A modification of the relativistic energy-momentum relation

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A modification of the accepted relativistic energy momentum relation is suggested. The new relation allows massive particles to have a maximum velocity $c(m)$ greater than the velocity of light c. The effect of the modification suggested here would be most apparent for objects with masses nearly equal to the Planck mass.

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Recently the OPERA collaboration reported the detection of superluminal neutrinos[1]. Motivated by this we tried to modify the standard energy ,mass and momentum relationship,namely

$$E^2 = m^2c^4 + p^2c^2, \tag{1}$$

to

$$E^2 = m^2c(m)^4 + p^2c(m)^2. \tag{2}$$

In terms of the velocity v,

$$E = \frac{mc(m)^2}{\sqrt{1 - v^2/c(m)^2}}. \tag{3}$$

Here $c(m)$ is a function of a dimensionless variable ζ which depends on the mass m and is defined below.We have simply replaced c by $c(m)$ in the usual relation.

Any modification,in our view, must satisfy the constraint that for $m = 0$, we should have $E = pc$.This means that we require $c(0) = c$,so that for $m = 0$ we have the usual energy-momentum relation for electromagnetic radiation.

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For non-relativistic velocities, our modification gives the Newtonian expression for kinetic energy. However,it modifies the rest energy for a massive object.

The crucial question is what is the variable ζ and the function $c(m)$ .Massive particles have gravitational interaction and it is interesting ,to note that one can define a mass,the Planck mass P_M in terms of the gravitational constant G_N ,namely

$$P_M = \sqrt{\hbar c/G_N} \approx 1,22 \times 10^{19} GeV/c^2 \approx 2.18 \times 10^{-5} gr. \tag{4}$$

Here h=Planck's constant and c is the velocity of light.The dimensionless variable ζ is defined to be

$$\zeta = m/P_M \tag{5}$$

We define $c(m)$ to be of the form

$$c(m) = c[1 + F(\zeta)] \tag{6}$$

where F is a positive analytic function of ζ with $F(0) = 0$ .Positivity of F is necessary for matter to have superluminal velocities.From the definition it is clear that ζ is extremely small for sub-atomic particles (electron ,proton etc.) and is very large for ordinary masses.For example, for $m = 20gr$, $\zeta \sim 10^6$ and enormous for ordinary and stellar objects.Since,the value of $F(\zeta)$ will affect the rest mass of the object,one has to choose it carefully.From the practical point of view,we consider

$$F(\zeta) = \zeta^n exp(-\zeta^n), \tag{7}$$

where n is a positive real number. This function has a maximum value $e^{-1} = 0.367879..$ for $\zeta = 1$ independent of the value of n. It is centered around $\zeta = 1$. As n increases the height remains the same, but it becomes narrower and narrower. For extremely large n (tending to infinity), this sequence of functions tends to a vertical line of height e^{-1} located at $\zeta = 1$.In this limit, it is like a "finite Dirac δ function" only non-zero for $\zeta = 1$. The function is plotted in Fig.1 for $n = 50, 100$ and 1000. One can see clearly how the function shrinks rapidly as n increases and will be practically narrow as a straight line located at $\zeta = 1$ for much larger n.

Thus, for extremely large n, the value of the usual rest mass mc^2 will not be affected for bodies which are very light(subatomic particles) or very heavy (eg.planets) compared to

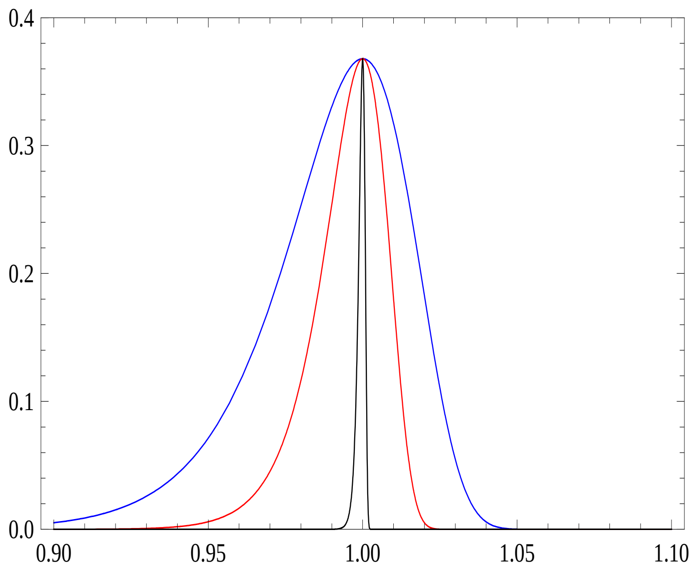

FIG. 1: The function $F(x) = x^n e^{x^n}$ $(x \equiv \zeta)$ for $n = 50, 100, 1000$ in color blue, red and black respectively. The function narrows rapidly with increasing n . The maximum height at $x = 1$ is independent of n.

the Planck mass, but will affect those with masses of the order of the Planck mass. Such a choice for n is necessary, since our particle accelerators work and we understand planetary motion. These provide extremely stringent tests for the usual energy-momentum relation. However, we do not have stringent tests for masses of the order of the Planck mass. It would be interesting to have experimental tests of the energy-momentum relation for this mass range.

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[1] OPERA Collaboration preprint ,submitted to JHEP, arXiv.1109.4897.This note was inspired by this report. However,the modification suggested here does not explain their result.In any case,after more stringent analysis they have withdrawn the original claim recently.

[2] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).