Neutrino Self-Energy In A Magnetized Plasma

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Plan of the talk

The format of the talk is:

- Motivation.
- Some comments on the general form of the neutrino self-energy in a magnetized medium.
- The Feynmann diagrams of the calculation up to one-loop.
- The one-loop self energy expressions.
- Discussion some points about the calculation.
- Showing the results for a charge symmetric plasma.
- Conclusion.

Motivation

The idea of calculating the neutrino self-energy in a medium seeded with a uniform classical magnetic field stems from the fact that:

- most of the astrophysical objects have some magnetic fields associated with them. The neutron star core can sustain magnetic field of the order of $10^{15}\,\mathrm{Gauss}$ and higher magnetic fields are expected in magneters.
- Presence of magnetic field in active galactic nuclei as well as accretion disk of merging objects and progenotors of Gamma Ray Bursts (GRBs)are obvious.
- It has been seen that the presence of magnetic field in the sun can also affect the neutrino propagation and helicity conversion.
- There were many neutrinos in the time of big-bang nucleosynthesis. There may have been some possible magnetic field also at that time.

So it is important to study the combined effect of both matter and magnetic field on neutrino propagation as neutrinos are produced in the core of the supernovas, acrive galactic nuclei and all other possible astrophysical objects.

General form of the neutrino self-energy

In this talk we assume the neutrinos to be Chiral fermions and consequently the self energy of a neutrino in vacuum is of the form,

$$\Sigma(k) = R\,[a\gamma^\mu k_\mu + b]\,\gamma_\mu L\,,$$

the above relation is true for any flavour of the neutrinos. a and b are constants. R and L are the chiral projection operators.

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In a magnetized medium we have 4-vectors u and b where,

- u stands for the 4-velocity of the centre-of-mass of the medium, its form in the rest frame of the medium is:

$$u^{\mu} = (1, 0, 0, 0).$$

- b is the 4-vector designating the magnetic field in the z-direction and its form in the rest frame of the medium is:

$$b^{\mu} = (0, 0, 0, 1).$$

These 4-vectors specify the medium and the magnetic field effects on the self-energy.

General form of the neutrino self-energy

In a magnetized medium the general form of the neutrino self-energy can be written as:

$$\Sigma(k) = R\Big(a_{\|} k_{\|}^{\mu} + a_{\perp} k_{\perp}^{\mu} + b u^{\mu} + c b^{\mu}\Big)\gamma_{\mu} L \,.$$

where $a_{\|}$, $a_{\perp}$, b and c are constants. More over:

$$k_{\|}^{\mu} = (k_0, k_3) \;\;, \;\; k_{\perp}^{\mu} = (k_1, k_2)\,.$$

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where $a_{\parallel}$, $a_{\perp}$, b and c are constants. More over:

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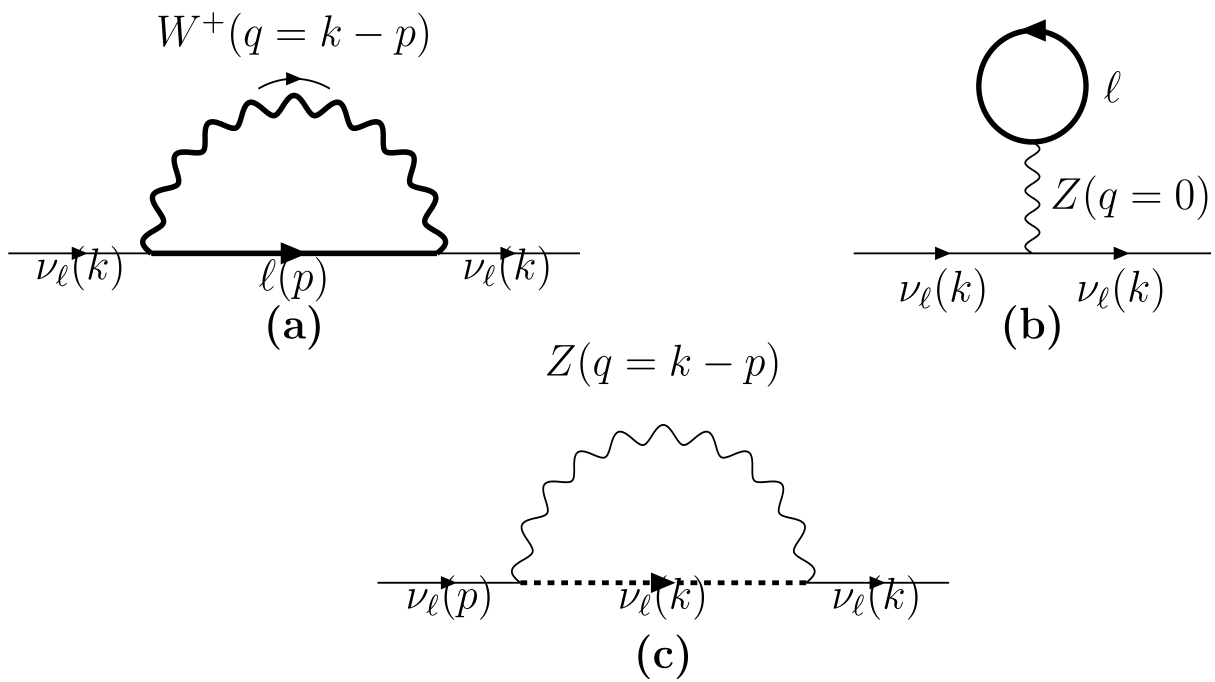
The above self-energy gives the following dispersion relation:

$$(1 - a_{\|})E_{\nu_\ell} = \pm\left[\big((1-a_{\|})k_3 + c\big)^2 + (1-a_{\perp})k_{\perp}^2\right]^{1/2} + b\,,$$

where $k_{\perp}^2 = k_1^2 + k_2^2$.

The Feynman diagrams – One-loop

The neutrino self-energy is calculated in the Unitary Gauge where the unphysical Higgs contribution is not present.



The bold lines in fig. (a) and (b) represent propagators of the charged particles in a magnetic field. And the dashed line in fig. (c) stands for the thermal propagator of the neutrino in the medium.

One-loop self-energy expressions

The contributions of the diagrams are as follows. From fig. (a) and (b) we have:

$$-i\Sigma^W(k) = \int \frac{d^4p}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2}}\right) \gamma_\mu \, L \, iS_\ell(p) \left(\frac{-ig}{\sqrt{2}}\right) \gamma_\nu \, L \, iW^{\mu\nu}(q) \, ,$$

$$-i\Sigma^T(k) = -\left(\frac{g}{2\cos\theta_W}\right)^2 R \, \gamma_\mu \, iZ^{\mu\nu}(0) \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\nu \left(c_V + c_A\gamma_5\right) iS_\ell(p)\right] \, ,$$

and fig. (c),

$$-i\Sigma^Z(k) = \int \frac{d^4p}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2}\cos\theta_W}\right) \gamma_\mu \, L \, iS_{\nu_\ell}(p) \left(\frac{-ig}{\sqrt{2}\cos\theta_W}\right) \gamma_\nu \, L \, iZ^{\mu\nu}(q) \, .$$

where g is the $SU(2)$ coupling constant, $\cos\theta_W$ is the Weinberg angle. The quantities c_V and c_A are the couplings which come in the neutral-current interaction of various particles with the Z boson.

$S_\ell(p)$ is lepton propagator in a magnetized plasma, $S_{\nu_\ell}(p)$ is the neutrino propagator in a medium, $W^{\mu\nu}(q)$ is the W boson propagator in a magnetic field and $Z^{\mu\nu}(q)$ is the Z boson propagator.

Some points about the calculation

In the present calculation it is assumed:

- the neutrinos are moving in a medium composed of charged leptons, nucleons and possible other neutrinos in thermal and chemical equilibrium. The W and the Z bosons are not in thermal equilibrium with the other particles, they only appear as virtual states in the one-loop diagram of the self-energy.
- Due to thermal equilibrium all the constituents in the plasma share the same temperature T and due to chemical equilibrium the chemical potentials of the particles are negative of the chemical potential of the anti-particles.
- The magnetic field strength is much smaller compared to the critical field strength($\sim 10^{20}\,\mathrm{Gauss}$) of the W bosons. Consequently only linear order corrections, with respect to the magnetic fields, are included in the W-propagators.
- The electron propagator gets all order contributions from the magnetic fields.
- Due to the presence of the magnetic field the energy of the electrons ceases to be continuous. The transvers (to the field direction) kinetic energy of the electrons becomes Landau quantized. But this phenomenon does not happen for the heavy W bosons.

Some points about the calculation

The energy of the leptons in presence of the magnetic field is,

$$E_{\ell,\,n} = \sqrt{m_\ell^2 + p_3^2 + \mathcal{H}}\,, \text{ where } \mathcal{H} = e\mathcal{B}(2n + 1 - \lambda)\,.$$

and,

- $\mathcal{B}$ is the magnitude of the magnetic field,
- n is the Landau level number, which is a positive integer including zero.
- λ are numbers designating the spin of the leptons and takes values $\lambda = \pm 1$

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With the Landau levels the distribution function of various charged particles in the plasma becomes modified to:

$$f_\ell = \frac{1}{e^{\beta(E_{\ell,\,n} - \mu_\ell)} + 1}\,,\ \bar{f}_\ell = \frac{1}{e^{\beta(E_{\ell,\,n} + \mu_\ell)} + 1}\,,$$

where β, μ_ℓ are the temperature and chemical potentials. The number densities of uncharged neutrinos, N_{ν_ℓ}, and their energies, k_0, remains unmodified.

The results for a charge symmetric plasma

For a charge symmetric plasma the coefficients in the self-energy comes out as:

$$
\begin{aligned}
b &= \frac{4g^2 k_0}{3M_W^2 M_Z^2} \langle E_{\nu_\ell^{\rm B}} \rangle N_{\nu_\ell} \\
&- \frac{2e\mathcal{B}g^2}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=\pm 1} \left[\frac{k_3}{E_{\ell,\, n}} \left(p_3^2 + \frac{m_\ell^2}{2} \right) \delta_{\lambda,1}^{n,0} + k_0 E_{\ell,\, n} \right] f_\ell \,, \\
c &= -\frac{2e\mathcal{B}g^2}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=\pm 1} \left[k_0 \left(E_{\ell,\, n} - \frac{m_\ell^2}{E_{\ell,\, n}} \right) \delta_{\lambda,1}^{n,0} + \frac{k_3 p_3^2}{E_{\ell,\, n}} \right] f_\ell \,. \\
a_\perp &= -\frac{2g^2 e\mathcal{B}}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=\pm 1} \left(\frac{\mathcal{H}}{2E_{\ell,\, n}} + \frac{m_\ell^2}{E_{\ell,\, n}} \right) f_\ell + \frac{g^2}{3M_W^4} \langle E_{\nu_\ell^{\rm B}} \rangle N_{\nu_\ell} \,, \\
a_\parallel &= -\frac{2g^2 e\mathcal{B}}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=\pm 1} \frac{m_\ell^2}{E_{\ell,\, n}} f_\ell + \frac{g^2}{3M_W^4} \langle E_{\nu_\ell^{\rm B}} \rangle N_{\nu_\ell} \,.
\end{aligned}
$$

Here $m_\ell,\, M_W,\, M_Z$ are the lepton, $W,\, Z$ boson masses and $\langle E_{\nu_\ell^{\rm B}} \rangle$ is the average thermal energy of the background neutrinos. The integrals are over the third component of the loop momenta

It is important to note that all the coefficients are of the order of M_W^{-4}.

The neutrino dispersion relation

To order of g^2 the neutrino dispersion relation is:

$$E_{\nu_\ell} = |\mathbf{k}| - c\cos\theta + (a_{||} - a_{\perp})|\mathbf{k}|\sin^2\theta + b\,,$$

where $k^3 = k_z = |\mathbf{k}|\cos\theta$

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magnetized medium the effective-potential acting on the neutrinos is of the form,

$$V_{\rm eff} = b - c\cos\theta + (a_{||} - a_{\perp})|\mathbf{k}|\sin^2\theta\,.$$

With the form of the effective potential the problem of neutrino oscillations in the **CP** symmetric magnetized plasma in the early universe can be tackled.

Conclusion

In this work we calculated the neutrino self-energy in a magnetized plasma,

- in the unitary gauge,
- using the fully modified electron propagators and slightly modified charged gauge boson propagator.
- The magnetic field is assumed to be smaller than the critical field corresponding to the W-boson mass.
- The resultant dispersion relation to order g^2 is seen to be proportional to M_W^{-4}
- The result is important for neutrino oscillation studies of the early universe or inside Gamma Ray Bursts, where it is expected that the plasma is magnetized and more over to a great extent charge-symmetric