Phase diagram of hot QCD in an external magnetic field

possible splitting of deconfining and chiral transition

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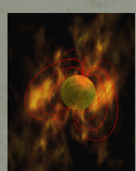
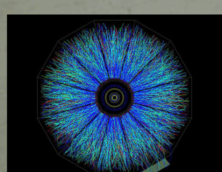
Motivation

Strong interactions under strong magnetic fields can be found in nature in:

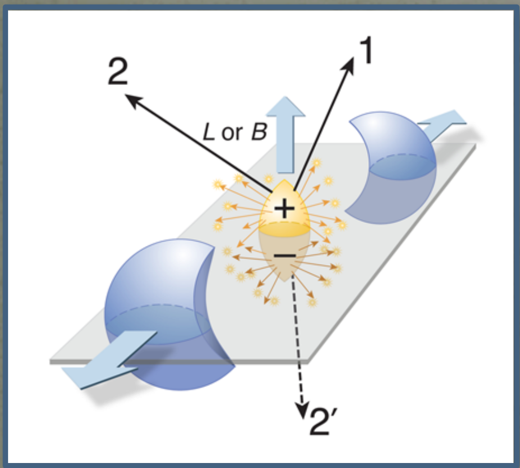
- Magnetars
- The early universe
- Non-central heavy ion collisions

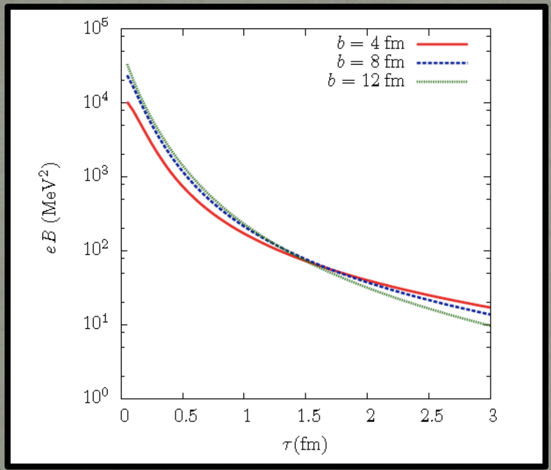
	The earth's magnetic field	0.6 Gauss
	A common hand-held magnet	100 Gauss
	The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
	Surface field of magnetars	10^{15} Gauss
	Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory	10^{18} Gauss

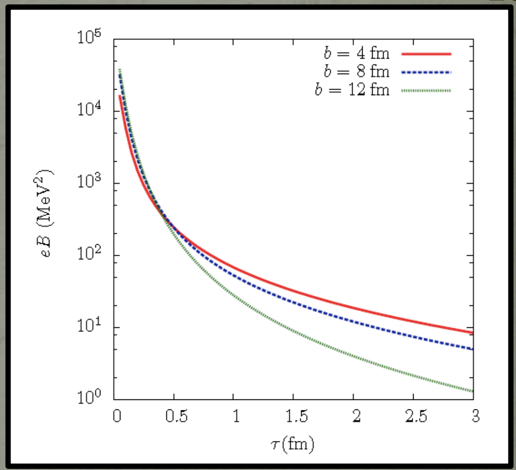
If topological charge is there how to see it?

THE CHIRAL MAGNETIC EFFECT

[Kharzeev, McLerran & Warringa, 2008]







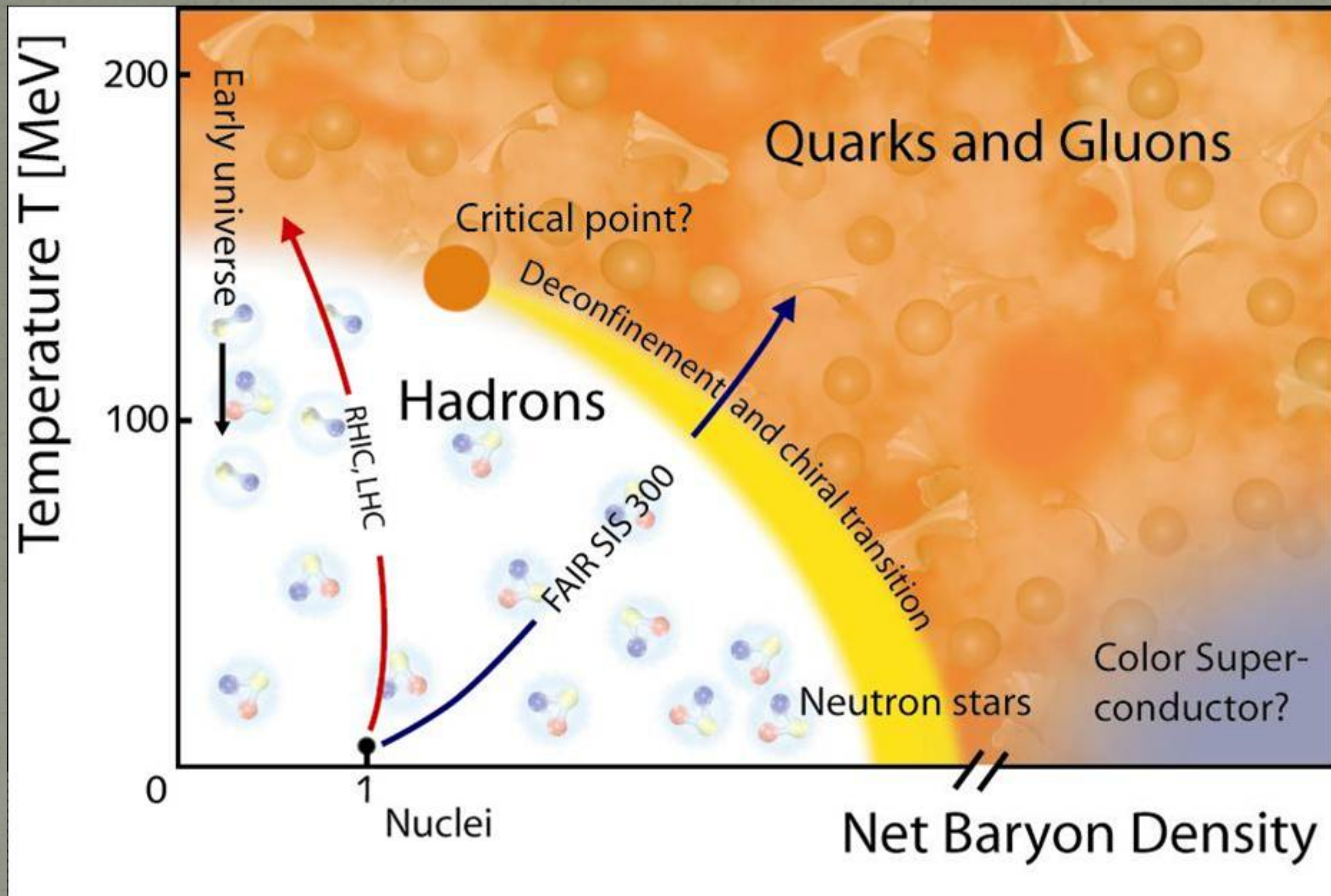
Au-Au, 62 GeV

Au-Au, 200 GeV

non-trivial gauge field configurations

strong magnetic field

Charge separation

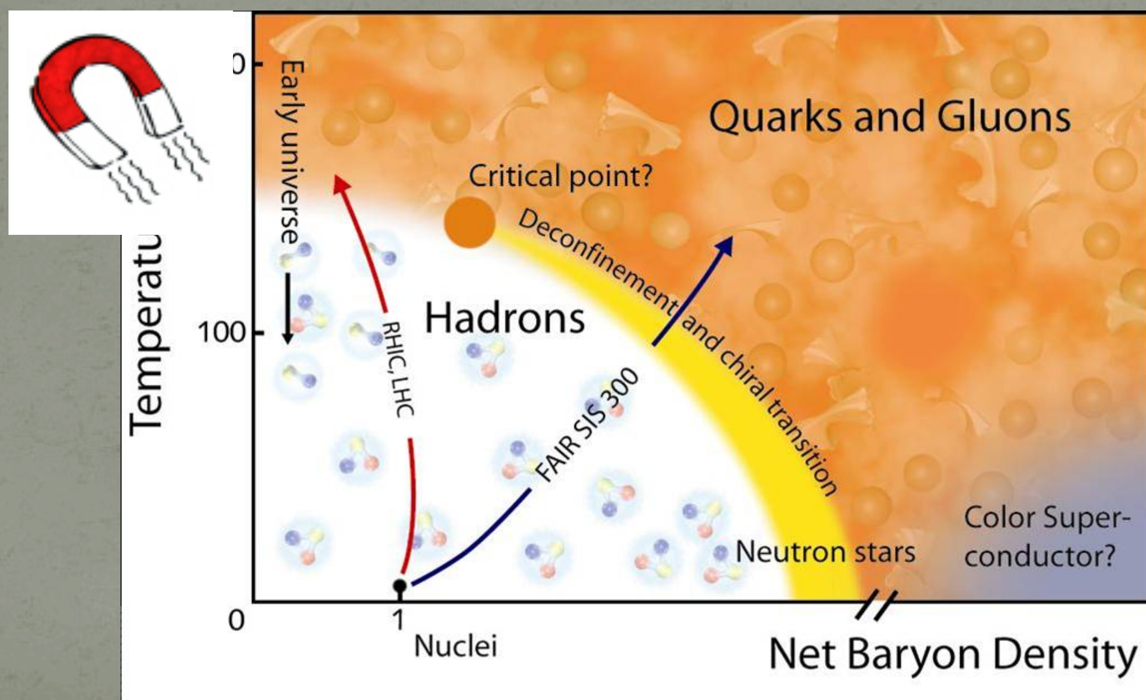

Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
FAIR SIS 300
Quarks and Gluons
Critical point?
Hadrons
Deconfinement and chiral transition
Neutron stars
Color Super-
conductor?

• How does the QCD diagram look like including another external control parameter, the magnetic field B?

• Are there modifications in the nature of the phase transition?

• How do the chiral and deconfinement transitions react to this magnetic field?

• How is the interplay between this two transitions?



Other approaches (most concerned about vacuum effects):

NJL:

- Gusynin, Miransky & Shovkovy (1994/1995)
- Klimenko et al. (1998-2008)
- Boer & Boomsma (2009)
- Ruggieri & Gatto (2010-2011)

LSM:

- Ayala (2009)
- Andersen (2010)

χPT:

- Shushpanov & Smilga (1997)
- Agasian & Shushpanov (2000)
- Cohen, McGady & Werbos (2007)
- Agasian & Fedorov (2008)
- Andersen (2011-2012)

Large-N QCD:

- Miransky & Shovkovy (2002)

Sakai-Sugimoto:

- Dudal & Callebaut (2011)

Lattice:

- D'Elia et al (2010)
- Chernodub et al (2010-2011)

Step by step

- Effect of a magnetic background on the chiral transition
  - Linear sigma model at finite temperature
  - How to introduce the magnetic field?

- Effective theory for the chiral and deconfining transitions:
  - The linear sigma model coupled to quarks and the Polyakov loop

- Incorporating an external magnetic field

- Free energy at one loop

- Phase structure

Effective theory

[AJM, M. Chernodub & E.S.Fraga (2010)]

A. Degrees of freedom and approximate order parameters

Chiral field: $\phi = (\sigma, \vec{\pi})\,, \qquad \vec{\pi} = (\pi^+, \pi^0, \pi^-)$

Quark spinors: $$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Polyakov loop: $$L(x) = \frac{1}{3}\mathrm{Tr}\,\Phi(x)\,, \quad \Phi = \mathcal{P}\exp\Big[i\int_0^{1/T} \mathrm{d}\tau A_4(\vec{x},\tau)\Big]$$

$$\text{Chiral symmetry}: \begin{cases} \langle\sigma\rangle \neq 0 \,, & \text{low } T \\ \langle\sigma\rangle = 0 \,, & \text{high } T \end{cases}$$

$$\text{Confinement}: \begin{cases} \langle L\rangle = 0 \,, & \text{low } T \\ \langle L\rangle \neq 0 \,, & \text{high } T \end{cases}$$

Confinement: the Polyakov loop

The Polyakov loop is related to the free energy of an infinitely heavy test quark

$$\langle L \rangle = exp\left(-F_{teste}/T\right)$$

McLerran & Svetitsky (1981)

In case of deconfinement the Lagrangian remains Z(N) invariant, but the Polyakov loop is not: spontaneous symmetry breaking.

$$\langle L \rangle = e^{2\pi i j / N} L_0$$

Confinement implies restaured Z(N): $\langle L \rangle = 0$

High temperature = low coupling constant : $\langle L \rangle \rightarrow 1$

$$l_0 = 0, \quad T < T_c \quad ; \quad l_0 > 0, \quad T > T_c$$

Polyakov loop models

- R. Pisarki, A Dumitru, O. Scavenius, J. Lenaghan, A. Jackson (2001-2006): Polyakov loop model

- R. Pisarki, A Dumitru, D. Zschiesche: matrix model

- K. Fukushima (2004-today): Haar measure of SU(3)

- C. Ratti et al (2004-today): Polyakov loop model / Fukushima model

Models fit lattice data

Bernd-Jochen Schaefer, Mathias Wagner, Jochen Wambac (2009): which model is in the best agreement with lattice -> Ratti.

D. Confining potential

$$\frac{V_L(L,T)}{T^4} = -\frac{1}{2}a(T)\,L^*L + b(T)\,\ln\left[1 - 6\,L^*L + 4\left(L^{*3} + L^3\right) - 3\,(L^*L)^2\right]$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2,$$

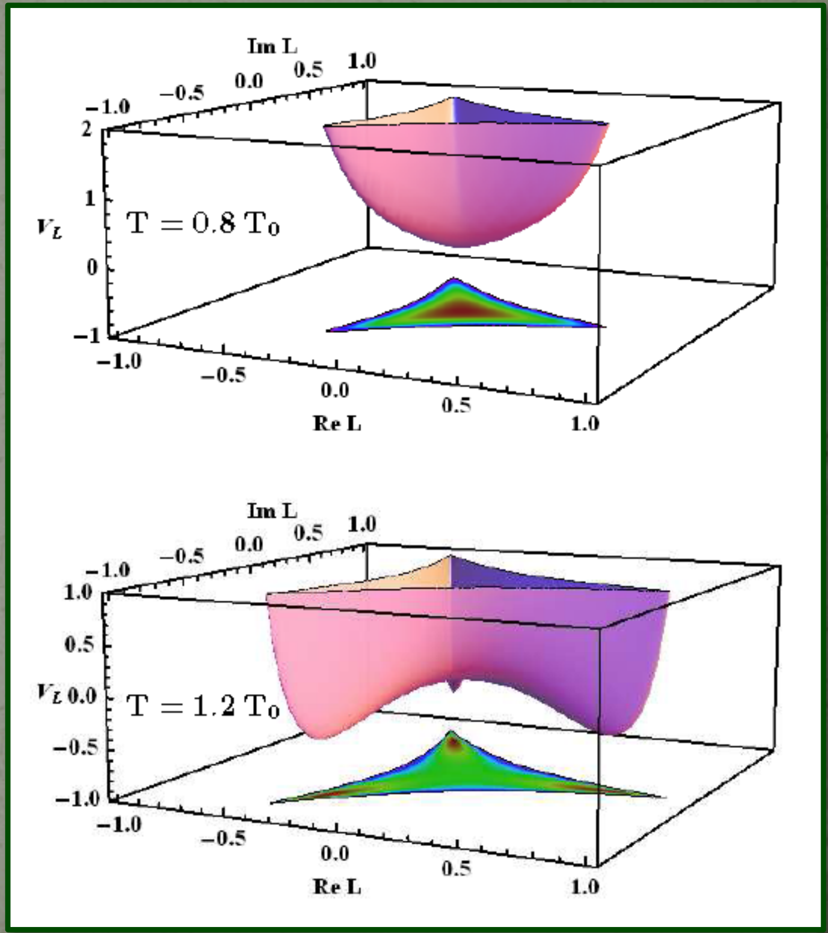
$$b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

$$\mathcal{L}_L = -V_L(L,T)$$

Parameters fix demanding:

[Roessner et al. (2008)]

- Stefan-Boltzmann limit reached at T -> ∞
- first order transition happens at $T=T_0$
- the potential fits lattice data for thermodynamical quantities (pressure, energy density and entropy)



Effective theory for the chiral transition (LσM)

[Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

- Symmetry: for massless QCD, the action is invariant under $SU(N_f)_L \times SU(N_f)_R$
- "Fast" degrees of freedom: quarks
  "Slow" degrees of freedom: mesons
- Typical energy scale: hundred of MeV
- We choose $SU(N_f=2)$, for simplicity: we have pions and the sigma
- $SU(2) \otimes SU(2)$ spontaneously broken in the vacuum
- Also accommodates explicit breaking by massive quarks

B. Chiral Lagrangian

$$L = \overline{\psi}\left[i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\gamma^{5}\vec{\tau}\cdot\vec{\pi})\right]\psi + \frac{1}{2}\left[\partial_{\mu}\sigma\,\partial^{\mu}\sigma + \partial_{\mu}\vec{\pi}\cdot\partial^{\mu}\vec{\pi}\right] - V\left(\sigma,\vec{\pi}\right)$$

Quark kinetic term

Fermion-meson interaction

Mesons kinetic term

$$V\left(\sigma,\pi\right) = \frac{\lambda}{4}\left(\left(\pi^{2}+\sigma^{2}\right) - v^{2}\right)^{2} - h\sigma$$

Spontaneously symmetry breaking

Explicit symmetry breaking

The effective potential

[E.S.Fraga & AJM, PRD78,025016 (2008); NPA 820, 103C (2009)]

Mean field treatment with the following assumptions:

Quarks constitute a thermalized gas that acts as a thermal bath in which the chiral fields evolve.

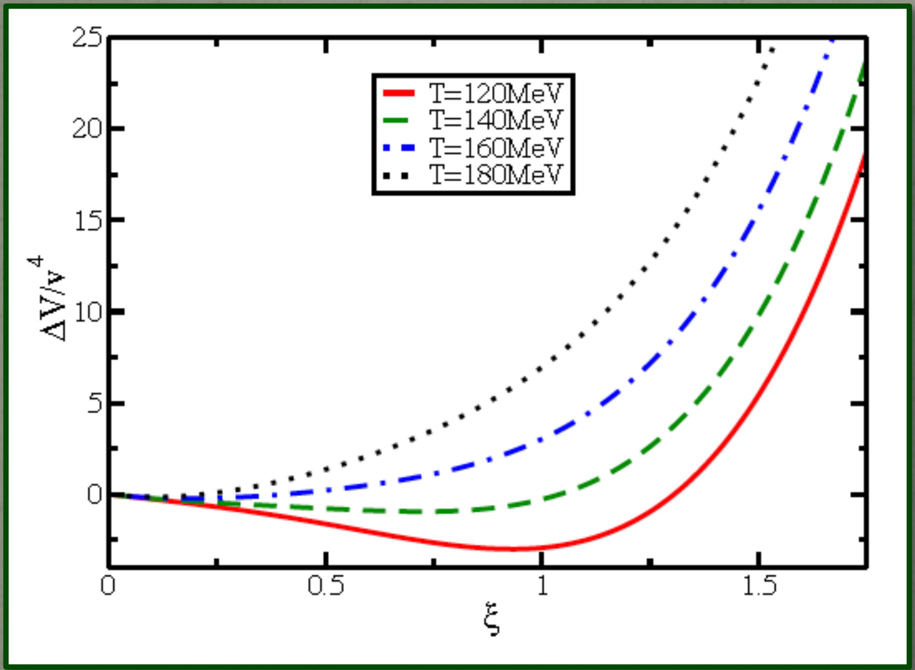
T=0 (vacuum: broken χ symmetry; reproduces LσM and χPT)

- Heavy σ meson ($M_\sigma \sim 600$ MeV), treated classically.
- $SU(2)_L \otimes SU(2)_R$ is spontaneously broken in the vacuum, with $\langle\sigma\rangle = f_\pi$, $\langle\pi\rangle = 0$
- h should be related to the nonzero pion mass
- f_π = 93 MeV is the pion decay constant, determined experimentally. It comes about when one computes the weak decay of the pion, which is proportional to the amplitude
- Quark degrees of freedom: its presence in the vacuum brings remarkable consequences.

The fermions provide a thermal bath for the long wavelength chiral fields. Integrating over quarks:

$$\Omega(T,\mu,\phi) = V(\phi) - \frac{T}{\nu} \ln \det \left[\frac{\left(G_E^{-1} + M(\phi)\right)}{T} \right]$$

effective potential



Effective potential for the chiral field σ for different values of the temperature. For low temperature the expected value of σ is non-zero. As the temperature increases it approaches to zero restoring the chiral symmetry.

[Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

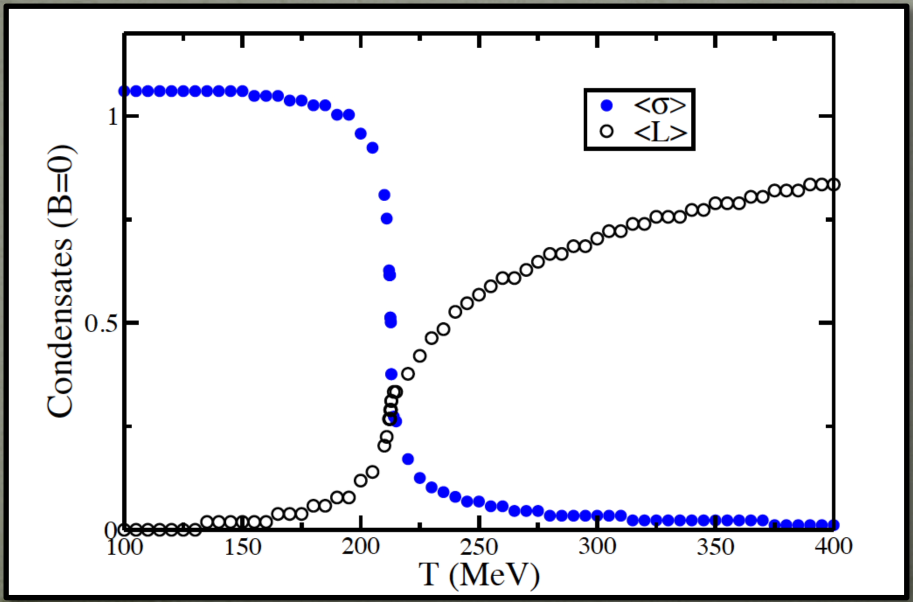
$\xi = \sigma / v$ (v used as mass scale)

The interaction with the Polyakov loop is implemented via the field A_μ in the covariant derivative

$$\mathcal{L}_q = \bar{\psi}\left[i\not{D} - g(\sigma + i\gamma_5\vec{\tau}\cdot\pi)\right]\psi$$

$$\begin{cases} \not{D} = \gamma^\mu D_\mu^{(q)} \\ D_\mu^{(q)} = \partial_\mu - iA_\mu \end{cases}$$

Interaction between the mesons and the Polyakov loop only via quarks: minimal coupling



- Crossover for both transitions.
- With no magnetic field the critical temperature is the same.

Including an external magnetic field

[E.S.Fraga & AJM, 2008]

For simplicity we assume a magnetic field that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{Gauge choice}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

Inserted via gauge field in the covariant derivative. For systems containing only chiral fields:

- charged mesons (new dispersion relations):

$$(\partial^2 + m^2)\phi = 0$$
$$\partial_\mu \to \partial_\mu + iqA_\mu$$

$$\Rightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

Landau levels:

$$\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B$$

$$\omega_B \equiv \frac{|q|B}{m}$$

- quarks (new dispersion relations):

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$
$$\partial_\mu \to \partial_\mu + iqA_\mu$$

$$\Rightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1-2s)|q|B$$

For the quarks interacting with the gauge filed:

$$D_\mu^{(q)} = \partial_\mu - iQa_\mu - iA_\mu$$

Abelian: magnetic field

Non-Abelian: Polyakov loop

Integrating over fermions

Integration length:

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \sum_l \int \frac{d^3k}{(2\pi)^3}$$

Finite temperature

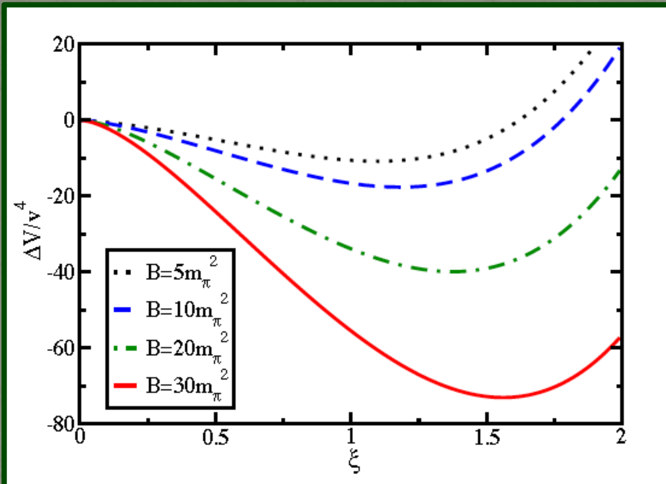
$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi} \sum_{n=0}^{\infty} \int \frac{dk_0}{2\pi} \frac{dk_z}{2\pi}$$

External magnetic field

l: Matsubara index
n: Landau level index

$$T \sum_\ell \int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi} \sum_\ell \sum_{n=0}^{\infty} \int \frac{dk_z}{2\pi}$$

Magnetic catalysis

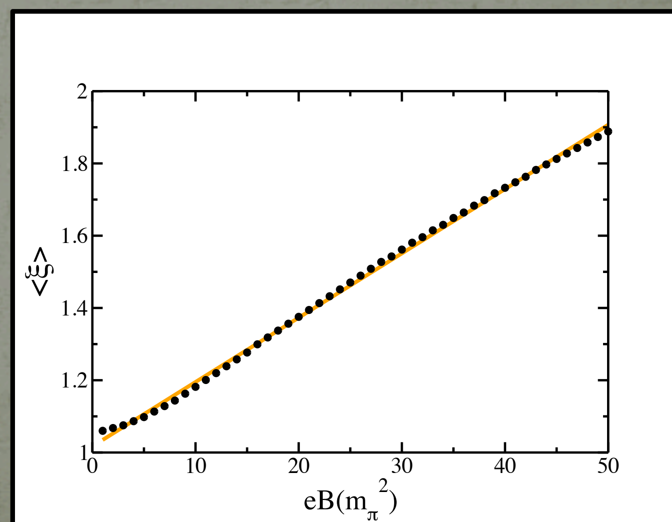


In the vacuum the value of the condensate increases as the magnitude of the magnetic field is increased.

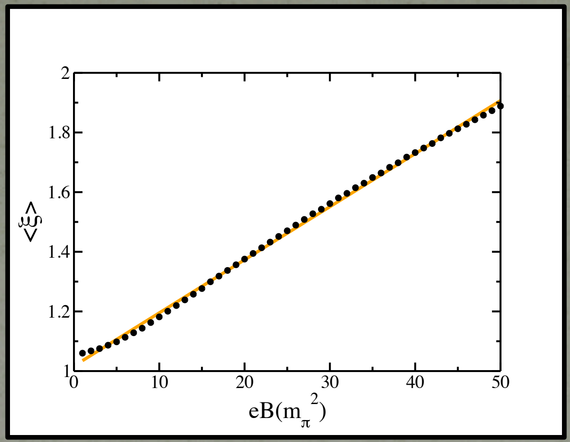
Reinforcement of the chiral symmetry breaking

Dependence of the condensate on the field: approximately linear and quadratic for small B

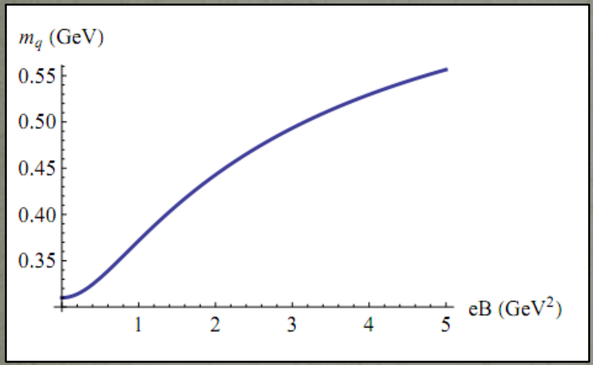
AJM, E. Fraga, M. Chernodub (2011)



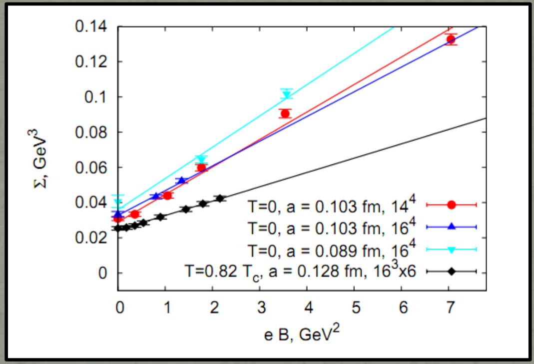
Other results for the condensate dependence on B



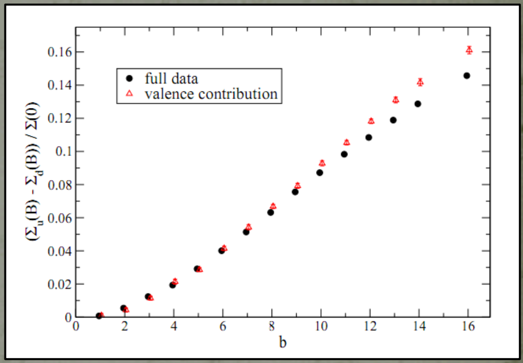
AJM, E. Fraga, M. Chernodub (2011)



N. Callebault and D. Dudal (2011): Sakai-Sugimoto (AdS/CFT)



P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya, M. I. Polikarpov (2010): lattice



M. D'Elia, F. Negro (2011): lattice

Paramagnetically-induced breaking of Z(3)

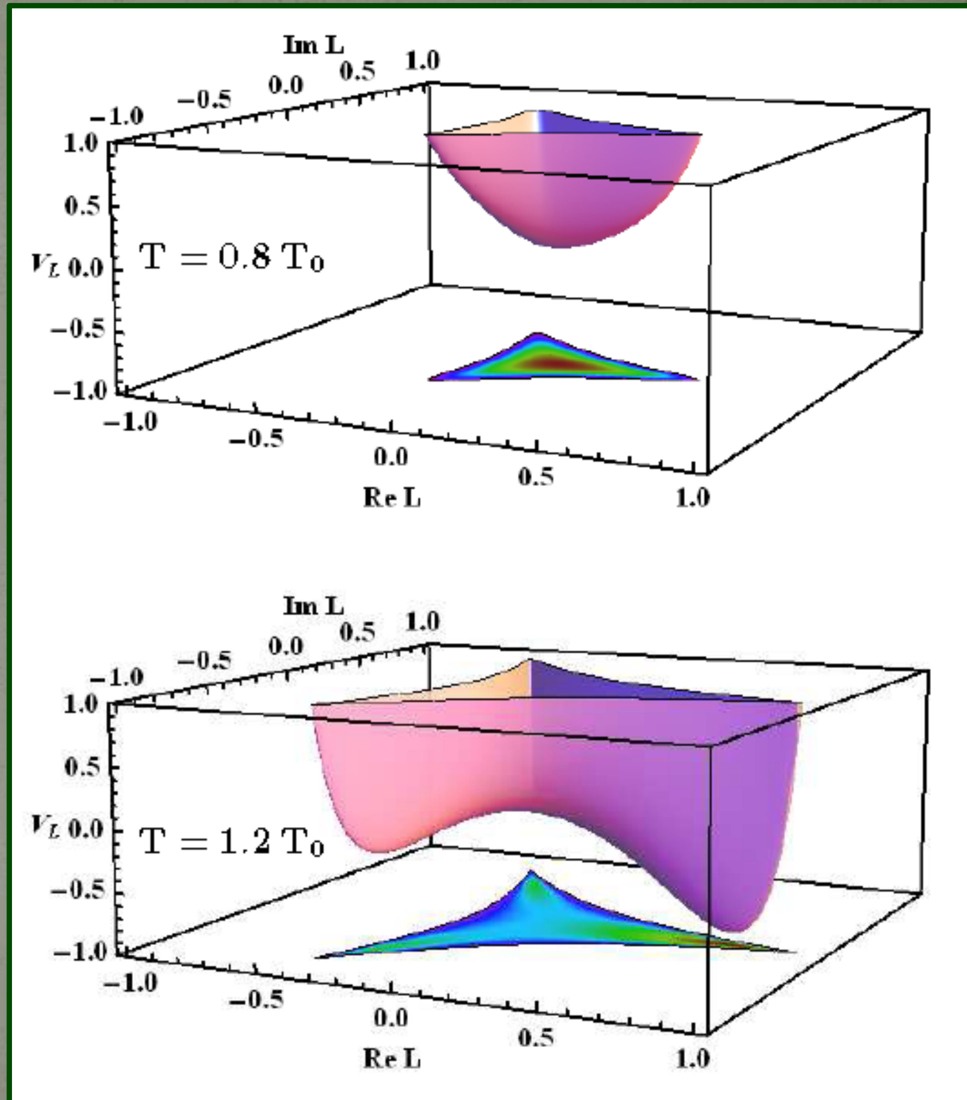
[AJM, Chernodub & E.S.Fraga (2010)]

The magnetic field drastically affects the potential for the Polyakov loop. For very large fields $|q|B \gg m_q^2$:

$$\Omega_q^{\rm para} = -3\frac{g\sigma|q|BT}{\pi^2}K_1\left(\frac{g\sigma}{T}\right)\mathrm{Re}\,L$$

(not Z(3) invariant)

The magnetic field reinforces the breaking of Z(3) that occurs in the presence of fermions, forcing <L> to be real-valued!

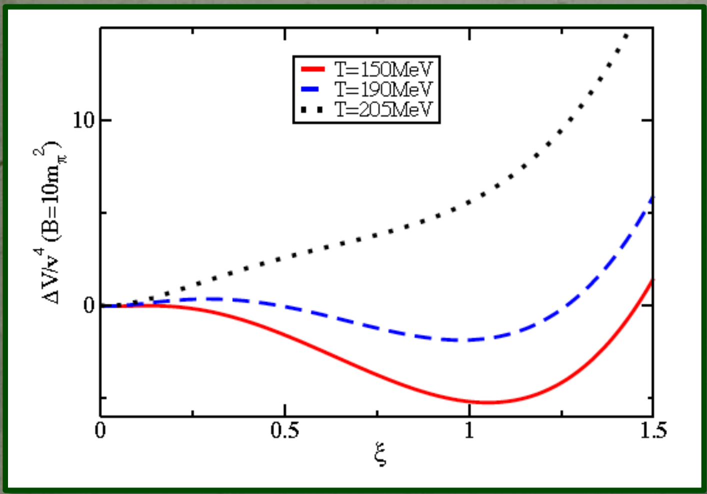


$$\sqrt{eB} = 3T$$

Effective potential

(i) Chiral condensate direction:

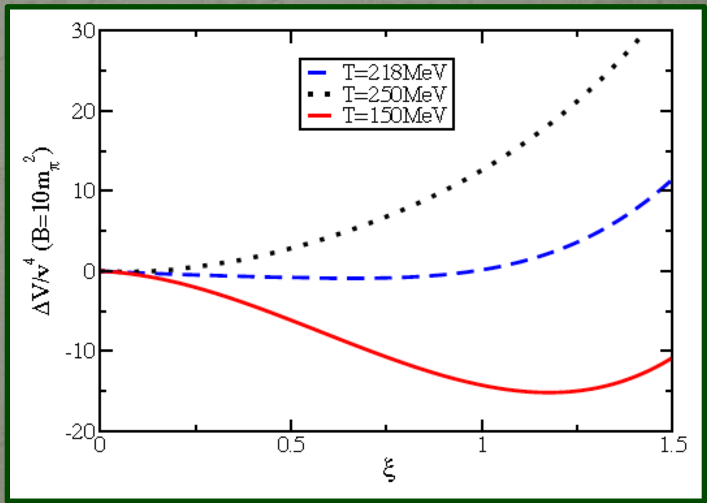
Without vacuum corrections



• A barrier is formed: 1st order chiral transition.

• Part of the system kept in the false vacuum: some bubbles and spinodal instability, depending on the intensity of supercooling.

With vacuum corrections



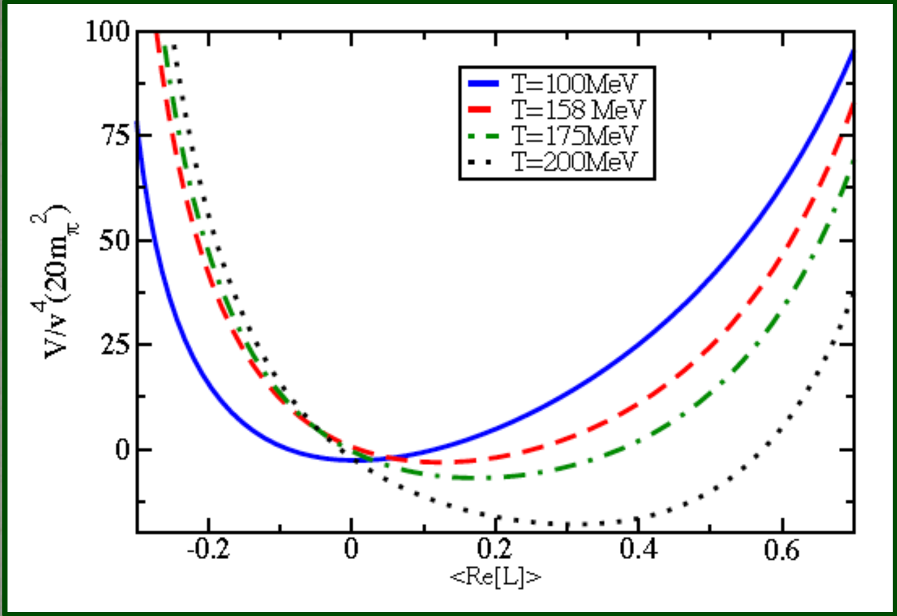
• No barrier: crossover for the chiral transition.

• System smoothly drained to the true vacuum: no bubbles or spinodal instability.

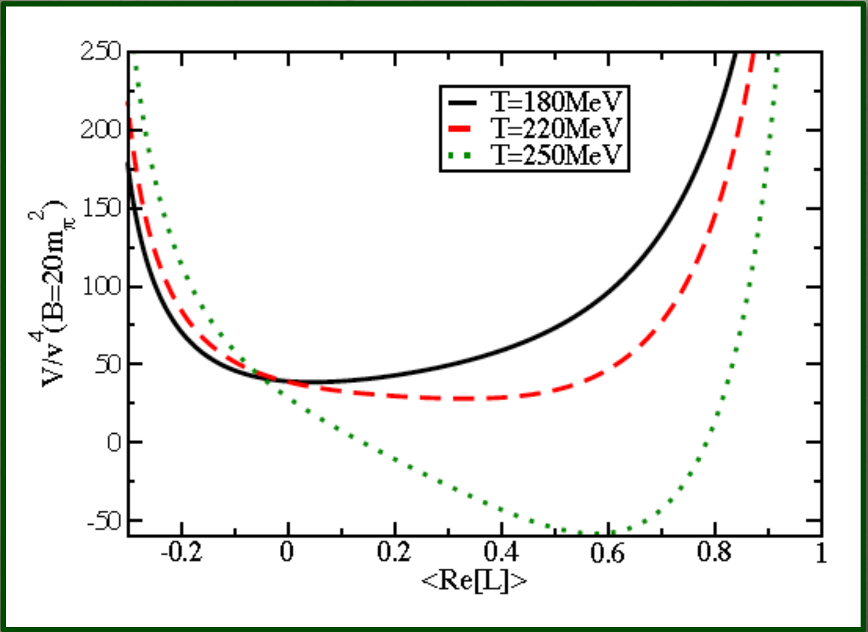
(ii) Re[L] direction:

- Jump in the evolution of the effective potential with T – 1st order transition.
- σ is at the minimum for each temperature.
- Jump in σ.

Without vacuum corrections



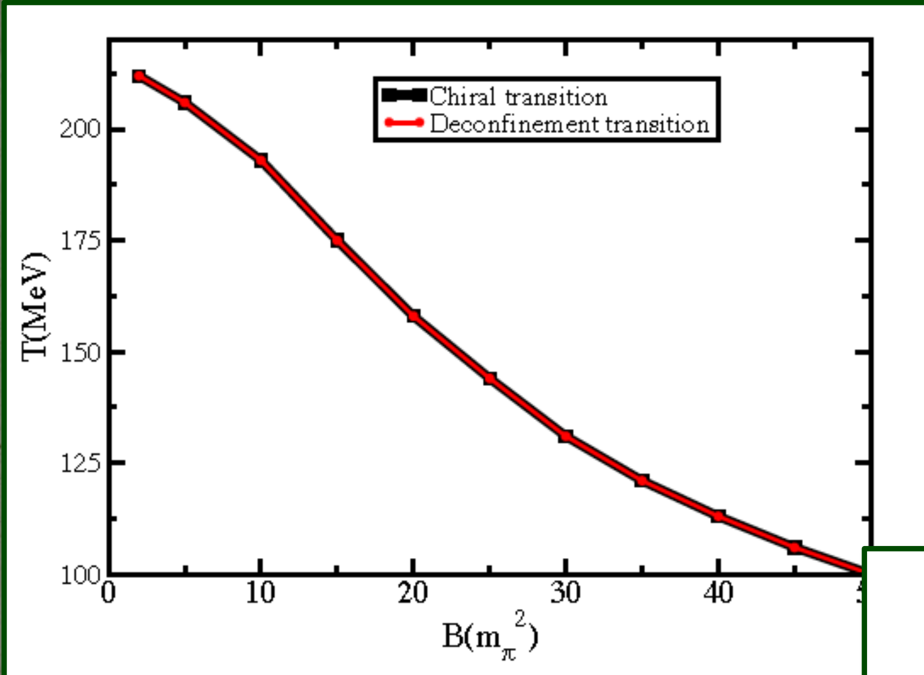
With vacuum corrections



- Smooth modification of the effective potential (no jumps) – crossover.
- σ is at the minimum for each temperature.
- No jump in σ.

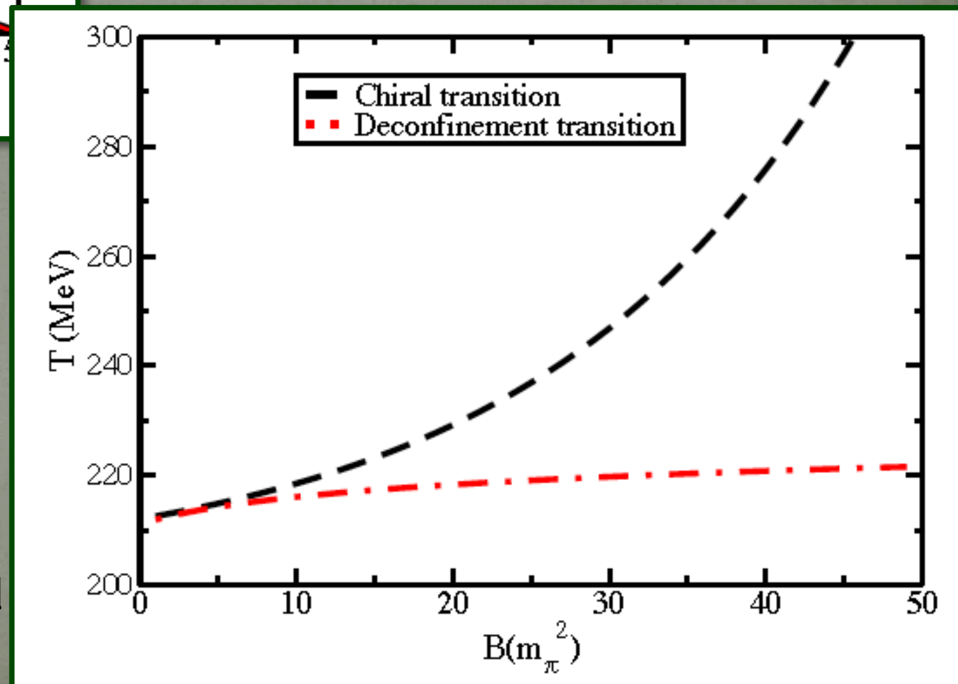
Phase diagrams

Without vacuum corrections



- Chiral and deconfinement lines coincide.
- The transitions are 1st order: very weak for B=0 (artifact), strong for large B (physical).
- Magnetic catalysis reproduced in the vacuum. [ESF & A.J. Mizher (2008)]

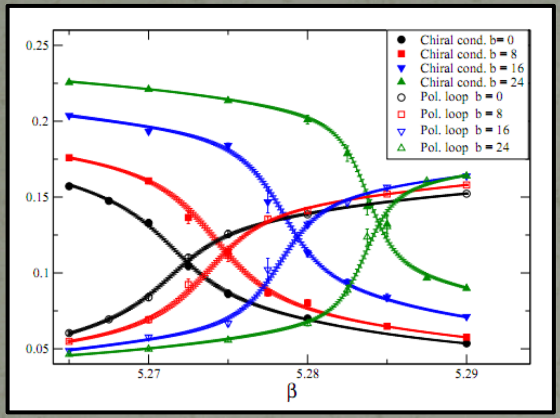
With vacuum corrections

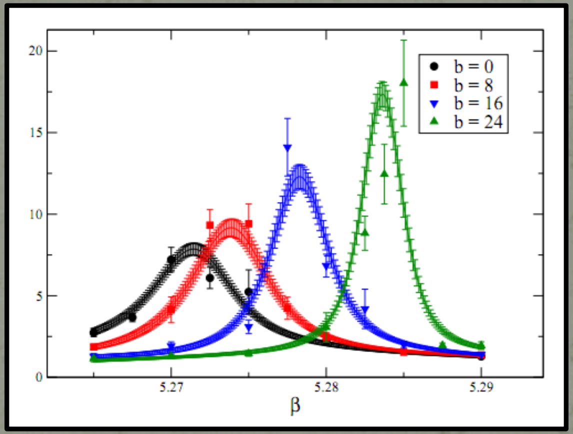


- Chiral and deconfinement (crossover) lines initially coincide, then split (3 phases).
- The deconfinement line flattens out for high enough B (does not go to zero).
- Chiral restoration becomes more and more difficult for high B.

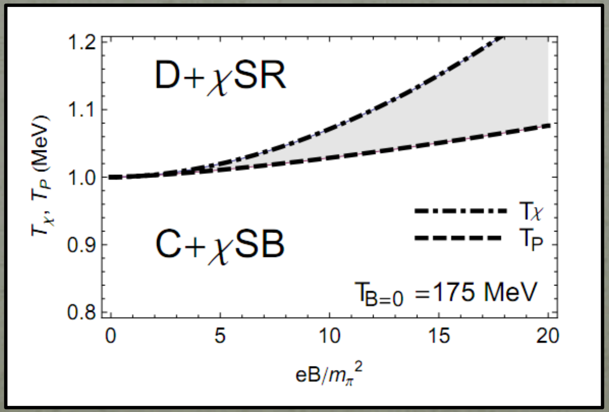
Results in other approaches

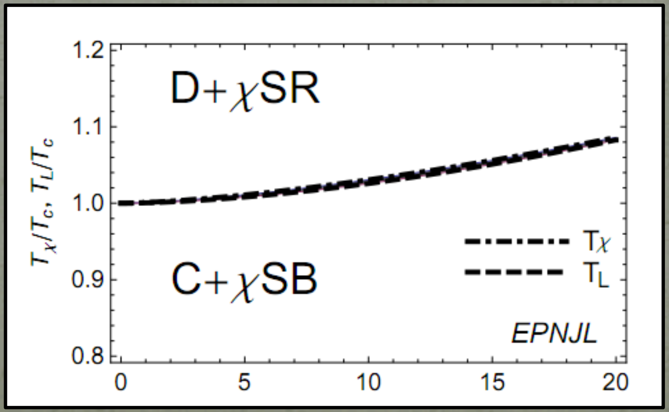
M. D'Elia, S. Mukherjee, F. Sanfilippo (2010): lattice





M. Ruggieri, R. Gatto (2010-2011): NJL and extensions





Final remarks

• Strong magnetic fields can modify the nature and the lines of the chiral and the deconfining transitions, opening new possibilities in the study of the phase diagram of QCD.

• Break of Z(3) reinforced by the magnetic field.

• Our approach indicates an approximately linear dependence of the chiral condensate on the magnetic field in the vacuum, in accordance with recent lattice calculations.

• Perhaps the two transition lines split for high values of B. In the effective theory we consider, that depends on including or not vacuum contributions.

• Either scenario is exciting and brings new possibilities: 1st order transition, splitting of lines, magnetic breaking of Z(3), …

• Results in the framework of the PNJL model reproduce the case of the 2nd scenario (Ruggieri): NJL naturally incorporate fermionic vacuum corrections. However, extensions of the PNJL tend to supress the split.

Further investigation

- Different behavior of u and d quarks.
- Magnetic field inhomogeneous in space.
- Dynamical studies: time scales, transition rates, etc. -> magnetic field inhomogeneous in time.
- Application to the early universe.
- Compact stars: dense systems.
- Interplay with topological effects [AJM, E. S. Fraga (2009)].
- Hidrodynamics in the presence of a magnetic background.
- Still a lot to understand from what model and lattice results have given so far.

In progress:

- comparison with results obtained using Schwinger proper-time method.

- weak field limit (based in previous work by A. Ayala et al) applied in theories of the $\lambda \phi^4$.

- Cosmology – inflation.

Muchas gracias a el departamento de altas energias y a todo el ICN por recibirme tan bien!

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Back up

Confinement: the Polyakov loop

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McLerran & Svetitsky (1981)

In case of deconfinement the Lagrangian remains Z(N) invariant, but the Polyakov loop is not: spontaneous symmetry breaking.

$$\langle L \rangle = e^{2\pi i j/N} L_0$$

Confinement implies restaured Z(N): $\langle L \rangle = 0$

High temperature = low coupling constant : $\langle L \rangle \rightarrow 1$

$$l_0 = 0, \quad T < T_c \quad ; \quad l_0 > 0, \quad T > T_c$$

Free energy at one loop and some results

[A.J. Mizher, M. Chernodub & ESF (2010)]

A. Vacuum contribution

The vacuum contribution can be expressed as the following Heisenberg-Euler energy density:

$$\Omega_q^{\rm vac}(B) = \frac{1}{iV_{4d}} \log\left[\frac{\det(i\not{D}^{(q)} - m_q)}{\det(i\not{\partial} - m_q)}\right] = N_c \cdot \frac{(qB)^2}{8\pi^2} \int_0^\infty \frac{ds}{s^3}\left(\frac{s}{\tanh s} - 1 - \frac{s^2}{3}\right) e^{-s\, m_q^2/(qB)}$$

Equivalently, we can also compute its contribution to the effective potential in the usual MSbar scheme

$$\frac{V_{\rm vac}(\xi,b)}{v^4} = -\frac{N_c b^2}{2\pi^2} \sum_{f=u,d} r_f^2 F\left(\frac{g^2\xi^2}{2r_f b}\right)$$

$$F(x) \equiv \zeta'(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\log x_f + \frac{x_f^2}{4}$$

where $\xi \equiv \frac{\sigma}{v}\,, \quad b \equiv \frac{eB}{v^2}\,, \quad t \equiv \frac{T}{v} \qquad x_f = \frac{m_f^2}{2|q_f|B}$

(v used as mass scale)

B. Paramagnetic contribution

- Computed in an analogous fashion.

- However, more involved: sums over Matsubara frequencies and Landau levels, SU(3) field, ...

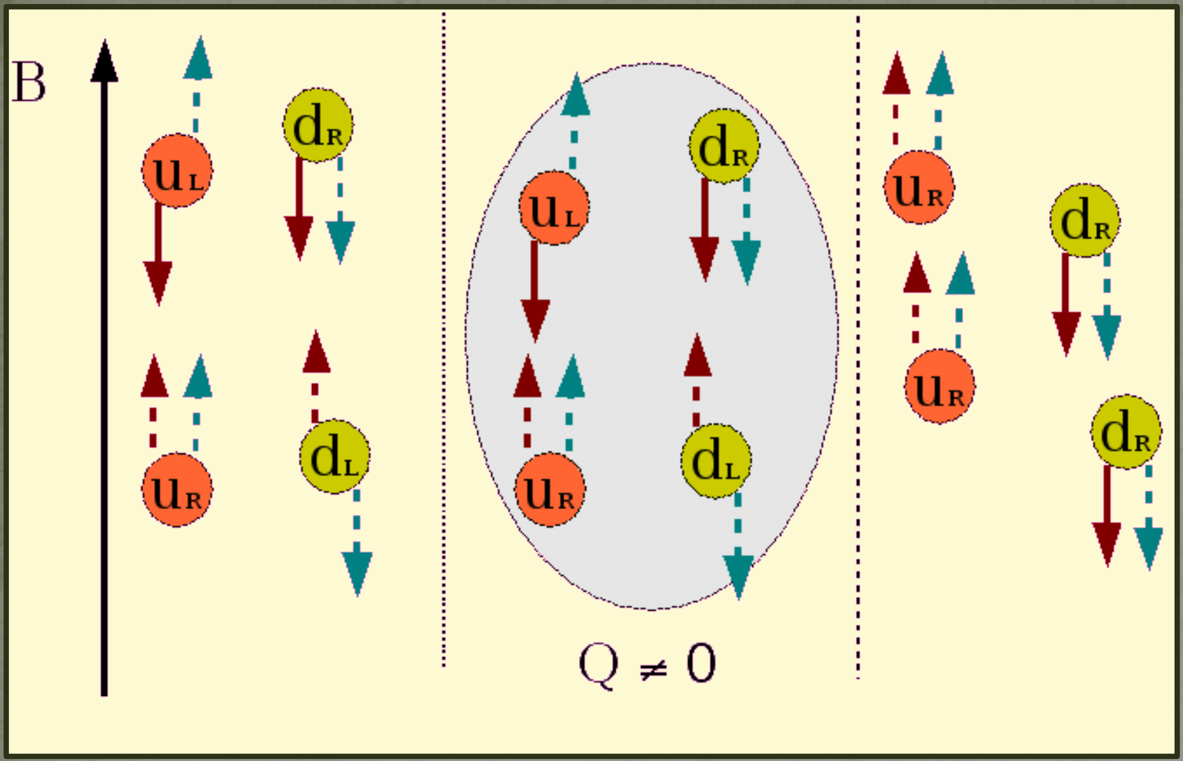
The final result can be written as:

$$\frac{V^{\rm para}(\xi,\phi_1,\phi_2,b,t)}{v^4} = -\frac{bt^2}{2\pi^2}K(b/t^2,\xi/t,\phi_1,\phi_2)$$

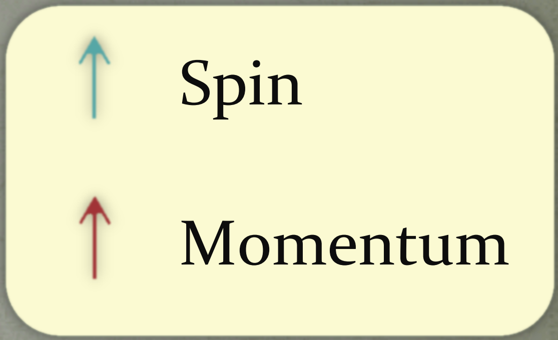
$$K(\xi,\phi_1,\phi_2,b,t) = \sum_{f=u,d}\sum_{s=\pm 1/2}\sum_{n=0}^{\infty}\sum_{i=1}^{3}\int_0^\infty dx\,\log\left(1+e^{-2\sqrt{x^2+\tilde{\mu}_{snf}(\xi,b)/t}}+2e^{-\sqrt{x^2+\tilde{\mu}_{snf}(\xi,b)/t}}\cos\phi_i\right)$$

$$\tilde{\mu}_{snf} = \left[g^2\xi^2+(2n+1-2s)r_f b\right]^{1/2}$$

$$\xi \equiv \frac{\sigma}{v}\,, \quad b \equiv \frac{eB}{v^2}\,, \quad t \equiv \frac{T}{v} \qquad\qquad q_f = r_f eB\ sgn(q_f)$$



$$h = \vec{p} \cdot \vec{s}$$



The interaction of the quarks with the non-trivial gauge fields gives rise to a difference between the number of quarks left and right. In the presence of the magnetic field it generates a current in its direction and a charge difference between the two hemisphere opposite to the reaction plane.