

Neutrino masses and mixings and FCNC's in a minimal S_3 –invariant extension of the Standard Model

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Neutrino Oscillations I

Neutrinos are emitted or absorbed in one of three well defined **flavour** eigenstates. Each flavour eigenstate is a superposition of three mass eigenstates

$$|\nu_\alpha(o)\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle \quad \alpha = e, \mu, \tau.$$

After a time t , the neutrino travels a distance L ,

$$|\nu_i(t)\rangle = e^{-i(E_i t - p_i L)} |\nu_i(o)\rangle, \quad L \text{ distance from the source}$$

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p}, \quad p^2 \gg m_i^2$$

The probability of detecting the neutrino emitted as $|\nu_\alpha\rangle$ in an eigenstate $|\nu(\beta)\rangle$ of flavour β is

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= |\langle \nu_\beta | \nu_\alpha(L) \rangle|^2 \\ &= \delta_{\alpha\beta} - 4 \sum_{i,j=1}^3 \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2(1.27 \Delta m^2 (L/E)) \\ &\quad + 2 \sum_{i,j=1}^3 \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2(2.54 \Delta m^2 (L/E)) \end{aligned}$$

$U_{\alpha i}$ is the mixing matrix.

Neutrino Oscillations II

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta) &= | \langle \nu_\beta | \nu_\alpha(L) \rangle |^2 \\
 &= \delta_{\alpha\beta} - 4 \sum_{i,j=1}^3 \text{Re}[U_{\alpha_i}^* U_{\beta_i} U_{\alpha_j} U_{\beta_j}^*] \sin^2(1.27 \Delta m_{i,j}^2 (L/E)) \\
 &+ 2 \sum_{i,j=1}^3 \text{Im}[U_{\alpha_i}^* U_{\beta_i} U_{\alpha_j} U_{\beta_j}^*] \sin^2(2.54 \Delta m_{ij}^2 (L/E))
 \end{aligned}$$

$\Delta m_{ij}^2 = m_i^2 - m_j^2$, L distance in **Km** from the source, E energy in **Gev**.

Table 1: (1 March 2012)

Parameter	Best fit
$\Delta m_{21}^2 (10^{-5} \text{ eV}^2)$	$7.65^{+0.23}_{-0.20}$
$ \Delta m_{31}^2 (10^{-3} \text{ eV}^2)$	$2.4^{+0.12}_{-0.11}$
$\sin^2 \theta_{12}$	$0.304^{+0.022}_{-0.016}$
$\sin^2 \theta_{23}$	$0.50^{+0.07}_{-0.06}$
$\sin^2 \theta_{13}$	$0.092^{+0.016}_{-0.016}$

Massive neutrinos in the theory

- In the Standard Model of fundamental interactions lepton flavour is conserved and neutrinos are massless
- To accommodate neutrino masses in the theory, in a systematic and consistent way, we formulated a Minimal, S_3 —invariant Extension of the Standard Model (J. Kubo, A. Mondragón, M. Mondragón and E. Rodríguez-Jáuregui, Prog. Theor. Phys. **109**, (5) 795-807 (2003))
- The extended Higgs sector gives mass to all fermions in the theory and makes the theory more flavour symmetric
- The S_3 flavour permutational symmetry allows for a systematic treatment of masses and mixings of all particles (quarks, leptons and Higgs') while reducing drastically the number of free parameters in the theory
- A number of predictions in other sectors of the theory is made (**FCNC's and contributions of the neutral currents to the magnetic anomaly of the muon**) in agreement with experiment.

Flavour permutational symmetry

- Prior to the introduction of the Higgs boson and mass terms, the Lagrangian of the Standard Model is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. $G_F \sim S_{3L} \otimes S_{3R}$
- Charged currents J_μ are invariant under G_F if the d and u -type fields are transformed with the same family group matrix

$$J_\mu = -i\bar{u}_L\gamma_\mu d_L + h.c. \Rightarrow G_F \sim S_3 \subset S_{3L} \otimes S_{3R}$$

- When $\langle 0|\Phi_H|0 \rangle \neq 0$, the Yukawa couplings give mass to quarks and leptons, if we assume that the S_3 permutational symmetry is not broken

$$M_q = m_{3q} \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

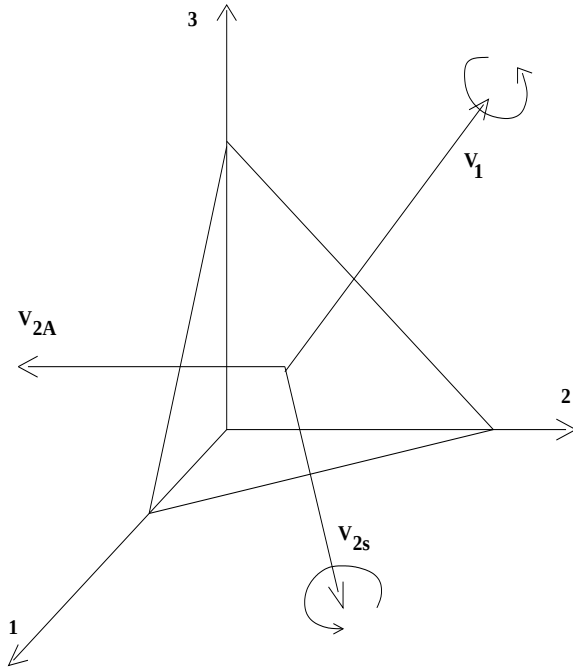
$$\begin{aligned} m_t \neq 0, \quad m_e = m_\mu = 0; \quad m_b \neq 0, \quad m_s = m_d = 0 \\ m_\tau \neq 0, \quad m_\mu = m_e = 0; \quad m_{\nu_\tau} = m_{\nu_\mu} = m_{\nu_e} = 0 \end{aligned}$$

$$\mathbf{V} = \mathbf{1}$$

There is no mixing nor CP- violation.

The Group S_3

The group S_3 of permutations of three objects



Permutations

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \iff$$

Rotations

a 120° – rotation around the
invariant vector $\mathbf{V}_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \iff$$

a 180° rotation around the
invariant vector $\mathbf{V}_{2s}$

Symmetry adapted basis

$$|v_{2A}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad |v_{2s}\rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad |v_1\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Irreducible representations of S_3

The group S_3 has two one-dimensional irreps (singlets) and one two-dimensional irrep (doublet)

- one dimensional: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- Two - dimensional: $\mathbf{2}$ doublet

Direct product of irreps of S_3

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \quad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}$$

the direct (tensor) product of two doublets

$$\mathbf{p}_D = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \quad \text{and} \quad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

has two singlets, r_s and r_A , and one doublet r_D^T

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2} \quad \text{is invariant}, \quad r_A = p_{D1}q_{D2} - p_{D2}q_{D1} \quad \text{is not invariant}$$

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

A Minimal S_3 invariant extension of the SM

The Higgs sector is modified,

$$\Phi \rightarrow H = (\Phi_1, \Phi_2, \Phi_3)^T$$

H is a reducible $\mathbf{1}_s \oplus \mathbf{2}$ rep. of S_3

$$H_s = \frac{1}{\sqrt{3}}(\Phi_1 + \Phi_2 + \Phi_3)$$

$$H_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Quark, lepton and Higgs fields are

$$Q^T = (u_L, d_L), u_R, d_R, \quad L^T = (\nu_L, e_L), e_R, \nu_R, \quad H$$

All these fields have three species (flavours) and belong to a reducible $\mathbf{1} \oplus \mathbf{2}$ rep. of S_3

Leptons' Yukawa interactions

Leptons

$$\begin{aligned}\mathcal{L}_{Y_E} &= -Y_1^e \bar{L}_I H_S e_{IR} - Y_3^e \bar{L}_3 H_S e_{3R} - Y_2^e [\bar{L}_I \kappa_{IJ} H_1 e_{JR} + \bar{L}_I \eta_{IJ} H_2 e_{JR}] \\ &- Y_4^e \bar{L}_3 H_I e_{IR} - Y_5^e \bar{L}_I H_I e_{3R} + h.c.,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \bar{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \bar{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &- Y_2^\nu [\bar{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \bar{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR}] \\ &- Y_4^\nu \bar{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \bar{L}_I (i\sigma_2) H_I^* \nu_{3R} + h.c.\end{aligned}$$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad I, J = 1, 2$$

Furthermore, the Majorana mass terms for the right handed neutrinos are

$$\mathcal{L}_M = -\nu_{IR}^T C \mathbf{M}_I \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R},$$

C is the charge conjugation matrix.

Mass matrices

We will assume that

$$\langle H_{D1} \rangle = \langle H_{D2} \rangle \neq 0 \quad \text{and} \quad \langle H_3 \rangle \neq 0$$

and

$$\langle H_3 \rangle^2 + \langle H_{D1} \rangle^2 + \langle H_{D2} \rangle^2 \approx \left(\frac{246}{2} \text{GeV} \right)^2$$

Then, the Yukawa interactions yield mass matrices of the general form

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$M_\nu = M_{\nu D} \tilde{M}^{-1} (M_{\nu D})^T \quad \text{with} \quad \tilde{M} = \text{diag}(M_1, M_2, M_3)$$

Mixing matrices

The mass matrices are diagonalized by unitary matrices

$$U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} \mathbf{U}_{d(u,e)R} = \text{diag}\left(m_{d(u,e)} m_{s(c,\mu)} m_{b(t,\tau)}\right)$$

and

$$U_\nu^T M_\nu U_\nu = \text{diag}\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right)$$

The masses can be complex, and so, U_{eL} is such that

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}\left(|m_e|^2, |m_\mu|^2, |m_\tau|^2\right), \quad \text{etc.}$$

The quark mixing matrix is

$$V_{CKM} = U_{uL}^\dagger U_{dL}$$

and, the neutrino mixing matrix is

$$\mathbf{V}_{MNS} = U_{eL}^\dagger U_\nu$$

Masses and mixings in the quark sector

The mass matrices for the quark sector take the general form

$$\mathbf{M}_{u(d)} = \begin{pmatrix} \mu_1^{u(d)} + \mu_2^{u(d)} & \mu_2^{u(d)} & \mu_5^{u(d)} \\ \mu_2^{u(d)} & \mu_1^{u(d)} - \mu_2^{u(d)} & \mu_5^{u(d)} \\ \mu_4^{u(d)} & \mu_4^{u(d)} & \mu_3^{u(d)} \end{pmatrix}$$

$$U_{u(d)L}^\dagger M_{u(d)} M_{u(d)}^\dagger U_{u(d)L} = \text{diag}\left(|m_{u(d)}|^2, |m_{c(s)}|^2, |m_{t(b)}|^2\right),$$

$$V_{CKM} = U_{uL}^\dagger U_{dL}$$

The set of dimensionless parameters

$$\begin{aligned} \mu_1^u/\mu_0^u &= -0.000293, & \mu_2^u/\mu_0^u &= -0.00028, & \mu_3^u/\mu_0^u &= 1, \\ \mu_4^u/\mu_0^u &= 0.031, & \mu_5^u/\mu_0^u &= 0.0386, \\ \mu_1^d/\mu_0^d &= 0.0004, & \mu_2^d/\mu_0^d &= 0.00275, & \mu_3^d/\mu_0^d &= 1 + 1.2I, \\ \mu_4^d/\mu_0^d &= 0.283, & \mu_5^d/\mu_0^d &= 0.058, \end{aligned}$$

The quark mixing matrix

Yields the mass hierarchy and the mixing matrix

$$\begin{aligned} m_u/m_t &= 2.5469 \times 10^{-5}, & m_c/m_t &= 3.9918 \times 10^{-3}, \\ m_d/m_b &= 1.5261 \times 10^{-3}, & m_s/m_b &= 3.2319 \times 10^{-2}, \end{aligned}$$

The computed mixing matrix is

$$\mathbf{V}_{CKM} = \begin{pmatrix} 0.968 + 0.117I & 0.198 + 0.0974I & -0.00253 - 0.00354I \\ -0.198 + 0.0969I & 0.968 - 0.115I & -0.0222 - 0.0376I \\ 0.00211 + 0.00648I & 0.0179 - 0.0395I & 0.999 - 0.00206I \end{pmatrix}$$

$$|\mathbf{V}_{CKM}^{th}| = \begin{pmatrix} 0.975 & 0.221 & 0.00435 \\ 0.221 & 0.974 & 0.0437 \\ 0.00682 & 0.0434 & 0.999 \end{pmatrix}$$

which should be compared with

$$|V_{CKM}^{exp}| = \begin{pmatrix} 0.97383 \pm 0.00024 & 0.2272 \pm 0.0010 & (3.96 \pm 0.09) \times 10^{-3} \\ 0.2271 \pm 0.010 & 0.97296 \pm 0.00024 & (42.21 \pm 0.45 \pm 0.09) \times 10^{-3} \\ (8.14 \pm 0.5) \times 10^{-3} & (41.61 \pm 0.12) \times 10^{-3} & 0.9991 \pm 0.000034 \end{pmatrix}$$

The Jarlskog invariant is

$$J = \text{Im} [(V_{CKM})_{11}(V_{CKM})_{22}(V_{CKM}^*)_{12}(V_{CKM}^*)_{21}] = 2.9 \times 10^{-5}, \quad J^{\text{exp}} = (3.0 \pm 0.3) \times 10^{-5}$$

The leptonic sector

To achieve a further reduction of the number of parameters, in the leptonic sector, we introduce an additional discrete Z_2 symmetry

—	+
H_I, ν_{3R}	$H_S, L_3, L_I, e_{3R}, e_{IR}, \nu_{IR}$

then,

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$$

Hence, the leptonic mass matrices are

$$M_e = \begin{pmatrix} \mu_2^e & \mu_2^e & \mu_5^e \\ \mu_2^e & -\mu_2^e & \mu_5^e \\ \mu_4^e & \mu_4^e & 0 \end{pmatrix} \quad \text{and} \quad M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}$$

The Mass Matrix of the charged leptons as function of its eigenvalues

The mass matrix of the charged leptons is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}.$$

$$x = m_e/m_\mu, \tilde{m}_\mu = m_\mu/m_\tau \text{ and } \tilde{m}_e = m_e/m_\tau$$

This expression is accurate to order 10^{-9} in units of the τ mass

There are no free parameters in $\mathbf{M}_e$ other than the Dirac Phase $\delta!!$

The Unitary Matrix U_{eL}

The unitary matrix U_{eL} is calculated from

$$U_{eL}^\dagger M_e M_{eL}^\dagger U_{eL} = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

We find

$$U_{eL} = \Phi_{eL} O_{eL}, \quad \Phi_{eL} = \text{diag}[1, 1, e^{i\delta_D}]$$

and

$$O_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix},$$

$$x = m_e/m_\mu, \quad \tilde{m}_\mu = m_\mu/m_\tau \text{ and } \tilde{m}_e = m_e/m_\tau$$

The neutrino mass matrix I

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$\mathbf{M}_\nu = \mathbf{M}_{\nu D} \tilde{\mathbf{M}}_R^{-1} \mathbf{M}_{\nu D}^T$$

with

$$\tilde{\mathbf{M}}_R = \text{diag}[M_1, M_2, M_3] \quad M_1 \neq M_2 \neq M_3$$

and

$$\mathbf{M}_{\nu D} = \begin{pmatrix} \mu_2 & \mu_2 & 0 \\ \mu_2 & -\mu_2 & 0 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

Then

$$M_\nu = \begin{pmatrix} \frac{2\mu_2^2}{\bar{M}} & \lambda \frac{2\mu_2^2}{\bar{M}} & \frac{2\mu_2\mu_4}{\bar{M}} \\ \lambda \frac{2\mu_2^2}{\bar{M}} & \frac{2\mu_2^2}{\bar{M}} & \lambda \frac{2\mu_2\mu_4}{\bar{M}} \\ \frac{2\mu_2\mu_4}{\bar{M}} & \lambda \frac{2\mu_2\mu_4}{\bar{M}} & \frac{2\mu_4^2}{\bar{M}} + \frac{\mu_3^2}{\bar{M}} \end{pmatrix}.$$

$$\frac{1}{\bar{M}} = \frac{1}{2} \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \quad \text{and} \quad \lambda = \frac{M_2 - M_1}{M_1 + M_1}$$

The neutrino mass matrix II

When $\bar{M}_R$ is real, the phases in M_ν may be factorized out as

$$M_\nu = P_\nu \bar{M}_\nu P_\nu \quad (1)$$

where

$$P_\nu = e^{i\phi} \text{diag}[1, 1, e^{i\delta_\nu}] \quad (2)$$

Then, the real symmetric matrix $\bar{M}_\nu$ may be diagonalized by means of an orthogonal matrix O_ν

$$M_\nu = P_\nu O_\nu [\text{diag}(|m_{1\nu}|, |m_{2\nu}|, |m_{3\nu}|)] O^T P_\nu \quad (3)$$

Therefore, the unitary matrix

$$U_\nu = P_\nu O_\nu \quad (4)$$

diagonalizes the neutrino mass matrix M_ν .

The neutrino mass matrix III

The reduced neutrino mass matrix $\bar{M}_\nu$ is reparametrized in terms of the neutrino masses $|m_i|$ and two free parameters μ_o and d , as

$$\bar{M}_\nu = \begin{pmatrix} \mu_o + \frac{1}{2}d & \frac{1}{2}d & \frac{1}{\sqrt{2}}(a + c) \\ \frac{1}{2}d & \mu_o + \frac{1}{2}d & -\frac{1}{\sqrt{2}}(a - c) \\ \frac{1}{\sqrt{2}}(a + c) & -\frac{1}{\sqrt{2}}(a - c) & b + \mu_o \end{pmatrix}$$

where

$$\begin{aligned} a^2 &= -\frac{(|m_1| - \mu_o)(|m_2| - \mu_o)(|m_3| - \mu_o)}{d}, & \mu_o &= \frac{2|\mu_2^2|}{\bar{M}}(1 - \lambda) \\ b &= |m_1| + |m_2| + |m_3| - d - 3\mu_o & d &= \lambda \frac{2|\mu_2|^2}{\bar{M}} \\ c^2 &= \frac{1}{d}[(d - |m_1|)(d - |m_2|)(d - |m_3|)] \end{aligned}$$

The Orthogonal Matrix O_ν

The real orthogonal matrix that diagonalizes $\bar{M}_\nu$ and M_ν is obtained from the eigenvectors of $\bar{M}_\nu$

$$O_\nu = \left[|m_1 \rangle, |m_2 \rangle, |m_3 \rangle \right]$$

$$|m_i \rangle = \begin{pmatrix} \left[\frac{-\sigma_{i+1}\sigma_{i+2}(\sigma_i-d)}{d(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \\ \left[\frac{\sigma_i(\sigma_i-d)}{(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \\ \left[\frac{\sigma_i(d-\sigma_{i+1})(d-\sigma_{i+2})}{d(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \end{pmatrix}, \quad i = 1, 2, 3, \dots \text{mod}(3)$$

where $\sigma_i = (m_i - \mu_o)$.

The unitary matrix U_ν that diagonalizes M_ν is

$$U_\nu = P_\nu O_\nu$$

The neutrino mixing matrix I

$$V_{PMNS}^{th} = U_{eL}^\dagger U_\nu.$$

The theoretical mixing matrix V_{PMNS}^{th} is

$$V_{PMNS}^{th} = O_{eL}^T P_{\nu-e} O_\nu K$$

where $P_{\nu-e}$ is a diagonal matrix of phases

$$P_{\nu-e} = \text{diag}[1, 1, e^{i(\delta_\nu - \delta_e)}]$$

Hence

$$\left(V_{PMNS}^{th}\right)_{ij} = \sum_{r=1}^3 \left(O_{eL}\right)_{ri} \left(P_{\nu-e}\right)_r \left(O_\nu\right)_{rj}$$

From a comparison of V_{PMNS}^{PDG} with V_{PMNS}^{th} , we obtain the neutrino mixing angles as functions of the lepton masses.

The neutrino mixing matrix II

From a comparison of V_{PMNS}^{th} with V_{PMNS}^{PDG} , we obtain the neutrino mixing angles as function of the lepton masses

$$\sin^2 \theta_{12}^\nu = \frac{|(V_{PMNS})_{12}|^2}{1 - |(V_{PMNS})_{13}|^2}$$

$$\sin^2 \theta_{23}^\nu = \frac{|(V_{PMNS})_{23}|^2}{1 - |(V_{PMNS})_{13}|^2}$$

$$\sin^2 \theta_{13}^\nu = |(V_{PMNS})_{13}|^2$$

Neutrino Mixing Angles I

The solar angle θ_{12} is strongly dependent on the neutrino masses but depends only very weakly on the charged lepton masses

$$\tan \theta_{12}^2 = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2 \phi_\nu)^{1/2} - |m_{\nu_3}| |\cos \phi_\nu|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2 \phi_\nu)^{1/2} + |m_{\nu_3}| |\cos \phi_\nu|}.$$

the numerical value of $\tan^2 \theta_{12}$ fixes the scale of the neutrino masses.

The atmospheric mixing angle θ_{23} depends mostly on the charged lepton masses

$$\sin \theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}} = 0.7071$$

$$x = m_e/m_\mu = 4.84 \times 10^{-3}, \quad \tilde{m}_\mu = m_\mu/m_\tau = 5.95 \times 10^{-2}$$

Neutrino Mixing Angles II

The reactor mixing angle θ_{13} is mostly determined by the interplay of the S_3 symmetry and the mass splitting of the right-handed neutrinos in the see saw mechanism plus a very small contribution from the charged leptons,

$$\begin{aligned} \sin \theta_{13} \approx & \frac{2(\lambda\mu)m_{\nu 3}}{m_{\nu 1} - m_{\nu 2}} \left(1 - \sqrt{\frac{(m_{\nu 2} - m_{\nu 3})}{m_{\nu 3} - m_{\nu 1}}} \right) \left(\cos \eta - \sqrt{\left(1 - \frac{m_{\nu 1}}{m_{\nu 3}} \right) \left(\frac{m_{\nu 2}}{m_{\nu 3}} - \frac{m_{\nu 1}}{m_{\nu 3}} \right)} \sin \eta \right) \\ & + \frac{1}{\sqrt{2}} \frac{m_e}{m_\mu} \frac{\left(1 + 4\left(\frac{m_e}{m_\mu}\right)^2 - \left(\frac{m_\mu}{m_\tau}\right)^4 \right)}{\sqrt{1 + \left(\frac{m_\mu}{m_\tau}\right)^2 + 5\left(\frac{m_e}{m_\mu}\right)^2 - \left(\frac{m_\mu}{m_\tau}\right)^4}} \end{aligned}$$

where

$$\cos \eta = \sqrt{\frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 2} - m_{\nu 1}}}, \quad \sin \eta = \sqrt{\frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 1}}}$$

We get

$$\sin \theta_{13}^{th} \approx 0.151 \quad \text{with} \quad (\lambda\mu) \approx 0.02$$

Neutrino Mixing Angles: Theory vs Experiment

The most recent experimental values of the neutrino mixing angles θ_{13} and θ_{23} (T. Schwetz, M. Tortola and J.W.F. Valle, New J. Phys. 13, 063004 (2011) and arXiv: 1108.137 v1[hep-ph] 4 Aug 2011; G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, arXiv: 1106.6028 v2 [hep-ph] 26 Aug 2011 and Daya Bay Collaboration arXiv: 1203.1669 v1 [hep-ex] 8 Mar 2012)

$$\bar{\theta}_{13}^{o\ exp} = 8.7_{-1.2}^{+1.2} \rightarrow \sin^2 2\bar{\theta}_{13}^{o\ exp} = 0.092_{-0.016}^{+0.016}; \text{ (T2K, MINOS, Daya Bay (8 Mar 2012))}$$

$$\bar{\theta}_{23}^{o\ exp} = 46_{-3}^{+3} \quad \sin^2 \bar{\theta}_{23}^{o\ exp} = 0.50_{-0.06}^{+0.06}$$

Our theoretical values are (Phys. Rev. D **76**, 076003 (2007) and this work)

$$\theta_{13}^{o\ th} = 8.67_{-2.0}^{+2.0} \quad \sin^2 \theta_{13}^{o\ th} = 0.022$$

$$\theta_{23}^{o\ th} = 44.97_{-1.2}^{+1.2} \quad \sin^2 \theta_{23}^{o\ th} = 0.50$$

in very good agreement with the experimental values !!!

The neutrino mass spectrum I

In the present model, the experimental restriction

$$|\Delta m_{21}^2| < |\Delta m_{23}^2|$$

implies an inverted neutrino mass spectrum $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$

From our previous expressions for $\tan \theta_{12}$

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2 \cos \phi_\nu \tan \theta_{12}} \frac{1 - \tan^4 \theta_{12} + r^2}{\sqrt{1 + \tan^2 \theta_{12}} \sqrt{1 + \tan^2 \theta_{12} + r^2}},$$

where $r = \Delta m_{21}^2 / \Delta m_{23}^2$.

Then, the mass $|m_{\nu_3}|$ is approximately given by

$$|m_{\nu_3}| \approx \frac{1}{2} \frac{\sqrt{\Delta m_{13}^2}}{\tan \theta_{12}} (1 - \tan^2 \theta_{12})$$

Neutrino mass spectrum II

- We wrote the neutrino mass differences, $m_{\nu_i} - m_{\nu_j}$, in terms of the differences of the squared masses $\Delta_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$ and one of the neutrino masses, say m_{ν_3} .
- The mass m_{ν_2} was taken as a free parameter in the fitting of our formula for $\tan \theta_{12}$ to the experimental value
- with

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} eV^2 \qquad \Delta m_{13}^2 = 2.4 \times 10^{-3} eV^2$$

and

$$\tan \theta_{12} = 0.696$$

we get

$$|m_{\nu_3}| \approx 0.019 \text{ eV} \implies |m_{\nu_2}| \approx 0.053 \text{ eV} \quad \text{and} \quad |m_{\nu_1}| \approx 0.052 \text{ eV}$$

- The neutrino mass spectrum has an inverted hierarchy of masses

FCNC I

In the Standard Model the FCNC at tree level are suppressed by the GIM mechanism.

Models with more than one Higgs $SU(2)$ doublet have tree level FCNC due to the exchange of scalar fields. The mass matrix written in terms of the Yukawa couplings is

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0,$$

FCNC processes:

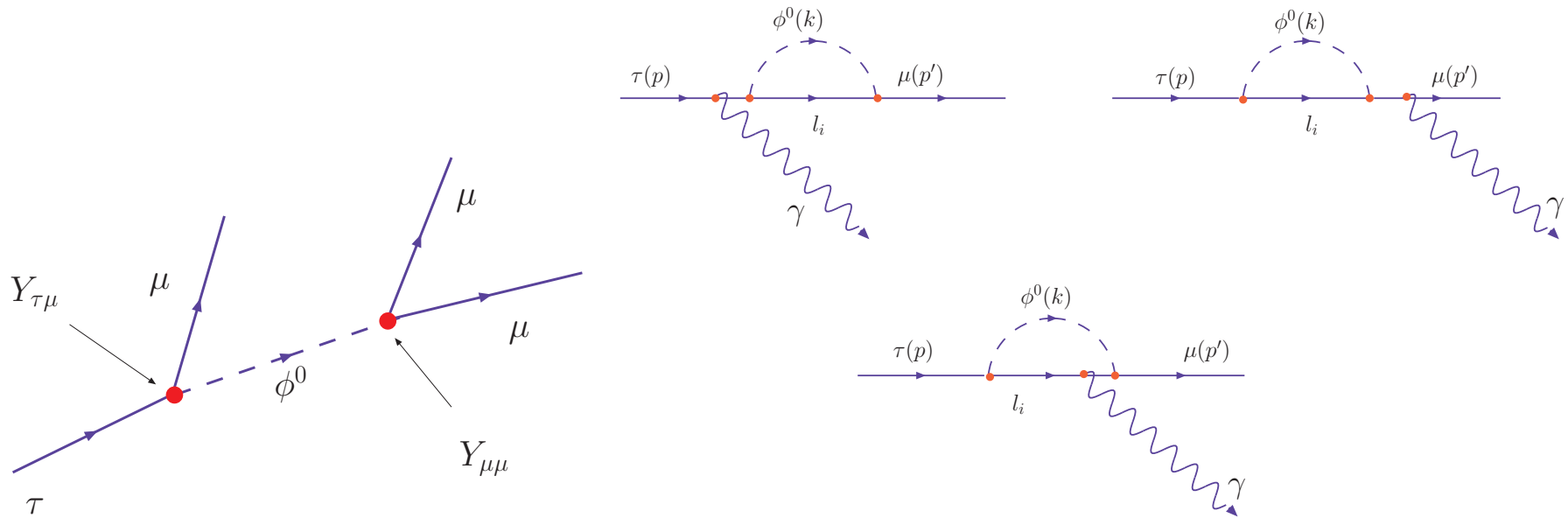


Figure 1: The diagram in the left contributes to the process $\tau^- \rightarrow 3\mu$. The three diagrams in the right contribute to the process $\tau \rightarrow \mu\gamma$.

The Yukawa matrices

The Yukawa matrices in the weak basis are

$$Y_w^{E1} = \frac{m_\tau}{v_1} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 & 0 \end{pmatrix}$$

and

$$Y_w^{E2} = \frac{m_\tau}{v_2} \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ 0 & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}.$$

Yukawa matrices in the mass representation

The Yukawa matrices in the mass basis defined by

$$\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$$

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m,$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m,$$

all off diagonal terms give rise to FCNC processes!!

Branching ratios

We define the partial branching ratio (only leptonic decays)

$$Br(\tau \rightarrow \mu e^+ e^-) = \frac{\Gamma(\tau \rightarrow \mu e^+ e^-)}{\Gamma(\tau \rightarrow e \nu \bar{\nu}) + \Gamma(\tau \rightarrow \mu \nu \bar{\nu})}, \quad \Gamma(\tau \rightarrow \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10} \pi^3} \frac{(Y_{\tau\mu}^{1,2} Y_{ee'}^{1,2})^2}{M_{H_{1,2}}^4}$$

thus

$$Br(\tau \rightarrow \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2} \right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}} \right)^4,$$

Similar computations lead to

$$Br(\tau \rightarrow e \gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H} \right)^4,$$

$$Br(\tau \rightarrow \mu \gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau} \right)^2 \left(\frac{m_\tau}{M_H} \right)^4,$$

$$Br(\tau \rightarrow 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H} \right)^4,$$

$$Br(\mu \rightarrow 3e) \approx 18 \left(\frac{m_e m_\mu}{m_\tau^2} \right)^2 \left(\frac{m_\tau}{M_H} \right)^4,$$

$$Br(\mu \rightarrow e \gamma) \approx \frac{27\alpha}{64\pi} \left(\frac{m_e}{m_\mu} \right)^4 \left(\frac{m_\tau}{M_H} \right)^4.$$

Numerical results

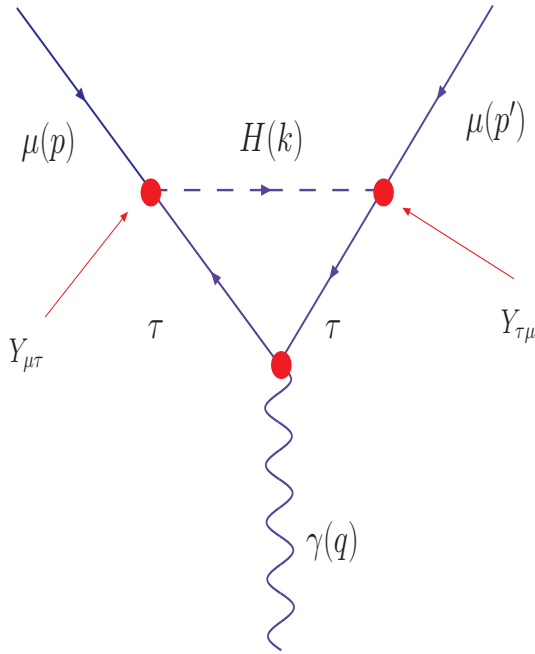
Table 2: Leptonic processes via FCNC

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \rightarrow 3\mu$	8.43×10^{-14}	5.3×10^{-8}	B. Aubert <i>et al.</i> (2007)
$\tau \rightarrow \mu e^+ e^-$	3.15×10^{-17}	8×10^{-8}	B. Aubert <i>et al.</i> (2007)
$\tau \rightarrow \mu \gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert <i>et al.</i> (2005)
$\tau \rightarrow e \gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert <i>et al.</i> (2006)
$\mu \rightarrow 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt <i>et al.</i> (1998)
$\mu \rightarrow e \gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks <i>et al.</i> (1999)

Small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the **gravitational core collapse and shock generation** in the explosion stage of a supernova

Muon Anomalous Magnetic Moment

The anomalous magnetic moment of the muon is related to the gyroscopic ratio by



$$a_\mu = \frac{\mu_\mu}{\mu_B} - 1 = \frac{1}{2}(g_\mu - 2)$$

In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour changing neutral scalars also contribute to the anomalous magnetic moment of the muon

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu} m_\mu m_\tau}{16\pi^2 M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right)$$

From our results: $Y_{\mu\tau} Y_{\tau\mu} = \frac{m_\mu m_\tau}{4v_1 v_2}$

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246 \text{ GeV})^2} \frac{(2 + \tan^2 \beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right), \quad \tan \beta = \frac{v_s}{v_1}$$

From the experimental upper bound on $(\mu \rightarrow 3e)$, we get $\tan \beta \leq 14$, Hence

$$\delta a_\mu = 1.7 \times 10^{-10}$$

Contribution to the anomaly of the muon's magnetic moment

The difference between the experimental value and the Standard Model prediction for the anomaly is

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}$$

$$\Delta a_\mu \sim 3\sigma \text{ (three standard deviations) !!}$$

But, the uncertainty in the computation of higher order hadronic effects is large

$$\delta a_\mu^{LBL}(3, had) \approx 1.59 \times 10^{-9}; \quad \delta a_\mu^{VP}(3, had) \approx -1.82 \times 10^{-9}$$

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx \frac{1.7}{28} \approx 6\% \quad \text{and} \quad \delta a_\mu^{(H)} < \delta a_\mu(3, had)$$

The contribution of the exchange of flavour changing scalars to the anomaly of the muon's magnetic moment, $\delta a_\mu^{(H)}$, is small but not negligible, and it is compatible with the best, state of the art, measurements and theoretical predictions.

Summary

- By introducing three $SU(2)_L$ Higgs doublet fields, in the theory, we extended the concept of flavour and generations to the Higgs sector and formulated a minimal S_3 —invariant Extension of the Standard Model
- The neutrino mixing angles θ_{12} , θ_{23} and θ_{13} , are determined by an interplay of the $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy
- The fit of $\sin^2 \theta_{13}^{th}$ to $\sin^2 \theta_{13}^{exp}$ breaks the mass degeneracy of the right handed neutrinos.
- The solar mixing angle, θ_{12} , fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values

$$|m_{\nu_2}| \approx 0.053eV, \quad |m_{\nu_1}| \approx 0.052eV, \quad |m_{\nu_3}| \approx 0.019eV$$

- The branching ratios of all flavour changing neutral processes in the leptonic sector are strongly suppressed by the $S_3 \times Z_2$ symmetry and powers of the small mass ratios m_e/m_τ , m_μ/m_τ , and $\left(m_\tau/M_{H_{1,2}}\right)^4$, but could be important in astrophysical processes
- The anomalous magnetic moment of the muon gets a small but non-negligible contribution from the exchange of flavour changing scalar fields

Conclusions

- The Minimal S_3 -Invariant Extension of the Standard Model **describes successfully** masses and mixings in the quark and leptonic sectors with a small number of free parameters.
- It predicted the numerical values of θ_{13} and θ_{23} neutrino mixing angles, as well as, all flavour changing neutral current processes in the leptonic sector in excellent agreement with experiment.
- The solar mixing angle, θ_{12} , fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values

$$|m_{\nu_2}| \approx 0.054 \text{ eV}, \quad |m_{\nu_1}| \approx 0.053 \text{ eV}, \quad |m_{\nu_3}| \approx 0.021 \text{ eV}$$

- The exchange of flavour changing scalar fields gives a small but non-negligible contribution to the anomaly of the magnetic moment of the muon.