Cosmological Perturbations in Massive Gravity

GUSTAVO NIZ

University of Guanajuato

PASCOS

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with K. Koyama & G. Tasinato

Arxiv:

Plan of the talk

1. Motivation
2. Fierz - Pauli
3. Spherically symmetric solutions (self-acceleration)
4. Perturbations & new symmetries
5. Ghost-free massive gravity
6. Spherically symmetric solutions (self-acceleration)
7. Conclusions

Why a massive graviton?

$h_{\mu\nu}$ m $v_{h_{\mu\nu}} < c$

Natural IR modification of GR which *may* explain present acceleration of the Universe

Fierz-Pauli mass term (1939)

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - m^2\left(\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu} - \mathcal{H}^2\right)\right]$$

$g_{\mu\nu} \updownarrow$

$$\mathcal{H}_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta}$$

unitary gauge $\quad \phi^\mu = x^\mu \Rightarrow \mathcal{H}_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \equiv h_{\mu\nu}$

Properties

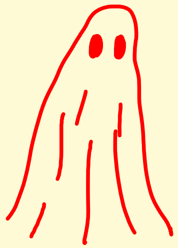
① To linear order in $h_{\mu\nu}$

- the theory is unique
- propagates 5 d.o.f. (2 T + 2 V + 1 S)

ADM: No momentum constraint.

- Solutions do not reduce to GR in massless limit. (van Dam, Veltman, Zakharov, '70)

② Higher orders in $h_{\mu\nu}$

- A 6^{th} scalar d.o.f appears which is a  around flat space

(Boulware & Deser, '72)

ADM: No Hamiltonian constraint

- GR solutions may be recovered

(Vainshtein, '72)

Static Spherically Symmetric Solutions

$$ds^2 = -C(r)\,dt^2 + A(r)\,dr^2 + B(r)\,d\Omega_2^2 + 2D(r)\,dr\,dt$$

Gauge $\phi^\mu = x^\mu$

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - m^2\underbrace{\left(H_{\mu\nu}H^{\mu\nu} - H^2\right)}\right]$$

$$\boxed{G_{\mu\nu} = T^{\Lambda}_{\mu\nu} + T^{m}_{\mu\nu} = T^{\Lambda+m}_{\mu\nu}}$$

$$D(r)\,G_{tt} + C(r)\,G_{tr} = 0 \quad \Rightarrow \quad D(r)\,T^{\Lambda+m}_{tt} + C(r)\,T^{\Lambda+m}_{tr} = 0$$

$$\Rightarrow \quad D(r)\left(\sqrt{2}\,r - \sqrt{3B(r)}\right) = 0$$

Two branches of solutions

(No Birkhoff's theorem)

Salam & Strathdee, 77

Non-diagonal branch

- $B(r) = \frac{3}{2} r^2$

- Analytic solutions : Schwarzschild-de Sitter metric

- Two integration constants (cf. Schwarzschild mass)

  Mass M + "vectorial" charge Q_0

- If $M = 0 = Q_0 \Rightarrow$ de Sitter

  ↳ Can be written as a FRWL (flat slicing) but need to transform the ϕ's

Self-accelerating solutions

$$ds^2 = -dt^2 + e^{2\tilde{m}t}\left(dx_1^2 + dx_2^2 + dx_3^2\right)$$

$$\phi^0 = \sqrt{\frac{3}{2}}\,\frac{1}{\tilde{m}}\left[\,\mathrm{Tanh}^{-1}\left(\frac{2\,\mathrm{Sinh}(\tilde{m}t) + \tilde{m}r^2 e^{\tilde{m}t}}{2\,\mathrm{Cosh}(\tilde{m}t) - \tilde{m}r^2 e^{\tilde{m}t}}\right) - \tilde{m}\, r\, e^{\tilde{m}t} + \tanh^{-1}\left(\tilde{m}\, r\, e^{\tilde{m}t}\right)\right]$$

$$\phi^i = \sqrt{\frac{3}{2}}\, e^{\tilde{m}t}\, x^i \qquad (i = 1, 2, 3)$$

$$\tilde{m}^2 = \frac{m^2}{4} + \frac{\Lambda}{3}$$

Salam & Strathdee, 77

Koyama, G.N. & Tasinato

Koyama, G.N. & Tasinato

New symmetries

Cosmological perturbations in simplest self-accelerating background

Tune Λ to get $\tilde{m}^2 = \frac{m^2}{4} + \frac{\Lambda}{3} = 0 \Rightarrow \phi^\mu = \sqrt{\frac{3}{2}}\, x^\mu$

ADM : $ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

$\Rightarrow$ Decompose N, N^i and γ_{ij} in

scalar, vector and tensor modes

Tensors & vectors

Exactly the <u>same</u> action as in GR <u>without</u> Λ

$\Rightarrow$ No dynamical tensor modes

Scalars

More complicated ...

Hamiltonian $\left\{ \begin{array}{l} 3 \text{ dynamical d.o.f.} \\ 3 \text{ } 1^{st} \text{ class constraints} \end{array} \right.$

$\therefore$ No dynamical scalars !

Results Summary

	Metric	ϕ's	Λ	$\mathcal{H}_{\mu\nu}$	1st order Perturbations
Minkowski	$g_{\mu\nu} = \eta_{\mu\nu}$	$\phi^\mu = x^\mu$	$\Lambda = 0$	$\mathcal{H}_{\mu\nu} = 0$	2 tensors 2 vectors 1 scalar
Self-accelerating background	$g_{\mu\nu} = \eta_{\mu\nu}$	$\phi^\mu = \sqrt{\frac{3}{2}}\, x^\mu$	$\Lambda = -\frac{3}{4} m^2$	$\mathcal{H}_{\mu\nu} = -\frac{1}{2}\eta_{\mu\nu}$	2 tensors

Cannot be distinguished from pure GR

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New Symmetry

$$\mathcal{L}^{(2)}_{FP+\Lambda} = \frac{9}{32} m^2 \left(h^\mu{}_\mu - \sqrt{\frac{8}{3}}\, \partial_\mu \phi^\mu \right)^2$$

$$\delta\phi^\mu \to \delta\phi^\mu - \sqrt{\frac{3}{2}}\,\xi^\mu + \chi^\mu \qquad \text{with} \qquad \partial_\mu \chi^\mu = 0$$

$$h_{\mu\nu} \to h_{\mu\nu} - \xi_{(\mu,\nu)}$$

Usual diffemorphism invariance

New symmetry

- ¿Does this symmetry hold beyond linear order? (Ghost?)

Back to the Ghost

Can add higher order terms in $\mathcal{H}_{\mu\nu}$ to the Fierz-Pauli term to get rid of the ghost

$\mathcal{O}(H^3)$

$$a_1 \mathcal{H}^\alpha{}_\mu \mathcal{H}^\mu{}_\nu \mathcal{H}^\nu{}_\alpha + a_2 \mathcal{H}_{\mu\nu} \mathcal{H}^{\mu\nu} \mathcal{H} + a_3 \mathcal{H}^3$$

Tune them to avoid the ghost.

It can be done to all orders ...

Ghost-free Massive Gravity

(de Rham, Gabadadze, Tolley, 2010)

$$\mathcal{L} = \frac{M_{pl}}{2}\sqrt{-g}\left[R - 2\Lambda - m^2\left(\mathcal{L}^{(2)}(K) - \alpha_3\mathcal{L}^{(3)}(K) - \alpha_4\mathcal{L}^{(4)}(K)\right)\right]$$

where

$$K_\mu{}^\nu = \delta_\mu{}^\nu - \sqrt{\partial_\mu\phi^\alpha\partial^\nu\phi^\beta\eta_{\alpha\beta}}$$

$$\langle K^n_{\mu\nu}\rangle \equiv \underbrace{K_\alpha{}^{\alpha_1}K_{\alpha_1}{}^{\alpha_2}\ldots K_{\alpha_n}{}^{\alpha}}_{n\ K\text{'s}}$$

$$\begin{cases} \mathcal{L}^{(2)}(K) = \langle K^2_{\mu\nu}\rangle - \langle K\rangle^2 \\ \mathcal{L}^{(3)}(K) = \langle K\rangle^3 - 3\langle K^2_{\mu\nu}\rangle\langle K\rangle + 2\langle K^3_{\mu\nu}\rangle \\ \mathcal{L}^{(4)}(K) = \langle K\rangle^4 + \ldots \\ \mathcal{L}^{(n)}(K) = 0 \qquad n>4 \end{cases}$$

Static Spherically Symmetric Solutions

$$ds^2 = -C(r)\,dt^2 + A(r)\,dr^2 + B(r)\,d\Omega_2^2 + 2D(r)\,dr\,dt$$

gauge: $\phi^\mu = x^\mu$

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - m^2 \underbrace{\left(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4\right)}\right]$$

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Gruzinov, Mirbabayi

$$\boxed{G_{\mu\nu} = T^{\Lambda}_{\mu\nu} + T^{m}_{\mu\nu} = T^{\Lambda+m}_{\mu\nu}}$$

$$D(r)\,G_{tt} + C(r)\,G_{tr} = 0 \quad \Rightarrow \quad D(r)\,T^{\Lambda+m}_{tt} + C(r)\,T^{\Lambda+m}_{tr} = 0$$

$$\Rightarrow \quad D(r)\left(f(\alpha_3,\alpha_4)\,r - \sqrt{B(r)}\right) = 0$$

FP: $\sqrt{2/3}$

Two branches of solutions again

Non-diagonal branch

- $B(r) = \dfrac{(1+3\alpha_3+\alpha_5)^2}{(2+3\alpha_3+\alpha_5)^2} r^2$; $\alpha_5 = \pm\sqrt{1+3\alpha_3+9\alpha_3^2-12\alpha_4}$

- Analytic solutions : Schwarzschild -(anti) de Sitter metric

- Two integration constants M, Q_0

- If $M = 0 = Q_0 \Rightarrow$ de Sitter or Anti de Sitter

(Can rewrite as FRWL)

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Self-accelerating solutions

$$ds^2 = -dt^2 + e^{2\tilde{m}t}\left(dx_1^2 + dx_2^2 + dx_3^2\right)$$

$$\phi^0 = f(t, r)$$

$$\phi^i = \chi_\pm \, e^{\tilde{m}t} \, x^i \qquad (i = 1, 2, 3)$$

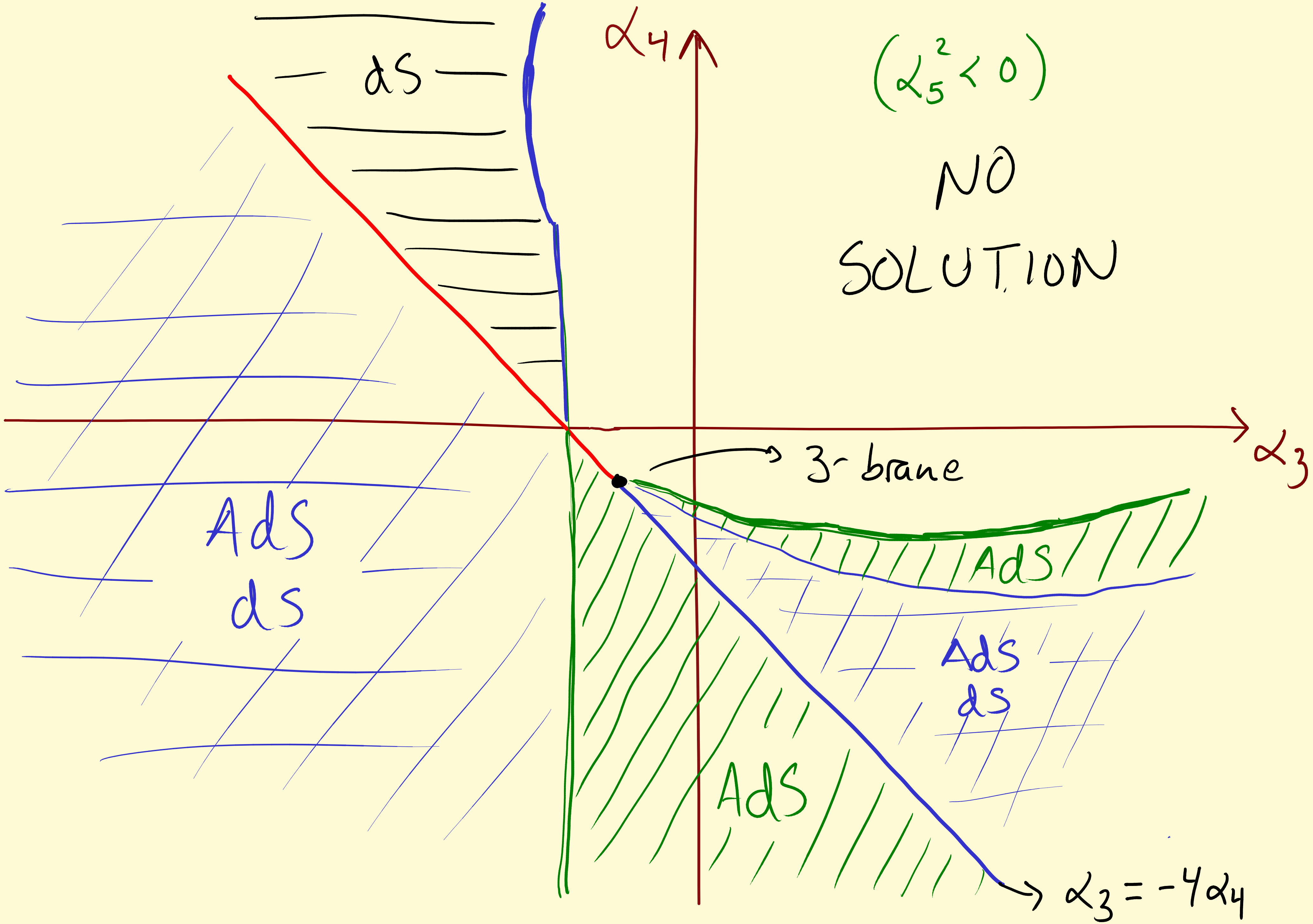
$$\chi_\pm = \frac{(1 + 6\alpha_3 + 12\alpha_4 \pm \alpha_5)}{(3\alpha_3 + 12\alpha_4)}$$

$\to \sqrt{\frac{3}{2}}$ in FP

$$\tilde{m}^2 = \frac{\Lambda}{3} + \frac{(1 + 3\alpha_3 + 2\alpha_5)}{3(2 + 3\alpha_3 + \alpha_5)^2} m^2$$

$$\alpha_5 = \pm\sqrt{1 + 3\alpha_3 + 9\alpha_3^2 - 12\alpha_4}$$

Koyama, Niz & Tasinato



α_4
$(\alpha_5^2 < 0)$
NO SOLUTION
dS
AdS
dS
3-brane
α_3
AdS
AdS
dS
AdS
$\alpha_3 = -4\alpha_4$

A more general solution...

$$ds^2 = -N^2(r,t)\,dt^2 + a^2(r,t)\left(dr^2 + r^2 d\Omega_2^2\right)$$

$$\phi^0 = f(r,t)$$

$$\phi^i = g(r,t)\, x^i$$

$$G_{\mu\nu} = T^{\Lambda + m}_{\mu\nu} \longrightarrow$$

No kinetic terms for ϕ's !

$$g(t,r) = X_\pm\, a(r,t)$$

(as before)

$$\left.\begin{aligned} T^m_{00} &= \frac{1}{2}\tilde{m}\, N^2(r,t) \\ T^m_{rr} &= -\frac{1}{2}\tilde{m}\, a^2(r,t) \\ \frac{T^m_{\varphi\varphi}}{\sin^2\theta} = T^m_{\theta\theta} &= -\frac{1}{2}\tilde{m}\, a^2(r,t) \end{aligned}\right\}$$

Same structure as $T^\Lambda_{\mu\nu}$ alone

Hu et al

(Nieuwenhuizen

D'Amico et al

Mukohyama et al)

$$\tilde{m}^2 = \frac{\Lambda}{3} + \frac{(1+3\alpha_3+2\alpha_5)}{3(2+3\alpha_3+\alpha_5)^2}\, m^2$$

Cosmology

If $a(r,t) = a(t)$ one recovers

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(T_{00}^{\Lambda+m} + \rho\right)$$

additional $T_{\mu\nu}^{matter}$

Perturbations

- To first order in perturbations the action seems the same as $GR + \Lambda_{eff.}$ in the scalar and vector sector.
- Only tensor modes may be different.

Crisostomi et al
Hu et al
Mukohyama et al

Conclusions

- Massive gravity naturally contains:
  - Schwarzschild-(Anti) de Sitter
  - Self-accelerating solutions

  } with $\Lambda_{eff} \sim m^2 \, (+\Lambda_0)$

- Linear perturbations around self-accelerating solutions are identical to GR plus an effective cosmological constant in the scalar and vector sectors. Tensors may be the only way to discriminate between massive gravity & ΛCDM

- ¿What happens beyond linear order?