Generalized Higgs inflation

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arXiv:1012.4238, PRD 83, 083515 (2011), K. Kamada, T. Kobayashi, MY, J. Yokoyama

arXiv:1203.4059, to appear in PRD, K. Kamada, T. Kobayashi, T. Takahashi, MY, J. Yokoyama

$c = \hbar = M_G = 1$

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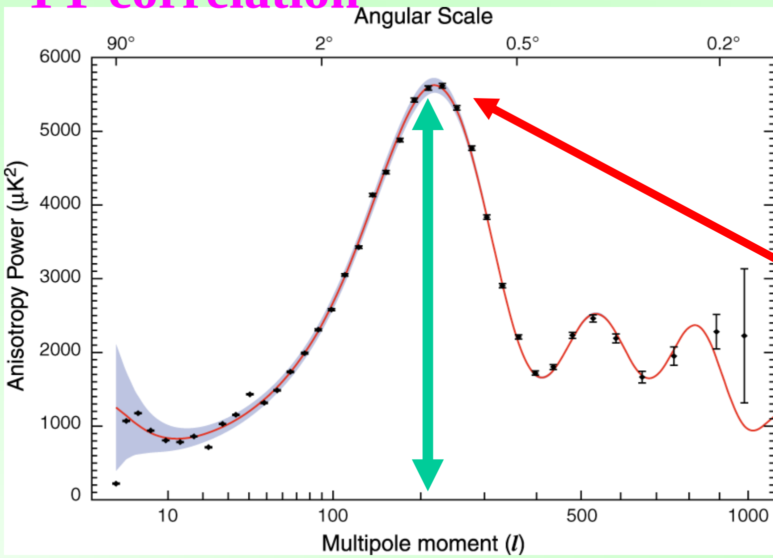
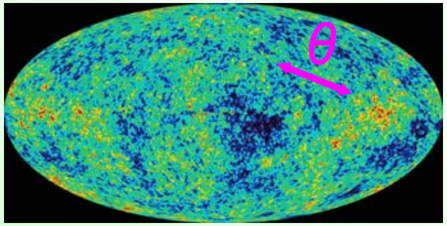
Introduction

WMAP & Inflation

TT correlation



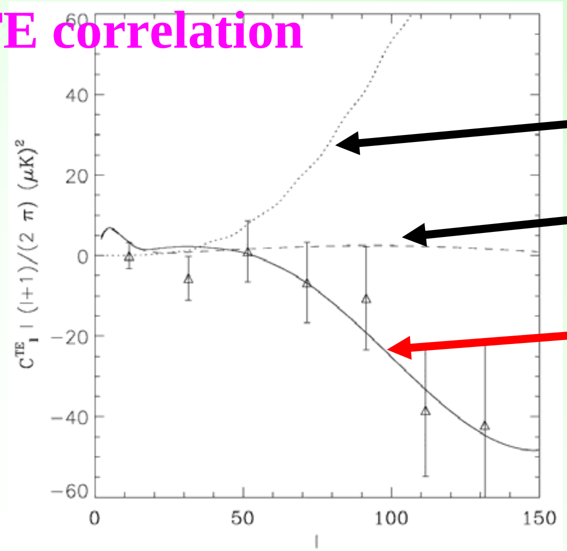


Angle $\theta \sim 180^\circ / l$

$\Omega_{total} \simeq 1.0$

⇒ **spatially flat** **(as predicted by inflation)**

TE correlation



Causal seed models

Inflation models (isocurvature)

Inflation models (adiabatic)

WMAP strongly supports inflation !!

Inflation and scalar field

- An (effective) scalar field is required to cause inflation.
- The standard model of particle physics includes a unique scalar field, i.e. Higgs field, ${}^t\mathcal{H} = (0, v+\phi)/\sqrt{2}$.

$$\Rightarrow \quad V(\phi) = \lambda\left(\mathcal{H}^2 - v^2\right)^2 \simeq \frac{\lambda}{4}\phi^4, \quad \text{for} \quad \phi \gg v.$$

The Higgs field may play a role of inflaton (Higgs inflation).

Collider constraint : $\lambda = \mathcal{O}(0.1)$

Cosmological constraint : $\lambda = \mathcal{O}(10^{-13})$

⇒ **Conflict !!**

(More precisely, chaotic inflation with quartic potential predicts too large tensor perturbations.)

We need to modify gravitational or kinetic sector to realize Higgs inflation.

Various Higgs inflation models I

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R + X - V(\phi) + \Delta\mathcal{L}, \quad X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

Modify the gravitational sector :

- **(original) Higgs inflation :** Bezrukov and Shaposhnikov 2008
Barvinsky, Kamenshchik, Starobinsky. 2008, and many others

Large and negative non-minimal coupling, $\Delta\mathcal{L} = -\frac{\xi}{2}\phi^2 R, \quad \xi < 0.$

⇒ **The effective Planck scale becomes larger during inflation so that curvature perturbations are suppressed. (λ = O(0.1) can fit the WMAP data.)**

- **new Higgs inflation :** Germani and Kehagias 2008, 2010

Derivative coupling to the Einstein tensor, $\Delta\mathcal{L} = \frac{1}{2\mu^2} G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$

⇒ **This coupling changes the normalization of the Higgs field so that quantum fluctuations are suppressed.**

Various Higgs inflation models II

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}R + X - V(\phi) + \Delta\mathcal{L}, \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

Modify the kinetic sector :

- **running kinetic inflation :** Nakayama and Takahashi 2010, 2011

Non-trivial kinetic term, $\Delta\mathcal{L} = \kappa\phi^{2n}X.$

⇒ **This coupling changes the normalization of the Higgs field so that quantum fluctuations are suppressed.**

- **Higgs G-inflation :** Kamada, Kobayashi, MY, Yokoyama 2011

Higher derivative interaction, $\Delta\mathcal{L} = \frac{\phi}{M^4}X\Box\phi.$

⇒ **This term acts as an additional friction term so that the potential is effectively smoothened and curvature perturbations are suppressed.**

Unification of Higgs inflation models

The following question may arise :

- **Can we understand all of the proposed Higgs inflation models in a unified way ?**

  We would like to single out the model from the future experimental and observation data like the LHC experiment and the Planck satellite.

  But, without such a unified model, we cannot perform the model (parameter) selection using the MCMC method.

- **Is there another type of Higgs inflation ?**

Generalized Higgs inflation

Galileon field

Nicolis et al. 2009
Deffayet et al. 2009

These Lagrangians are invariant under Galilean shift symmetry in flat space :

$$\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu$$

$$\begin{cases} \mathcal{L}_1 = \phi \\ \mathcal{L}_2 = (\partial\phi)^2 \\ \mathcal{L}_3 = (\partial\phi)^2 \, \Box\phi \\ \mathcal{L}_4 = (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] \\ \mathcal{L}_5 = (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3\right] \end{cases}$$

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

Lagrangian has higher order derivatives, but EOM is second order.

Generalized Galileon: covariantization of Galileon field

Horndeski 1974
Deffayet et al. 2009, 2011

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$X = -\frac{1}{2}(\nabla\phi)^2, \qquad G_{iX} \equiv \partial G_i/\partial X.$$

This is the most general non-canonical and non-minimally coupled single-field model which yields second-order equations.
These Lagrangians were found by Horndeski in 1974 and recently rediscovered by Deffayet et al. in the present form.

NB : ● $G_4 = M_G^2 / 2$ yields the Einstein-Hilbert action
● $G_4 = f(\phi)$ yields a non-minimal coupling of the form $f(\phi)R$

Classification of various Higgs inflation models

Expand the functions in terms of X :

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots,$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots,$$

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$\begin{cases} g_3(\phi)\Box\phi = 2g_3'X + (\text{t.d.}), \\ g_5(\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = -g_5'\left[XR + (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + 3g_5''X\Box\phi - 2g_5'''X^2 + (\text{t.d.}), \end{cases} \Rightarrow \begin{cases} g_3 = 0, \\ g_5 = 0. \end{cases}$$

- **(original) Higgs inflation :** $\Delta\mathcal{L} = -\frac{\xi}{2}\phi^2 R.$ ⇒ $g_4(\phi) := g(\phi) = -\frac{\xi}{2}\phi^2 \left(+\frac{M_{\rm pl}^2}{2}\right).$
- **new Higgs inflation :**

  $\Delta\mathcal{L} = \frac{1}{2\mu^2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -\frac{1}{2\mu^2}\phi G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \frac{1}{2\mu^2}\left[XR + (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right].$ ⇒ $h_4(\phi) = \frac{1}{2\mu^2}.$
- **running kinetic inflation :** $\Delta\mathcal{L} = \kappa\phi^{2n}X.$ ⇒ $\mathcal{K}(\phi) = \kappa\phi^{2n}\,(+1).$
- **Higgs G-inflation :** $\Delta\mathcal{L} = \frac{\phi}{M^4}X\Box\phi.$ ⇒ $h_3(\phi) = \frac{\phi}{M^4}.$

All of the Higgs inflation models proposed thus far can be understood as potential driven cases of the Generalized G(alileon)-inflation.

Running Einstein Higgs inflation

The case with h5(ϕ) ≠ 0 has not yet been considered.

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots,$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots,$$

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$\Rightarrow \quad \Delta\mathcal{L} = h_5(\phi)XG_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}h_5(\phi)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].$$

This kind of kinetic and derivative coupling to Einstein tensor should also be discussed as possibility of Higgs inflation.

We name it running Einstein Higgs inflation and investigate all of the five models (and their arbitrary combination) on equal footing.

Dynamics of generalized Higgs inflation

- **Assuming the following slow-roll conditions :**

$$\epsilon := -\frac{\dot{H}}{H^2} \ll 1, \quad \eta := -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1, \quad \delta := \frac{\dot{g}}{Hg} \ll 1,$$

$$\alpha_2 := \frac{\dot{\mathcal{K}}}{H\mathcal{K}} \ll 1, \quad \alpha_i := \frac{\dot{h}_i}{Hh_i} \ll 1 \ \ (i = 3, 4, 5).$$

$$\Rightarrow \begin{cases} H^2 \simeq \dfrac{V}{6g}, \\ 3HJ \simeq -V' + 12H^2 g'. \end{cases}$$

$g = M_G^2/2$, $\mathcal{K} = 1$, hi=0
➔ standard Friedmann Eq.
standard EOM

$$J \simeq \mathcal{K}\dot{\phi} + 6\left(Hh_3X + H^2h_4\dot{\phi} + H^3h_5X\right).$$

Which term ($\mathcal{K}$, hi) dominates the dynamics ?

Powerspectrum of primordial fluctuations

Primordial tensor perturbations

Kobayashi, MY, Yokoyama 2011

Perturbed metric :

$$\begin{cases} ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right) \\ h_{ii} = 0 = h_{ij,j} \end{cases}$$

⇒

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i,$$

$$\begin{cases} \mathcal{L}_2 = K(\phi, X), \\ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 = \underline{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 = \underline{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ \qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{cases}$$

$$S_T^{(2)} = \frac{1}{8}\int dt d^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right].$$

$$\begin{cases} \mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \\ \mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right], \\ c_T^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}. \end{cases}$$

For $G_{4X} \neq 0$ or $G_{5\phi} \neq 0$ or $G_{5X} \neq 0$,
the sound velocity squared c_T^2 can deviate from unity.

No ghost & gradient instabilities ⇔ $\mathcal{F}_T > 0, \quad \mathcal{G}_T > 0.$

Powerspectrum of tensor perturbations

Mode functions : $v_{ij} = \frac{\sqrt{\pi}}{2}\sqrt{-y_T}H^{(1)}_{\nu_T}(-ky_T)e_{ij}$ ← **polarization tensor**

$$\Rightarrow \quad h_{ij}(\mathbf{k}, y_T) = \frac{v_{ij}(y_T)}{z_T}\hat{a}_{\mathbf{k}} + \frac{v^*_{ij}(y_T)}{z_T}\hat{a}^\dagger_{-\mathbf{k}}$$

Commutation relations : $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$

$$\Rightarrow \quad \mathcal{P}_T = \frac{k^3}{2\pi^2}\left|\frac{v_{ij}}{z_T}\right|^2 = 2^{2\nu_T}\left|\frac{\Gamma(\nu_T)}{\Gamma(3/2)}\right|^2 \frac{(1-\epsilon-s_T)^2}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\bigg|_{ky_T=-1}.$$

$$\Rightarrow \quad n_T = 3 - 2\nu_T = -\frac{2\epsilon + 3s_T + g_T}{1-\epsilon-s_T} = -\frac{4\epsilon + 3f_T - g_T}{2(1-\epsilon-s_T)}.$$

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad f_T := \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T := \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}, \quad \left(s_T := \frac{\dot{c}_T}{Hc_T} = \frac{1}{2}(f_T - g_T)\right)$$

Note that the blue spectrum $n_T > 0$ can be easily obtained as long as $4\epsilon + 3f_T - g_T < 0$.

Primordial scalar perturbations

Kobayashi, MY, Yokoyama 2011

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\zeta)d\boldsymbol{x}^2$$

Unitary gauge : $\phi = \phi(t), \quad \delta\phi = 0.$

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\zeta)^2\right]$$

Hamiltonian & Momentum constraints

$$\begin{aligned}
\mathcal{F}_S &:= \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \\
\mathcal{G}_S &:= \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T, \\
c_s^2 &:= \frac{\mathcal{F}_S}{\mathcal{G}_S}.
\end{aligned}$$

$$\begin{aligned}
\Sigma := \;& XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
& +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
& +6[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
& -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)] \\
& +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
& +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X(6G_{5\phi} \\
& +9XG_{5\phi X} + 2X^2G_{5\phi XX}) \\
= \;& X\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial X} + \frac{1}{2}H\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial H}.
\end{aligned}$$

$$\begin{aligned}
\Theta := \;& -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
& -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
& -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
& +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right) \\
= \;& -\frac{1}{6}\sum_{i=2}^{5}\frac{\partial\mathcal{E}_i}{\partial H}.
\end{aligned}$$

No ghost & gradient instabilities $\Leftrightarrow$ $\mathcal{F}_S > 0, \quad \mathcal{G}_S > 0.$

Powerspectrum of scalar perturbations

Mode functions : $u_k = \frac{\sqrt{\pi}}{2}\sqrt{-y_S}H_q^{(1)}(-ky_S)$

$$\Rightarrow \quad \zeta(\mathbf{k}, y_S) = \frac{u_k(y_S)}{z_S}\hat{a}_{\mathbf{k}} + \frac{u^*_{-k}(y_S)}{z_S}\hat{a}^\dagger_{-\mathbf{k}}$$

Commutation relations : $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$

$$\Rightarrow \quad \mathcal{P}_\zeta = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z_S}\right|^2 = 2^{2q-3}\left|\frac{\Gamma(\nu_S)}{\Gamma(3/2)}\right|^2 \frac{(1-\epsilon-s_S)^2}{4\pi^2}\frac{H^2}{2\mathcal{F}_S c_s}\bigg|_{ky_S=-1}.$$

$$\Rightarrow \quad n_S = 3 - 2\nu_S = -\frac{2\epsilon + 3s_S + g_S}{1-\epsilon-s_S} = -\frac{4\epsilon + 3f_S - g_S}{2(1-\epsilon-s_S)}.$$

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad f_S := \frac{\dot{\mathcal{F}}_S}{H\mathcal{F}_S}, \quad g_S := \frac{\dot{\mathcal{G}}_S}{H\mathcal{G}_S}, \quad \left(s_S := \frac{\dot{c}_S}{Hc_S} = \frac{1}{2}(f_S - g_S)\right)$$

Note that almost scale invariance requires 2 ε + 3s$_S$ + g$_S$ << 1, while each slow-roll parameter can be large.

Tensor-to-scalar ratio : $r := \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = 16\left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2}\left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2} = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}.$

Powerspectrum of perturbations of Generalized Higgs inflation

Powerspectrum of primordial tensor fluctuations

Slow-roll EOM :

$$\begin{cases} H^2 \simeq \dfrac{V}{6g}, \\ 3HJ \simeq -V' + 12H^2 g'. \end{cases}$$

$$\begin{cases} K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots, \\ G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots, \end{cases}$$

⇒ $\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g.$ $\left(S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right].\right)$

⇒ $c_T^2 \simeq 1.$

⇒
$$\begin{cases} \mathcal{P}_T \simeq \dfrac{4}{g}\left(\dfrac{H}{2\pi}\right)^2 \quad \left(\simeq \dfrac{8}{M_G^2}\left(\dfrac{H}{2\pi}\right)^2 \text{ for } g = \dfrac{M_{\rm pl}^2}{2}\right), \\ n_T = -\dfrac{2\epsilon + g_T}{1-\epsilon} \simeq -(2\epsilon + g_T) \quad (\epsilon \ll 1) \end{cases}$$

Powerspectrum of primordial scalar fluctuations

$$\begin{cases} \mathcal{F}_S \simeq \dfrac{X}{H^2}\left(\mathcal{K} + 6H^2 h_4\right) + \dfrac{4\dot{\phi}X}{H}\left(h_3 + H^2 h_5\right), \\ \mathcal{G}_S \simeq \dfrac{X}{H^2}\left(\mathcal{K} + 6H^2 h_4\right) + \dfrac{6\dot{\phi}X}{H}\left(h_3 + H^2 h_5\right). \end{cases}$$

$$\left(S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2} (\nabla\zeta)^2 \right] . \right)$$

- **K or h4 term domination**

(running kinetic or new Higgs inflation)

$$\mathcal{F}_S \simeq \mathcal{G}_S \simeq g(2\epsilon + g_T).$$

$$\Rightarrow \begin{cases} c_S^2 \simeq 1. \\ \mathcal{P}_S \simeq \dfrac{1}{g(2\epsilon + g_T)} \dfrac{H^2}{8\pi^2}. \end{cases}$$

$$\Rightarrow \quad r = 8(2\epsilon + g_T) = -8n_T.$$

- **h3 or h5 term domination**

(Higgs G or running Einstein inflation)

$$\mathcal{F}_S \simeq \frac{2}{3}\mathcal{G}_S \simeq \frac{4}{3} g(2\epsilon + g_T).$$

$$\Rightarrow \begin{cases} c_S^2 \simeq \dfrac{2}{3}. \\ \mathcal{P}_S \simeq \dfrac{1}{g(2\epsilon + g_T)} \dfrac{3\sqrt{6}H^2}{64\pi^2}. \end{cases}$$

$$\Rightarrow \quad r = \frac{32\sqrt{6}}{9}(2\epsilon + g_T) = -\frac{32\sqrt{6}}{9} n_T.$$

Thus, consistency relations may be useful to discriminate which Higgs inflation model is realized.

Non-Gaussianity of Generalized Higgs inflation

Bispectrum of curvature perturbations :

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3).$$

$$B_\zeta = \frac{(2\pi)^4\mathcal{P}_\zeta^2}{4k_1^3k_2^3k_3^3}\left[6\mathcal{C}_1\frac{(k_1k_2k_3)^2}{K^3}+\frac{\mathcal{C}_2}{K}\left(2\sum_{i>j}k_i^2k_j^2-\frac{1}{K}\sum_{i\neq j}k_i^2k_j^3\right)+\mathcal{C}_3\left(\sum_i k_i^3+\frac{4}{K}\sum_{i>j}k_i^2k_j^2-\frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\right)\right.$$
$$\left.+\frac{\mathcal{C}_4}{K}\left(\sum_i k_i^4-2\sum_{i>j}k_i^2k_j^2\right)\left(1+\frac{1}{K^2}\sum_{i>j}k_ik_j+\frac{3k_1k_2k_3}{K^3}\right)\right].$$

(K=k1+k2+k3)

$$\begin{cases}\mathcal{C}_1 = (3-c_s^2)\left(\dfrac{1}{c_s^2}-1\right),\\ \mathcal{C}_2 = 3\left(1-\dfrac{1}{c_s^2}\right),\\ \mathcal{C}_3 = \dfrac{1}{2}\left(\dfrac{1}{c_s^2}-1\right),\\ \mathcal{C}_4 = -\dfrac{1}{2c_s^2}\left(\dfrac{1}{c_s^2}-1\right).\end{cases}$$

→

$$f_{\rm NL} = \frac{5}{1944}\left(\frac{1}{c_s^2}-1\right)^2(73+39c_s^2).$$

$$\left(\frac{2}{3}\leq c_s^2\leq\infty\right)$$

Non-Gaussianity of the curvature perturbations of generalized Higgs inflation cannot be large.

Summary

- We have presented a unified treatment of all of the Higgs inflation models proposed so far in the context of the generalized G(alileon)-inflation, which is the most general single field inflation model with second order equation of motion.
- We find yet another class of Higgs inflation model, running Einstein Higgs inflation.
- The general formula for powerspectra of tensor and scalar perturbations are derived. Non-Gaussianity of curvature perturbations of generalized Higgs inflation is small.
- For more quantitative analysis, we need to take quantum effects into account. But, since generalized Higgs inflation includes couplings to the Einstein tensor and higher derivative terms, it is a challenge.