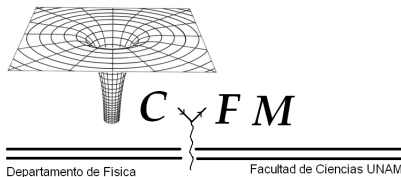


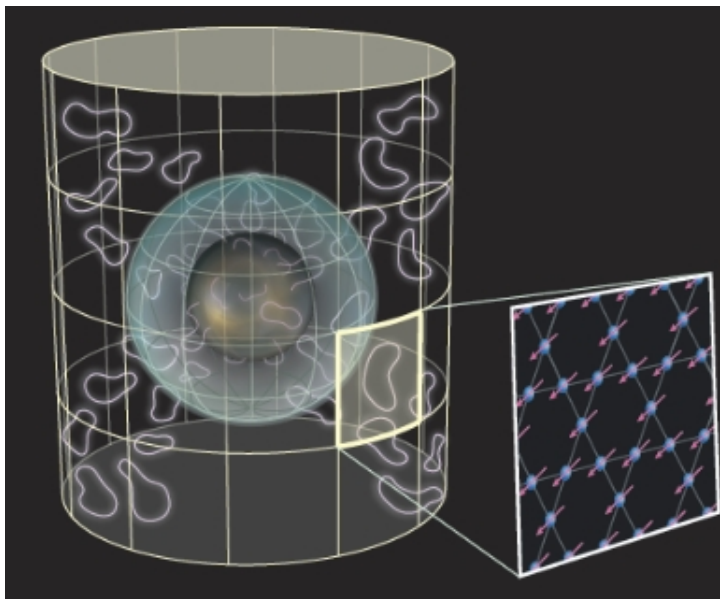
# Renormalización holográfica y mezcla de operadores

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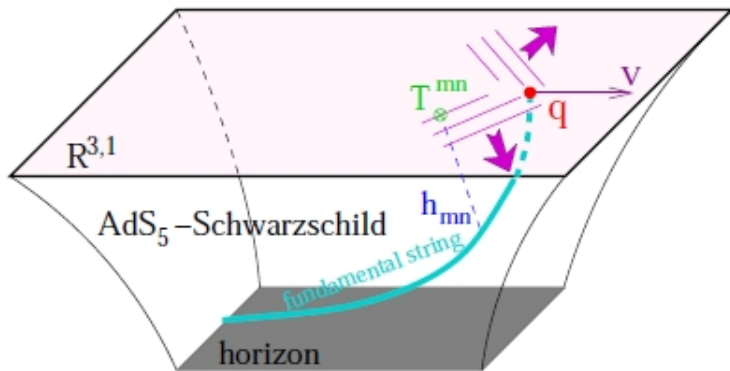
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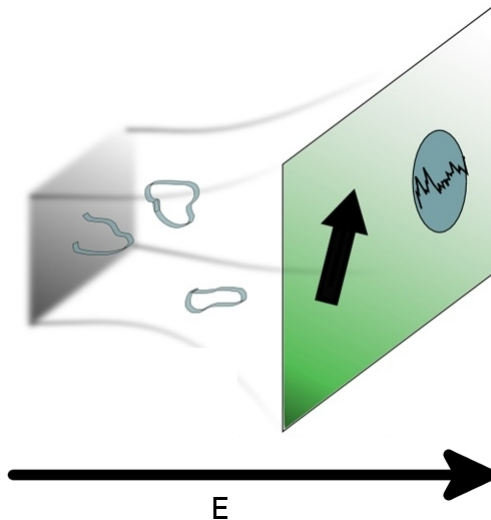
# About the gauge/gravity correspondence



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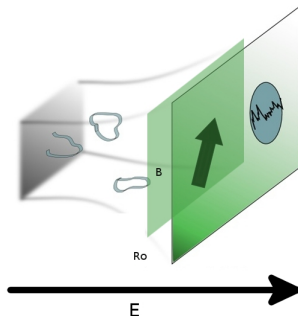
# Holographic renormalization/Operator Mixing

The procedure that regularizes the gravity action turns out to be exactly what is necessary to renormalize the field theory results.

All the aspects of renormalization are recovered, including in particular things like the operator mixing with the renormalization flow.

# Holographic renormalization/Operator Mixing

$$S = \int_0^{R_0} \mathcal{L} + \int_B CT$$



$\partial_{R_0} S = 0$  is the Callan-Symanzik equation!

# Not quite QCD

The matter content of the theories we will be working with consists of both, fundamental and adjoint fields..

The former include  $N_f$  flavors of fermions  $\Psi^a$  and scalars  $\Phi^a$ ,  $a = 1, \dots, N_f$ , to which we will refer indistinctly as 'quarks'.

To be as close as possible to QCD, we will couple only the fundamental fields to electromagnetism and set the electric charge of the adjoint fields to zero.

We do this by replacing the  $SU(N_c)$ -covariant derivative  $D_\mu$  by

$$\mathcal{D}_\mu = D_\mu - ie\mathcal{A}_\mu$$

when acting on the fundamental fields, and adding a kinetic term for the photon.

We obtain a  $SU(N_c) \times U(1)_{\text{em}}$  gauge theory whose Lagrangian is

$$\mathcal{L} = \mathcal{L}_{SU(N_c)} - \frac{1}{4} \mathcal{F}_{\mu\nu}^2 + e \mathcal{A}^\mu J_\mu^{\text{EM}},$$

with  $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ , and the electromagnetic current given by

$$J_\mu^{\text{em}} = \bar{\Psi} \gamma_\mu \Psi + \frac{i}{2} \Phi^* \mathcal{D}_\mu \Phi - \frac{i}{2} (\mathcal{D}_\mu \Phi)^* \Phi.$$



# Not quite QCD

In this theory the photon production is given by the expression

$$\frac{d\Gamma}{d\vec{k}} = \frac{e^2}{(2\pi)^3 2|\vec{k}|} n_B(k^0) \sum_{s=1,2} \epsilon_{(s)}^\mu(\vec{k}) \epsilon_{(s)}^\nu(\vec{k}) \chi_{\mu\nu}(k) \Big|_{k^0=|\vec{k}|},$$

with  $\chi_{\mu\nu}(k) = -2 \operatorname{Im} G_{\mu\nu}^R(k)$  the spectral density, and

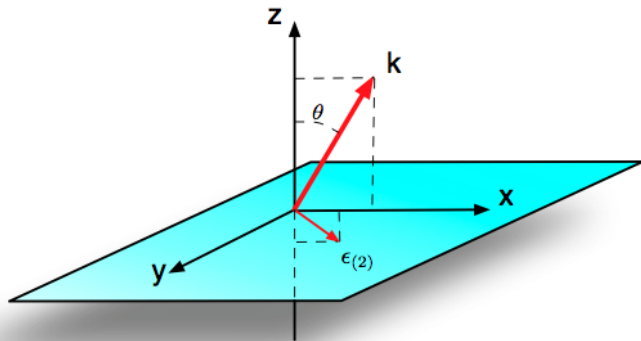
$$G_{\mu\nu}^R(k) = -i \int d^4x e^{-ik \cdot x} \Theta(t) \langle [J_\mu^{EM}(x), J_\nu^{EM}(0)] \rangle$$

the retarded correlator of two electro-magnetic currents.

Each term of the sum above stands for the number of photons emitted with polarization vectors  $\vec{\epsilon}_{(s)}$ .

# Not quite QCD

These two vectors are orthogonal to  $\vec{k}$ , which we can choose to lie in the  $\hat{x}\hat{z}$ -plane



$$\vec{k} = k_0(\sin \theta, 0, \cos \theta).$$

So we are free to choose

$$\vec{\epsilon}_{(1)} = (0, 1, 0),$$

and

$$\vec{\epsilon}_{(2)} = (\cos \theta, 0, -\sin \theta).$$

# Not quite QCD

The production of photons with polarization  $\vec{\epsilon}_{(1)}$  is then proportional to

$$\chi_{yy} \sim \text{Im} \langle J_y^{EM} J_y^{EM} \rangle,$$

whereas for those with polarization  $\vec{\epsilon}_{(2)}$  it is proportional to

$$\epsilon_{(2)}^\mu \epsilon_{(2)}^\nu \chi_{\mu\nu} = \cos^2 \theta \chi_{xx} + \sin^2 \theta \chi_{zz} - 2 \cos \theta \sin \theta \chi_{xz}.$$

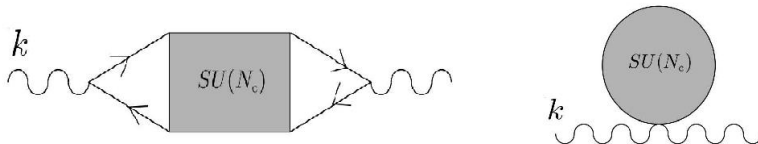
expression from which we see that we need to compute the three correlators

$$\chi_{xx} \sim \text{Im} \langle J_x^{EM} J_x^{EM} \rangle, \quad \chi_{zz} \sim \text{Im} \langle J_z^{EM} J_z^{EM} \rangle, \quad \chi_{xz} \sim \text{Im} \langle J_x^{EM} J_z^{EM} \rangle$$

# Not quite QCD

At first sight it would seem like we need to compute this correlators in the full  $SU(N_c) \times U(1)_{\text{em}}$  gauge theory.

It turns out that to leading order in  $e$ , only the diagrams



contribute, so the correlator can be computed on the  $SU(N_c)$  theory.

# Field/Operator correspondence

To motivate the field/operator correspondence, I would like to point out that, in the context of the gauge/gravity correspondence, the coupling constant in SYM is related to the coupling constant in string theory as  $g_{YM}^2 \propto g_s$ , and in due course  $g_s$  corresponds to the value  $e^{\phi_\infty}$ .

In general we could think of deforming the action by adding terms of the form

$$S \rightarrow S + \int d^4x \phi(x) \mathcal{O}(x),$$

where  $\varphi(x)$  is a source and  $\mathcal{O}$  a gauge invariant operator.

We could therefore expect/hope that any source in the field theory corresponded to the boundary value of a string theory field in  $AdS$ , i.e.,  $\phi(x) = \Phi(r, x)|_{r \rightarrow \infty}$ .

The anisotropic plasma can be modeled holographically by the following background metric in string frame

$$ds^2 = \frac{1}{u^2} \left( -\mathcal{B}\mathcal{F} dt^2 + dx^2 + dy^2 + \mathcal{H}dz^2 + \frac{du^2}{\mathcal{F}} \right) + e^{\phi/2} d\Omega_5^2,$$

with  $\mathcal{H} = e^{-\phi}$  and  $d\Omega_5^2 = d\vartheta^2 + \cos^2 \vartheta d\varphi^2 + \sin^2 \vartheta d\Omega_3^2$ .

Besides the metric and the dilaton we have

$$F_5 = 4 (\Omega_5 + \star \Omega_5) , \quad F_1 = a dz ,$$

where  $a$  parametrizes the degree of anisotropy.

We will introduce  $N_f$  flavours of massless quarks by embedding  $N_f$  equatorial D7-branes.

For any temperature different from zero the quarks will be in a deconfined phase.

The induced metric over the D7's will be the original metric with a three-sphere instead of a five-sphere.



# Back to the source

Global symmetries of the gauge theory are in one-to-one correspondence with gauge symmetries on the gravity side.

Conserved currents of the gauge theory are dual to gauge fields on the gravity side.

Let  $A_m$  ( $m = 0, \dots, 7$ ) be the gauge field associated to the overall  $U(1)$  gauge symmetry on the D7-branes. Upon dimensional reduction on the 3-sphere wrapped by the D7-branes,  $A_m$  gives rise to a massless gauge field  $\{A_\mu, A_u\}$ , three massless scalars, and a tower of massive Kaluza-Klein (KK) modes.

# Back to the source

All these fields propagate on the five non-compact dimensions of the D7-branes.

We will work in the gauge  $A_u = 0$ , and consistently set to zero the scalars and the higher KK modes.

The gauge field  $A_\mu$  is the desired dual to the conserved electromagnetic current  $J_\mu^{\text{em}}$  of the gauge theory.

Correlation functions of  $J_\mu^{\text{em}}$  can be calculated by varying the string partition function with respect to the value of  $A_\mu$  at the boundary of the spacetime.

We may proceed by assuming the equatorial embedding of the D7-branes to be fixed without considering the back reaction due to  $A_\mu$  and then solve for the gauge field on that embedding.

Because of the particular orientation of the branes they do not couple to the WZ term.

There could be a coupling to the background  $F_1 = dC_0$

$$\int_{D7} P[C_0] \wedge e^{2\pi\alpha' F},$$

but this would be quartic in the  $U(1)$  field strength  $F = dA$ .

It is enough to consider the Dirac-Born-Infeld action:

$$S = -N_f T_{D7} \int_{D7} d^8x e^{-\phi} \sqrt{-\det(g + 2\pi\ell_s^2 F)},$$

and given that we only need to compute two point correlators, we can keep up to quadratic order in  $F$

$$S = -N_f T_{D7} \int_{D7} d^8x e^{-\phi} \sqrt{-\det g} \left( \frac{(2\pi\ell_s^2)^2}{4} F^2 \right).$$

In the particular case at hand the action reduces to

$$S = \tilde{N}_{D7} \int dt d\vec{x} du \frac{e^{-3\phi/4}}{u^5} F^2,$$

where  $\tilde{N}_{D7} = -2\pi^4 N_f T_{D7} \ell_s^4$ .

# Computing $A_\mu$

We Fourier-decompose the components of the gauge field as

$$A_\mu(t, \vec{x}, u) = \int \frac{dk^0 d\vec{k}}{(2\pi)^4} e^{-ik^0 t + i\vec{k} \cdot \vec{x}} A_\mu(k^0, \vec{k}, u).$$

end their equations of motion are

$$\partial_u (M g^{uu} g^{yy} \partial_u A_y) - M g^{yy} (g^{tt} k_0^2 + g^{xx} k_x^2 + g^{zz} k_z^2) A_y = 0$$

for  $A_y$ , and for the remaining modes as the coupled system

$$\begin{aligned} \partial_u (M g^{uu} g^{tt} \partial_u A_t) - M g^{tt} [g^{xx} k_x (k_x A_t - k_0 A_x) + g^{zz} k_z (k_z A_t - k_0 A_z)] &= 0, \\ \partial_u (M g^{uu} g^{xx} \partial_u A_x) - M g^{xx} [g^{tt} k_0 (k_0 A_x - k_x A_t) + g^{zz} k_z (k_z A_x - k_x A_z)] &= 0, \\ \partial_u (M g^{uu} g^{zz} \partial_u A_z) - M g^{zz} [g^{tt} k_0 (k_0 A_z - k_z A_t) + g^{xx} k_x (k_x A_z - k_z A_x)] &= 0, \end{aligned}$$

$$\text{where } M \equiv \frac{e^{-\frac{3}{4}\phi} \sqrt{B}}{u^5}.$$

# Holographic correlators

We need to write the on-shell action as a boundary term and vary this with respect to the values of  $A_\mu$  at  $u = 0$ .

The relevant quantities will be the limit  $\epsilon \rightarrow 0$  of the expressions derived from the surface action at  $u = \epsilon$

$$S_\epsilon = 2\tilde{\mathcal{N}}_{\text{D7}} \int_{\partial\mathcal{M}_\epsilon^5} dt d\vec{x} \frac{e^{-\frac{3}{4}\phi} \sqrt{\mathcal{BF}}}{\epsilon} \left[ -\frac{1}{\mathcal{BF}} A_t A'_t + A_x A'_x + A_y A'_y + e^\phi A_z A'_z \right]$$

where  $\partial\mathcal{M}_\epsilon^5$  is the hypersurface at  $u = \epsilon$  and primes denote derivatives with respect to  $u$ .

# The $\chi_{yy}$ spectral function

The correlator  $\chi_{yy}$  can be done independently of all the others given that it e.o.m. is not coupled to the remaining ones there is no mixing term in the boundary action.

We can use the expression

$$G_{yy}^R(\omega) = -\tilde{\mathcal{N}}_{D7} \lim_{u \rightarrow 0} \frac{2Q(u)A_y^*(\omega, u)\partial_u A_y(\omega, u)}{A_y^*(\omega, 0)A_y(\omega, 0)},$$

where

$$Q(u) = \frac{e^{-3\phi/4}\sqrt{B}\mathcal{F}}{u},$$

$\omega \equiv k^0/2\pi T$  and  $A_y(\omega, u)$  is a solution of the equation of motion obeying the incoming-wave boundary condition at the horizon.

# Picking the ingoing solution

Now we need to numerically integrate, so we perturbatively solve the equation of motion close to the horizon.

Near the horizon  $A_y$  behaves like  $(u - u_h)^\beta (u + u_h)^\alpha \tilde{A}_y$ .

The zeroth order shows that

$$\beta = \pm i \frac{4e^{\phi_h/2} k_0 u_h}{a^{2/7} (16 + e^{7\phi_h/2} u_h^2) \sqrt{\mathcal{B}_h}}.$$

To impose the ingoing wave condition we choose the negative sign for  $\beta$  and for latter convenience we let

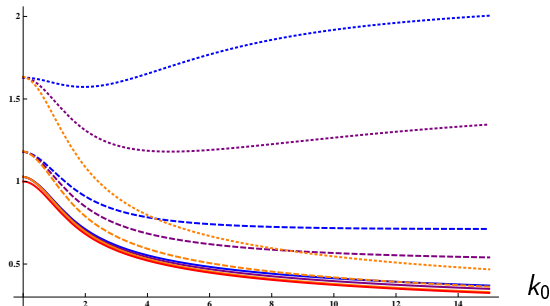
$$\alpha = - \frac{4e^{\phi_h/2} k_0 u_h}{a^{2/7} (16 + e^{7\phi_h/2} u_h^2) \sqrt{\mathcal{B}_h}}.$$



## Results for $\chi_{yy}$

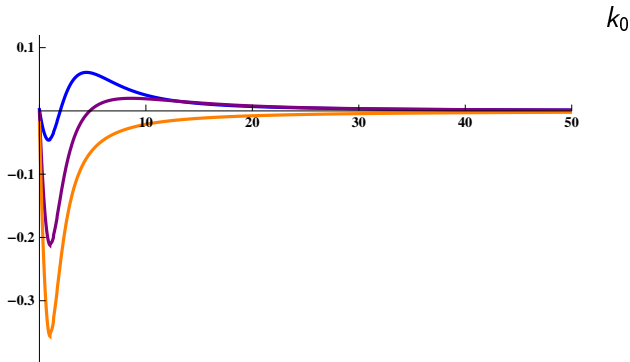
Now we carry out the numerical integration for a number of values of the anisotropy parameter  $a/T$  and of the angle  $\theta$ .

The results are depicted in the following figure



**Figure:** Plots of the spectral function  $\chi_{yy}/2\tilde{N}_{D7}k_0$ . Here  $k_0 = \omega/2\pi T$ . Blue, purple, and orange correspond respectively to the angles  $\theta = 0, \pi/4, \pi/2$ . Continuous, dashed, and dotted correspond to  $a/T = 1.38, 4.41, 12.2$ . The solid red line at the bottom corresponds to

# Results for $\chi_{yy}$



**Figure:** Plot of the derivative  $\frac{\partial(\chi_{yy}/\tilde{N}_{D7} k_0)}{\partial k_0}$  for the case of large anisotropy,  $a/T = 12.2$ . The color code is as in the previous graph (blue, purple, and orange correspond respectively to  $\theta = 0, \pi/4, \pi/2$ ).

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

The procedure to find the remaining correlators is clearer if it is done in terms of the gauge invariant fields  $E_i \equiv \partial_i A_0 - \partial_0 A_i$ . With the aid of the constraint

$$-\frac{1}{\mathcal{BF}} A'_t + \sin \theta A'_x + \cos \theta e^\phi A'_z = 0,$$

the equations become

$$E''_x + \left[ \partial_u \left( \log \frac{e^{-3\phi/4} \sqrt{\mathcal{BF}}}{u} \right) + \frac{k_x^2}{\bar{k}^2} \partial_u (\log \mathcal{BF}) \right] E'_x + \frac{\bar{k}^2}{\mathcal{F}} E_x + \frac{e^\phi k_x k_z}{\bar{k}^2} \partial_u (\log \mathcal{BF}) E'_z = 0,$$

$$E''_z + \left[ \partial_u \left( \log \frac{e^{\phi/4} \sqrt{\mathcal{BF}}}{u} \right) + \frac{e^\phi k_z^2}{\bar{k}^2} \partial_u (\log e^\phi \mathcal{BF}) \right] E'_z + \frac{\bar{k}^2}{\mathcal{F}} E_z + \frac{k_x k_z}{\bar{k}^2} \partial_u (\log e^\phi \mathcal{BF}) E'_x = 0,$$

where  $\bar{k}^2 \equiv \frac{k_0^2}{\mathcal{FB}} - k_x^2 - e^\phi k_z^2$ .

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

The action at  $u = \epsilon$  can be written in terms of these fields as

$$s_\epsilon = \frac{2\tilde{\mathcal{N}}_{D7}}{\epsilon} \int_{\partial\mathcal{M}_\epsilon^5} dt d\vec{x} \frac{e^{-\frac{3}{4}\phi} \sqrt{\mathcal{BF}}}{k_0^2 \bar{k}^2} \left[ \left( \frac{k_0^2}{\mathcal{BF}} - e^\phi k_z^2 \right) E_x E'_x + \left( \frac{k_0^2}{\mathcal{BF}} - k_x^2 \right) e^\phi E_z E'_z \right. \\ \left. + e^\phi k_x k_z (E_x E'_z + E_z E'_x) + \bar{k}^2 E_y E'_y \right].$$

We will be interested in the behaviour of this action close to the boundary.

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

We use the expansion of the metric functions around  $u = 0$  to solve perturbatively and find

$$\begin{aligned}E_x &= E_x^{(0)} + E_x^{(2)} \cos \theta u^2 - \frac{1}{24} \left( \frac{3}{4} E_x^{(0)} k_0^2 \cos \theta + 5 E_x^{(2)} \right) \cos \theta a^2 u^4 + O(u^6) , \\E_z &= E_z^{(0)} - E_x^{(2)} \sin \theta u^2 + E_z^{(4)} u^4 - \frac{\cos^2 \theta a^2 k_0^2}{16} \left( E_x^{(0)} \tan \theta + E_z^{(0)} \right) u^4 \log u + O(u^6) ,\end{aligned}$$

and...

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

$$S_\epsilon = 2\tilde{N}_{D7} \int_{\partial\mathcal{M}_\epsilon^5} dt d\vec{x} [\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \dots + O(u^2)],$$

where

$$\begin{aligned}\mathcal{L}_1 &= -\frac{3}{4} \sin^2 \theta E_x^{(0)2} - \frac{1}{4} \cos^2 \theta E_z^{(0)2} - \cos \theta \sin \theta E_x^{(0)} E_z^{(0)}, \\ \mathcal{L}_2 &= \frac{1}{3k_0^2} \left[ \frac{1 + 5 \cos 2\theta}{\cos \theta} E_x^{(0)} E_x^{(2)} + \frac{48}{a^2} \tan \theta E_x^{(0)} E_z^{(4)} - 10 \sin \theta E_z^{(0)} E_x^{(2)} + \frac{48}{a^2} E_z^{(0)} E_z^{(4)} \right], \\ \mathcal{L}_3 &= -\left( E_x^{(0)} \sin \theta + E_z^{(0)} \cos \theta \right)^2 \log u.\end{aligned}$$

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

We notice that  $\mathcal{L}_3$  diverges as we take the  $\epsilon \rightarrow 0$  limit, which implies that the boundary action is sensitive to the anomaly of the background.

From what we have seen, the contribution of this divergent term to the production of photons with polarization  $\vec{\epsilon}_{(2)}$  is proportional to

$$\cos^2 \theta \frac{\delta^2 \mathcal{L}_3}{\delta E_x^{(0)2}} + \sin^2 \theta \frac{\delta^2 \mathcal{L}_3}{\delta E_z^{(0)2}} - 2 \cos \theta \sin \theta \frac{\delta^2 \mathcal{L}_3}{\delta E_x^{(0)} \delta E_z^{(0)}} ,$$

which vanishes identically.

This means that we only need to consider the contribution coming from  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , which is indeed finite.

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

This turns out to be

$$\frac{G_{xx}^R}{\tilde{\mathcal{N}}_{D7}} = -\frac{3}{4}k_0^2 \sin^2 \theta + \frac{1 + 5 \cos 2\theta}{3 \cos \theta} \frac{\delta E_x^{(2)}}{\delta E_x^{(0)}} + \frac{16 \tan \theta}{a^2} \frac{\delta E_z^{(4)}}{\delta E_x^{(0)}},$$

$$\frac{G_{zz}^R}{\tilde{\mathcal{N}}_{D7}} = -\frac{1}{4}k_0^2 \cos^2 \theta - \frac{10 \sin \theta}{3} \frac{\delta E_x^{(2)}}{\delta E_z^{(0)}} + \frac{16}{a^2} \frac{\delta E_z^{(4)}}{\delta E_z^{(0)}},$$

$$\begin{aligned} \frac{G_{xz}^R + G_{zx}^R}{\tilde{\mathcal{N}}_{D7}} &= -k_0^2 \cos \theta \sin \theta + \frac{1 + 5 \cos 2\theta}{3 \cos \theta} \frac{\delta E_x^{(2)}}{\delta E_z^{(0)}} \\ &\quad + \frac{16 \tan \theta}{a^2} \frac{\delta E_z^{(4)}}{\delta E_z^{(0)}} - \frac{10 \sin \theta}{3} \frac{\delta E_x^{(2)}}{\delta E_x^{(0)}} + \frac{16}{a^2} \frac{\delta E_z^{(4)}}{\delta E_x^{(0)}}. \end{aligned}$$



# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

But not to despair, because if we sum them as we know we should we get

$$\epsilon_{(2)}^{\mu} \epsilon_{(2)}^{\nu} \chi_{\mu\nu} = 4\tilde{\mathcal{N}}_{D7} \text{Im} \left( \cos \theta \frac{\delta E_x^{(2)}}{\delta E_x^{(0)}} - \sin \theta \frac{\delta E_x^{(2)}}{\delta E_z^{(0)}} \right),$$

which is the quantity we wanted to compute.

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

Let's write the solutions  $E_x$  and  $E_z$  as a vector

$$\mathbf{E} = \begin{pmatrix} E_x(u) \\ E_z(u) \end{pmatrix}.$$

and suppose I can pick the two specific solutions that at the boundary reach the values

$$\lim_{u \rightarrow 0} \mathbf{E}_1 \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \lim_{u \rightarrow 0} \mathbf{E}_2 \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

In terms of this base the general ingoing solution can be written as

$$\mathbf{E} = E_x^{(0)} \mathbf{E}_1 + E_z^{(0)} \mathbf{E}_2,$$

which is precisely suitable for doing variations with respect to  $E_x^{(0)}$  and  $E_z^{(0)}$ .

For the general solution the coefficient  $E_x^{(2)}$  that we are interested in is given by

$$E_x^{(2)} = E_x^{(0)} E_{x,1}^{(2)} + E_z^{(0)} E_{x,2}^{(2)}.$$

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ spectral functions

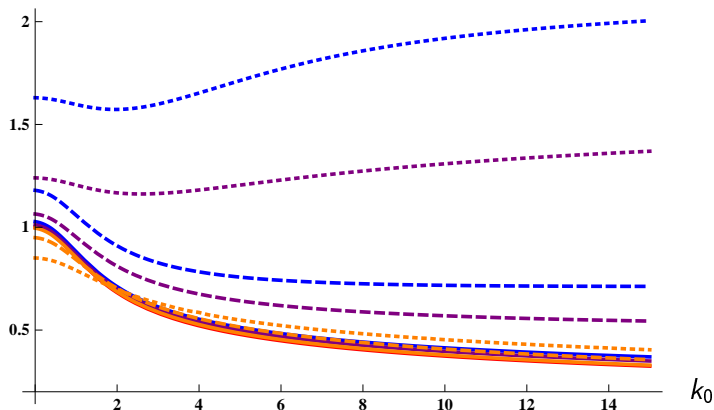
Using any base  $\mathbf{E}_a$  and  $\mathbf{E}_b$  we can construct  $\mathbf{E}_1$  and  $\mathbf{E}_2$  as the linear combinations

$$\mathbf{E}_1 = \frac{E_{z,b}^{(0)} \mathbf{E}_a - E_{z,a}^{(0)} \mathbf{E}_b}{E_{x,a}^{(0)} E_{z,b}^{(0)} - E_{x,b}^{(0)} E_{z,a}^{(0)}}, \quad \mathbf{E}_2 = \frac{E_{x,a}^{(0)} \mathbf{E}_b - E_{x,b}^{(0)} \mathbf{E}_a}{E_{x,a}^{(0)} E_{z,b}^{(0)} - E_{x,b}^{(0)} E_{z,a}^{(0)}},$$

expressions from which follows that

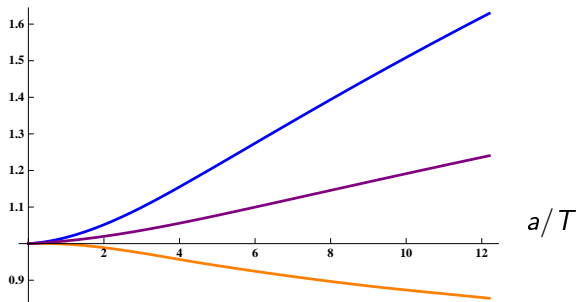
$$E_{x,1}^{(2)} = \frac{E_{z,b}^{(0)} E_{x,a}^{(2)} - E_{z,a}^{(0)} E_{x,b}^{(2)}}{E_{x,a}^{(0)} E_{z,b}^{(0)} - E_{x,b}^{(0)} E_{z,a}^{(0)}}, \quad E_{x,2}^{(2)} = \frac{E_{x,a}^{(0)} E_{x,b}^{(2)} - E_{x,b}^{(0)} E_{x,a}^{(2)}}{E_{x,a}^{(0)} E_{z,b}^{(0)} - E_{x,b}^{(0)} E_{z,a}^{(0)}}.$$

# The $\chi_{xx}$ , $\chi_{zz}$ , and $\chi_{xz}$ results



**Figure:** Plots of the correlator  $\chi_{\text{mixed}} \equiv \epsilon_{(2)}^\mu \epsilon_{(2)}^\nu \chi_{\mu\nu}$ . Color code as above (blue, purple, and orange correspond respectively to  $\theta = 0, \pi/4, \pi/2$ , while continuous, dashed, and dotted correspond to  $a/T = 1.38, 4.41, 12.2$ ). We have checked that the  $\theta = 0$  curves (blue ones) are identical to the  $\theta = 0$  curves for the  $\chi_{yy}$  correlator, as it should

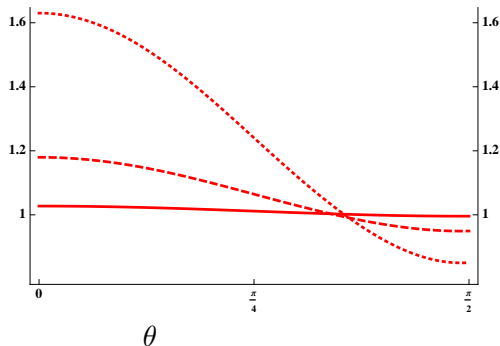
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**Figure:** Plot of the conductivity  $\sigma_{\epsilon(2)}$  as a function of  $a/T$ . As above, blue, purple, and orange correspond, respectively, to  $\theta = 0, \pi/4, \pi/2$ .

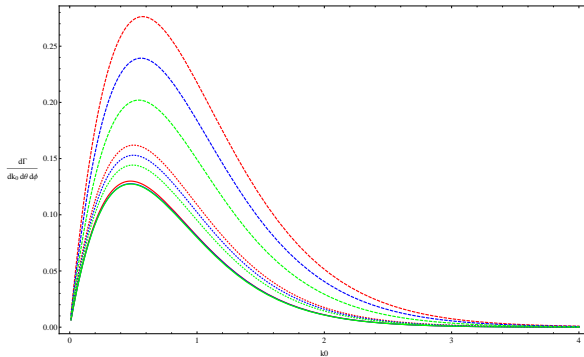
# Photon production

xed



**Figure:** Plot of the conductivity  $\sigma_{\epsilon(2)}$  as a function of  $\theta$ . As above, continuous, dashed and dotted curves correspond, respectively, to  $a/T = 1.38, 4.41, 12.2$ . Note that there is (seems to be?) an angle ( $\theta \simeq 1.25$ ) at which the conductivity is  $a/T$ -independent.

# Photon production



**Figure:** Plot of the differential production of photons per unit of time and volume against frequency. The solid, dotted and dashed lines stand for  $a/T = 1.38, 4.41, 12.2$  respectively. The colors red, blue and green correspond to angles between  $\mathbf{k}$  and the direction  $\hat{z}$  of  $\pi/80, \pi/4$  and  $39\pi/80$  respectively.