrm{cm}^2 < -1.98\times 10^{-37} \ \mathrm{e}\cdot\mathrm{cm}^2. \tag{43}$$

A. The top quark EDM

Once Eq. (28) is numerically evaluated, one obtains

$$\begin{aligned} d_t &= (3.08 - 5.73) \times 10^{-19}\, Im\,(a_L a_R^*) \text{ e} \cdot \text{cm}, \\ &= -2.65 \times 10^{-19}\, Im\,(a_L a_R^*) \text{ e} \cdot \text{cm}, \end{aligned} \tag{46}$$

where the positive (negative) contribution corresponds to the the Feynman diagram where the photon emerges from the boson (quark) line. It should be mentioned that d_t develops an imaginary part, which is almost twice larger than the real one. The appearance of an imaginary (absortive) part is not usual in the static electromagnetic properties of light particles, but in this case it arises as a consequence of the fact that, being the top quark so heavy, $m_t > m_W + m_b$.

Bearing in mind the constraints of Eqs. (35) and (37), we will explore the two scenarios discussed above. We obtain in the first scenario

$$|d_t| \lesssim 2.65 \times 10^{-22} \text{ e} \cdot \text{cm}, \tag{47}$$

whereas the latter scenario leads to

$$|d_t| \lesssim 7.95 \times 10^{-23} \text{ e} \cdot \text{cm}. \tag{48}$$

In this case, the constraint $\kappa_R < 10^{-2}$ was used.

We also would like to compare our results with those obtained within the framework of other theories. As already mentioned, in the SM the top quark EDM arises first at three loops and it has been estimated to be of the order of 10^{-30} e·cm [25, 32]. Beyond the SM, the top quark EDM has also received some attention. For instance, in multi–Higgs models, values for d_t lying in the range $10^{-20} - 10^{-21}$ e·cm have been estimated [13]. We can thus conclude that our prediction, which is compatible with the constraints imposed by B meson physics, is about eight orders of magnitude larger than the SM one but one or two orders of magnitude smaller than that obtained in multi–Higgs models.

Conclusions:

The impact of the coupling tbW on the CP-odd electromagnetic properties of the W and the top quark was studied.

The electric dipole (EDM) and magnetic quadrupole moments of the W are 7 and 14 orders of magnitude larger than the SM prediction.

The quark top EDM, 8 orders of magnitude larger than the SM contribution.

Our results are one order of magnitude smaller than theories with multi-Higgs doublets.

The direct measurement of the CP-odd structure of the WWgamma vertex might be in range of sensitivity of NLC o CLIC.