Unitarity Bounds in Models of Particle Physics and Inflation

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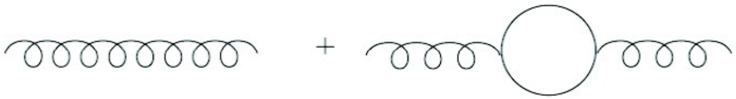
Outline

- Perturbative unitarity as a tool in particle physics
- Unitarity and Higgs inflation
- Unitarity in models of particle physics and extra dimensions

Strength of Gravity

- When does gravity become strong?
- One loop graviton correction goes as $$\frac{1}{16\pi^2}\left(\frac{E}{\bar{M}(E)}\right)^2$$
- Can also consider running of Planck mass:



[F. Larsen & F. Wilczek]

$$\bar{M}_P(\mu)^2 = \bar{M}_P(0)^2 - \frac{1}{96\pi^2}\mu^2 N_l \qquad N_l = N_s + N_\psi - 4N_V$$

- Gravity becomes strong at a scale $\mu_\star$ if: $\mu_\star \gtrsim 4\pi \bar{M}(\mu_\star)$

i.e. loop correction
is of order 1

Question?

- Does this provide all the information to know when the effective theory breaks down?

- A further (better) tool to investigate this is the breakdown of perturbative unitarity...

Unitarity in Quantum Field Theory

- Follows from the conservation of probability, i.e. unitarity of the S-matrix: $S^\dagger S = 1$

- Implies that amplitudes do not grow too fast with energy.

- Can derive a bound on the size of the partial wave amplitudes:

$$\mathcal{A} = 16\pi \sum_j (2j+1) P_j(\cos\theta)\, a_j$$

$$|\mathrm{Re}\, a_j| \leq \frac{1}{2} \qquad\qquad |\mathrm{Im}\, a_j| \leq 1$$

- Well known example is the bound on the Higgs boson mass...

Unitarity in WW Scattering

With no Higgs we find the *j=0* partial wave grows with energy as

$$a_0 = \frac{g^2 s}{128\pi M_W^2}$$

and the maximum energy for perturbative unitarity is

$$E \lesssim 1.7 \text{ TeV}$$

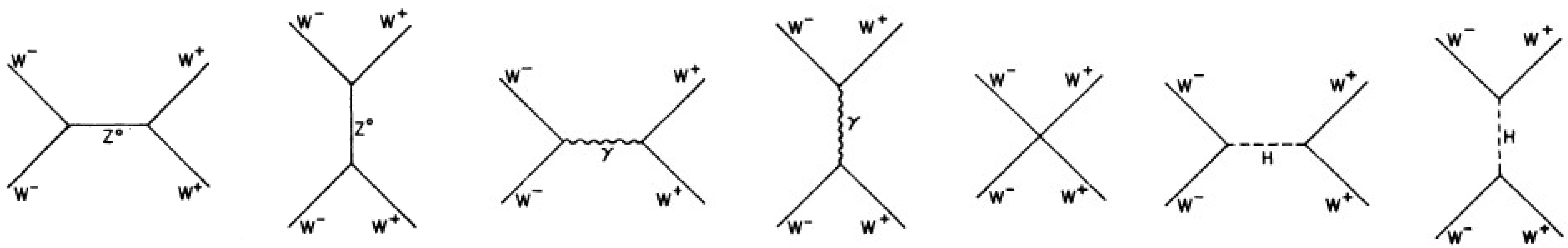
Including the Higgs we find the *j=0* partial wave is given by

$$a_0 = -\frac{G_F m_h^2}{4\sqrt{2}\pi}$$

and the maximum Higgs mass to maintain perturbative unitarity is

$$m_H \lesssim 790 \text{ GeV}$$

[Lee, Quigg & Thacker '77]



Higgs Inflation

The standard model Higgs potential is not flat $V = \lambda_H \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v_H^2}{2} \right)^2$

However, scalar fields can (should?) be non-minimally coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_P^2}{2} R - \xi \mathcal{H}^\dagger \mathcal{H} R + \mathcal{L}_{SM} \right]$$

Transform to the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \qquad\qquad \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}}$$

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

where the potential is $U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left(1 + \exp\left(-\frac{2\chi}{\sqrt{6} M_P} \right) \right)^{-2}$

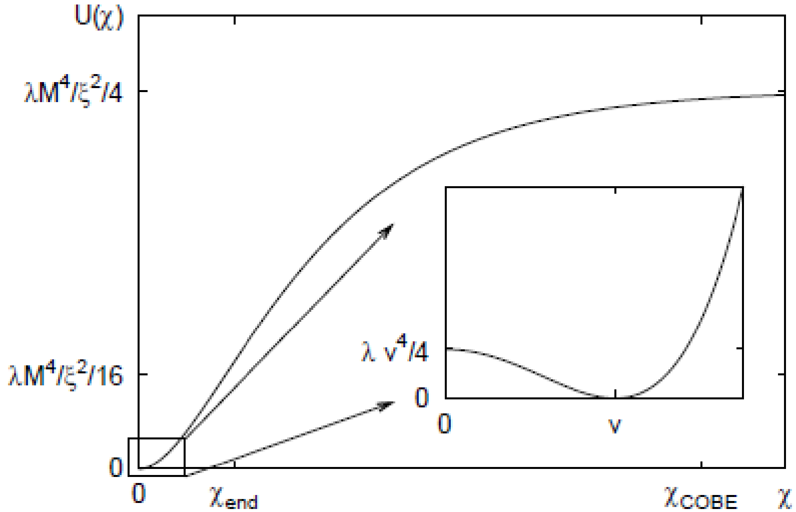
[F.L. Bezrukov & Mikhail Shaposhnikov]

Higgs Inflation

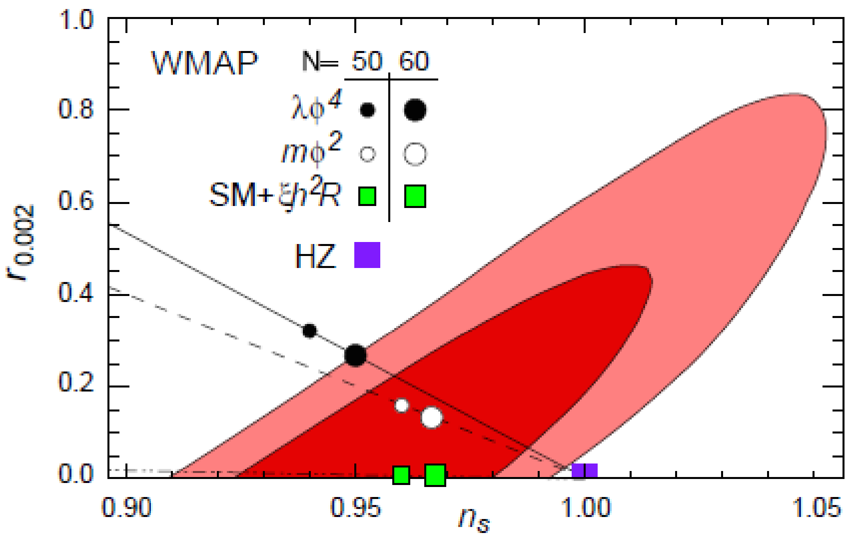
$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

When $\chi \gg M_P \; (h \gg M_P/\sqrt{\xi})$, the potential is flat and slow roll inflation can occur.

However it is found that we need $\xi \sim 10^4$ to obtain the correct amplitude of density fluctuations.



Potential in the Einstein frame.



The allowed WMAP region for inflationary parameters spectral index *n*, and the tensor to scalar ratio *r*.

[Source - arXiv:0710.3715]

Unitarity in Higgs Inflation

The large value of $\xi \sim 10^4$ might make one concerned from a particle physics perspective.

Let us consider gravitational scattering of Higgs bosons (we impose different in and out states – *s*-channel only) in the Jordan frame

$$a_0 = \frac{\pi}{3}\frac{s}{M_P^2}(1+12\xi)^2 \sim \frac{\xi^2}{M_P^2}s$$

for $\xi \gg 1$

$$|\mathrm{Re}\, a_j| \leq \frac{1}{2}$$

$$\Rightarrow \Lambda \lesssim \frac{M_P}{\xi}$$

[MA & X. Calmet] [Burgess, Lee & Trott] [Barbon & Epinosa]

But remember inflation takes place for $h \gg M_P/\sqrt{\xi}$ which is therefore above the regime of validity for the effective theory!

Background Dependence

[Bezrukov, Magnin, Shaposhnikov & Sibiryakov]

For the original Higgs inflation model we expanded around *φ=0*.
We could expand around inflating background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \,, \qquad \phi = \bar{\phi} + \delta\phi \,.$$

Then we find an interaction term

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2\Box\hat{h}$$

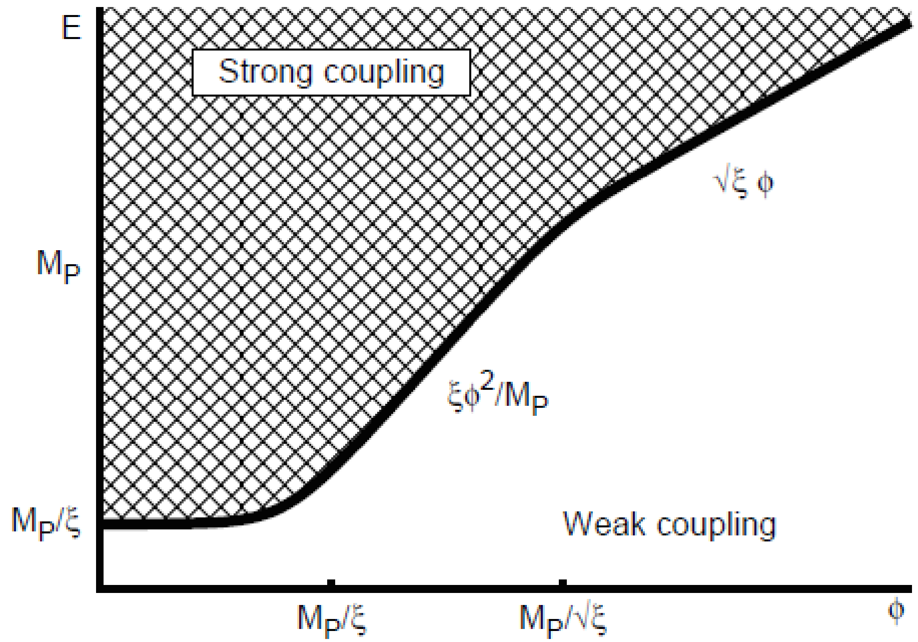
Leading to a $\bar{\phi}$ dependent cut-off $\Lambda^J(\bar{\phi}) = \dfrac{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}$

Small field: $\bar{\phi} \ll M_P/\xi \qquad \Lambda^J \simeq \dfrac{M_P}{\xi}$

Re-heating: $M_P/\xi \ll \bar{\phi} \ll M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \dfrac{\xi\bar{\phi}^2}{M_P}$

Inflation: $\bar{\phi} \gg M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \sqrt{\xi}\bar{\phi}$

New Physics or Strong Coupling?

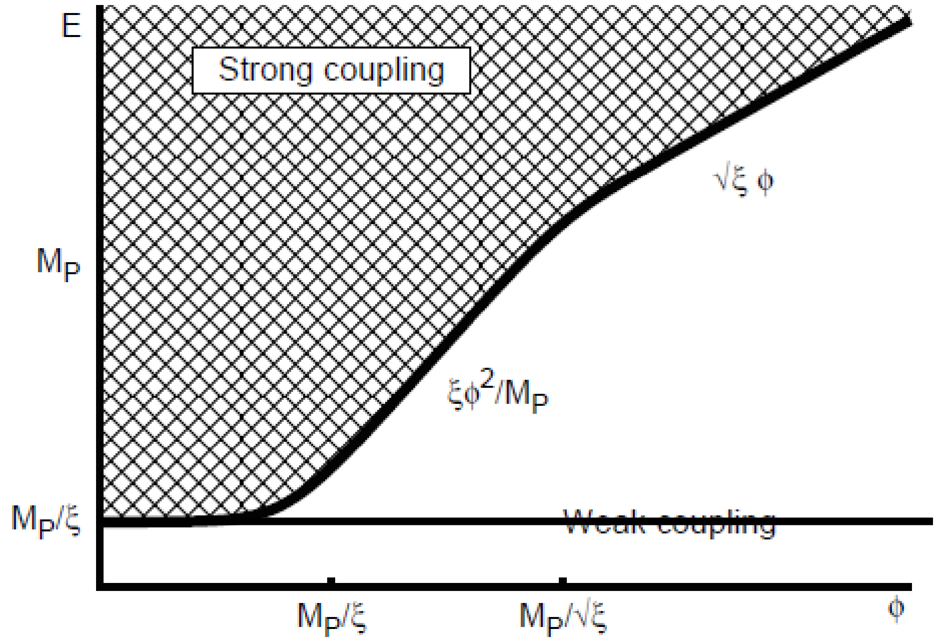


Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

New Physics or Strong Coupling?



Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

However if new physics is required to unitarise the theory at small background field values, potential must include the operators

$$\frac{(H^\dagger H)^n}{\Lambda_0^{2n-4}}$$

Appearing at $\Lambda_0 = \frac{M_P}{\xi}$. and spoiling the flat potential.

[MA & X Calmet]

New Model of Higgs Inflation

To get around the unitarity problems a new model of Higgs inflation was proposed [Germani & Kehagias]

$$S = \int d^4x\sqrt{-g}\left[\frac{\bar{M}_P^2}{2}R - \frac{1}{2}(g^{\mu\nu} - w^2 G^{\mu\nu})\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda}{4}\Phi^4\right]$$

where $G^{\mu\nu} = R^{\mu\nu} - \frac{R}{2}g^{\mu\nu}$ is the Einstein tensor.

Expanding around the inflating background we find an interaction

$$I \simeq \frac{1}{2H^2\bar{M}_P}\partial^2 h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

Which gives a cut-off

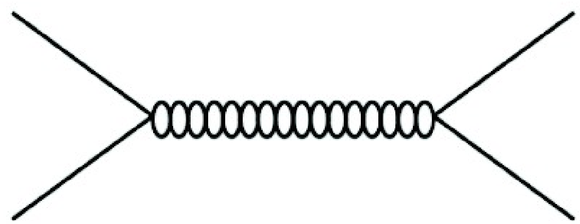
$$\Lambda \simeq (2H^2\bar{M}_P)^{1/3}. \simeq 2\times 10^{-3}\bar{M}_P.$$

But during inflation we have $2.1\times 10^{-2}\bar{M}_P < \Phi_0$

and so again the inflationary scale exceeds the realm of validity of the effective theory. [MA & X Calmet]

Unitarity of Linearised GR Coupled to Matter

Let us consider gravitational scattering of particles (we impose different in and out states – *s* channel only) [Han &Willenbrock 2004]



$\rightarrow$	$s'\bar{s}'$	$\psi'_+\bar{\psi}'_-$	$\psi'_-\bar{\psi}'_+$	$V'_+V'_-$	$V'_-V'_+$
$s\bar{s}$	$-2\pi G_N s(1/3 d^2_{0,0} - 1/3(1+12\xi)^2 d^0_{0,0})$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,1}$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,-1}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,2}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,-2}$
$\psi_+\bar{\psi}_-$	$-2\pi G_N s\sqrt{1/3}\ d^2_{1,0}$	$-2\pi G_N s d^2_{1,1}$	$-2\pi G_N s d^2_{1,-1}$	$-4\pi G_N s\ d^2_{1,2}$	$-4\pi G_N s\ d^2_{1,-2}$
$\psi_-\bar{\psi}_+$	$-2\pi G_N s\sqrt{1/3}\ d^2_{-1,0}$	$-2\pi G_N s d^2_{-1,1}$	$-2\pi G_N s d^2_{-1,-1}$	$-4\pi G_N s 2\ d^2_{-1,2}$	$-4\pi G_N s 2\ d^2_{-1,-2}$
V_+V_-	$-4\pi G_N s\sqrt{1/3}\ d^2_{2,0}$	$-4\pi G_N s\ d^2_{2,1}$	$-4\pi G_N s\ d^2_{2,-1}$	$-8\pi G_N s\ d^2_{2,2}$	$-8\pi G_N s\ d^2_{2,-2}$
V_-V_+	$-4\pi G_N s\sqrt{1/3}\ d^2_{-2,0}$	$-4\pi G_N s\ d^2_{-2,1}$	$-4\pi G_N s\ d^2_{-2,-1}$	$-8\pi G_N s\ d^2_{-2,2}$	$-8\pi G_N s\ d^2_{-2,-2}$

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'} \qquad |\mathrm{Re}\ a_J| \leq 1/2$$

Unitarity Bounds [MA & X. Calmet 2010]

Let us look at J=2 partial wave. One can diagonalize the partial wave matrix and apply the unitarity bound. One finds:

$$a_2 = -\frac{1}{320\pi}\frac{s}{\bar{M}_P^2}N \qquad N = 1/3N_s + N_\psi + 4N_V$$

Which gives us the bound $E^*_{\rm CM} \leq \bar{M}_P\sqrt{\frac{160\pi}{N}}$

For large N, unitarity can be violated well below the Planck mass. However the value of M_P depends on the particle content!

RG equation $\bar{M}_P(\mu)^2 = \bar{M}_P(0)^2 - \frac{1}{96\pi^2}\mu^2 N_l \qquad N_l = N_s + N_\psi - 4N_V$

The scale at which gravity becomes strong is

$$\bar{M}_P^2(\mu_*) \sim \mu_*^2 \qquad \Longrightarrow \qquad \mu_*^2 = \frac{\bar{M}_P(0)^2}{1 + \frac{N_l}{96\pi^2}}$$

Examples of Models

- SM: N_S=4, N_ψ=45 and N_V=12 thus $E^*=2.3M_P$, $\mu^*=M_P$
- MSSM: N_S=98, N_ψ=61 and N_V=12 thus $E^*=1.6M_P$, $\mu^*=0.95M_P$
- SUSY SO(10) GUT with Higgses in the 10, 16, $\overline{16}$ and 770: $E^*=0.41M_P$, $\mu^*=0.53M_P$
- Quantum gravity at 1 TeV requires:

Large hidden sector: $N_l=5.6\times10^{33}\sim N$

Large extra dimensions (ADD) $\sim 10^{33}$ KK gravitons

} Unitarity violated at $E^*\sim 0.5$ TeV

Large Extra Dimensions – AADD model

• Large extra dimensions offer a geometrical interpretation of the hierarchy problem. Fundamental scale M_D can be as low as a TeV.

$$M_P^2 = R^n M_D^{2+n}$$

• Compact dimensions → tower of massive Kaluza Klein gravitons.

$$m_n \simeq \frac{n}{R}$$

• Graviton exchange can occur via any of the KK gravitons

$$\sum_i \frac{1}{s - m_i^2 + i m_i \Gamma(m_i)}$$

• For *n>1* extra dimensions, KK sum is divergent. What scale does the theory break down?

• $|\mathrm{Re}\, a_j| \leq \frac{1}{2} \;\Rightarrow\; E < 0.5 M_D$ $|\mathrm{Im}\, a_j| \leq 1 \;\Rightarrow\; E < 0.8 M_D$

[I. Antoniadis, MA & X. Calmet]

AADD Requires Low String Scale

- New physics (string theory) must appear below M_D to fix unitarity
- Relation between string scale M_S and M_D is model dependent, however we can use the unitarity bounds to translate experimental bounds on M_S to bounds on M_D
- *CMS search* for the production of dijet resonances:

$$M_S > 4.00\text{TeV}$$

$$\left.\begin{array}{l} |\text{Re}\, a_j| \leq \frac{1}{2} \;\Rightarrow\; M_S < 0.5 M_D \\[2em] |\text{Im}\, a_j| \leq 1 \;\Rightarrow\; M_S < 0.8 M_D \end{array}\right\} \quad \begin{array}{l} M_D > 8\text{TeV} \\[2em] M_D > 5\text{TeV} \end{array}$$

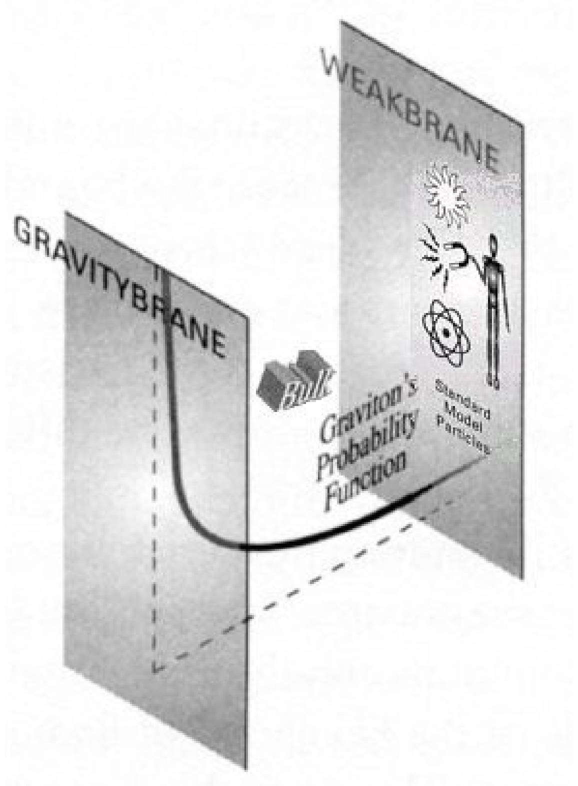
[I. Antoniadis, MA & X. Calmet]

Linear Dilaton Model (LST dual)

[Antoniadis, Arvanitaki, Dimopoulos & Giveon]

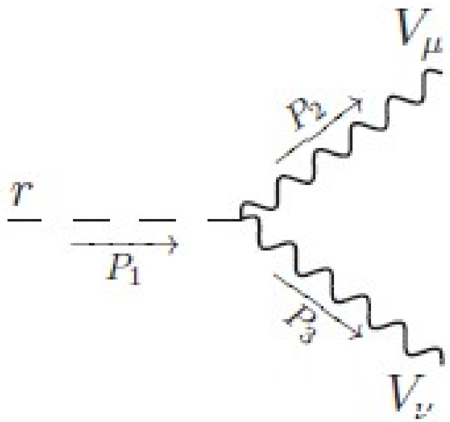
$$S_{bulk} = \int d^5x\sqrt{-g}\left[M^3\left(\mathcal{R} - \frac{1}{3}(\nabla\varphi)^2\right) - e^{\frac{2}{3}\varphi}\Lambda\right],$$

$$S_{vis(hid)} = \int d^4x\sqrt{-\hat{g}}e^{\frac{1}{3}\varphi}\left(\mathcal{L}_{vis(hid)} - V_{vis(hid)}\right),$$



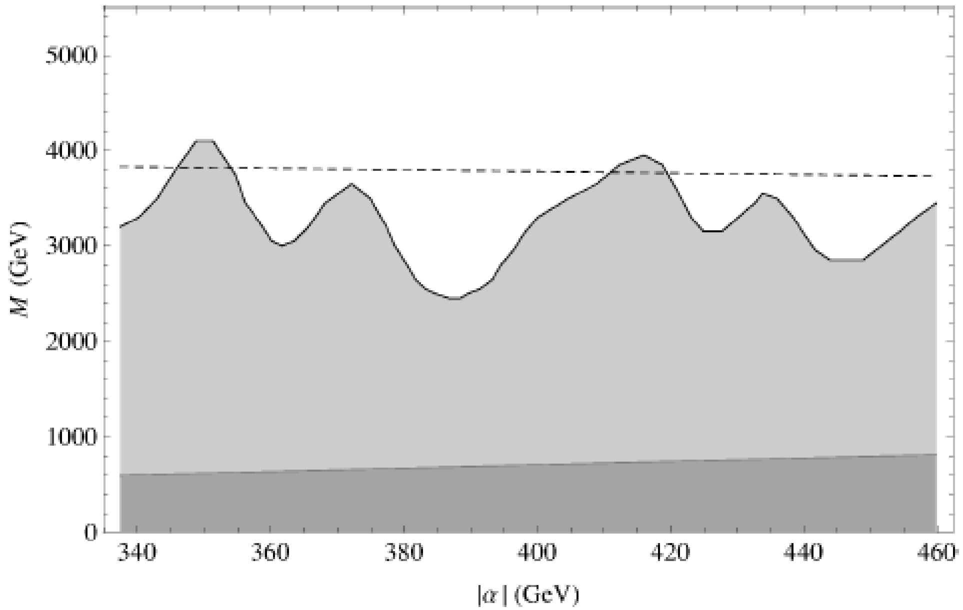
$$\phi(z) = \alpha|z|,$$

$$ds^2 = e^{-\frac{2}{3}\alpha|z|}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right),$$



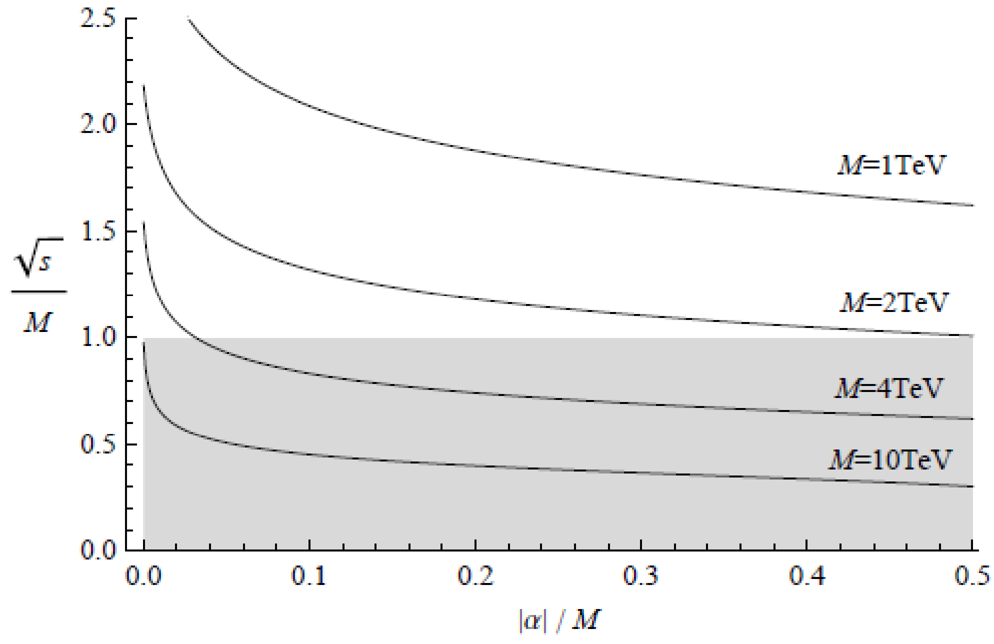
$$\frac{ib_1\kappa_\phi}{M}\left(P_2{\cdot}P_3\eta^{\mu\nu} - P_2^\mu P_3^\nu\right)$$
$$+ 2im_V^2\left(\frac{b_1}{M}\left(\frac{\kappa_\phi}{2} - \kappa_\Phi\right) + \frac{a_1}{v}\right)\eta^{\mu\nu}$$

Linear Dilaton Model (LST dual)



Direct search at LHC excludes M < 3TeV

[Cox & Gherghetta]



Unitarity bounds exclude M > 3TeV!

[MA – work in progress]

Conclusions

- Perturbative unitarity is a powerful tool to investigate realm of validity for effective theories
- The Higgs inflation models (old and new) suffer from unitarity problems.
- Models with large numbers of fields can violate unitarity well below the Planck scale
- Large extra dimensions require a low string scale
- Linear dilaton model is highly constrained by unitarity

Backup slides...

RG Higgs Self Coupling [arXiv:1112.3022]

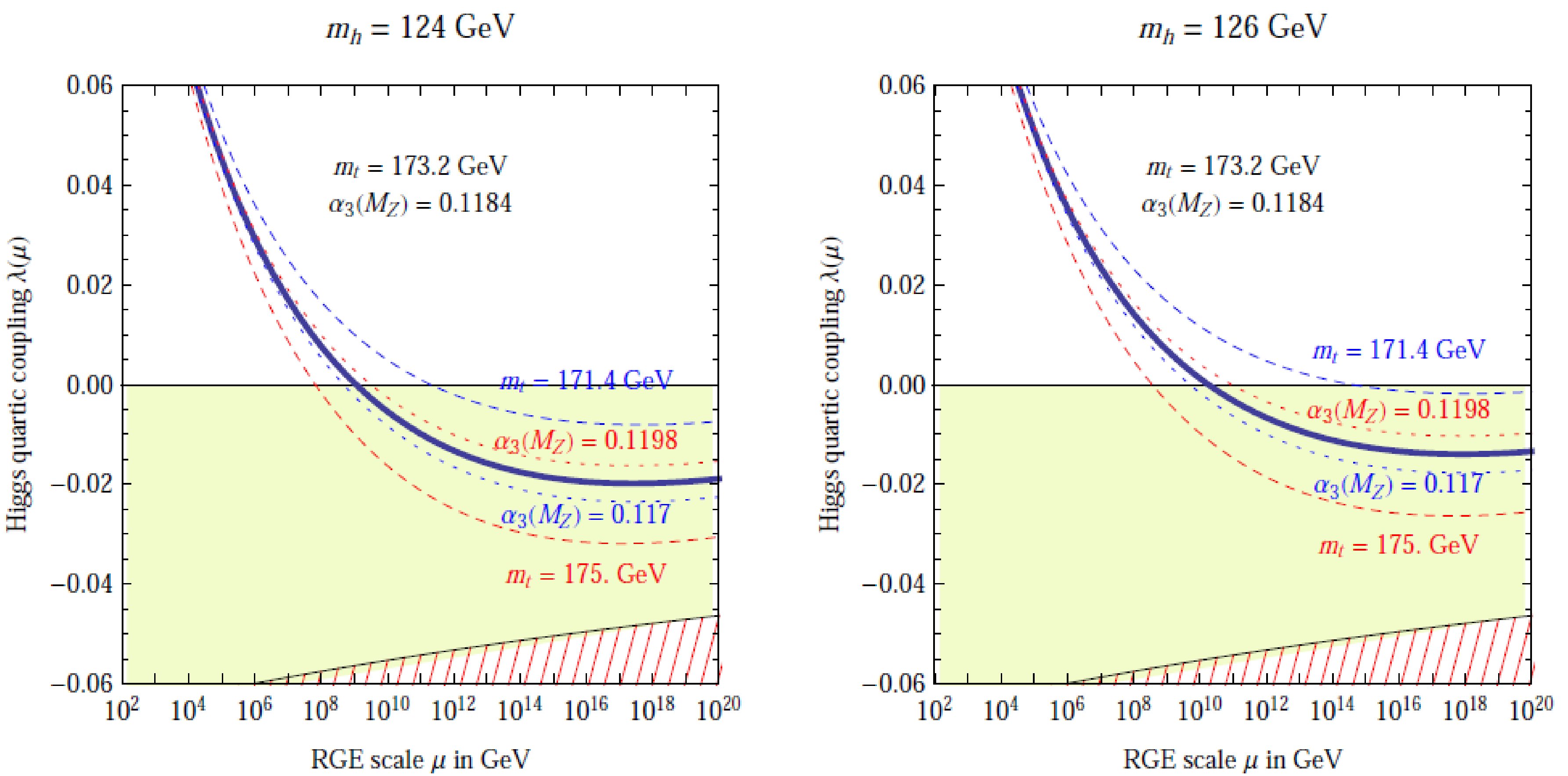
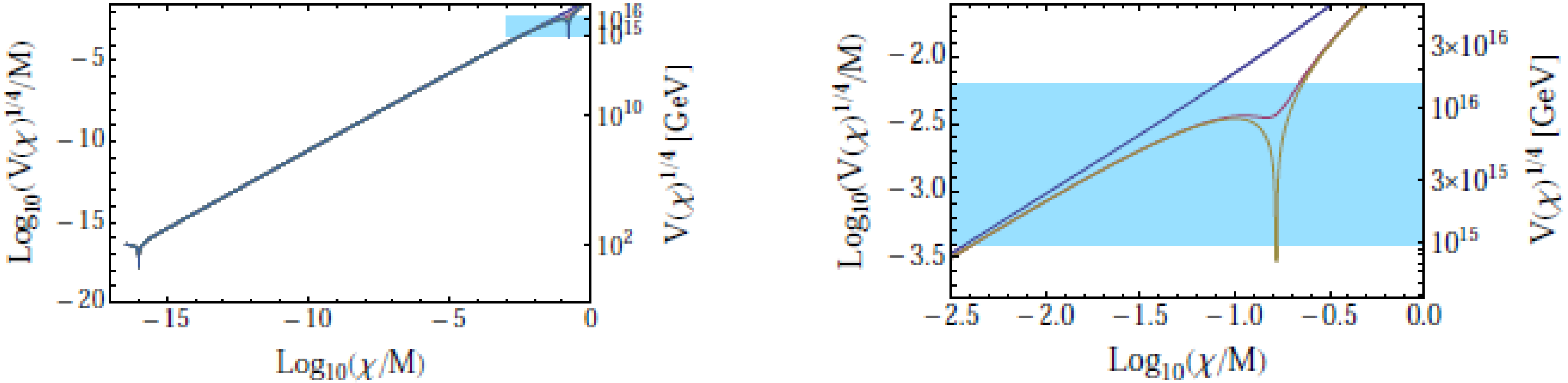


Figure 1: *RG evolution of the Higgs self coupling, for different Higgs masses for the central value of m_t and α_s, as well as for $\pm 2\sigma$ variations of m_t (dashed lines) and α_s (dotted lines). For negative values of λ, the life-time of the SM vacuum due to quantum tunneling at zero temperature is longer than the age of the Universe as long as λ remains above the region shaded in red, which takes into account the finite corrections to the effective bounce action renormalised at the same scale as λ (see [11] for more details).*

SM False Vacuum Inflation [Masina & Notari]

• Stability of the Higgs effective potential is extremely sensitive to the value of the Higgs and top quark masses. Can obtain a local minima at large field values.



Higgs potential as function of Higgs field value, mt=171.8 GeV, mh=125.2, 125.158, 125.1577 GeV

• Higgs sitting in this false minima would provide exponential inflation and could then tunnel at end of inflation.

• To match amplitude of density perturbations with mt=173.2 GeV find

$$m_H = (126.0 \pm 3.5)\,\mathrm{GeV}\ .$$

Einstein vs. Jordan Frame [Hertzberg] [Burgess, Lee, Trott]

The cut off (Λ) should be the same in both frames. But if we look at the Einstein frame action again

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

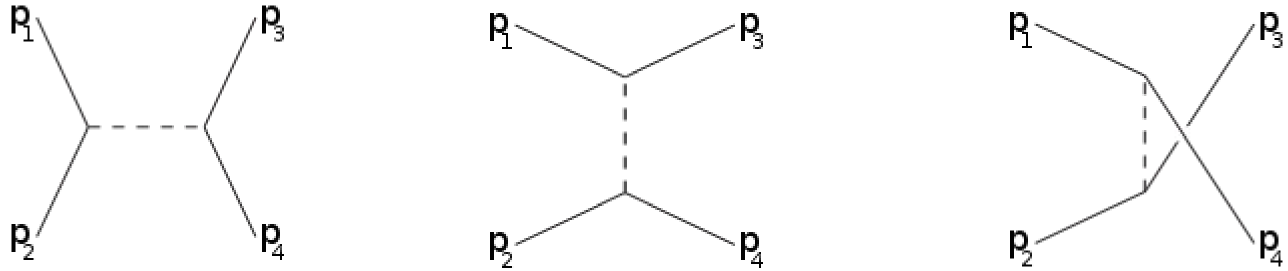
we see that the cut off is just the usual gravitational cut off ($\Lambda = M_P$).

Einstein Frame

Cannot canonically normalise all the fields of the Higgs doublet so cannot actually get Einstein frame potential with multiple scalars.

Jordan Frame

If only have a single field need to include *s, t* and *u* channels. Then we find a cancellation between the three diagrams leaving ($\Lambda = M_P$).



Three More Scenarios

1. Asymptotic Safety [MA & X Calmet]

The theory is non-perturbatively renormalisable and approaches a non trivial fixed point in the UV, no new physics is required.

2. Unitarising Higgs Inflation [Giudice & Lee]

Can remove unitarity problem by introducing a massive σ field in analogy with the non-linear sigma model. But in reality it is this new field that then drives inflation.

3. Composite Inflation [Channuie, Jørgensen, Sannino]

The inflaton emerges as a composite field of a strongly interacting gauge theory.

$$\frac{\xi}{2}\frac{(QQ)^{\dagger}QQ}{\Lambda_{ECI}^{4}}R$$

SM False Vacuum Inflation

• To end inflation need tunnelling rate $\Gamma \simeq H^4$ ut if both parameters are constant then inflation ends too quickly.

• Solution – introduce a Brans-Dicke scalar which provides a time dependent Planck mass.

$$-S = \int d^4x\sqrt{-g}\left[\mathcal{L}_{SM} + \frac{(\partial_\mu\phi\partial^\mu\phi)}{2} - \frac{M^2}{2}f(\phi)R\right] \qquad f(\phi) \simeq 1 + \beta\left(\frac{\phi}{M}\right)^2 + \gamma\left(\frac{\phi}{M}\right)^4 + \ldots\ ,$$

• If $f(\phi)$ is a monotonic increasing function it will grow during inflation and the effective Planck mass grows. Gravity becomes weaker and so the Hubble parameter decreases with time until $\Gamma \simeq H^4$ and the Higgs field tunnels efficiently.

• BSM physics can change the Higgs effective potential. I have found that inflation is still viable in the presence of seesaw neutrino masses with reasonable bounds on the neutrino mass parameters, although the false minimum moves closer towards the Planck mass (work in progress....)