Phase structure of $\lambda\phi^4$ model in the presence of a magnetic background

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Motivation

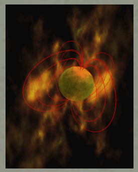
Strong interactions under magnetic fields can be found in nature in:

- Magnetars
- The early universe
- Non-central heavy ion collisions

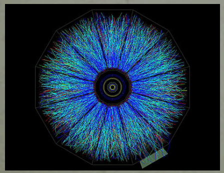
The earth's magnetic field 0.6 Gauss

A common hand-held magnet 100 Gauss

The strongest steady magnetic fields achieved so far in the laboratory 4.5×10^5 Gauss



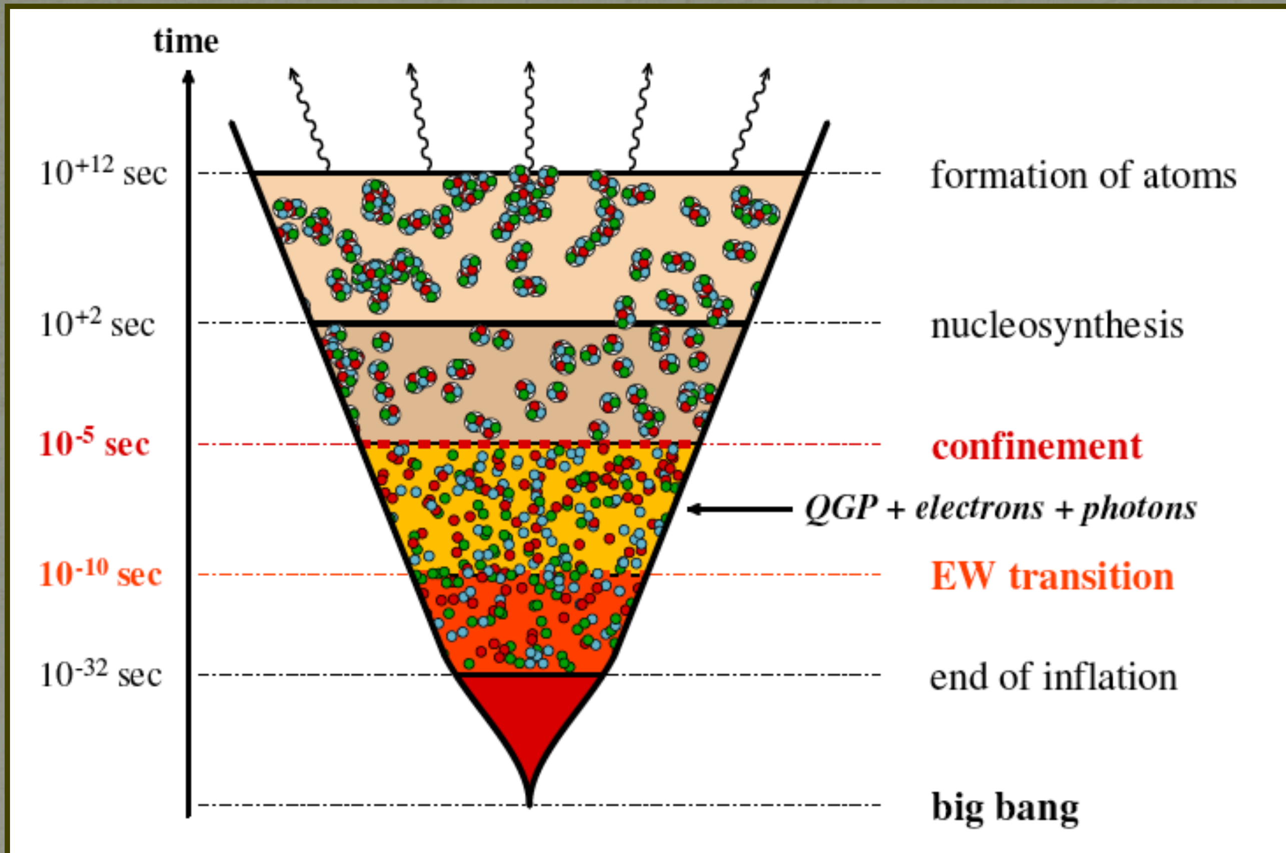
Surface field of magnetars 10^{15} Gauss



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory 10^{18} Gauss

Kharzeev, QM2009

Heavy ion collision: why?



High temperature plasma – above 4 trillion Kelvin

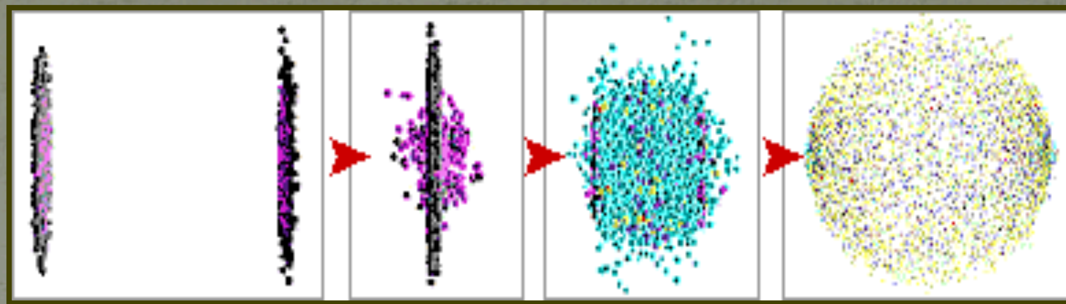
Heavy ion collision: where?

Relativistic heavy ion collider (RHIC)

200 GeV/nucleon

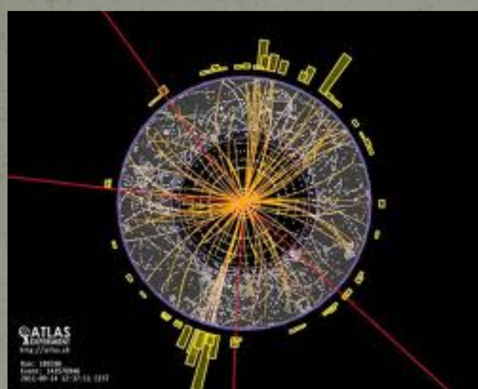
Large Hadron Collider (LHC) (~ 1 month a year)

2.76 TeV/nucleon



Proton-proton

Heavy ions



vs

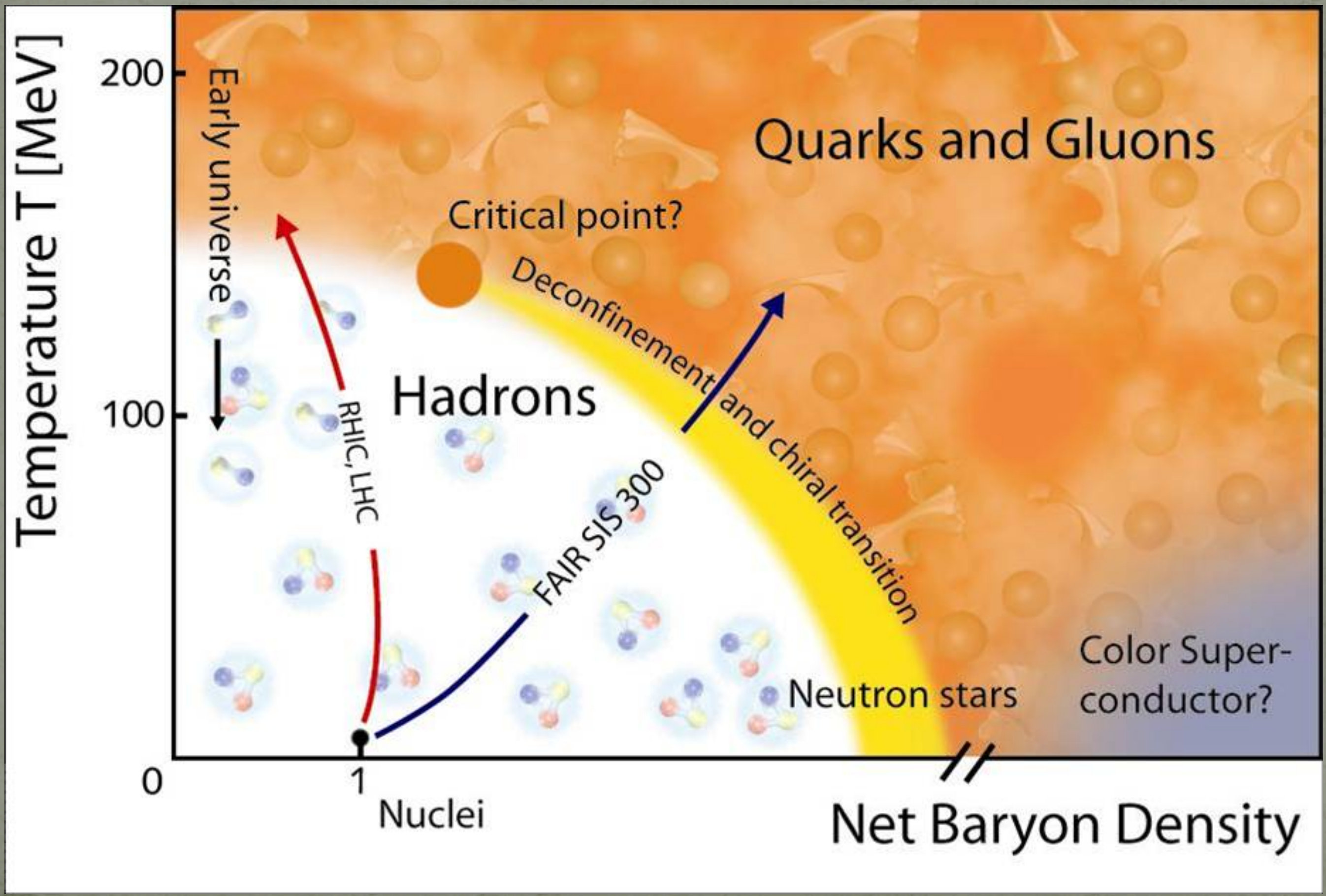
~ 20 particles

~ 4800 particles in
central collisions

High temperatures and densities yield to phase transitions

Confinement – Deconfinement

Chiral symmetry restoration



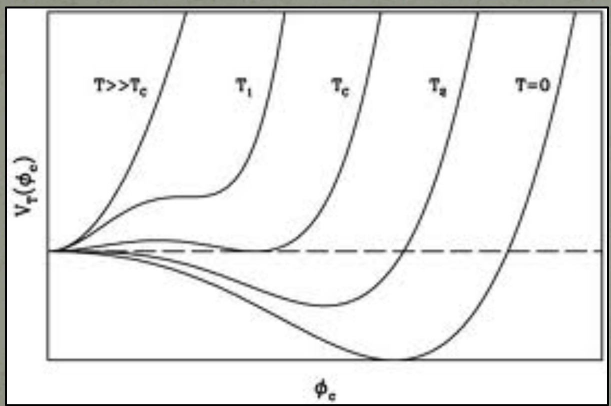
Phase transitions

Broken x restored phase:

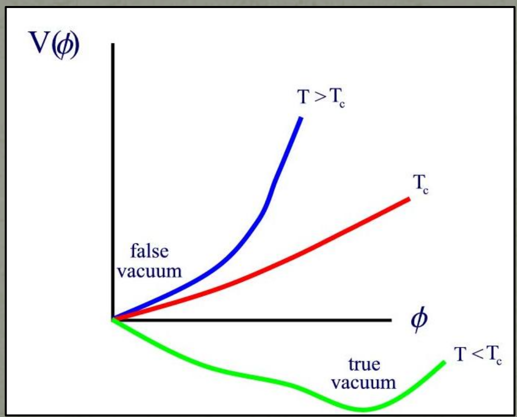
The order parameter carries information about the symmetries of the system.

$$\text{Chiral symmetry}: \begin{cases} \langle \sigma \rangle \neq 0 \quad , \quad \text{low } T \\ \langle \sigma \rangle = 0 \quad , \quad \text{high } T \end{cases}$$

$$\text{Confinement}: \begin{cases} \langle L \rangle = 0 \quad , \quad \text{low } T \\ \langle L \rangle \neq 0 \quad , \quad \text{high } T \end{cases}$$



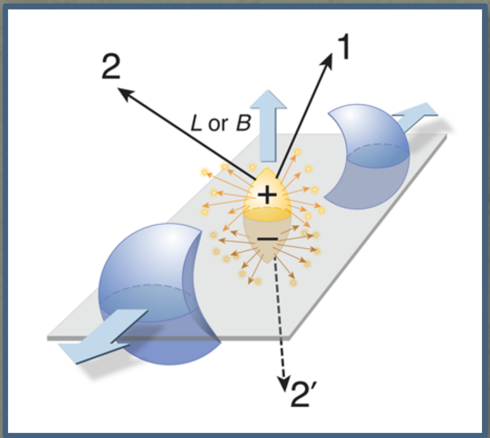
x

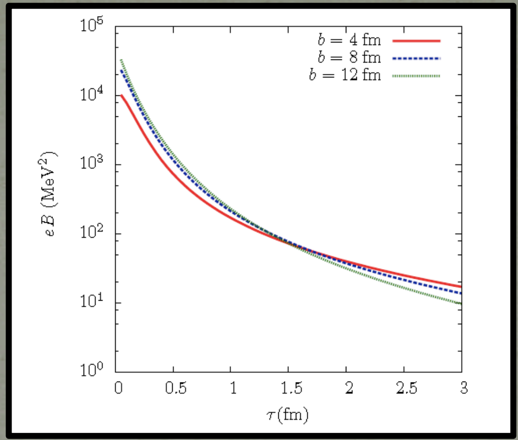


1st order

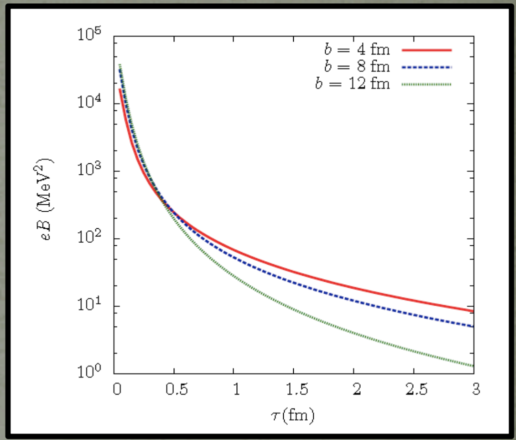
2nd order

Magnetic fields in heavy ion collisions





Au-Au, 62 GeV



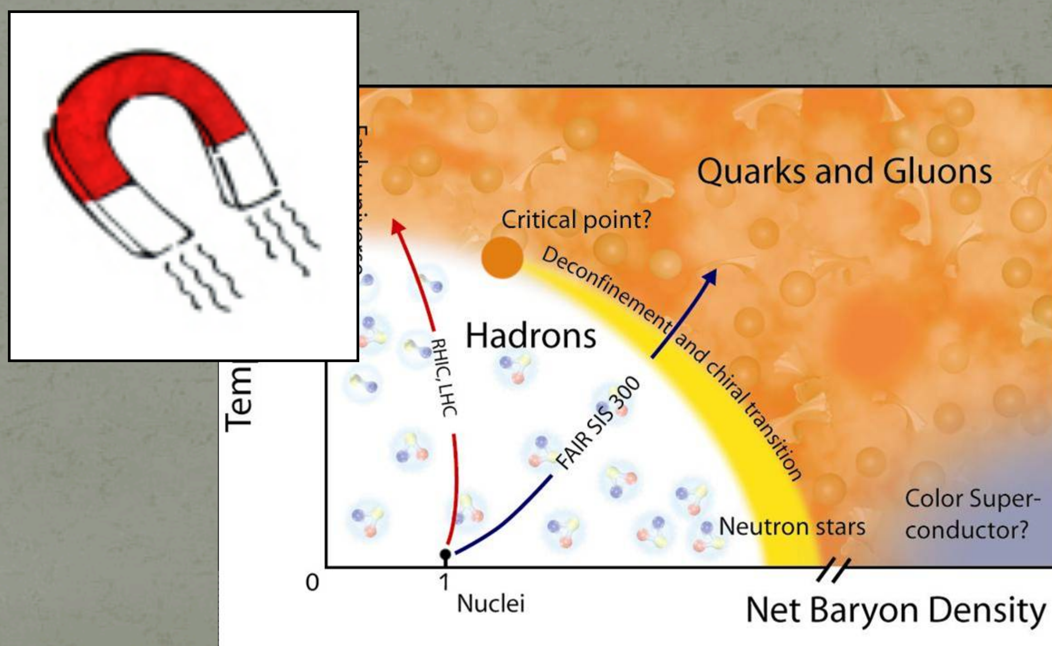
Au-Au, 200 GeV

THE CHIRAL MAGNETIC EFFECT [Kharzeev, Warringa, McLerran]

non-trivial gauge field configurations ⇄ **strong magnetic field**

Charge separation

- How does the QCD diagram look like including another external control parameter, the magnetic field B?
- Are there modifications in the nature of the phase transition?
- How do the chiral and deconfinement transitions react to this magnetic field?
- How is the interplay between this two transitions?



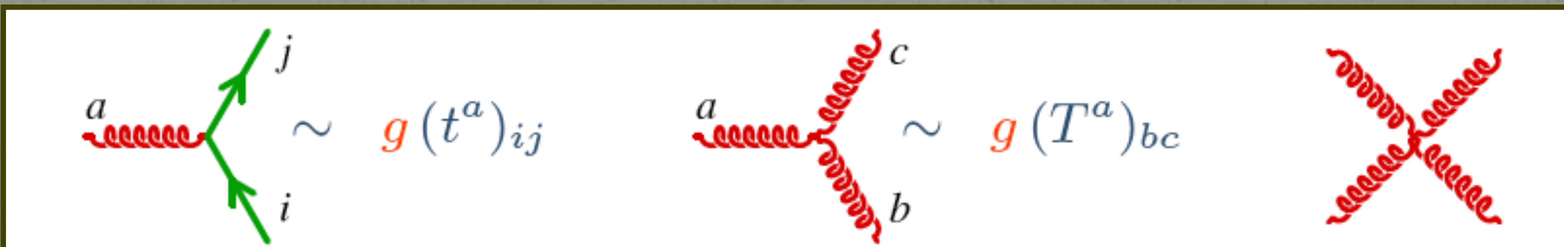
Quark-gluon plasma: how to deal with it?

QCD:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \overline{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A^a_\mu \tau^a) \psi_f$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Dynamic fields: N_f flavors of quarks, N_c colors and (N_c^2-1) gluons;
SU(N) symmetry: non-Abelian, gluons interact with themselves.



Lattice QCD: full numerical simulations.

Sign problem, ambiguities in fermion definitions, computationally expensive.

Asymptotic freedom ⇒ **perturbation theory**

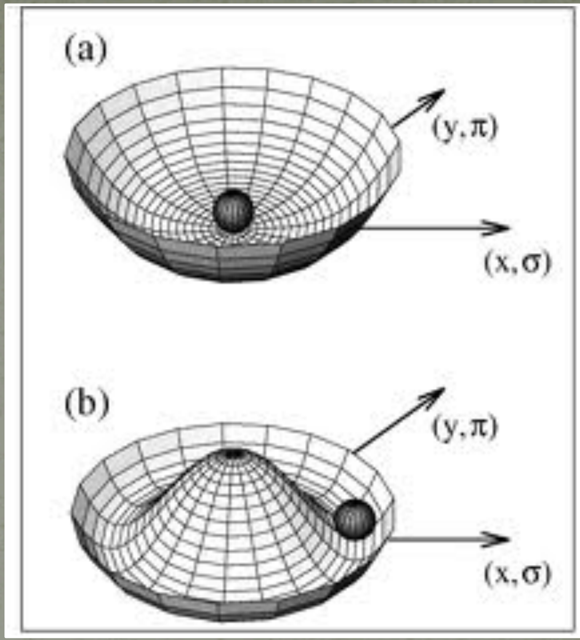
AdS/CFT: formal solutions of a ‘cousin’ theory.

Effective models: not the whole story. Mimic of a sector of the theory.

Combination of all this can lead to a more robust characterization of the plasma!

Effective models

Chiral sector: spontaneous symmetry breaking



σ Linear model:

$$\mathcal{L} = \overline{q}\left[i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\right] q + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi}\cdot\partial^\mu \vec{\pi}\right) - U(\sigma, \vec{\pi})$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - H\sigma$$

NJL:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu \partial_\mu - m_0\right) q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

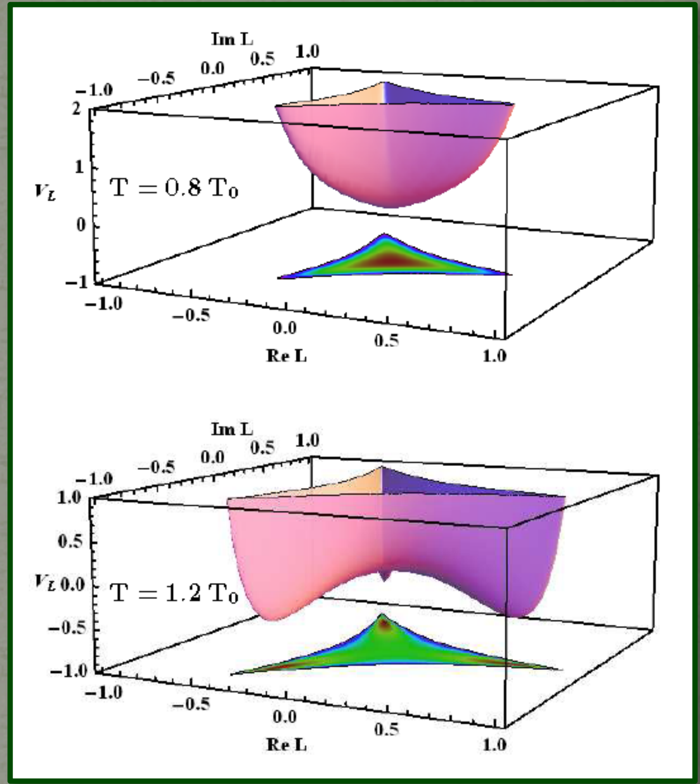
Gauge sector

Polyakov loop models:

$$\ell(x) = \frac{1}{N}\,\mathrm{tr}\left(\mathcal{P}\exp\left(ig\int_0^{1/T} A_0(x,\tau)\,d\tau\right)\right)$$

Parameters fix demanding:

- Stefan-Boltzmann limit reached at T -> ∞
- first order transition happens at $T=T_0$
- the potential fits lattice data for thermodynamical quantities (pressure, energy density and entropy)



$$\frac{V_L(L,T)}{T^4} = -\frac{1}{2}a(T)\,L^*L + b(T)\,\ln\left[1 - 6\,L^*L + 4\left(L^{*3} + L^3\right) - 3\,(L^*L)^2\right]$$

Finite temperature formalism

Quantum mechanics

$$\hat{\rho} = e^{-\beta(\hat{H} - \mu_i \hat{N}_i)}$$

$$Z = Z(V, T, \mu_1, \mu_2, ...) = Tr\ \hat{\rho}$$

Field theory – Matsubara frequencies

$$Z(\beta) = \int [\mathcal{D}q(\tau)] e^{-S_E(\beta)}$$

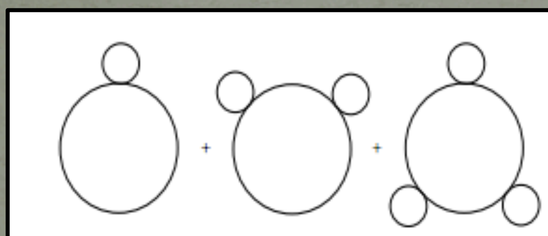
$$S_E = \int_0^\beta d\tau \int d^3x \left[\frac{1}{2}\phi\left(-\frac{\partial^2}{\partial\tau^2} - \nabla^2 + m^2\right)\phi + V(\phi)\right]$$

$$S_E = \frac{1}{2}\beta^2 \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} [(\omega^2 + \omega_n^2)\phi_n(k)\phi_{-n}(-n)]$$

$$\log(Z(\beta)) \quad = \quad -\frac{1}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \log(\omega^2 + \omega_n^2)$$

Ring diagrams

In thermal field theory the contribution after 1-loop order is not 2-loops. A sum of a class of diagrams converge to a contribution of order $\lambda^{3/2}$



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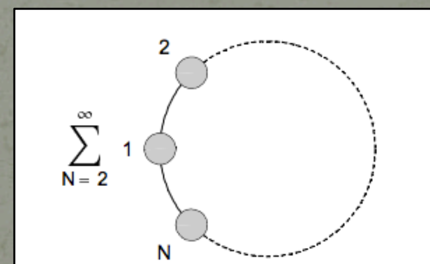

IR regularization for massless theories!

Including an external magnetic field

For simplicity we assume a magnetic field that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{Gauge choice}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

Scalars ➔ $$(\partial^2 + m^2)\phi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu$$

Fermionic ➔ $$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu$$

- charged scalars: propagator in the presence of a magnetic field

$$D = \int_0^\infty \frac{ds}{\cosh(qBs)} e^{-s\left[(\omega_n - i\mu)^2 + p_z^2 + m^2 + p_\perp^2 \frac{\tanh(qBs)}{qBs}\right]}$$

The $\lambda\ \phi^4$ model

$$L = \frac{1}{2}\left[\partial_\mu \phi \partial^\mu \phi\ \right] - m^2|\phi|^2 + \lambda|\phi|^4 + \delta L$$

Kinetic term

Spontaneously symmetry breaking

Counterterms

Weak field approximation

$$iD^B(k) = \frac{i}{k_\parallel^2 - k_\perp^2 - m^2}\left\{1 - \frac{(eB)^2}{(k_\parallel^2 - k_\perp^2 - m^2)^2} - \frac{2(eB)^2 k_\perp^2}{(k_\parallel^2 - k_\perp^2 - m^2)^3}\right\}$$

Effective potential

1-loop:

$$V^{(1)} = -\frac{\pi^2 T^4}{90} + \frac{m^2 T^2}{24} - \frac{m^3 T}{12\pi} - \frac{m^4}{32\pi^2} ln\left(\frac{m}{4\pi T}\right) + \mathcal{O}(m^4)$$

Ring diagrams contribution

$$V^{(ring)} = \frac{T}{12\pi}\left[m^3 - (m^2 + \Pi_\beta^B)^{3/2}\right] +$$

$$\frac{T}{8\pi}(eB)^2 \frac{\Pi_\beta}{(\Pi_\beta^B)^2}\left[(m^2 + \Pi_\beta^B)^{1/2} - \frac{m^2 + \Pi_\beta^B}{m} + \cdots\right]$$

Vacuum contribution: IR divergences

$$= \int dm^2\, T \sum_{n=-\infty}^{\infty} \int \frac{d^d p}{(2\pi)^d} D(\boldsymbol{k}, \omega_n)$$

Renormalization!

$$\begin{aligned} V_B = &- \left[\frac{\mu^2}{4} - \frac{18\lambda^2 v_0^2}{32\pi^2} \psi \left(\frac{1}{2} + \frac{3\lambda v_0^2 - \mu^2}{2qB} \right) \right. \\ &\left. + \frac{2qB}{(4\pi)^2} 3\lambda \left\{ \ln\Gamma \left(\frac{1}{2} + \frac{3\lambda v_0^2}{2qB} \right) - \frac{1}{2}\ln(2\pi) \right\} \right] v^2 \\ &+ \left[\frac{\mu^2}{8 v_0^2} - \frac{9\lambda^2}{32\pi^2} \psi \left(\frac{1}{2} + \frac{3\lambda v_0^2 - \mu^2}{2qB} \right) \right] v^4 \\ &+ \left(\frac{2qB}{4\pi} \right)^2 \int dm^2 \left(\ln\Gamma \left(\frac{1}{2} + \frac{3\lambda v_0^2}{2qB} \right) - \frac{1}{2}\ln(2\pi) \right) \end{aligned}$$

Put all together and study the parameter space

Final remarks

QCD in the presence of magnetic fields is a subject that has been calling attention in the last years.

Contributions from different fields: effective models, lattice, AdS/CFT, data analysis, etc.

Magnetic backgrounds can change remarkably the QCD phase diagram: 2nd to 1st phase transitions, change of the critical temperature, split of the transition lines, etc.

Mix of different areas and techniques: thermal field theory, electrodynamics, heavy ion phenomenology, phase structures, etc...

Recent area: still a lot to explore and to be understood.