# Dark energy from inflation

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# Outline

**The accelerated eras of the universe**
- $\Lambda$CDM cosmology
- Unaddressed questions
- Primordial inflation
- Interpretations of the current acceleration
- Freezing quintessence models

**Dark energy from primordial inflation**
- Massive scalar field
- Posterior probability distributions
- During primordial inflation
- Field variance and inhomogeneities
- Inferring the energy scale of inflation

**Conclusion**

CR, T. Suyama, T. Takahashi, M. Yamaguchi, S. Yokoyama:
`arXiv:1006.0368`

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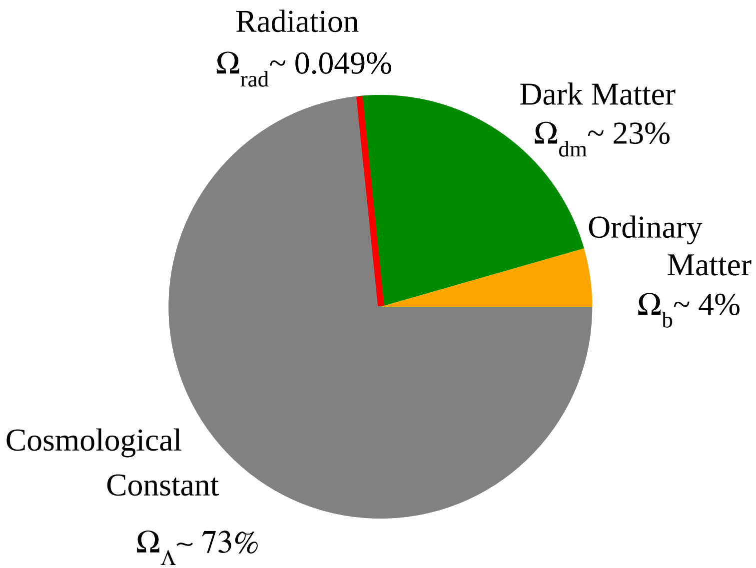
# ΛCDM cosmology

- Fits well all current observations: background + perturbations




  - Gaussian perturbations
  - No evidence for non-adiabaticity
  - Accelerating flat universe today

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \simeq 0.73$$

$$\Omega_K \equiv \frac{-K}{H_0^2} = 0 \pm 0.01$$

- Almost scale invariant primordial power spectra

$$n_S - 1 \simeq -0.036 \pm 0.013$$

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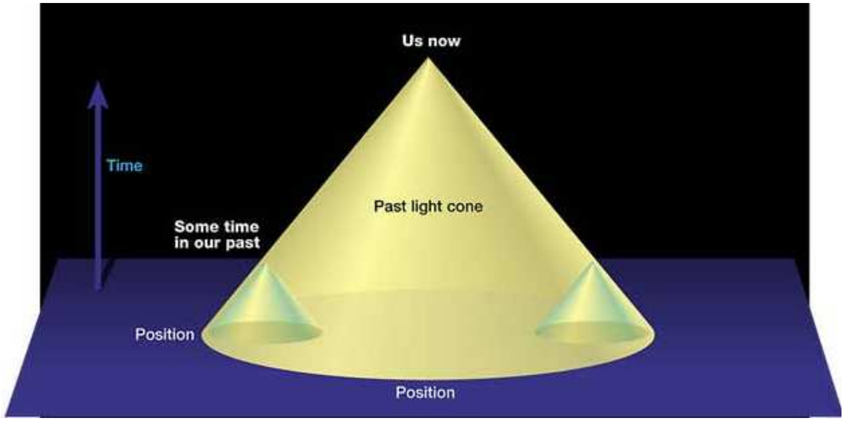
# Unaddressed questions in $\Lambda$CDM cosmology

- Dark sector
  - $\Omega_{\rm dm}$: dark matter of unknown origin and nature (WIMP?)
  - $\Omega_{\Lambda} \neq 0$: see later
- Decelerating issues: flatness is unstable and horizon problems




- Fine-tuned parameters and IC?
  - $\Omega_{\rm K} \simeq 0$, $n_{\rm S} \lesssim 1$, adiabatic IC, Gaussianity

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# Primordial inflation

- Accelerated expansion at early times
  - Can be triggered by a cosmological fluid $P = w\rho$ with $w < -1/3$
  - Scalar field: $w \simeq -1 \Rightarrow H^2 = H_{\mathrm{inf}}^2$ is almost constant
- Answers flatness, horizon problems

$$a(t) \propto e^{H_{\mathrm{inf}} t}, \quad \Omega_{\mathrm{K}}(t) \propto e^{-2H_{\mathrm{inf}} t}, \quad d_{\mathrm{h}} \simeq \frac{e^{H_{\mathrm{inf}} t}}{H_{\mathrm{inf}}}$$

- Answers $n_{\mathrm{S}} \simeq 1$ and Gaussianity from quantum fluctuations

$$A_{\mathrm{S}} = \frac{\kappa^2 H_{\mathrm{inf}}^2}{8\pi^2 \epsilon_{1*}}, \quad n_{\mathrm{S}} = 1 - 2\epsilon_{1*} - \left.\frac{\mathrm{d}\ln\epsilon_1}{\mathrm{d}N}\right|_* \lesssim 1$$

- A scalar field decaying into cosmological fluids yields adiabaticity

$$\delta n_{\mathrm{dm}}/n_{\mathrm{dm}} = \delta n_\gamma/n_\gamma = \delta n_{\mathrm{b}}/n_{\mathrm{b}}$$

- Could inflation say something on $\Omega_\Lambda$?

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# Interpretations of the current acceleration

- Standard approach: GR has two fundamental constants $M_{\rm Pl}$ and $\Lambda_{\rm GR}$

$$M_{\rm Pl} \simeq 10^{19}\mathrm{GeV}, \qquad \rho_\Lambda^{1/4} \simeq (\rho_{\rm c}\Omega_\Lambda)^{1/4} \simeq 10^{-3}\mathrm{eV}$$

- No reason of having an intrinsic space-time tension $\Lambda_{\rm GR} = 0$
  - $\Lambda$ is $\langle T_{\mu\nu}\rangle$ induced by the zero point energy of QFTs
    - It is absurdly large in the current Standard Model and extensions
    - Once GR will be quantized, unified... $\Rightarrow \Lambda = \Lambda_{\rm predicted}$
  - $\Lambda$ is induced by a new cosmological fluid: dark energy
    - Assumes that "$\Lambda_{\rm GR} - \langle T_{\mu\nu}\rangle = 0$"
    - It is dynamical: coincidence problem + extra parameters
  - Various proposals of dark energy
    - Modification of GR, extra-dimensions, a late time dominating scalar field: quintessence

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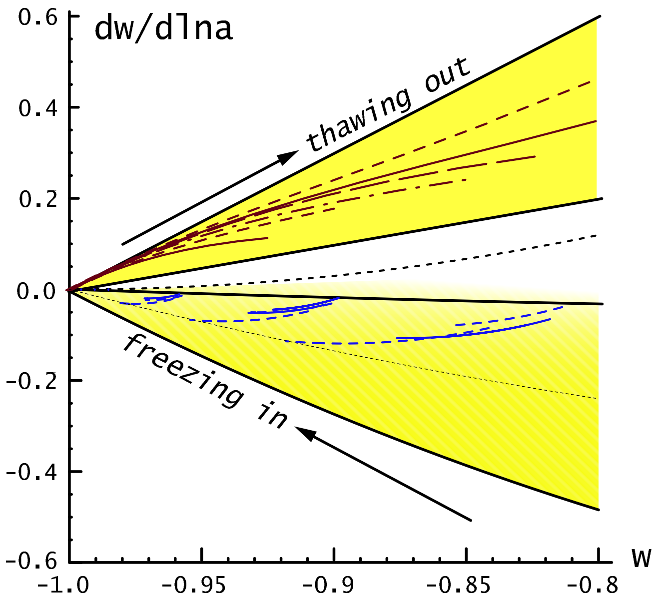
# Freezing quintessence models

- Scalar field $Q$ in potential domination + tracking
  - Ratra–Peebles potential $V(Q) = M^{4+\alpha} Q^{-\alpha}$

$$Q \propto a^{3(1+w)/(\alpha+2)}, \quad \rho_Q \propto \frac{1}{a^{3(1+w_Q)}}, \qquad w_Q \equiv \frac{\alpha w - 2}{\alpha + 2}$$

  - Dominates over matter now: $z + 1 = (\Omega_Q / \Omega_{mat})^{(\alpha+2)/6}$
- Testable [Caldwell 05] and SUGRA motivated [Brax 99]




- Freezing models: $w_Q > -1$ and $w_Q \to -1$
- Tracking ensures IC are washed out!

  $\to$ one observable $M$

- Thawing models are exactly as inflation
  - How to start? Which IC? 3 params

# Current acceleration from a free scalar field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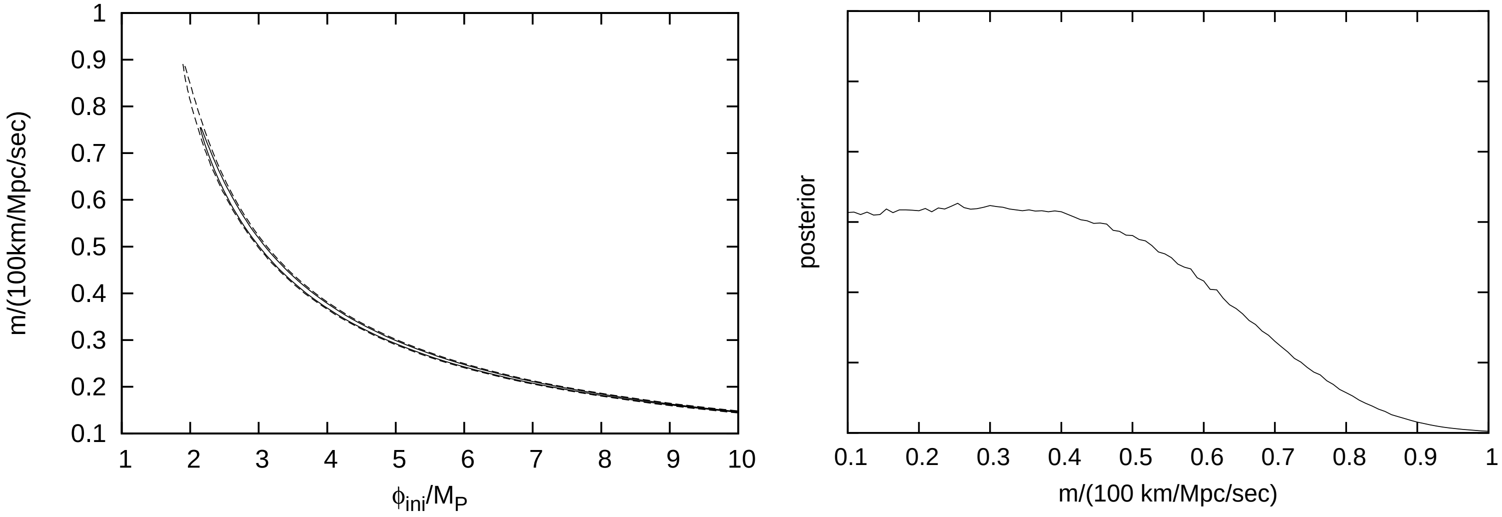
- Simplest thawing model: $V(\phi) = \dfrac{1}{2}m^2\phi^2$

$$H^2 = \frac{\kappa^2}{3}\left[\rho_{\mathrm{rad}}(t) + \rho_{\mathrm{mat}}(t) + \rho_\phi(t)\right], \qquad \frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} + 3H\frac{\mathrm{d}\phi}{\mathrm{d}t} + m^2\phi = 0$$

- Extra model parameters: $\phi_{\mathrm{ini}}$, $\dot{\phi}_{\mathrm{ini}}$, $m$
  - $\dot{\phi}_{\mathrm{ini}}$ has no effect: relaxation towards slow-roll (see inflation)
  - For $\rho_\phi$ small: $\phi(t) \propto HJ_1(m/H)$ which is constant $m \ll H$
  - Starts to roll-down when $H \simeq m$
- Fitting current data
  - Type Ia supernovae + BAO + CMB peak [Amanullah 10, Komatsu 10, Percival 10]
  - Gridding over $H_0$, $\phi_{\mathrm{ini}}$, $m$ + Bayesian statistics

# Posterior probability distributions

- From prior choices
  - Jeffreys' prior on $\phi_{\text{ini}} \geq M_{\text{Pl}}$ (necessary to trigger acceleration)
  - Flat prior on $m < 100$ km/s/Mpc
  - Large flat prior on $H_0$ around $70$ km/s/Mpc
- Marginalised posteriors for $\phi_{\text{ini}}$ and $m$ (over $H_0$)




- $95\%$ confidence intervals: $m < 75\,\mathrm{km/s/Mpc} \Rightarrow$ ultra light field

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# During primordial inflation

- Thawing field quantum fluctuates (test field): $\varphi(t, \boldsymbol{x}) = \phi(t) + \delta\phi(t, \boldsymbol{x})$
  - Homogeneous part

$$\ddot{\phi} + 3H_{\mathrm{inf}}\dot{\phi} + m^2\phi = 0 \Rightarrow \phi(N) = \phi_0 e^{-Nm^2/(3H_{\mathrm{inf}}^2)} \to 0$$

  - Fluctuations in Fourier space: $\mu \equiv a\delta\phi_{\boldsymbol{k}}$ $(aH_{\mathrm{inf}} = -1/\eta)$

$$\delta\ddot{\phi}_{\boldsymbol{k}} + 3H_{\mathrm{inf}}\delta\dot{\phi}_{\boldsymbol{k}} + \left(\frac{k^2}{a^2} + m^2\right)\delta\phi_{\boldsymbol{k}} = 0 \Rightarrow \mu'' + \left(m^2 + k^2 - \frac{2}{\eta^2}\right) = 0$$

  - Free field quantization: positive energy waves for $k\eta \gg 1$

$$\mu = e^{i(\nu+1/2)\pi/2}\sqrt{\frac{\pi}{4k}}\sqrt{k\eta}H_\nu^1(k\eta), \quad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H_{\mathrm{inf}}^2}}$$

- Power spectra after Hubble exit: $\mathcal{P}_{\delta\phi} = \lim_{k\eta\ll 1} \dfrac{k^3}{2\pi^2}\left|\dfrac{\mu}{a}\right|^2$

# Field variance and inhomogeneities


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- Primordial inhomogeneities as small as tensor modes

$$\mathcal{P}_{\delta\phi} \simeq \frac{H_{\rm inf}^2}{4\pi^2}\left(\frac{k}{aH_{\rm inf}}\right)^{2m^2/(3H_{\rm inf}^2)} = \frac{H_{\rm inf}^2}{4\pi^2} + \dots$$

- Field variance in physical space after $N$ e-folds

$$\langle \delta\phi^2 \rangle = \int_{a_i H_{\rm inf}}^{a H_{\rm inf}} \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \left|\delta\phi_{\boldsymbol{k}}\right|^2 = \frac{3H_{\rm inf}^4}{8\pi^2 m^2}\left[1 - e^{-N(2m^2)/(3H_{\rm inf}^2)}\right] \to \frac{3H_{\rm inf}^4}{8\pi^2 m^2}$$

- Universal energy density expectation value

$$\langle V(\phi) \rangle = \frac{1}{2} m^2 \langle \delta\phi^2 \rangle = \frac{3H_{\rm inf}^4}{16\pi^2}$$

- This is dark energy provided: $H_{\rm inf} = (\Omega_\Lambda)^{1/4}\sqrt{4\pi H_0 M_{\rm Pl}}$

$$H_{\rm inf} \simeq 6\times 10^{-3}\,\mathrm{eV}, \quad \rho_{\rm inf}^{1/4} = (3M_{\rm Pl}^2 H_{\rm inf}^2)^{1/4} \simeq 5\,\mathrm{TeV}$$

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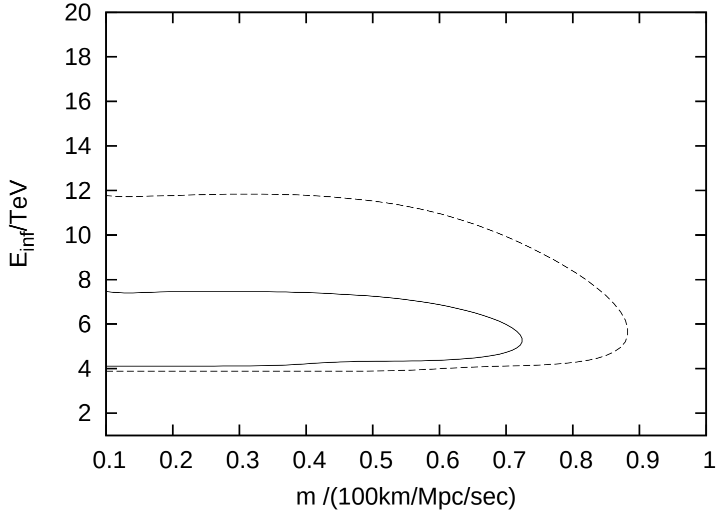
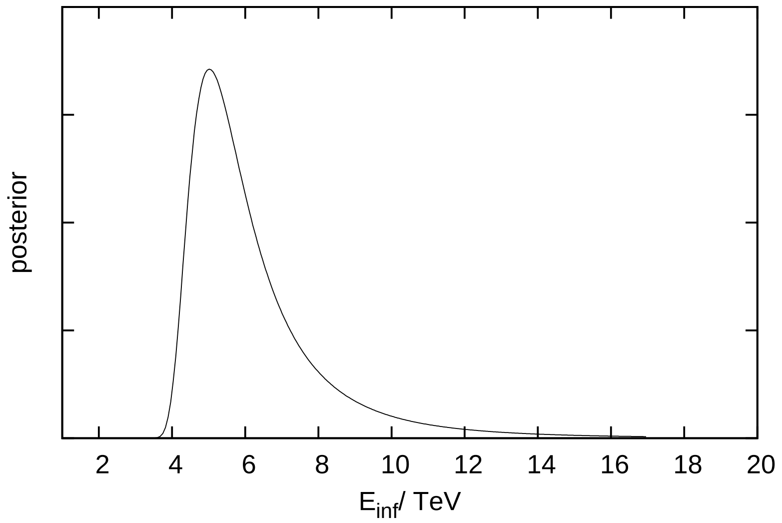
# Inferring the energy scale of inflation

- $\phi_{\rm ini}$ is Gaussian distributed with $\sigma^2 = 3H_{\rm inf}^2/(8\pi^2 m^2)$

$$P(H_{\rm inf}, m|D, I) \propto P(H_{\rm inf}, m|I) \iint \mathrm{d}H_0 \frac{\mathrm{d}\phi_{\rm ini}}{\sqrt{2\pi\sigma}} e^{-\phi_{\rm ini}^2/(2\sigma^2)} \times P(H_0|I)\mathcal{L}(D, m, \phi_{\rm ini}, H_0, I)$$

- Marginalised posteriors for $E_{\rm inf} = \rho_{\rm inf}^{1/4}$




- 95% confidence level: $3.8\,\mathrm{TeV} < E_{\rm inf} < 12.1\,\mathrm{TeV}$

# Conclusion

- Dark energy as a new inflationary era triggered by the primordial one
  - Needs ultra light field $m < 10^{-33}\,\mathrm{eV}$ (and $N$ huge)
  - Makes predictions: primordial inflation occured at $\mathrm{TeV}$ scale
  - $\Lambda =$ energy density typical of $\mathrm{TeV}$ quantum fluctuations.
- Actually works for any field potential $V(\phi)$ having a minimum
  - Stochastic inflation formalism

$$P(\phi|H_{\mathrm{inf}}) \propto \exp\left[-\frac{8\pi^2}{3H_{\mathrm{inf}}^4}V(\phi)\right] \Rightarrow \langle V \rangle \simeq \frac{3H_{\mathrm{inf}}^4}{8\pi^2}$$

  - Needs however a frozen field during radiation/matter (light)