A finite volume gluon dressing function for dynamical quark masses

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Motivation

- 99% of the mass of the visible universe owes itself to the strong interactions of QCD
- Dynamical quark masses and confinement are crucial to understand hadrons
- Schwinger-Dyson equations have acquired predictive capabilities over the past two decades
- These predictions complement those coming from lattice simulations and/or effective theories

Motivation

- QCD (almost) admits a global conformal symmetry in the light flavor sector
- AdS/CFT has been implemented in the light-front QCD to describe hadrons
- In particular, AdS_5/CFT_4 has been used to model spectra for N and Δ from quark-diquark system
- Idea: put the q-(qq) system on the boundary of AdS_5 and then compactify to $S^3 \otimes R^1$

AdS5

- Defined as an R^{2+4} subspace according to

$$u^2 + v^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = \rho^2,$$

- The boundary at infinity is identified with the "cone"

$$u^2 + v^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = 0$$

- Minkowski (4D) space is identified as

$$v^2 - x_4^2 = u^2 - x_1^2 - x_2^2 - x_3^2 = 0,$$

M. Kirchbach and C. Compean, PRD82, 034008 (2010).

AdS5

- Alternatively, we identify the Minkowski space as emerging from the AdS$_5$ cone as

$$u^2 + v^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2, \quad R \neq 0,$$

- Then, the compactification is to $S^3 \otimes S^1$ which is parametrized as

$$u + iv = Re^{i\tau} \qquad x_1 + ix_2 = R \sin\chi \sin\theta e^{i\varphi}, \quad x_3 = R\sin\chi\cos\theta,$$
$$x_4^2 + \mathbf{r}^2 = R^2, \qquad r = |\mathbf{r}| = R\sin\chi, \quad \chi = \sin^{-1} r\sqrt{\kappa}, \quad \kappa = \frac{1}{R^2},$$

M. Kirchbach and C. Compean, PRD82, 034008 (2010).

AdS5

- With this map, we have

$$ds^2 = \Omega^{-2}(-\mathrm{d}\tau^2 + \mathrm{d}\chi^2 + \sin^2\chi\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right))$$

- That is, $S^3 \otimes S^1 \simeq S^3 \otimes R^1$
- The free geodesic motion on S^3 is

$$-\hbar^2\widehat{\Box}\psi + \mu^2\psi = 0$$

$$\widehat{\Box} = -\frac{1}{R^2}\frac{\partial^2}{\partial\tau^2} - \frac{1}{R^2}\mathcal{K}^2,$$

$$-\mathcal{K}^2 = \left[\frac{1}{\sin^2\chi}\frac{\partial}{\partial\chi}\sin^2\chi\frac{\partial}{\partial\chi} - \frac{\mathbf{L}^2(\theta,\varphi)}{\sin^2\chi}\right]$$

- The conformal symmetry aspects of AdS/CFT are adequately captured by the κ^2 eigenvalue problem

M. Kirchbach and C. Compean, PRD82, 034008 (2010).

The “cot” potential

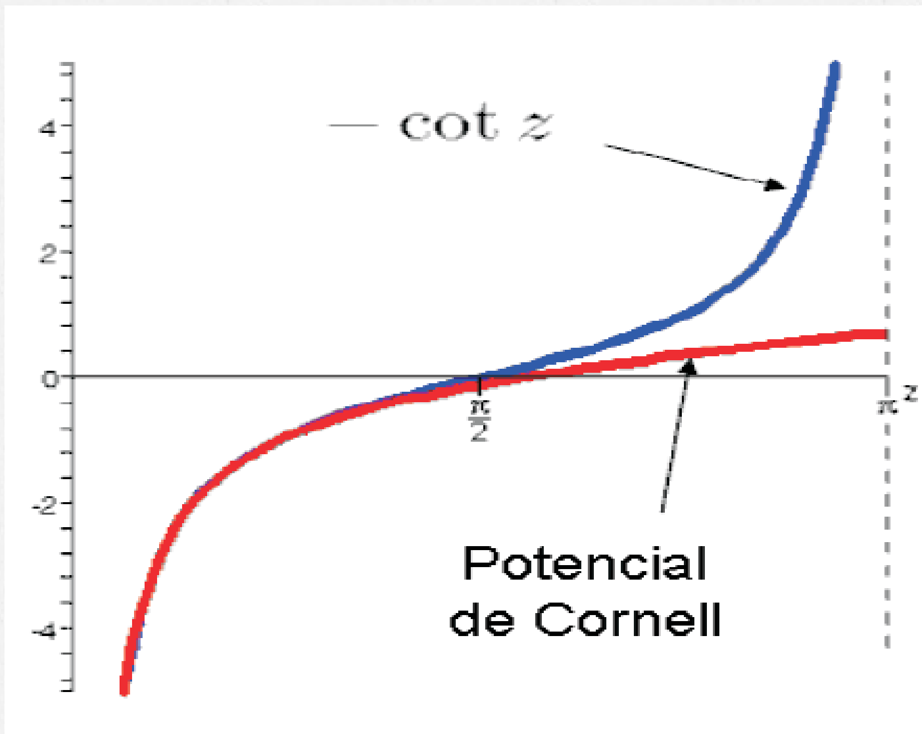
- Interactions on S^3 must satisfy the Laplace-Beltrami equation, i.e., the potential should be an harmonic function on the manifold
- The “curved” Coulomb potential becomes of finite range

$$\cot\chi = \frac{x_4}{r}, \quad r \in [0, R], \quad x_4 \in [-R, R],$$

$$-\cot\frac{r}{R}\pi = -\frac{d}{r} + \frac{1}{3}\frac{r}{d} + \frac{r^3}{45d^3} + \frac{2r^5}{945d^3} + \ldots, \quad \text{with} \quad d = \frac{R}{\pi},$$

M. Kirchbach and C. Compean, PRD82, 034008 (2010).

The “cot” potential



The “cot” potential

- In cartesian coordinates, the Fourier transform of the “cot” potential yields the *instantaneous gluon propagator*

$$4\pi\Pi(|\mathbf{q}|) = -2\mu\frac{(-2G\sqrt{\kappa})}{\hbar^2}\int_0^\infty d|x||x|^3\delta(|x|-R)\int_0^{2\pi}d\varphi\int_0^\pi d\theta\sin\theta \int_{0/\frac{\pi}{2}}^{\frac{\pi}{2}/\pi} d\chi\sin^2\chi e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}|\cos\theta}\cot\chi,$$

$$\Pi(|\mathbf{q}|) = c\frac{2\sin^2\frac{|\mathbf{q}|}{2\hbar\sqrt{\kappa}}}{\left(\frac{|\mathbf{q}|}{\hbar\sqrt{\kappa}}\right)^2},\quad c = \frac{4G\mu}{\hbar^2\kappa}$$

M. Kirchbach and C. Compean, PRD82, 034008 (2010).

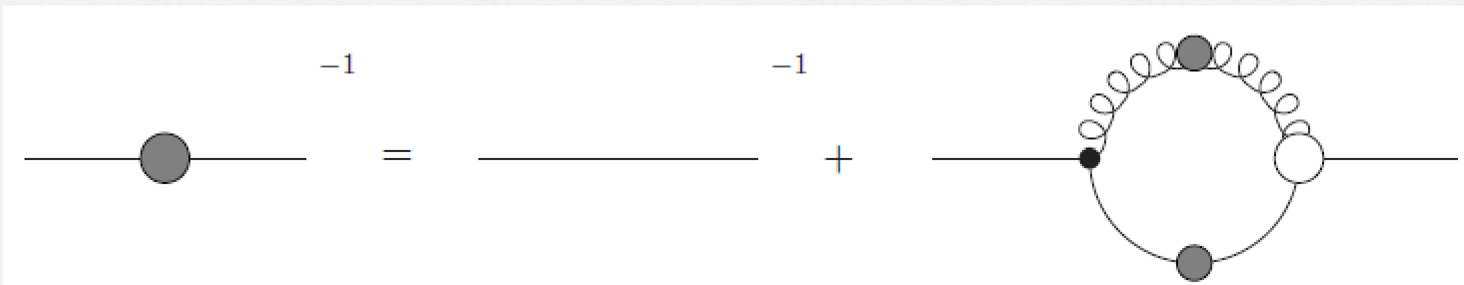
The "cot" potential

- Light baryon spectra
- Unflavored meson spectra
- N and Δ electric form factors
- Proton electric form factor and charge radius
- Roper resonance

M. Kirchbach and C. Compean, PRD82, 034008 (2010); M. Kirchbach. A. Pallares, C. Compean and AR, J. Phys. Conf. Ser. 378, 012036 (2012).

Dynamical masses *a la* Schwinger-Dyson

- The QCD gap equation is



- and corresponds to

$$[S_F(p)]^{-1} = Z_2 \left[S_F^{(0)}(p)\right]^{-1} - C_F \frac{Z_2 \widetilde{Z}_1}{\widetilde{Z}_3} \frac{g^2}{(2\pi)^4} \int d^4k \gamma^\mu S_F(k) \Gamma_q^\nu(k,p) D_{\mu\nu}(p-k) \,.$$

Dynamical masses *a la* Schwinger-Dyson

- The fermion propagator

$$S(p) = \frac{1}{\iota\gamma.pA(p^2) + B(p^2)} = \frac{F(p^2)}{\iota\gamma.p + M(p^2)} = \iota\gamma.p\sigma_v(p^2) + \sigma_s(p^2)$$

- The gluon propagator (in Landau gauge)

$$G^{ab}_{\mu\nu} = -i[(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\frac{G(q^2)}{q^2}]\delta^{ab}$$

Dynamical masses *a la* Schwinger-Dyson

- A common practice to truncate the SDE is to replace $\Gamma^\mu \rightarrow \gamma^\mu$ and

$$\frac{g^2}{4\pi}\frac{Z_2}{\tilde{Z}_3} D_{\mu\nu}(q) \rightarrow \alpha\left(q^2\right) D^{(0)}_{\mu\nu}(q)$$

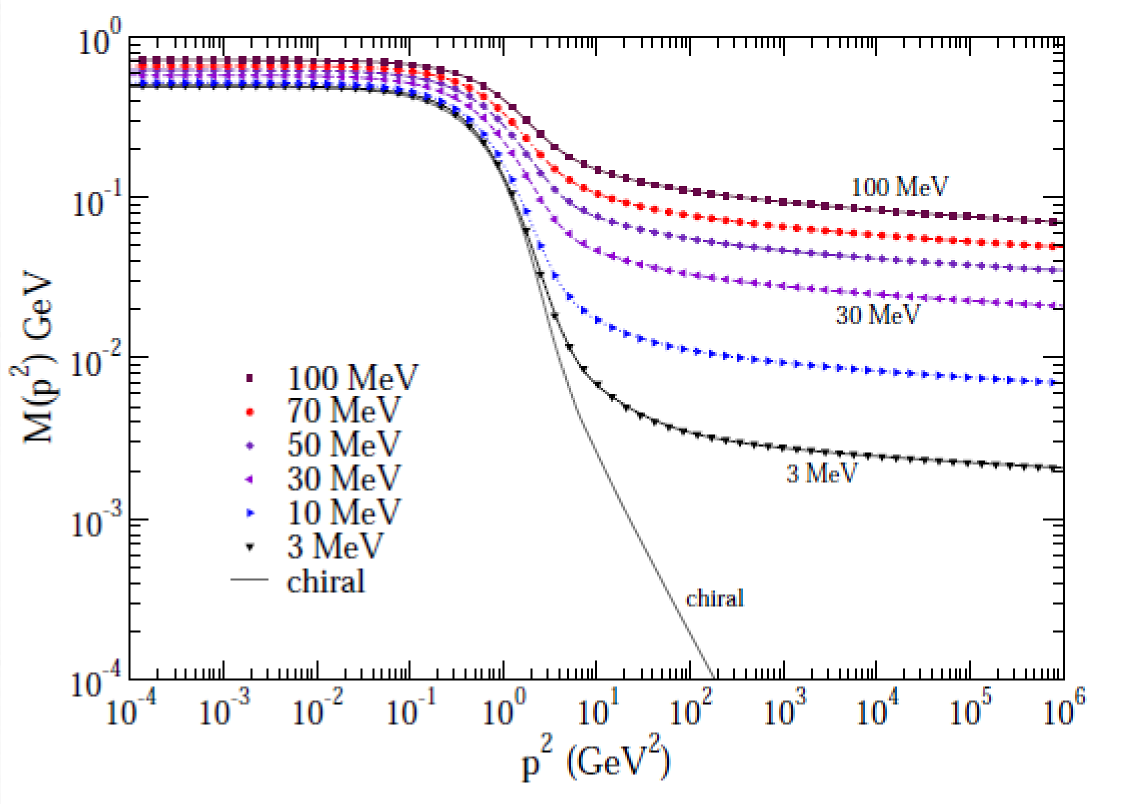
- The Maris-Tandy model consists in

$$\begin{aligned} \alpha\left(q^2\right) &= \frac{\pi}{\omega^6} D\, q^4 \exp(-q^2/\omega^2) \\ &+ \frac{2\pi\gamma_m}{\log\left(\tau + \left(1+q^2/\Lambda^2_{QCD}\right)^2\right)} \times \left[1 - \exp\left(-q^2/\left[4m_t^2\right]\right)\right] \end{aligned}$$

$m_t = 0.5\ \mathrm{GeV}$ $\tau = \mathrm{e}^2 - 1$ $\gamma_m = 12/(33-2N_f)$ $\Lambda_{QCD} = 0.234\,\mathrm{GeV}$

P. Maris and P.C. Tandy, PRC 60, 055214 (1999).

Dynamical masses *a la* Schwinger-Dyson



See S.H. Ortiz, K. Raya & A. Ahmad posters

P. Maris and P.C. Tandy, PRC 60, 055214 (1999).

Dynamical masses *a la* Schwinger-Dyson

- In order to test that the solutions to the mass function are consistent with Confinement, we check reflection positivity.
- Let's introduce the spatially averaged propagator

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4} \sigma_s(p^2)$$

- If it is positive definite, it describes a stable excitation. If not, it describes a confined particle

Dynamical masses *a la* Schwinger-Dyson

- For example, for a free fermion,

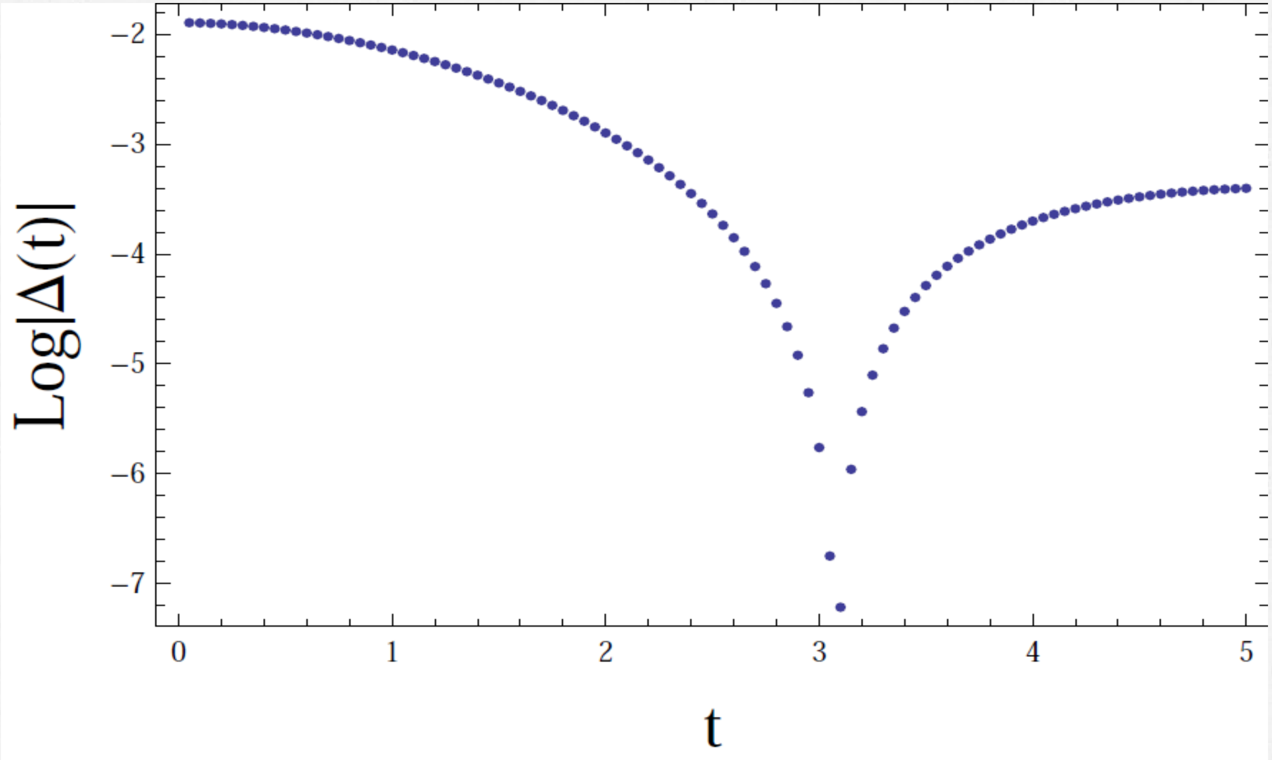
$$\sigma_s(p^2) = \frac{m}{p^2 + m^2} \quad \Longrightarrow \quad \Delta(t) = \frac{1}{2} e^{-m\,t}$$

- But if m develops an imaginary part, $m = a + ib$,

$$\Delta(t) = e^{-a\,t} \cos(b\,t + \delta)$$

See S. H. Ortiz & A. Ahmad posters for details

Dynamical masses *a la* Schwinger-Dyson



Our goal

- Use as an effective gluon dressing the function

$$\frac{G(\mathbf{q}^2)}{\mathbf{q}^2} = c\frac{2\sin^2\frac{|\mathbf{q}|}{2}}{\mathbf{q}^2},$$

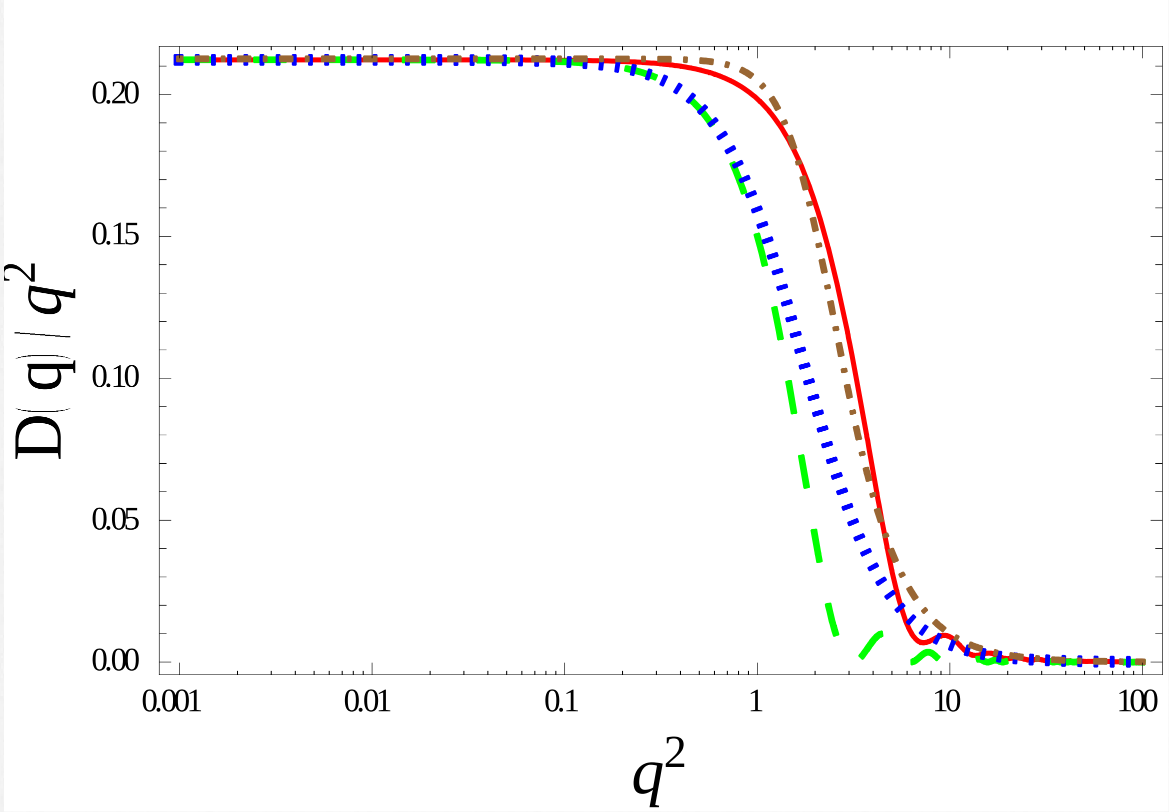
Adjusting *c* to solve the gap equation with similar overall behavior as in the Maris-Tandy model.

Effective gluon dressing

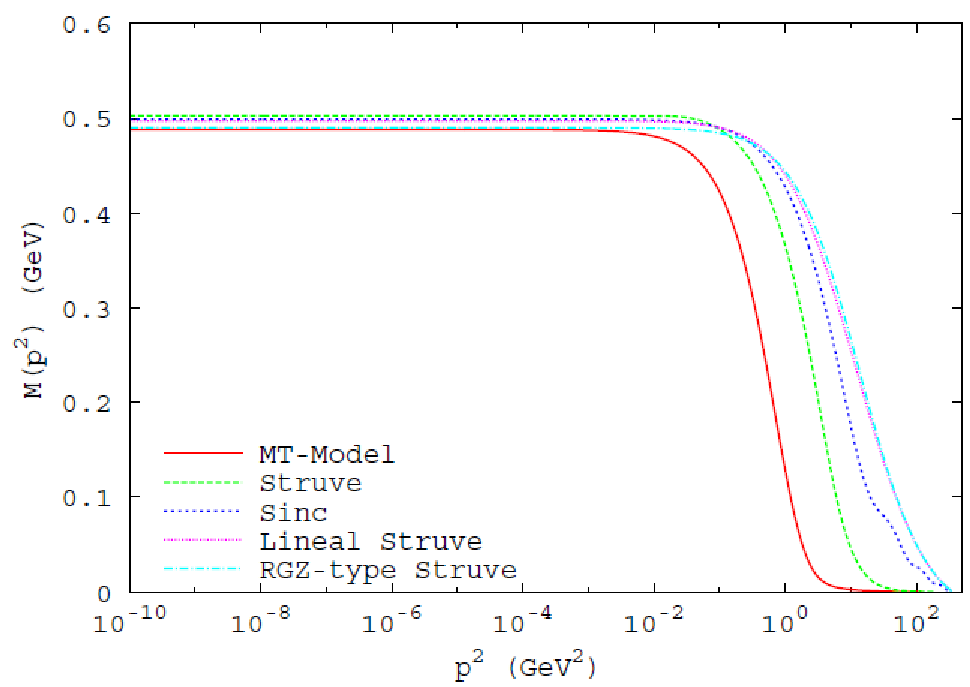
- Model I: $\frac{D(q)}{q^2} = c\ \frac{H_1(q)}{q^2}$ Struve
- Model II: $\frac{D(q)}{q^2} = c\ \frac{\sin(q)^2}{q^2}$ Sinc
- Model III: $\frac{D(q)}{q^2} = c\ \frac{2/\pi}{q^2+3}$ Struve linearized
- Model IV: $\frac{D(q)}{q^2} = c\ \frac{q^2+m^2}{q^4+q^2m^2+m^4}$ RGZ type

See A. Ahmad poster

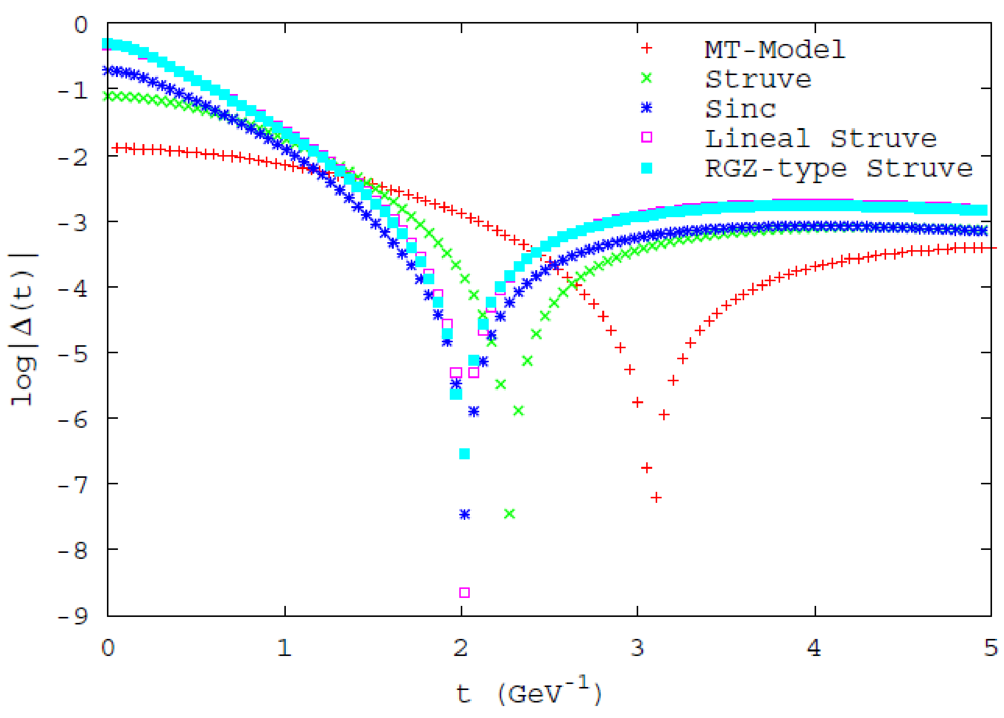
Effective gluon dressing



Effective gluon dressing



Effective gluon dressing



Final Remarks

Do

- Enhancement of quark masses in the IR
- Expected UV behavior of quark propagator
- Neat signals of confinement

Don't

- Perturbation theory settles at larger p^2
- Larger value of the chiral condensate
- Smaller confinement scale

Final Remarks

- Still to do
  - Check the role of the curvature κ
  - Calculate other hadron observables, f_π
  - Calculate bound-state properties
  - Add a thermal bath

  - ...and so on

Final Remarks

Thank you!