Toroidal Dipole Moment of the Neutralino in the cMSSM

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Why Supersymmetry?

The Standard Model (SM) needs an extension because:

- The Higgs mechanism which generates the masses of the weak gauge bosons has not been established yet.
- The couplings of the three gauge interactions do not unify at a high energy scale.
- There are contributions to the Higgs boson mass that diverge quadratically.
- The SM does not provide a candidate for CDM.

Why Supersymmetry?

If we introduce supersymmetry:

- ✓ New particles appear at the TeV scale modifying the β-functions of the three gauge couplings in such a way that they unify near the 1016 GeV.
- ✓ SUSY-GUT induce a dynamical electroweak symmetry breaking.
- ✓ By conecting fermions to bosons, the hierarchy problem is solved.
- ✓ The LSP, if stable, provides a good candidate for CDM.

Minimal Supersymmetric Standard Model (MSSM)

The supersymmetry is a space-time simmetry. The supersymmetric transformations change fermionic states into bosonic ones and viceversa.

$$\hat{Q}\,|Fermión\rangle = |Bosón\rangle \ , \qquad \hat{Q}\,|Bosón\rangle = |Fermión\rangle$$

For each particle there is another one, its "superpartner", whose spin differs by ½.

The MSSM requires twice the degrees of freedom of the SM, including two Higgs complex doublets, which is necessary to have the theory free of anomalies.

MSSM content of particles

Names		spin 0	spin 1/2
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$
Higgs, higgsinos	H_u	$(H^+_u \;\; H^0_u)$	$(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$
	H_d	$(H^0_d \;\; H^-_d)$	$(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$

Names	spin 1/2	spin 1
gluino, gluon	$\widetilde{g}$	g
winos, W bosons	$\widetilde{W}^\pm \;\; \widetilde{W}^0$	$W^\pm \;\; W^0$
bino, B boson	$\widetilde{B}^0$	B^0

The superpartners are not necessarily the masses eigenstates of the model. After electroweak and supersymmetry breakings, mixes of different gauginos and higgsinos can appear. In particular, the 4 neutralinos are linear combinations of the neutral higgsinos and gauginos.

R-Parity

$$R = (-1)^{3B+2S+L}$$

This is a new discrete simmetry which distinguishes between SM particles (R = 1) and their SUSY partners (R = -1).

In models that conserve R-parity, SUSY particles can only be produced/annihilated by pairs. Thus the LSP is stable.

cMSSM

SUSY has to be a broken symmetry.

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{c.c.} - (m^2)^i_j\phi^{j*}\phi_i$$

Supposedly, this breaking occurs in a hidden sector that communicates to the observable one via only gravitational interactions. Gauge interactions are unified.

Parameters of the model

The MSSM can then be described by only five additional parameters (in stead of more than 100 of the most general MSSM):

m_0 – the universal scalar mass at the scale of GUT

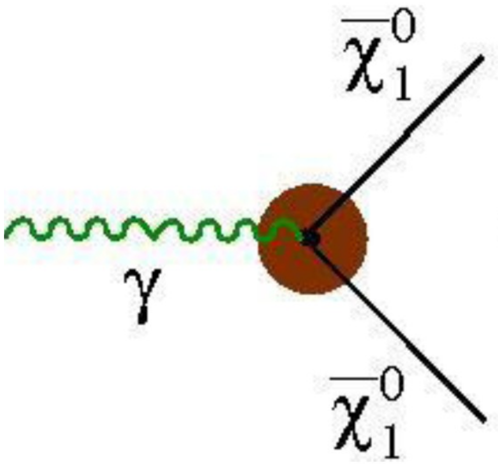
$m_{1/2}$ – the unified gaugino mass at the scale of GUT

A_0 – the value of universal trilinear coupling at the scale of GUT

$\tan\beta$ – ratio of the vacuum expectation of the two Higgses

sign μ – sign of Higgsino mass parameter

Electromagnetic vertex



The analysis of the electromagnetic properties of the neutralinos is very interesting then.

The amplitude of the interaction with an external electromagnetic field is:

$$M \propto eJ_\mu^{EM}(q)A^\mu(q)$$

with

$$\begin{aligned} J_\mu^{EM}(q)_{Dirac} &= [\bar{u}_f(p')\Gamma_\mu(q)u_i(p)] \ , \\ J_\mu^{EM}(q)_{Majorana} &= J_\mu^{EM}(q)_{Dirac} + [\bar{v}_i(p)\Gamma_\mu(q)u_f(p')] \equiv \\ & \bar{u}_f(p')\left[\Gamma_\mu(q) - (C^{-1}\Gamma_\mu(q)C)^T\right] u_i(p) \end{aligned}$$

Electromagnetic vertex

It is well known that the electromagnetic properties of a ½-spin particle are described by 4 form factors.

The electromagnetic vertex can be defined as follows:

$$\Gamma_\mu(q) = f_Q(q^2)\gamma_\mu + f_\mu(q^2) i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu q^\nu\gamma_\nu)\gamma_5$$

fQ – charge form factor

fμ – magnetic dipole form factor

fE – electric dipole form factor

fA – anapole form factor

Electromagnetic vertex

These form factors are physical observables when $q^2 \rightarrow 0$, and their combinations define the magnetic dipole (μ), electric dipole (d) and anapole (a) moments.

In the non-relativistic limit, the energy of interaction with an external electromagnetic field is:

$$\mathcal{H}_{int} \propto -\mu\,(\sigma \cdot B) - d\,(\sigma \cdot E) - a\,(\sigma \cdot \nabla \times B)$$

Anapole moment

The Majorana particles only have one electromagnetic property if CPT-invariance is to be preserved: the anapole moment

The anapole moment was introduced by Zel'dovich to describe a T-invariant interaction that does not conserve P- and C-parity individually.

But the anapole moment does not have a simple classical analogue, and that is why a more convenient quantity describing this kind of interaction was proposed: the toroidal dipole moment (TDM).

Toroidal Dipole Moment

The anapole and toroidal form factors are connected by:

$$f_A\left(q^2\right) = T\left(q^2\right) + \frac{m_i^2 - m_f^2}{q^2 - \Delta m^2}\left[D\left(q^2\right) - D\left(\Delta m^2\right)\right]$$

The toroidal dipole moment is given by:

$$\tau = T(0).$$

The corresponding interaction energy is:

$$Hint = -\boldsymbol{\tau}\cdot\boldsymbol{J}.$$

It has the moment of force:

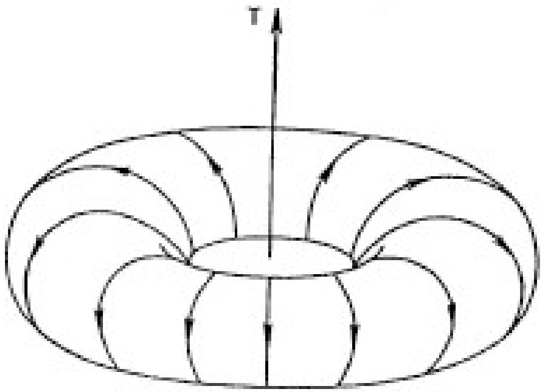
$$\boldsymbol{M} = \tau[\,\boldsymbol{\sigma} \times \boldsymbol{J}\,].$$

The particle can interact via its TDM with an external electromagnetic current, with the source of an inhomogeneous magnetic field and/or with the source of a time-dependent electric field.

TDM

The TDM is the first term of the third independent multipole family: the toroidal moments.

This type of static multipole moments does not produce any external fields in vacuum but generates a free-field (gauge invariant) potential, which is responsible for topological effects.



The simplest TDM model was given by Zel'dovich as a conventional solenoid rolled up in a torus and with only a poloidal current. For such stationary solenoid, without azimuthal components for the current or the electric field, there is only one magnetic azimuthal field different from zero inside the torus.

What we did

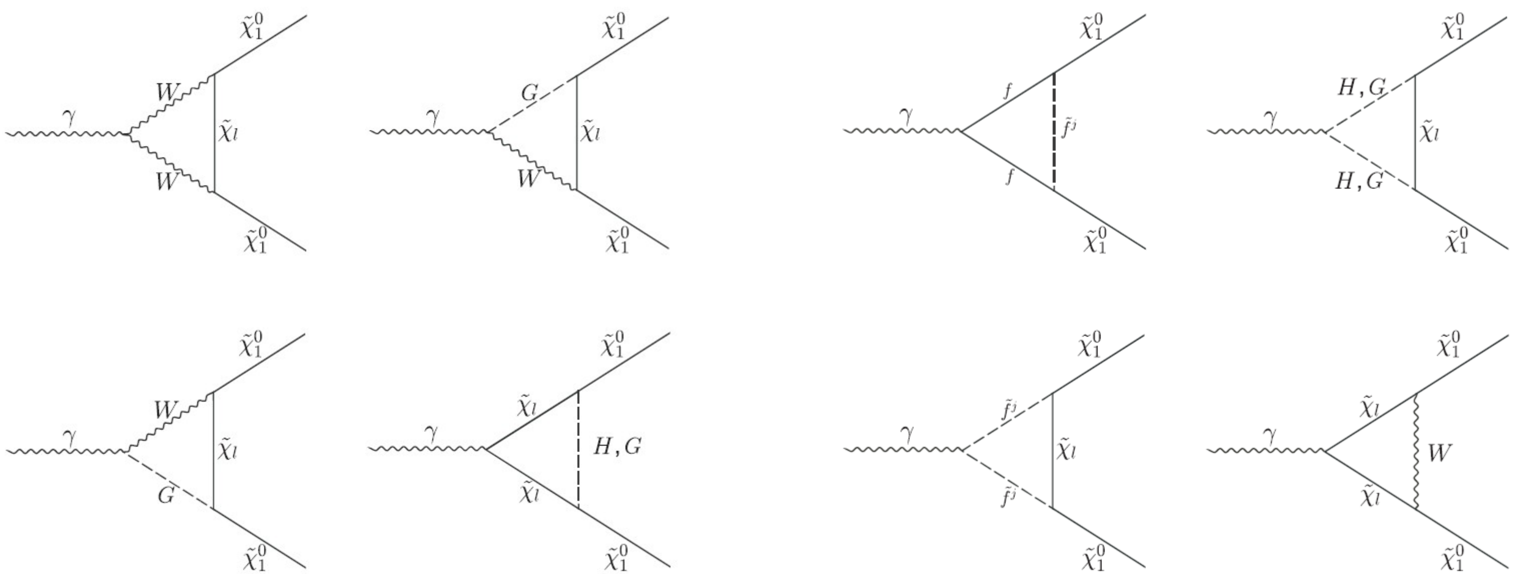
What we did was to analyze the electromagnetic vertex for each Feynman diagram that contributes to the calculation. In fact, we isolate the terms that have the Loretz structure $\gamma_\mu\gamma_5$. The sum of all this terms is:

$$\sum_i \Xi_i = f_A(q^2)q^2$$

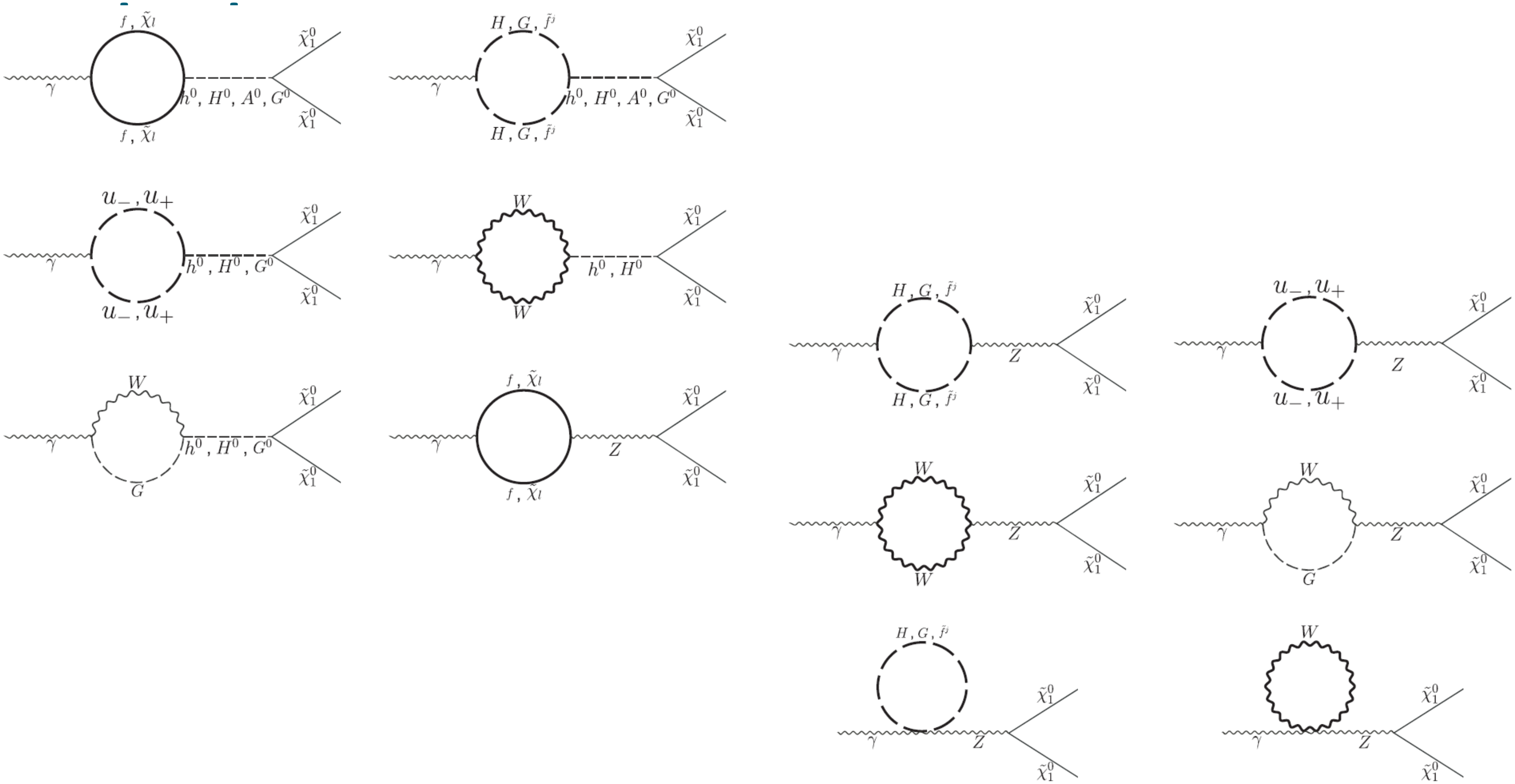
We get the TDM using l'Hopital rule.

$$\tau = f_A(0) = \lim_{q^2 \to 0} \frac{\sum_i \Xi_i}{q^2} = \frac{\partial \sum_i \Xi_i}{\partial q^2}$$

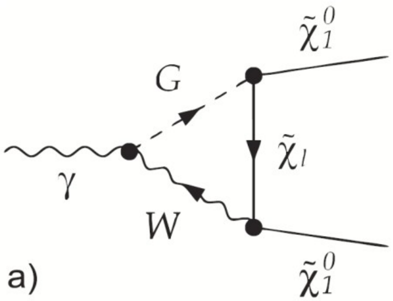
Diagrams of one-loop vertex corrections that contribute to the calculation



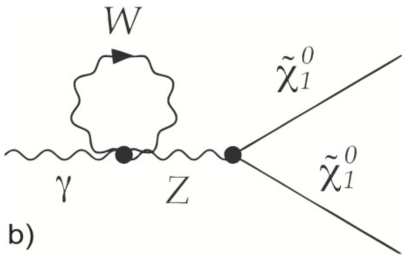
Diagrams of one-loop corrections to the self-energy that contribute to the



$$\Xi_1 = \sum_l \frac{e^3}{32\pi^2 c_W s_W^2 \left(q^2 - 4M_{\chi_1^0}^2\right)} \left[\alpha B_0\left(q^2, M_W^2, M_W^2\right) + \beta B_0\left(M_{\chi_1^0}^2, M_{\chi_l}^2, M_W^2\right) + \delta C_0\left(q^2, M_{\chi_1^0}^2, M_{\chi_1^0}^2, M_{\chi_l}^2, M_{\chi_l}^2\right)\right]$$



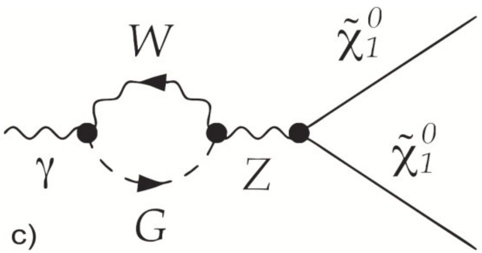
a)



b)

$$\Xi_2 = \frac{-M_W^2 e^3 \varepsilon}{16\pi^2 s_W^2 \left(q^2 - M_Z^2\right)} \left[3B_0\left(0, M_W^2, M_W^2\right) + 1\right]$$

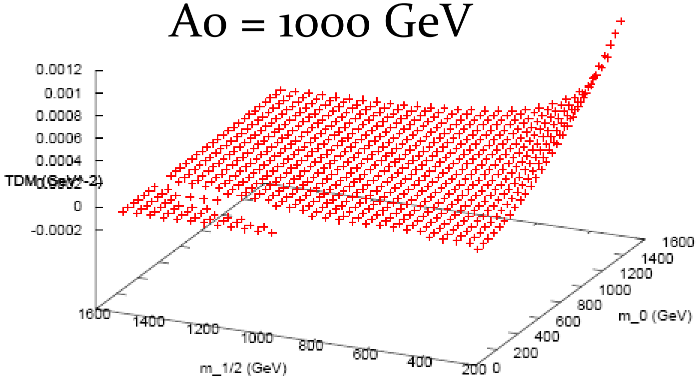
$$\Xi_3 = \frac{e^3 \zeta}{\pi^2 s_W^2 \left(q^2 - M_Z^2\right)} \left[-\frac{M_W^2}{192} B_0\left(0, M_W^2, M_W^2\right) + \frac{q^2 - 4M_W^2}{384} B_0\left(q^2, M_W^2, M_W^2\right) + \frac{q^2 - 9M_W^2}{576}\right]$$



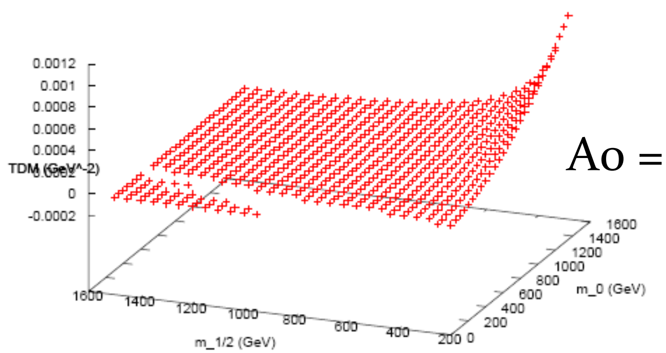
c)

Results

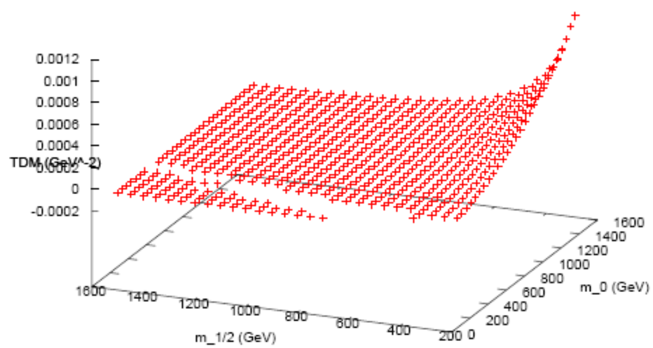
A0 = 1000 GeV



tan β = 10, sign μ = +



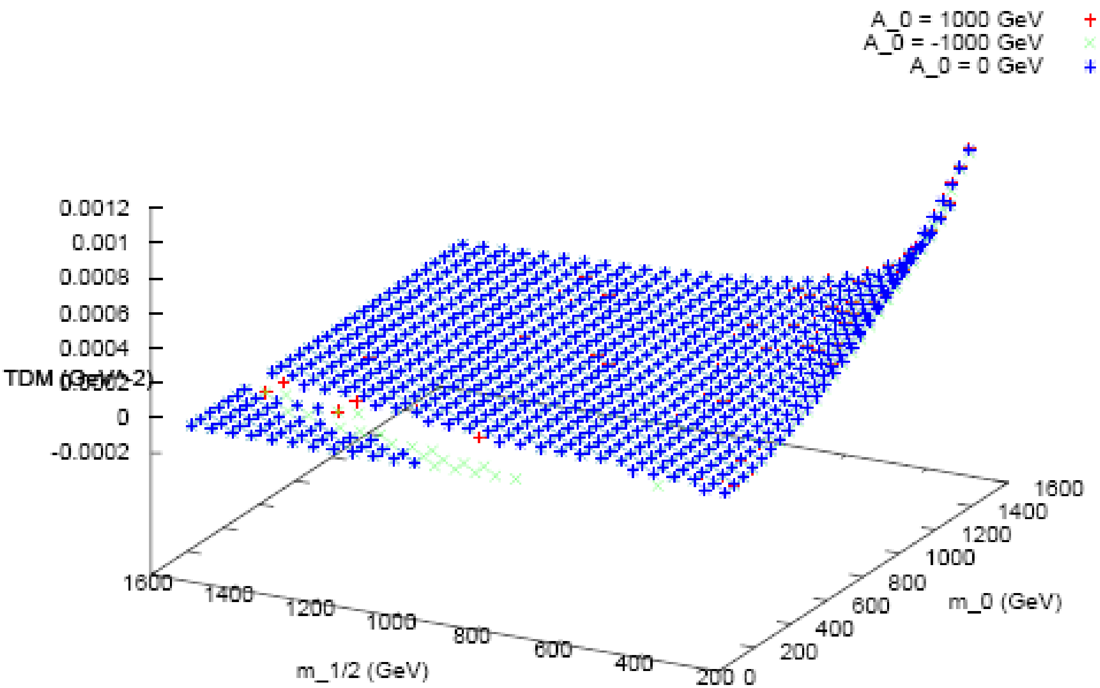
A0 = 0 GeV



A0 = -1000 GeV

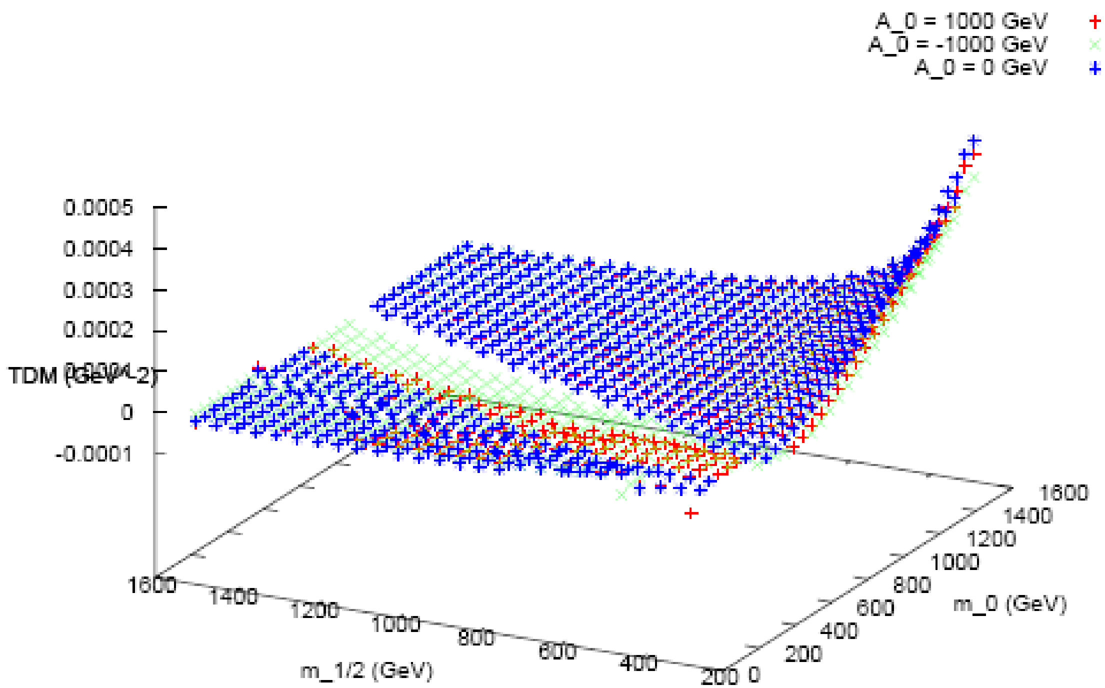
Results

tan β = 10, sign μ = +



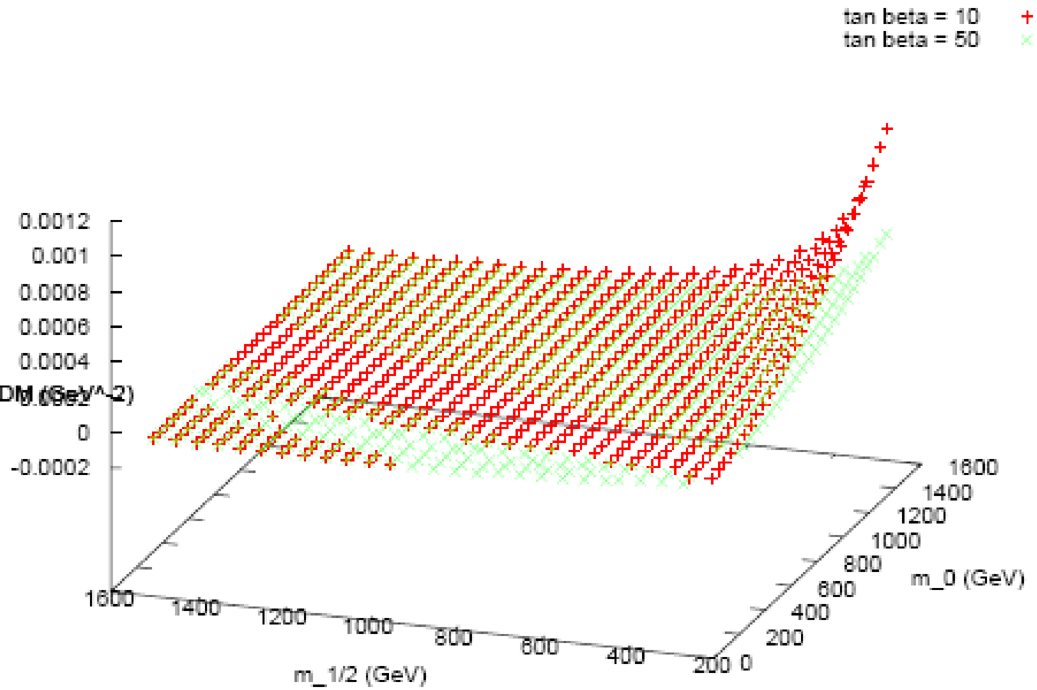
Results

tan β = 50, sign μ = +



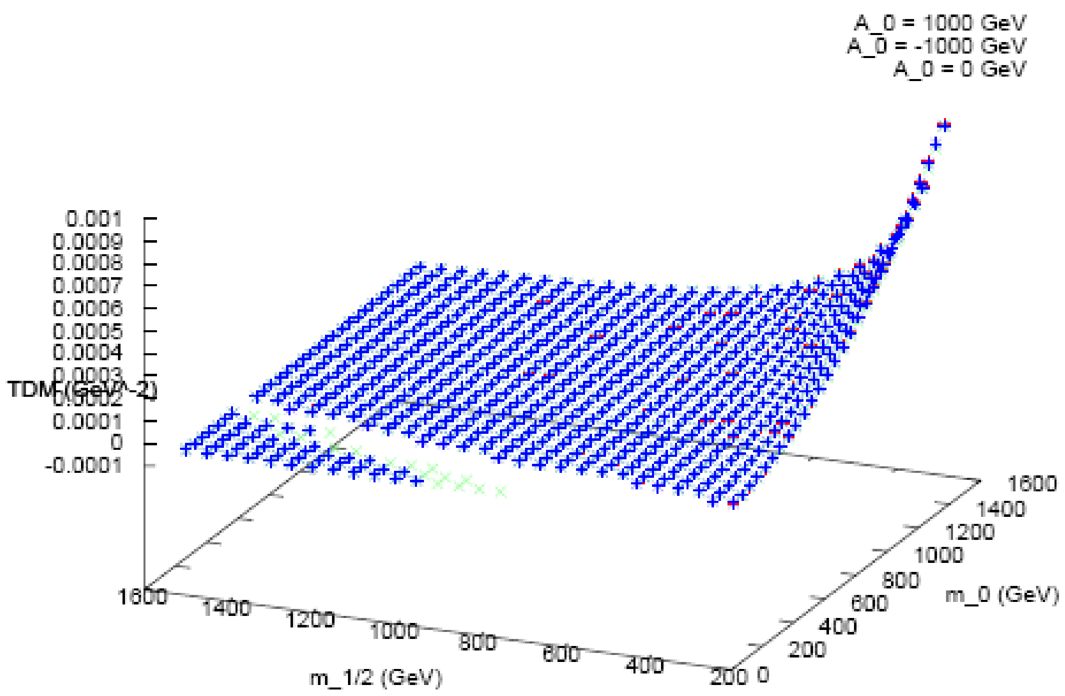
Results

Ao = o GeV, sign μ = +



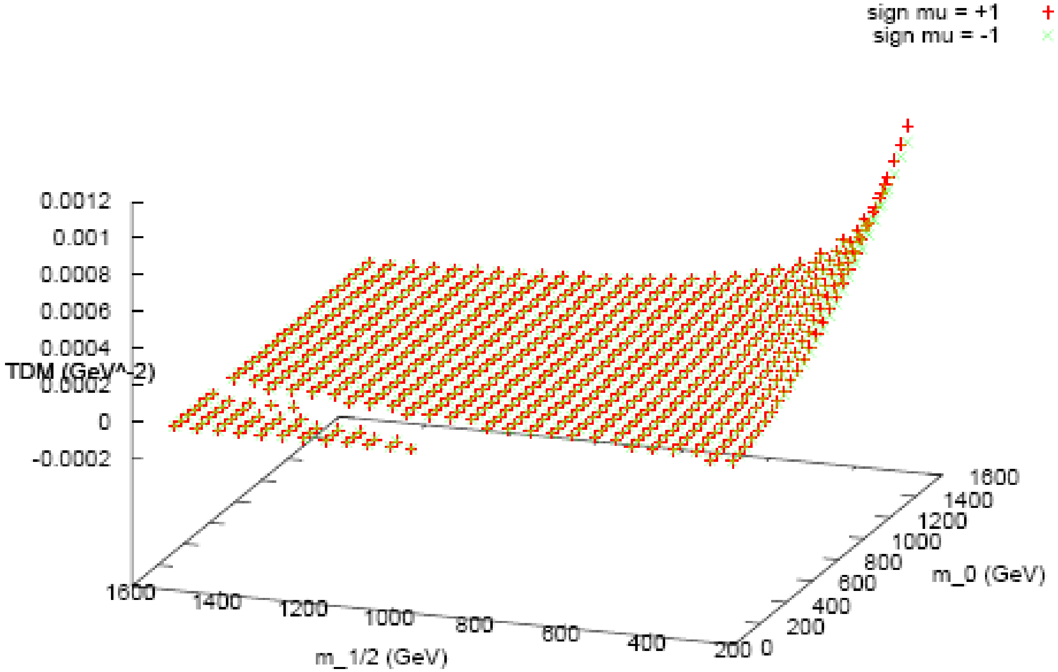
Results

tan β = 10, sign μ = -



Results

tan β = 10, A0 = 0 GeV

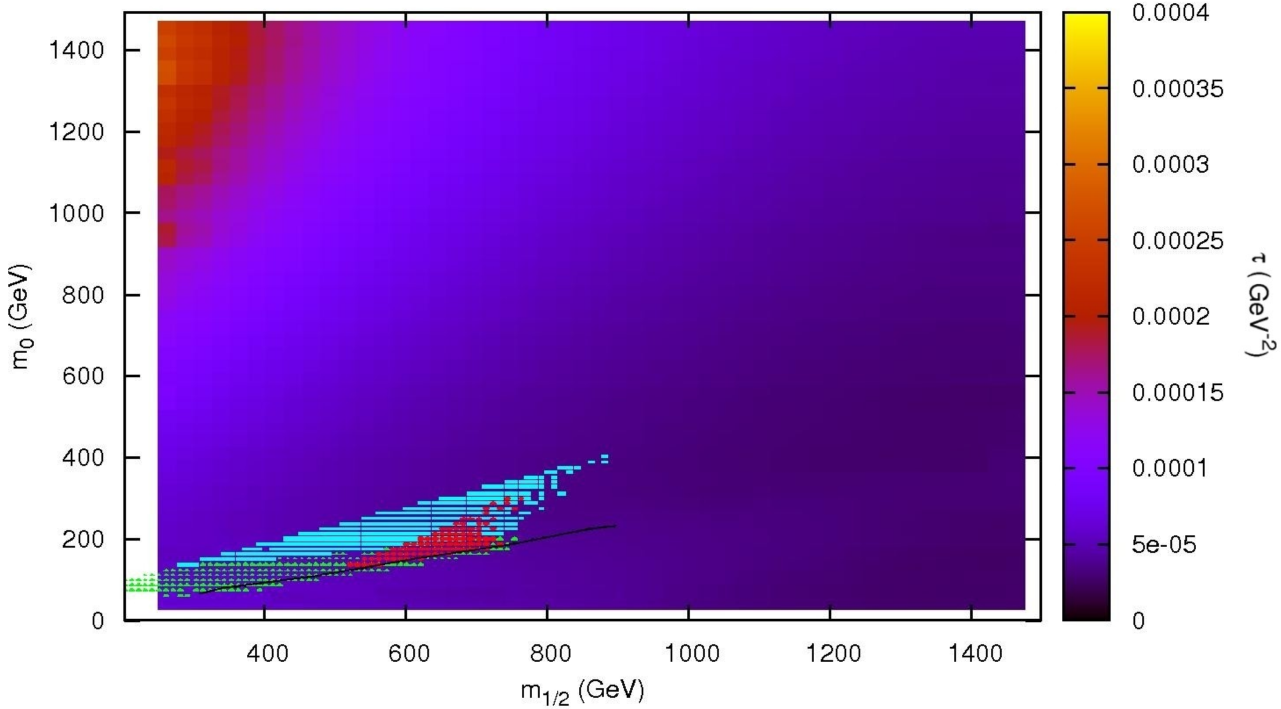


Experimental limits

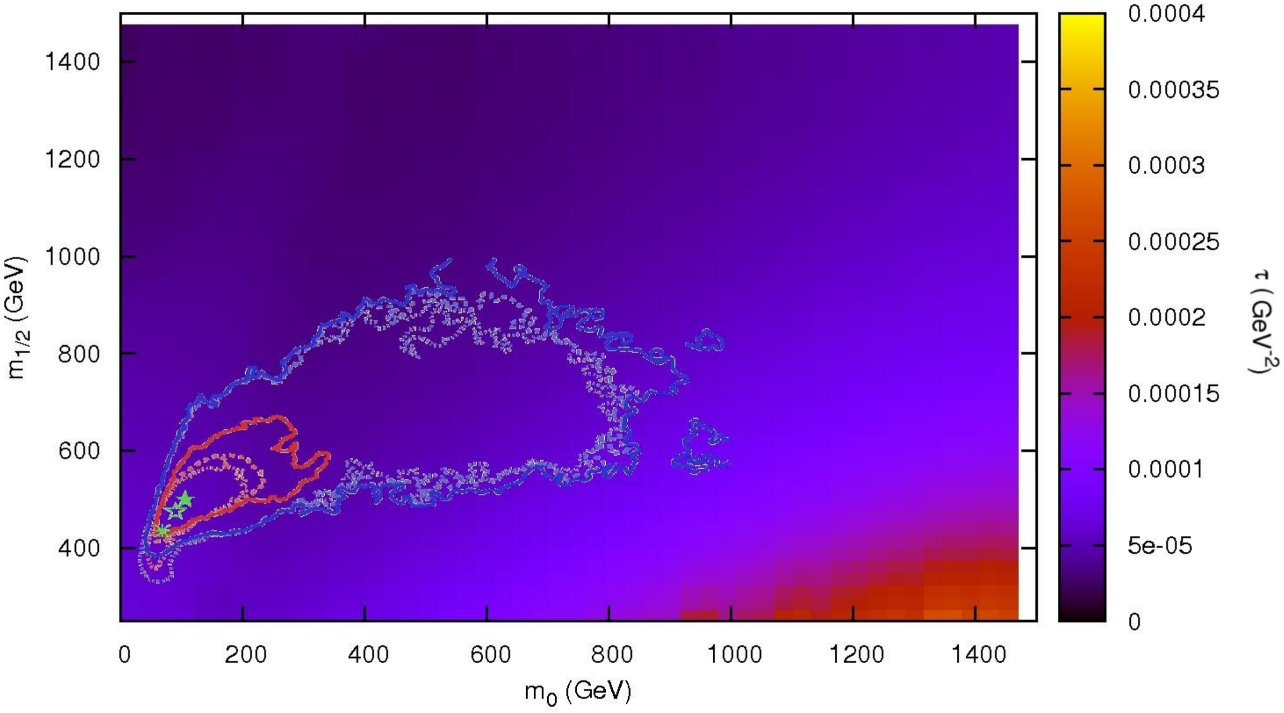
$$\sigma = \frac{1}{2\pi} Z^2 \alpha^2 \frac{M_R^2(N,D)}{4m_p^2} \left(\frac{a}{\mu_n}\right)^2 v^2 \frac{S+1}{3S}$$

	Na I	Ge
$\lvert\mu_D/\mu_n\rvert \sqrt{(S+1)/3S}$	$1.4\ 10^{-4}$	$--$
$\lvert d/e\rvert \sqrt{(S+1)/3S}$	$8\ 10^{-21}\mathrm{cm}$	$5\ 10^{-21}\mathrm{cm}$
$\lvert Q/e\rvert \sqrt{(S+1)(2S+3)/10S(2S-1)}$	$1\ 10^{-6}\mathrm{fm}^2$	$6\ 10^{-7}\mathrm{fm}^2$
$\boxed{\lvert a/\mu_n\rvert \sqrt{(S+1)/3S}}$	$4\ 10^{-2}\mathrm{fm}$	$3\ 10^{-2}\mathrm{fm}$
r_D^2	$1.4\ 10^{-6}\mathrm{fm}^2$	$9\ 10^{-7}\mathrm{fm}^2$
$\lvert\chi_E\rvert$	$5\ 10^{-8}\mathrm{fm}^3$	$7\ 10^{-8}\mathrm{fm}^3$

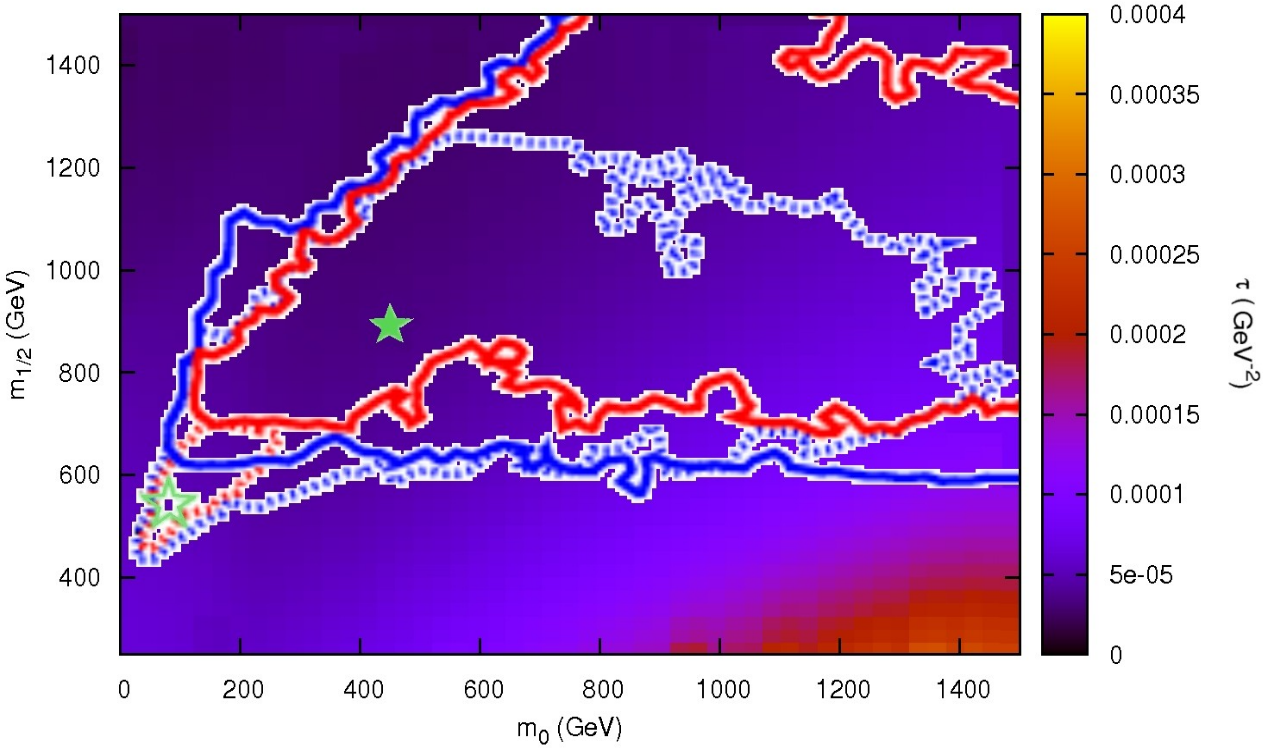
M. Pospelov and T. ter Veldhuis, Phys. Lett. B480, 181 (2000) [hep-ph/0003010]



L.S. Stark *et al.*, JHEP 08 (2005) 059



O. Buchmueller *et al.*, 2011, arXiv: 1102.4585v1



O. Buchmueller *et al.*, 2011, arXiv: 1110.3568v1

Conclusions

The TDM is the only electromagnetic property of the neutralino.

We found that the neutralino TDM is sensitive to m_0, $m_{1/2}$ y $\tan\beta$. The TDM we get is between 0-10^{-3} GeV^{-2}.

The TDM can be one more argument that helps us to discriminate among candidates and models for CDM. In case somebody can measure a TDM different from zero (10^{-3} - 10^{-4} GeV^{-2}) for WIMPs, this would indicate that the neutralino of our model is not (at least not the main) the component of CDM.

Thanks!