2. EFF. POTENTIAL APPLICATIONS

2. Eff. Potential Applications

2.1 Scalar Mass(es) from Veff

In the classical approximation, the tree-level potential $V_0(\phi)$ dictates the minimum of the theory, say at $\langle\phi\rangle = v_0$, and also gives information about the mass of the scalar field fluctuations around that minimum, $\varphi = \phi - v_0$:

$$V_0(\phi) = V_0(v_0) + \underbrace{\left.\frac{\partial V}{\partial \phi}\right|_0}_{0} \varphi + \frac{1}{2} \underbrace{\left.\frac{\partial^2 V}{\partial \phi^2}\right|_0}_{m_\varphi^2} \varphi^2 + O(\varphi^3)$$

with the mass-squared given simply by the 2nd derivative of the potential at the minimum. This can of course be generalized for several scalars ϕ_i, with the squared mass matrix $M_{ij}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}$.

The use of an effective potential that includes quantum corrections can be used to easily calculate radiative corrections to scalar masses. In fact, this has been applied with advantage to important particle physics models like the SM or the MSSM, to calculate radiative corrections to the Higgs mass. For this reason I will assume ϕ is the Higgs scalar from now on.

In Lect. 1 we understood the effective potential in different ways. In the particular application discussed in this lecture we will make use of the fact that $V_{eff}(\phi)$ is the generator of 1PI Green functions of ϕ at zero-external momentum, see page 1.22:

$$V(\phi) = -\tilde{\Gamma}_\phi^{(0)}(0)$$

Neglecting small width effects, the physical scalar (Higgs) mass is given by the pole of the propagator $\tilde{G}^{(2)}_\phi(p^2)$ or, what is the same, the zero of the inverse of the propagator $\tilde{\Gamma}^{(2)}_\phi(p^2)$ [see footnote in page 1.8 for $\tilde{G}^{(2)}(p^2) = -1/\tilde{\Gamma}^{(2)}(p^2)$]. We can write, up to convention dependent factors of i, -1 in the definition of $\tilde{G}^{(2)}(p^2)$,

$$\tilde{\Gamma}^{(2)}_\phi(p^2) = p^2 - m_0^2 - \Pi_{\phi\phi}(p^2)$$

where $\Pi_{\phi\phi}(p^2)$ is the ϕ 1PI self-energy, and m_0^2 is the tree-level mass (e.g. in the SM, $m_0^2 = m^2 + 3\lambda\phi^2$). The pole mass M_h^2 is therefore determined by the equation

$$\boxed{M_h^2 = m_0^2 + \Pi_{hh}(M_h^2)}$$

$\Pi_{hh}(p^2)$ starts at $O(\hbar)$ and the above equation can be solved perturbatively up to the desired order. At 1-loop order one has

$$M_{h,1}^2 = m_0^2 + \Pi^{(1)}_{hh}(m_0^2)$$

at two-loops:

$$M_{h,2}^2 = m_0^2 + \Pi^{(1)}_{hh}(M_{h,1}^2) + \Pi^{(2)}_{hh}(m_0^2)$$

and so on.

We can compare this pole mass with the mass approximation obtained from simply taking two-derivatives of the effective potential:

$$\frac{\partial^2 V_{eff}}{\partial \phi^2} = \underbrace{\frac{\partial^2 V_0}{\partial \phi^2}}_{\substack{m_0^2 \\ \text{(tree-level)}}} + \underbrace{\frac{\partial^2 \Delta V}{\partial \phi^2}}_{\substack{\text{radiative} \\ \text{correction}}}$$

$$= -\frac{\delta^2 \tilde{\Gamma}^{(0)}}{\delta \phi^2} = -\tilde{\Gamma}^{(2)}_\phi(0) = \left[-p^2+m_0^2+\Pi_\phi(p^2)\right]\Big|_{p^2=0}$$

$$= m_0^2 + \Pi_{hh}(0)$$

Comparing with the previous expression for the pole mass we see that the only difference is that the external momentum is now zero. Calling the previous mass $M_{h,V}^2$ (the "potential approximation" to the mass) we find the relation:

$$\boxed{M_h^2 = M_{h,V}^2 + \Delta\Pi_{hh}(M_h^2)} \quad \text{(M.1)}$$

where $\Delta\Pi_{hh}(M_h^2) = \Pi_{hh}(M_h^2) - \Pi_{hh}(0)$, corrects for the difference in external momentum.

The recipe (M.1) to calculate the Higgs mass deserves some comments:

1. It is convenient to trade the tree-level mass parameter m^2 by the expectation value of the Higgs:

$$\frac{1}{\phi}\frac{\partial V}{\partial \phi}\Big|_{\phi=v} = 0 = \frac{1}{\phi}\left(\frac{\partial V_0}{\partial \phi} + \frac{\partial \Delta V}{\partial \phi}\right)\Big|_v = m^2 + \lambda v^2 + \left(\frac{1}{\phi}\frac{\partial \Delta V}{\partial \phi}\right)_v$$

so that

$$M_{h,V}^2 = \frac{\partial^2 V}{\partial \phi^2}\Big|_v = m^2 + 3\lambda v^2 + \left(\frac{\partial^2 \Delta V}{\partial \phi^2}\right)_v = 2\lambda v^2 + \underbrace{\left(\frac{-1}{\phi}\frac{\partial}{\partial \phi} + \frac{\partial^2}{\partial \phi^2}\right)}_{\mathcal{D}_m^2}\Delta V\Big|_v$$

and noting that $\mathcal{D}_m^2 V_0 = 2\lambda v^2$, we get the simpler expression

$$M_{h,V}^2 = \partial_m^2 V_{eff}$$

which allows to extract $M_{h,V}^2$ without having to worry about the location of the minimum, simply taking derivatives of V_{eff}.

2. In many practical examples it is a good approximation to neglect the tree-level value of the Higgs mass (eg if the one-loop radiative corrections are dominated by some large coupling, like the top Yukawa coupling, while $M_{h,0}^2$ depends on smaller couplings, like g^2 in the MSSM). In such cases, $M_{h,V}^2$ can already capture the most important radiative corrections. It is obviously easier to calculate $M_{h,V}^2$ from the potential that M_h^2 diagrammatically.

3. Even if one wants to include subdominant corrections, it is advantageous to split the calculation in $M_{h,V}^2 + \Delta\Pi_{hh}(M_h^2)$. The potential piece is quite easy and $\Delta\Pi_{hh}(M_h^2)$ is much easier to compute diagrammatically that the full Π_{hh}.

4. It is easy to show that M_h^2 is scale-independent, as it should: From the scale independence of V_{eff} it is immediate that $\partial^2 V_{eff}/\partial\phi^2$ depends on the renormalization scale only through the anomalous dimension $d\phi/d\log Q = \gamma\phi$, so that

$$\frac{d}{d\log Q} M_{h,V}^2 = -2\gamma M_{h,V}^2$$

To see this, simply notice that

$$\frac{dV}{d\log Q} = 0 \quad \forall\phi \Rightarrow \frac{\partial^n}{\partial\phi^n}\frac{dV}{d\log Q} = 0$$

In particular:

$$\frac{\partial}{\partial\phi}\frac{dV}{d\log Q} = \frac{\partial}{\partial\phi}\left[\sum_i \frac{\partial V}{\partial\lambda_i}\beta_{\lambda_i} + \frac{\partial V}{\partial\phi}\gamma\phi\right]V = \underbrace{\sum_i \frac{\partial^2 V}{\partial\lambda_i\partial\phi}\beta_{\lambda_i} + \frac{\partial^2 V}{\partial\phi^2}\gamma\phi}_{\frac{d}{d\log Q}\frac{\partial V}{\partial\phi}} + \frac{\partial V}{\partial\phi}\gamma = 0$$

$$\Rightarrow \qquad \frac{d}{d\log Q}\frac{\partial V}{\partial\phi} = -\gamma\frac{\partial V}{\partial\phi}$$

that is, $\partial/\partial\phi$ and $d/d\log Q$ don't commute (as $\phi=\phi(Q)$).

In the same way, one arrives at

$$\frac{d}{d\log Q}\frac{\partial^n V}{\partial\phi^n} = -n\gamma\frac{\partial^n V}{\partial\phi^n}$$

with $n=2$ giving the result for the scale dep. of $M^2_{h,V}$ above.

On the other hand $\Delta\Pi_{hh}(p^2)$ can only depend on wave-function renormalization, as the p^2-independent pieces cancel out.

In more detail:

bare quantities

$$\mathcal{L} = \frac{1}{2}(\partial\phi_b)^2 - \frac{1}{2}m_b^2\phi_b^2 + \dots$$

renormalized quantities

$$= \frac{1}{2}(\partial\phi_R)^2 - \frac{1}{2}m_R^2\phi_R^2 + \frac{1}{2}\delta Z_H(\partial\phi_R)^2 - \frac{1}{2}\delta m^2\phi_R^2 + \dots$$

$$= \frac{1}{2}\underbrace{(1+\delta Z_H)}_{Z_H}(\partial\phi_R)^2 - \frac{1}{2}(m_R^2+\delta m^2)\phi_R^2 + \dots$$

We have

$$\phi_b^2 = Z_H\phi_R^2 \quad \text{and} \quad \frac{d\phi_b}{d\log Q} = 0 \Rightarrow \gamma \equiv \frac{1}{\phi_R}\frac{d\phi_R}{d\log Q} = -\frac{1}{2}\frac{d\log Z_H}{d\log Q}$$

and, for the inverse propagators:

$$\phi_b\underbrace{I_b(p^2)}_{p^2-m_b^2-\Pi_b(p^2)}\phi_b = \phi_R\underbrace{I_R(p^2)}_{p^2-m_R^2-\Pi_R(p^2)}\phi_R$$

so that

$$\frac{dI_b(p^2)}{d\log Q}=0 \;\Rightarrow\; \frac{d\Pi_R(p^2)}{d\log Q}=2\gamma\, I_R(p^2)-\beta_{m^2}$$

And we have all the ingredients to prove $dM_h^2/d\log Q=0$:

$$\begin{aligned}\frac{d}{d\log Q}M_h^2 &= \frac{d}{d\log Q}\left(M_{h,v}^2+\Delta\Pi(M_h^2)\right)\\ &= -2\gamma M_{h,v}^2+2\gamma I_R(M_h^2)-\cancel{\beta_{m^2}}\\ &\qquad -2\gamma I_R(0)+\cancel{\beta_{m^2}}+\frac{d\Delta\Pi}{dp^2}\cdot\frac{dM_h^2}{d\log Q}\\ &= -2\gamma\underbrace{\left(M_{h,v}^2-M_h^2+\Delta\Pi(M_h^2)\right)}_{0}+\frac{d\Delta\Pi}{dp^2}\cdot\frac{dM_h^2}{d\log Q}\end{aligned}$$

$$\Rightarrow \quad \frac{dM_h^2}{d\log Q}=0$$

5. One can also show that M_h^2 is gauge invariant, even if the explicit expression obtained from the potential can depend explicitly on the gauge fixing parameter ξ: $M_h^2(\phi,\xi)$. In the same manner that one could make use of Ward identities to derive (see sect. 1.8):

$$\xi\,\frac{\partial V(\phi,\xi)}{\partial\xi}+C(\phi,\xi)\,\frac{\partial V(\phi)}{\partial\phi}=0$$

one arrives at the equation

$$\xi\,\frac{\partial M_h^2(\phi,\xi)}{\partial\xi}+C(\phi,\xi)\,\frac{\partial M_h^2(\phi,\xi)}{\partial\phi}=0 \qquad \text{at } \frac{\partial V}{\partial\phi}=0$$

which, using $c(\phi,\xi) = d\phi/d\log\xi$, as in 1.41, leads directly to

$$\frac{dM_h^2(\phi,\xi)}{d\log\xi} = 0$$

which shows M_h^2 is really independent of ξ, as it should be the case of a physical mass.

2.1 Hierarchy Problem from Veff perspective

Consider the expression for the one-loop potential regularized with a momentum cutoff, page 1.29,

$$\delta_1 V(\phi) = \frac{\Lambda^2}{32\pi^2} \,\mathrm{Str}\, M^2(\phi) + \frac{1}{64\pi^2} \,\mathrm{Str}\, M^4(\phi) \left(\log\frac{M^2(\phi)}{\Lambda^2} - \frac{1}{2}\right)$$

with $\mathrm{Str}\, M^2(\phi) = \sum_\alpha N_\alpha M_\alpha^2(\phi)$, with α labelling different particle species with N_α degrees of freedom (negative for fermions) and ϕ-dependent masses $M_\alpha^2(\phi)$. Let us again consider the generic form $M_\alpha^2(\phi) = k_\alpha' + k_\alpha \phi^2$. Plugging this in $\delta_1 V(\phi)$ we see that 1-loop corrections induce a contribution to the Higgs mass term in the potential

$$\delta_1 V(\phi) \supset \frac{1}{2}\left(\frac{\Lambda^2}{16\pi^2}\sum_\alpha N_\alpha k_\alpha\right)\phi^2 = \frac{1}{2}\delta m_1^2 \phi^2$$

which is quadratically divergent. Such correction is at the root of the hierarchy problem that afflicts scalars. Although one could argue that this divergence can be simply renormalized away, if Λ is considered to represent the scale of a new sector coupled to the scalar ϕ, i.e. a physical threshold scale, then $\Lambda^2 \gg m^2$ will tend to increase the mass of ϕ up to Λ, unless this is prevented by fine-tuning the large Λ^2 correction against the tree-level mass m_0^2 of ϕ:

$$m_0^2 + \ast\, \Lambda^2/16\pi^2 \approx O(m_0^2)$$

You can convince yourself that this hierarchy problem does not go away eg in dimensional regularization by considering the simple example of a light scalar ϕ coupled to some other heavy scalar Φ as in:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \left[\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\kappa\phi^2\Phi^2\right]$$

with $M^2 \gg m^2$. We can look now at the effects of Φ on the scalar potential of the light ϕ using the generic result for $V_1(\phi)$, see page 1.29

$$V_1(\phi) = \frac{1}{64\pi^2}\sum_\alpha N_\alpha M_\alpha^4(\phi)\left[\log\frac{M_\alpha^2(\phi)}{Q^2} - c_\alpha\right]$$

where now the field-dependent masses are

$$M_\phi^2 = m^2 + 3\lambda\phi^2$$
$$M_\Phi^2 = M^2 + \kappa\phi^2$$

Plugging this in $V_1(\phi)$ above and expanding in ϕ^2/M^2 one finds

$$\delta_1 V(\phi) \supset \frac{1}{2}\underbrace{\left[\frac{M^2}{16\pi^2}\kappa^2\left(\log\frac{M^2}{Q^2} - 1\right)\right]}_{\delta_1 m^2}\phi^2$$

with M playing the role of Λ.

2.2 Stability of the Vacuum in the SM

With the quite recent discovery of the Higgs at the LHC (assuming it is the Higgs) we know the last parameter of the SM:

$$M_h = 125.7 \pm 0.4 \text{ GeV} \quad \Rightarrow \quad \lambda \simeq 0.126$$

[with λ normalized as $\delta_4 V = (\lambda/4)\phi^4$]. This means, in particular that the shape of the SM potential is now fixed (up to experimental and theoretical uncertainties, e.g. in the top Yukawa coupling, h_t, or the strong gauge coupling constant g_s). In principle this would just correspond to some particular example of a boring Mexican-hat potential, but it is well known since the late 70's that a low Higgs mass and a heavy top mass can destibilize the Higgs effective potential.

Let us have a closer look at the SM potential to understand this. At 1-loop, keeping only the most important top loop corrections, we have (sect. 1.6):

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4}\lambda\phi^4 - \frac{12}{64\pi^2}\left(\frac{1}{2} h_t^2 \phi^2\right)^2 \left[\log \frac{h_t^2 \phi^2}{2Q^2} - \frac{3}{2}\right] + \dots$$

Here Q is the renormalization scale and, as we discussed in Sect. 1.7, all parameters in $V(\phi)$ are evaluated at that same scale. If we choose Q at the electroweak scale, say $Q = M_t$, we know the values of these parameters directly, e.g. $\lambda \simeq 0.126$, as mentioned above. We then see that for $\phi \gg M_t$ the negative top loop contribution can overcome the positive value of λ and drive $V(\phi)$ to negative values, destabilizing the EW vacuum. But we have to worry about how reliable is this estimate as we are finding a 1-loop effect gaining over the tree level result. In other words, is the perturbative expansion reliable here? In principle, there is nothing wrong with a loop effect winning over a tree-level result. After all, the 1-loop correction depends on a large coupling h_t that did not appear at tree level, while the tree-level coupling λ is certainly small. We should rather worry about the possible effects of neglected higher order corrections, that can also be enhanced by large logarithms $\log(\phi^2/M_t^2)$.

The presence of such logarithms in the higher order corrections to $V(\phi)$ is easy to figure out as these logs give the explicit scale dependence of the potential with the renormalization scale (remember the discussion in sect. 1.7).

Assume for simplicity that h_t is the dominant coupling in the loop corrections and use $\alpha \equiv h_t^2/(4\pi^2)$ as expansion parameter, sometimes enhanced by powers of $\log \equiv \log(\phi^2/Q^2)$. We can display the structure of higher order loop corrections to $V(\phi)$ by looking eg. at the corrections they imply for a dimensionless quatity like the quartic coupling. This structure is schematically the following

"finite" pieces

$$\begin{array}{llllllll}
\text{tree:} & \hbar^0: & \alpha^0 & & & & & \\
\text{1-loop:} & \hbar: & \alpha\log & + & \alpha & & & \\
\text{2-loop:} & \hbar^2: & (\alpha\log)^2 & + & \alpha^2\log & + & \alpha^2 & \\
 & \vdots & & & & & & \\
\text{n-loop:} & \hbar^n: & (\alpha\log)^n & + & \alpha(\alpha\log)^{n-1} & + & \cdots + \alpha^n\log & + \;\alpha^n \\
 & & \text{Leading logs} & & \text{NLO} & & \cdots &
\end{array}$$

For large $\log \phi^2/M_i^2$, the dominant term at a given order is the leading log (LO) term, $(\alpha\log)^n$, followed by the next-to-leading log (NLO) term $\alpha(\alpha\log)^{n-1}$, and so on.

We can rewrite the iterative equation we obtained (p. 1.37) for the scale-dependence of $V_n(\phi)$:

$$\frac{\partial V_n}{\partial \log Q} + \mathcal{D}^{(1)} V_{n-1} + \ldots + \mathcal{D}^{(n)} V_0 = 0$$

where $\mathcal{D}^{(n)} \equiv \beta^{(n)}_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \gamma^{(n)} \frac{\partial}{\partial \phi}$, in terms of the hierarchical pieces of each loop contribution to V. Matching powers of logs and α we get:

$$\frac{\partial V_n^{LO}}{\partial \log Q} + \mathcal{D}^{(1)} V_{n-1}^{LO} = 0$$

$$\frac{\partial V_n^{NLO}}{\partial \log Q} + \mathcal{D}^{(1)} V_{n-1}^{NLO} + \mathcal{D}^{(2)} V_{n-2}^{LO} = 0$$

and so on.

From the first equation we see that V_n^{LO} can be obtained iteratively to <u>all orders</u> just from V_0 and $\mathcal{D}^{(1)}$, i.e. 1-loop RGEs. From the second we see that V_n^{NLO} can be obtained iteratively to <u>all orders</u> if, in addition we know V_1 and $\mathcal{D}^{(2)}$, i.e. the 1-loop potential and the 2-loop RGEs. You could explicitly check that this is indeed the case using the expression of pages 1.31-32 for V_2 in the SM.

This can be used with advantage to improve the calculation of the potential at large values of ϕ. If we were to include the whole series of LO terms, to all loops, with Q fixed to some particular value, this series would simply resum to the tree level potential with parameters evaluated at the scale Q and ran from M_t to Q with <u>1-loop</u> RGEs. This is the so-called RG-improved tree-level potential, and resums LO terms to all loops. The wisest choice of scale when one is interested in knowing the potential at some field value ϕ is simply $Q=\phi$. The RG-improved potential is therefore

$$\tilde{V}_0(\phi) = \frac{1}{2} m^2(\phi)\, \Phi^2 + \frac{1}{4} \lambda(\phi)\, \Phi^4$$

with $dm^2(\phi)/d\log\phi = \beta_{m^2}^{(1)}$; $d\lambda(\phi)/d\log\phi = \beta_\lambda^{(1)}$ and we have introduced $\Phi = \Phi(\phi)$ with $d\Phi(\phi)/d\log\phi = \gamma^{(1)}\Phi(\phi)$. One can see how this precisely corresponds to summing the whole series either explicitly :

$$\lambda(\phi) = \lambda(M_t) + \int_{Q=M_t}^{\phi} \beta_\lambda^{(1)}(t)\,\underbrace{dt}_{d\log Q}$$

$$= \lambda(M_t) + \int_{M_t}^{\phi} \Big[\beta_\lambda^{(1)}(\underset{\uparrow \log M_t}{t_0}) + (t - t_0)\,\frac{d\beta_\lambda^{(1)}}{dt}(t_0) + \dots \Big]\,dt$$

$$= \lambda(M_t) + \beta_\lambda^{(1)}(t_0)\log\frac{\phi}{M_t} + \frac{1}{2}\,\beta_\lambda^{(1)\prime}(t_0)\left(\log\frac{\phi}{M_t}\right)^2 + \dots$$

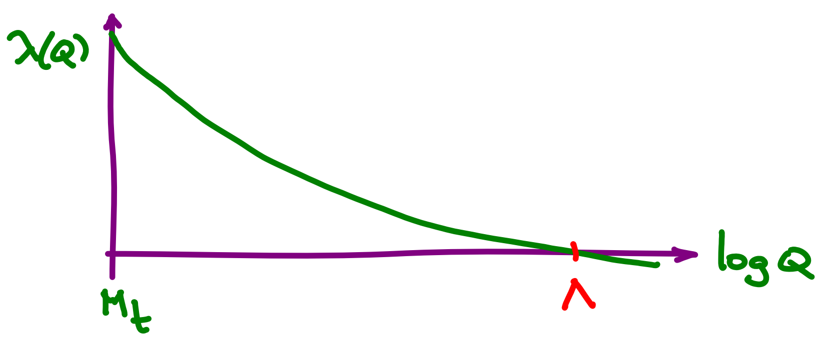
or just remembering that V is scale independent. A different choice of scale simply redistributes the weight of different loop-order contributions: a smart choice of Q moves the bulk of the corrections to the tree-level part.

In any case, with the RG-improved potential, the stability problem gets rephrased into the behaviour of the running $\lambda(\phi)$ at large values of ϕ (i.e. in the UV). The 1-loop beta function describing such running is

$$\beta_\lambda^{(1)} = \frac{1}{16\pi^2}\Big[-6h_t^4 + 12\lambda h_t^2 + \frac{3}{8}(2g^4 + G^4) - 3\lambda(2g^2 + G^2) + 24\lambda^2\Big]$$

and it is dominated by the negative top contribution. The origin of this contribution, a top loop, is of course exactly the same that gives the dangerous loop contribution in the potential.

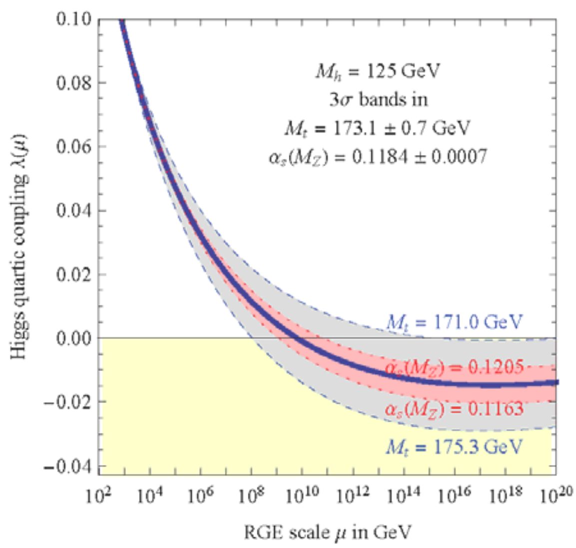
The running λ decreases in the UV:



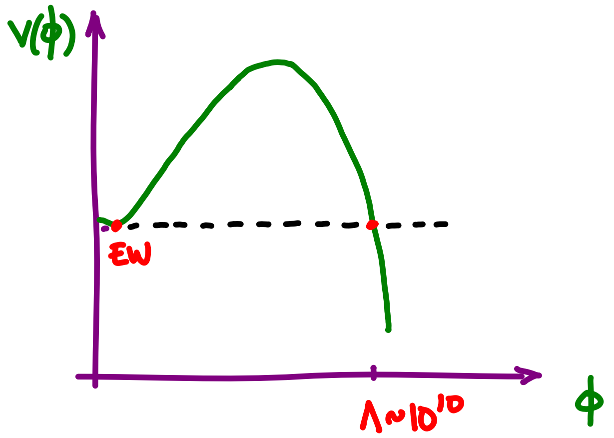
eventually turning negative at some scale Λ and destabilizing the potential at $\phi \sim \Lambda$, confirming the stability problem announced by the 1-loop potential.

One can go of course beyond the RG-improved tree-level potential, adding more subdominant corrections, but there is no qualitative change on the effect just described. The state-of-the-art calculation makes use of the 2-loop potential with RGEs running at 3-loops (so that NNLO corrections are resummed). It is also necessary to determine with precision the input values of λ and h_t at the EW scale, relating $\lambda(M_t)$ and $h_t(M_t)$ to the physical pole masses of the Higgs and top. To be consistent with the NNLO calculation, those matching relations for λ and h_t need to be known up to $O(\alpha^2)$, where α here is $h_t^2/(4\pi)$ or α_s.

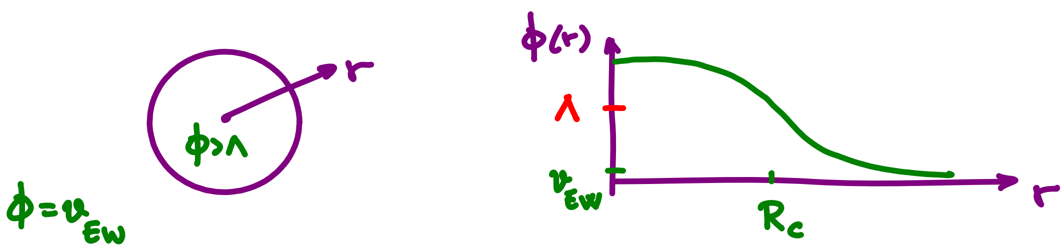
The running of λ at this level of precision for $M_h = 125$ GeV and $M_t = 173.1 \pm 0.7$ GeV, $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ is as indicated in this plot, showing an instability at $\Lambda \sim 10^{10}$ GeV (for the central M_t, α_s values). Stability of the potential all the way up to M_{Pl} seems still possible, but marginal.



We conclude from the previous result that most likely the SM Higgs potential has this shape:



(under the strong assumption that there is no physics beyond the SM below Λ!). If this is the case, we are living in a metastable vacuum and decay to a deeper vacuum at $\phi > \Lambda$ becomes possible (either by quantum tunneling through the huge barrier at $\phi < \Lambda$ or by thermal fluctuations over that barrier in the early universe). While the thermal tunneling depends on the early universe history (in particular on the reheating temperature) the likelihood of quantum tunneling decay only depends on the age (and size) of the universe. The probability of this non-perturbative process is controlled by the action of a field configuration that probes the instability region



Configurations like this, with size bigger than some critical

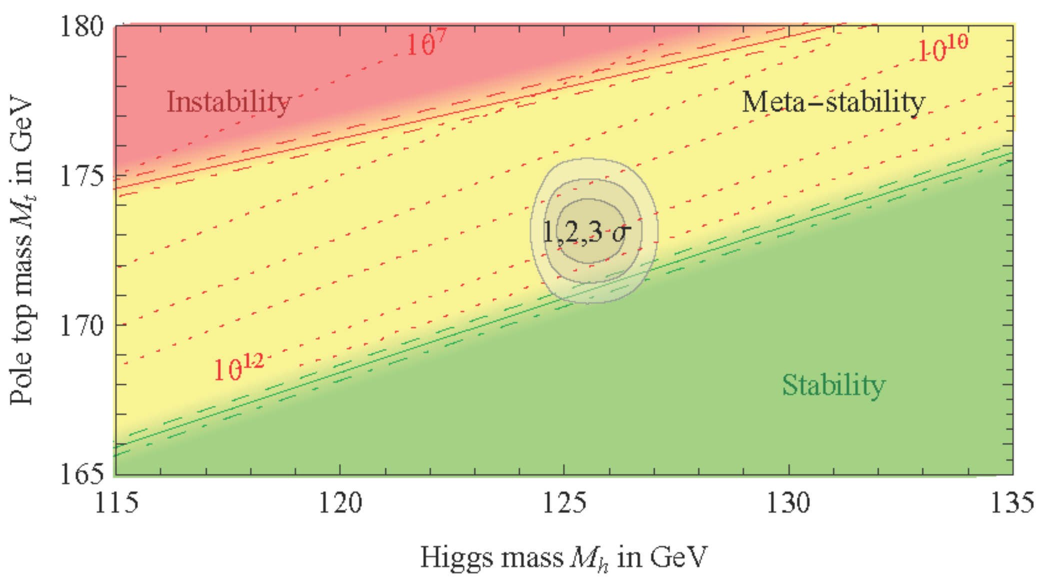
radius will grow and fill out the universe with the $\phi > \Lambda$ phase.
It can be easily shown that the action for the critical bubble, with a potential $V(\phi) = -\frac{|\lambda|}{4}\phi^4$, as in the SM, is simply $S_c = 8\pi^2/(3|\lambda|)$. As λ runs with scale, the tunneling will occur at the scale that minimizes S_c and one gets the tunneling probability:

only scale in the problem.

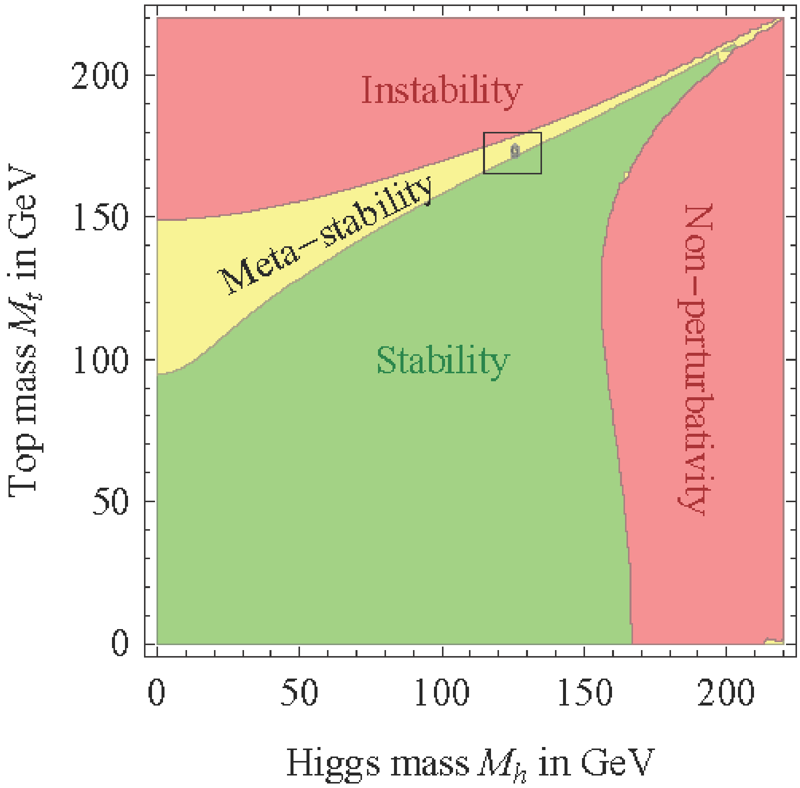
$$p = \underset{\phi > \Lambda}{\mathrm{Max}}\ V_U\,\phi^4 \exp\left[-\frac{8\pi^2}{3|\lambda(\phi)|}\right]$$

space-time volume of past light-cone.

It turns out that for $M_h \sim 125$ GeV, $|\lambda(\phi)|$ stays quite small (~ 0.01) for $\Lambda < \phi < M_{pl}$ and the lifetime of the EW vacuum is huge. Therefore, the metastability of the EW vacuum poses no problem for the SM.
The phase-diagram of the SM in the M_h-M_t plane looks like this:



Or, zooming out in the same plane :



If the SM is all there is (up to M_{Pl}), we live in a rather special location of parameter space.

Bibliography

The first calculation of 1-loop radiative corrections to the Higgs mass in the SM, diagrammatic, is in

- A. Sirlin, R. Zucchini, NPB266 (1986) 389.

The simpler effective potential approach can be found e.g. in

- A. Casas et al, NPB436 (1995) 3.

The state-of-the-art calculation is in

- G. Degrassi et al., JHEP08 (2012) 098.

where a detailed discussion of implications for vacuum stability is carried out.

For an effective potential calculation of the Higgs mass in the MSSM, see e.g.

- Y. Okada, M. Yamaguchi, T. Yanagida, PTP85 (1991) 1. J. Ellis, G. Ridolfi, F. Zwirner, PLB 257 (1991) 83, R. Barbieri, M. Frigeni, PLB 258 (1991) 395.

for the pioneering 1-loop calculations, and

- J.R. Espinosa, R.J. Zhang, JHEP03 (2000) 026

for the first 2-loop calculation.

For a discussion of the hierarchy problem based on the effective potential with aplications to SUSY and see-saw neutrino scenarios, see

- J.A. Casas, J.R. Espinosa, I. Hidalgo, JHEP11 (2004) 057.

The possibility that our EW vacuum might be metastable has a long history, with the first studies being

- N. Cabibbo et al., NPB 158 (1979) 295, P.Q. Hung, PRL 42 (1979) 873. M. Lindner, ZPC 31 (1986) 295. M. Sher Phys. Rep. 179 (1989) 273.

and a very long list of subsequent work on this, including:

- G. Altarelli, G. Isidori, PLB 337 (1994) 141, J.A. Casas, J.R. Espinosa, M. Quirós, PLB 342 (1995) 171, PLB 382 (1996) 374, G. Isidori, G. Ridolfi, A. Strumia, JHEP 01 (2002) 041, G. Isidori, V.S. Rychkov, A. Strumia, N. Tetradis, PRD 77 (2008) 025034, J. Ellis et al, PLB 679 (2009) 369, J. Elias-Miró et al, PLB 709 (2012) 222.

One particularly nice early paper on this topic is

- P. Arnold, PRD 40 (1989) 613.