On the dynamics of unified k-essence cosmologies

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Motivation

Scalar fields play an important role in cosmological models because, due to their simplicity and adaptability, they can account for different interesting phenomena, e.g. inflation, DE.

There is an increasing interest to study $F(X)$ models in cosmology. Recent works have studied the possibility that dark matter, dark energy, and inflation have a common origin.

Scalar fields

Lagrangian of a canonical scalar field

$$\mathcal{L} = X + V(\phi)$$

with $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

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Generalized Lagrangian

$$\mathcal{L} = P(X, \phi)$$

Different particular forms of the Lagrangians has been studied for different reasons.

Generalized kinetic Lagrangian $\mathcal{L} = F(X) + V(\phi)$

The FRW equation are:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}(2XF_X - F + V)$$

and

$$\frac{d}{dt}(2XF_X - F + V) + 6HXF_X = 0\,,$$

where $H = \dot{a}/a$ is the Hubble parameter and $M_{\rm Pl}^2 \equiv 1/8\pi G$. The density and pressure of the effective "fluid" given by:

$$\rho = 2XF_X - F, \qquad P = F.$$

Unified Lagrangian

One particular choice is made for the Lagrangian

$$F(X) = \frac{1}{(2\alpha - 1)}\left[(AX)^{\alpha} - 2\alpha\alpha_0\sqrt{AX}\right] + M\,, \quad (1)$$

$$V(\phi) = \frac{1}{2}m^2\phi^2\,. \quad (2)$$

This Lagrangian has the interesting properties to emulate the dark matter, to yield dark energy, and in the very early Universe to drive inflation under certain election of parameters.

If the kinetic part is much larger than the potential

$$\mathcal{L} \approx F(X)$$

that allows the integration

$$\rho = \left[\alpha_0 + \frac{c_0}{a^3}\right]^n - M$$

with $n = 2\alpha/(2\alpha - 1)$, we identify

$$\alpha_0^n - M = \rho_{\mathrm{de}0}$$

$$\frac{n c_0 \alpha_0^{n-1}}{a_0^3} = \rho_{\mathrm{dm}0}$$

Adjusting parameters

The energy density can be expanded as

$$\rho = \underbrace{\alpha_0^n - M}_{\text{dark energy}} + \underbrace{\frac{n c_0 \alpha_0^{n-1}}{a^3}}_{\text{dark matter}} + \sum_{k=2}^{n} \binom{n}{k} \alpha_0^{n-k} \left(\frac{c_0}{a^3}\right)^k .$$

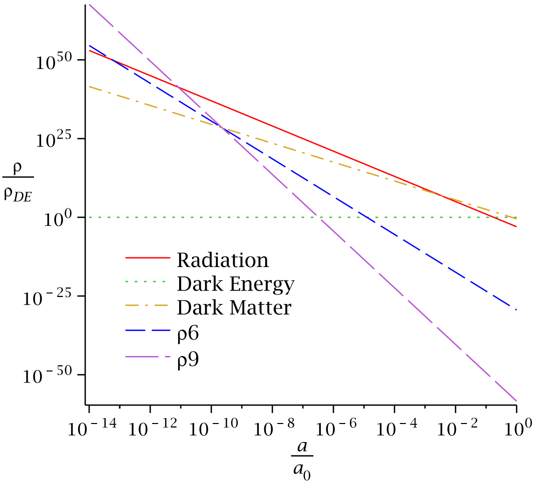
The extra terms should be small.

- If $M = 0$ entonces $\alpha \gg 10^{21}$
- If $\alpha = 1$ then $\alpha_0^2 \sim M$ y $M \gg 10^{24} \rho_{de0}$
- If $\alpha = 3/4$ then $M \gg 10^{28} \rho_{de0}$

Additionally, one demands $M < \rho_{\text{Planck}}$.

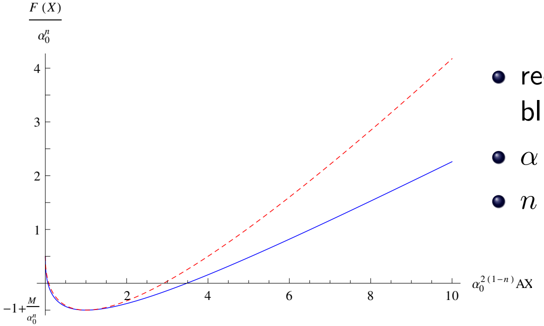
Example of evolution

For the case $\alpha = 3/4$



kinetic potential

$$F(X) = \frac{1}{(2\alpha - 1)} \left[(AX)^{\alpha} - 2\alpha\alpha_0 \sqrt{AX} \right] + M$$



- red curve $n = 2$, blue curve $n = 3$.
- α adimensional
- $n = \frac{2\alpha}{2\alpha - 1}$

Another kinetic model

Scherrer proposed the model

$$F(X) = F_m + F_2(X - X_m)^2,$$

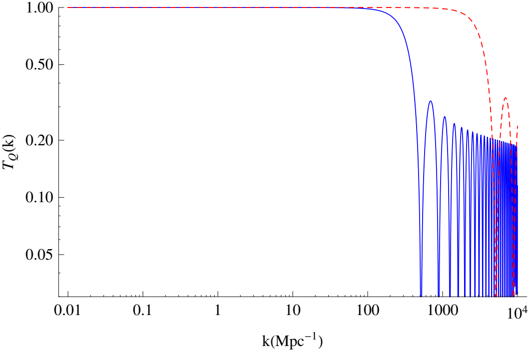
that for small deviations around the minimum it evolves as

$$\rho = -F_m + 4F_2X_m^2\epsilon_1(a/a_1)^{-3}$$

with $\epsilon = (X - X_m)/X_m$.
The transfer function of these models deviates from cold dark matter (CDM) as $T(k) = T_Q(k)T_{\rm CDM}(k)$, $T_Q(k)$ should be close to 1 and therefore ϵ_0 must be less than 10^{-16} (Giannakis & Hu 2005).

Transfer function

Around the minimum our model can be approximated as Scherrer's with a deviation $\epsilon_0 = (X_0 - X_m)/X_m$ less than 10^{-23} for $n = 2$ (blue line) and 10^{-26} for $n = 3$ (red line).
The transfer function do not deviate much from CDM's.



Inflation

At the beginning the potential term dominates over all F(X) terms to achieve inflation, if one has slow roll conditions. The evolution equations are:

$$3M_{\rm pl}^2 H^2 = 2XF_X - F + V, \tag{3}$$

$$(F_X\dot{\phi})^{\cdot} + 3HF_X\dot{\phi} + V' = 0, \tag{4}$$

In slow roll they become

$$3M_{\rm pl}^2 H^2 = V(\phi), \tag{5}$$

$$3HF_X\dot{\phi} + V'(\phi) = 0, \tag{6}$$

Observables from inflation

The spectral index $n_s = 1 - 0.3\sqrt{n-1}$
Ratio of scalar to tensorial perturbations $r = 0.15\sqrt{n-1}$
Initial energy for inflation $V_i = 5.9 \times 10^{-9} M_{\mathrm{Pl}}^4 \sqrt{n-1}$
At the end of inflation reheating could be produce via gravitational particle production $\rho_{rf} \sim 0.01 g H_f^2$
The system being dominated by the kinetic terms at the end of inflation, the field decays as a^{-3n}, whereas radiation decays as a^{-4}. Thus, the radiation begins to dominate over the kinetic terms after $25/(3n-4)$ e-folds. For $n = 2$, it needs 12 e-folds and for $n = 4$, it needs 3 e-folds.

Dynamical system, for $\alpha = 1$

Performing a change of variable to $z \equiv \dot{\phi}/\sqrt{A}$, let us to arrive to a system of first order autonomous equations

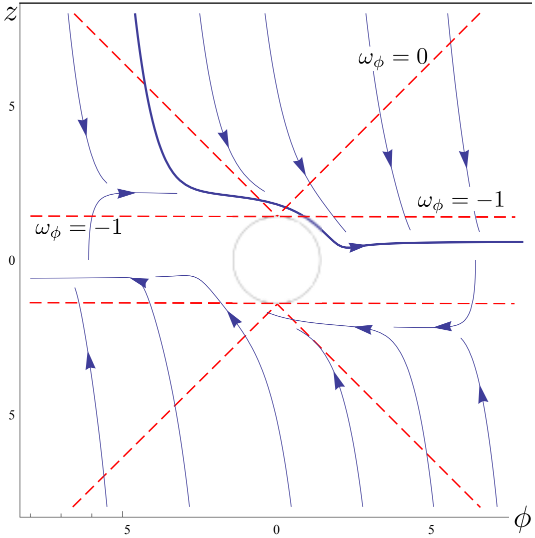
$$\dot{z} = -\frac{m^2\phi}{\sqrt{A}} + \frac{\sqrt{3}}{2M_P}\left(-\sqrt{2}z + 2sign(z)\right)\sqrt{z^2 + m^2\phi^2 - 2M}, \quad (7)$$

$$\dot{\phi} = \frac{z}{\sqrt{A}}. \quad (8)$$

With the equation of state of the field written in terms of these variables as

$$\omega_\phi = \frac{2M + z^2 - \sqrt{8}|z| - m^2\phi^2}{-2M + z^2 + m^2\phi^2}. \quad (9)$$

Phase space



Critical points

In general for the purely kinetic Lagrangian, we obtain the critical values of the system as $AX = 0$ and $AX = \alpha_0^{2/(2\alpha-1)}$, which correspond to the equation of state $-(2\alpha - 1)$ and -1, respectively. The first point is unstable and the second is stable. This is important because the dynamical evolution of the system will drive the field to a behaviour similar to a cosmological constant at late times.

Conditions for UDM

Now we study the conditions for a more general class of scalar fields to reproduce the same dynamical features. If the Lagrangian has a general form $\mathcal{L} = \mathcal{L}(X, \phi)$, its equation of state is

$$\omega = \frac{\mathcal{L}}{2X\mathcal{L}_X - \mathcal{L}}, \tag{10}$$

and the effective sound speed

$$c_s^2 = \frac{\mathcal{L}_X}{2X\mathcal{L}_{XX} + \mathcal{L}_X}. \tag{11}$$

A sufficient condition for the field to behave as dark matter is that both quantities be close to zero, leaving the conditions

$$\frac{\mathcal{L}}{X\mathcal{L}_X} \ll 1 \quad \text{and} \quad \frac{\mathcal{L}_X}{X\mathcal{L}_{XX}} \ll 1\,.$$

Conditions for UDM...

There are several Lagrangians that accomplish the above conditions and they have been proposed as models for unified dark matter models (UDM) meaning that they can behave as dark matter and, adding a constant to the Lagrangian, as a combination of dark matter and dark energy.

Eq. of state condition

An example proposed in the literature is Scherrer's model (2004):

$$\mathcal{L} = F(X) = F_0 + F_m(X - X_0)^2,$$

when the kinetic term is near the minimum $X \sim X_0$, it is known to behave as dark matter plus dark energy.

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An example proposed in the literature is Scherrer's model (2004):

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when the kinetic term is near the minimum $X \sim X_0$, it is known to behave as dark matter plus dark energy.
In general, when the system is around a minimum, the Lagrangian can be expanded as

$$\mathcal{L}(X, \phi) = \mathcal{L}_0 + \frac{1}{2}\mathcal{L}_2\delta^2 + \frac{1}{3!}\mathcal{L}_3\delta^3 \cdots$$

δ is the deviation from the minimum, $\delta = X - X_0$. The first condition is

$$\frac{\delta}{2X_0} - \frac{6\mathcal{L}_2 + X_0\mathcal{L}_3}{12X_0^2\mathcal{L}_2}\delta^2 + \cdots \ll 1\,,$$

that imposes the condition on δ/X_0 to be small. For Scherrer's model δ/X_0 has to be smaller than 10^{-16} at the present epoch to avoid discrepancies in the structure formation and CMB power spectrum in comparison with observations.

Our model

For our Lagrangian with $\alpha = 1$, the dynamics of the field after inflation leaves it very close to the minimum. We have shown that during the equality epoch the deviation δ/X_0 is of order 10^{-13} and at the present epoch of order 10^{-23}, resulting in an identical model to the standard model of cosmology.

Sound speed condition

For the field around a minimum the condition is written as

$$\frac{\delta}{X_0} - \frac{2\mathcal{L}_2 + \mathcal{L}_3}{2X_0^2\mathcal{L}_2}\delta^2 + \cdots \ll 1\,.$$

This condition is similar to the previous and, except for very particular Lagrangians, the accomplishment of the first equation will be enough, that is, the deviation from the minimum δ/X_0 must be small. This is achieved for the Scherrer's model, as well as for the model with $\alpha = 1$.

Conclusions 1/2

- We have presented a phase space analysis for a unified model of dark matter, dark energy, and inflation. We have shown that for a large set of the initial conditions $(\phi, \dot{\phi})$ a viable dynamics occurs in which inflation $(\omega_\phi = -1)$ happens first, followed by a period of dark matter domination $(\omega_\phi = 0)$, to finish with dark energy $(\omega_\phi = -1)$.
- Once inflation ends, the model is fully described by the purely kinetic Lagrangian. We have demonstrated that this system possesses a late time stable solution in which $\omega_\phi = -1$, that is dark energy. There is a range of parameters to achieve a successful cosmological model, and in the present work the dynamical analysis clearly shows why the system is tenable.

Conclusions 2/2

- We also presented the general features that are necessary to have a model that behaves as dark matter. If one adds a cosmological constant to this model, one ends with a unified dark matter and dark energy model, called generically UDM. There are two conditions that these models should fulfill playing the role of an effective fluid with small pressure and small speed of sound.
- We have analyzed some models studied in the literature that fulfill these conditions. In particular, $F(X)$ models that possess a minimum, as Sherrer's model or our model, when they are close enough to the minimum, they behave as dark matter. Departures from the minimum cause a change in the transfer function and therefore to a different growth history in comparison to the standard model of cosmology.
- Look at the works: De-Santiago et al, arXiv:1102.1777, arXiv:1204.3631.