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1/N Resolution to the η Problem

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Based on
A.A., U. Danielsson & M.M. Sheikh-Jabbari, arXiv:1112.2272 [hep-th]

η-Problem

- WMAP data, constraints the $\in$ & η parameter to be $\leq 10^{-2}$

$$\in = -\frac{\dot{H}}{H^2} \qquad\qquad \eta = \frac{\ddot{H}}{H\dot{H}} \approx \frac{m^2}{H^2}$$

- They are a measure of flatness of the potential.

- One should protect the potential and its flatness against the classical corrections of non-inflationary sector and also quantum gravity corrections.

- In top-down approaches: classical interference of moduli stabilization process with the inflaton moduli causes the η-problem.

In presence of vacuum energy, *V* ⟶ soft masses, including the inflaton's, receives

$$\Delta m^2 = \frac{V}{M_{\rm pl}^2} = 3H^2 \longrightarrow \eta\text{-problem}$$

McAllister (2005)

- η – problem can also arise due to the mixing of the inflaton and graviton at one-loop level

$$\xi \frac{\Lambda^2}{M_{\rm pl}^2} R\phi^2 \qquad\qquad \xi = O(1)$$

Sakharov (1968)
Adler (1982)

- If $\Lambda \approx M_{\rm pl}$, since $R \approx 12H^2$ ⟶ $\Delta m^2 \approx H^2$ ⟶ Infamous η-problem

η-problem from loop correction to the $h-\varphi-\varphi$ vertex

- Einstein-Hilbert Gravity+matter: a non-renormalizable theory, but one can still use the EFT Wilsonian techniques.

- Consider the action of N scalar fields minimally coupled to gravity

$$\mathcal{L} = -\frac{1}{2}M_{\rm pl}^2 R - \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a \boxed{- \frac{1}{2}M_a^2\phi_a^2 - V(\phi_a)} \longrightarrow \text{realizes slow-roll inflation}$$

- One or some of these scalars play the role of inflaton(s).

- The rest exhibit fields other than inflaton direction or also possible remnants of the underlying quantum gravity theory. $M_a < \Lambda_{\rm dressed}$

- Performing the one loop analysis: we have to find the propagators and vertices.

$$\bar{g}^{\mu\nu} = g^{1/2} g^{\mu\nu} \qquad \bar{g}_{\mu\nu} = g^{-1/2} g_{\mu\nu},$$

- The gravitational part of the action:

Goldberg (1958)

$$\frac{M_{\rm pl}^2}{16}\left(2\bar{g}^{\rho\sigma}\bar{g}_{\lambda\mu}\bar{g}_{\kappa\nu} - \bar{g}^{\rho\sigma}\bar{g}_{\mu\kappa}\bar{g}_{\lambda\nu} - 4\delta^\sigma_\kappa\delta^\rho_\lambda\bar{g}_{\mu\nu}\right)\bar{g}^{\mu\kappa}_{,\rho}\,\bar{g}^{\lambda\nu}_{,\sigma}\,.$$

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}, \qquad \text{where} \qquad \hat{h}_{\mu\nu} \equiv M_{\rm pl}^{-1} h_{\mu\nu}.$$

$$\bar{g}^{\mu\nu} = \eta^{\mu\nu} - \hat{h}^{\mu\nu} + \mathcal{O}(\hat{h}^2),$$

- Perturbing the action up to third order in $h_{\mu\nu}$ one obtains

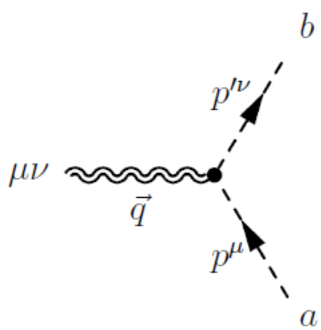
$$\mathcal{L} = -\frac{1}{2}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + \frac{1}{2}\phi_a(\Box - M_a^2)\phi_a + \frac{1}{2M_{\rm pl}}h_{\mu\nu}T^{\mu\nu} + \frac{1}{M_{\rm pl}}\mathcal{O}(h(\partial h)^2)$$

where

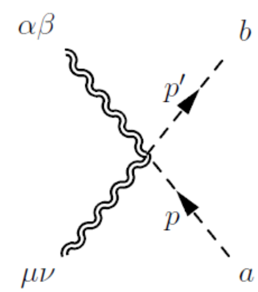
$$T^{\mu\nu} = \partial^\mu\phi_a\partial^\nu\phi_a - \frac{1}{2}\eta^{\mu\nu}\partial^\alpha\phi_a\partial_\alpha\phi_a + \eta^{\mu\nu}(\frac{1}{2}M_a^2\phi_a^2 + V(\phi_a))\,.$$

- Adding a gauge-fixing term,

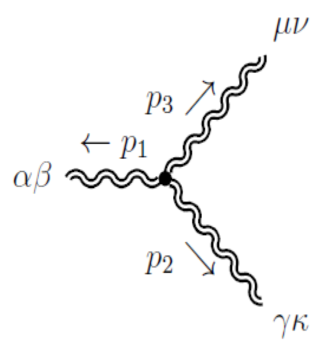
$$\mathcal{L}_{\rm g.f.} = \frac{M_{\rm pl}^2}{4}\bar{g}^{\mu\alpha}_{,\mu}\bar{g}^{\nu\beta}_{,\nu}\eta_{\alpha\beta}.$$



$= V^{ab}_{3\mu\nu}(p,p') \propto \frac{1}{M_{\rm pl}}\delta^{ab}$

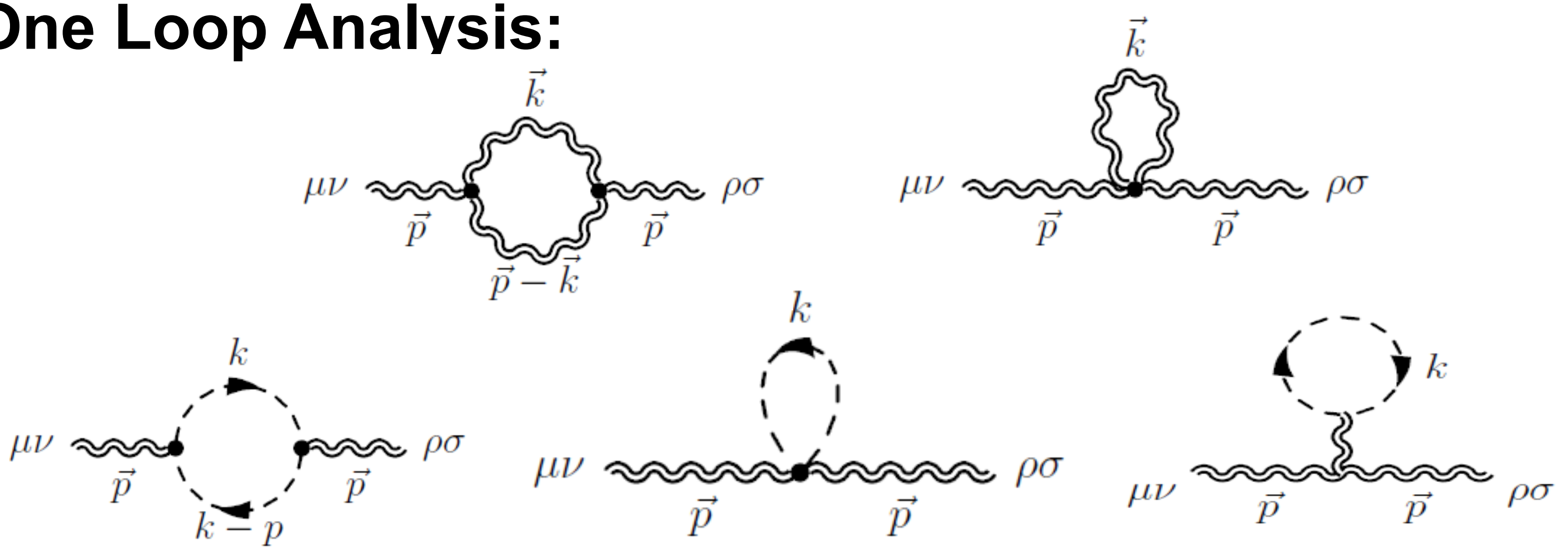


$= V^{ab}_{4\alpha\beta\mu\nu}(p,p') \propto \frac{1}{M_{\rm pl}^2}\delta^{ab}$



$= W_{3\alpha\beta\mu\nu\gamma\kappa}(p_2,p_3) \propto \frac{1}{M_{\rm pl}}$

One Loop Analysis:



(I) (II) (III)

- Only the last three diagrams are the one that has *N*-dependence.
- Diagram (II) and (III), even though proportional to *N,* has only quartic divergences.
- The leading order divergent term is $\Lambda^4\delta_{\mu\nu}\delta_{\alpha\beta}$ which is the cosmological constant term.
- The next-to-leading order divergent part of diagrams (I) is proportional to

$$N\left(\frac{\Lambda}{M_{\rm pl}}\right)^2 D_{\mu\nu\rho\sigma},$$

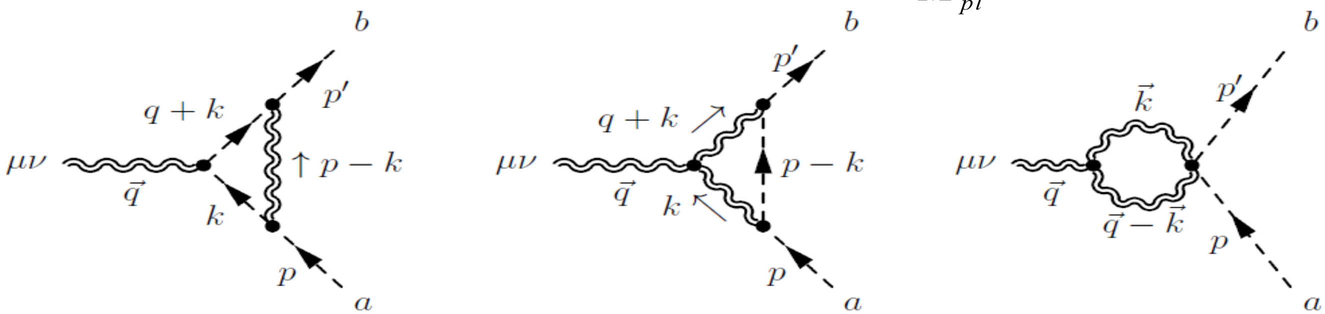
- Quantum gravity effects become important when

$$\Lambda^2_{\rm dressed} = \frac{M^2_{\rm pl}}{N}$$

Dvali (2007,2008)

One loop graviton-scalar vertex

We would like to calculate the correction to scalar-graviton three vertex at one loop level and from that calculate the coefficient of $R\phi_a^2$ or $\frac{1}{M_{pl}}(\partial^2 h)\phi^2$ at one loop level



- No *N*-dependence appears in any of the above diagrams.
- The coefficient of these one-loop diagrams is suppressed with respect to tree-level result $\propto \frac{1}{M_{Pl}}$ by a factor of

$$\left(\frac{\Lambda}{M_{\mathbf{pl}}}\right)^2$$

- This means that the coefficient in front of the generated $R\phi^2$ is suppressed by the above factor. So if Λ is suppressed with number of species lighter than the cutoff, the coefficient of conformal breaking term, $R\phi_a^2$, will be suppressed too.

$R\varphi^2$ term in inflationary background and resolution to the η-problem

- We have to extend our analysis to dS background.
- The relevant observation is that the modes contributing to *N* are the quantum modes.
- EOM of massive scalar field in an inflationary background is

$$\ddot{\phi}_a(k;t) + 3H\dot{\phi}_a(k;t) + (\frac{k^2}{a(t)^2} + M_a^2)\phi_a(k;t) = 0,$$

- Ignoring the $k^2/a(t)^2$ the solution takes the form:

$$\phi_a(k;t) = \phi_a^{(0)} e^{H\omega t} \qquad \omega = -3/2 \pm \sqrt{9/4 - M_a^2/H^2}$$

Requiring ω to be imaginary implies $M_a \geq 3H/2.$

- Inclusion of $k^2/a(t)^2$ will slightly changes this result:

  - Modes with $M_a > 3H/2$ always remain quantum mechanical.

  - Modes with $M_a < 3H/2$ are quantum mechanical, as long as they are sub-horizon $\frac{k}{a(t)} > \frac{3H}{2}$

- Interested only in the UV behavior ⟶ sub- & super-Hubble modes both contribute.

$$\Lambda^2_{\rm dressed} = \frac{M_{\rm pl}^2}{N},$$

$$L = L_{cl} + \frac{\xi}{N} R\phi_a^2 .$$

$m_a < \Lambda$

$H < \Lambda$

$\xi = O(1)$

With slow-roll parameter $\eta_{ab} \equiv M_{\rm pl}^2 \frac{V_{ab}}{V}$, where $V_{ab} = \frac{\partial^2 V}{\partial\phi_a \partial\phi_b}$

$$\eta_{ab} = \eta_{ab}^{cl} + \frac{\xi}{N}\frac{R}{3H^2}\delta_{ab} \simeq \eta_{ab}^{cl} + \frac{4\xi}{N}\delta_{ab} .$$

To have a successful inflationary scenario, $\eta \approx 0.01$. If $\eta^{cl} \approx 0.01$ one can suppress the quantum correction by having $N \geq \mathbf{few} \times \mathbf{1000}$

Examples:

- **N-flation:** S. Dimopoulos, S. Kachru et. al. (2005)

  - *O(N)* symmetric model, $V = \frac{m^2}{2}\sum_{i=1}^{N}\phi_i^2$:

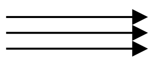
    * Few thousand fields will solve the quantum η problem.

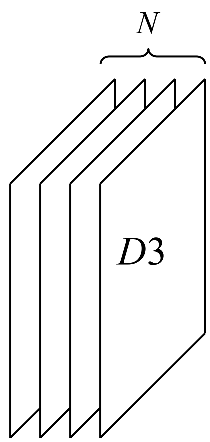
    * However this comes at the price of having "super-$\Lambda_{\rm dressed}$" excursions for the individual field.

- **Gauged M-flation**

A.A. & M.M. Sheikh-Jabbari, JCAP 1106 (2011) 014, arXiv:1101.0048 [hep-th]
A.A., H. Firouzjahi & M.M. Sheikh-Jabbari, JCAP 1005 (2010) 002, arXiv:0911.4284 [hep-th]
A.A., H. Firouzjahi & M.M. Sheikh-Jabbari, JCAP 0906:018,2009, arXiv:0903.1481 [hep-th],

$$C_{+123ij} = \frac{2\hat{\kappa}}{3}\varepsilon_{ijk}x^k$$





10-d IIB supergravity background

$$ds^2 = 2dx^+dx^- - \hat{m}^2\sum_{i=1}^{3}(x^i)^2(dx^+)^2 + \sum_{K=1}^{8}dx_K dx_K$$

$i, j = 1,2,3$ parameterize 3 out 6 dim $\perp$ to the D3-branes and x^K denotes 3 spatial dim along and five transverse to the D3-branes.

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s}\int d^4x\,\mathrm{STr}\left(1-\sqrt{-|g_{ab}|}\sqrt{|Q^I_J|} + \frac{ig_s}{4\pi\, l_s^2}\left[X^I, X^J\right]C^{(6)}_{IJ0123} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Myers (1999)

$$g_{ab} = G_{MN}\partial_a X^M \partial_b X^N \qquad M,N = 0,1,\ldots,9 \qquad I,J = 4,5,\ldots,9$$

$$a,b = 0,1,2,3$$

$$Q^{IJ} = \delta^{IJ} + \frac{i}{2\pi\, l_s^2}\left[X^I, X^J\right]$$

Matrix Inflation from String Theory

With $\hat{m}^2 = \frac{4g_s^2\hat{\kappa}^2}{9}$ the above background with constant dilaton is solution to the SUGRA

$$V = -\frac{1}{4(2\pi l_s^2)^2}[X_i, X_j][X_i, X_j] + \frac{ig_s\hat{\kappa}}{3.2\pi l_s^2}\varepsilon^{ijk}X_i[X_j, X_k] + \frac{1}{2}\hat{m}^2 X_i^2$$

Upon the field redefinition $\Phi_i \equiv \frac{X_i}{\sqrt{(2\pi)^3 g_s l_s^2}}$

$$V = \mathrm{Tr}\left(-\frac{\lambda}{4}[\Phi_i, \Phi_j][\Phi_i, \Phi_j] + \frac{i\kappa}{3}\varepsilon_{jkl}[\Phi_k, \Phi_l]\Phi_j + \frac{m^2}{2}\Phi_i^2\right)$$

$$\lambda = 8\pi g_s \qquad \kappa = \hat{\kappa}\, g_s \cdot \sqrt{8\pi\, g_s} \qquad \hat{m}^2 = m^2$$

From the brane-theory perspective, it is necessary to choose $\hat{m}$ and $\hat{\kappa}$ such that

$$\hat{m}^2 = \frac{4g_s^2\hat{\kappa}^2}{9}$$

In the stringy picture, we have *N* D3-branes that are blown up into a single giant D5-brane under the influence of *RR* 6-form. The inflaton corresponds to the radius of this two sphere.

Truncation to the SU(2) Sector:

Φ_i are *N X N* matrices and therefore we have $3N^2$ scalars. It makes the analysis very difficult

However from the specific form of the potential and since we have three Φ_i, it is possible to show that one can consistently restrict the classical dynamics to a sector with single scalar field:

$$\Phi_i = \hat{\phi}(t) J_i, \qquad i = 1,2,3$$

J_i are *N* dim. irreducible representation of the SU(2) algebra:

$$\left[J_i, J_j\right] = i\varepsilon_{ijk} J_k \qquad \mathrm{Tr}\left(J_i J_j\right) = \frac{N}{12}\left(N^2 - 1\right)\delta_{ij}$$

Plugging these to the action, we have:

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P}{2} R + \mathrm{Tr} J^2 \left(-\frac{1}{2}\partial_\mu \hat{\phi}\, \partial^\mu \hat{\phi} - \frac{\lambda}{2}\hat{\phi}^4 + \frac{2\kappa}{3}\hat{\phi}^3 - \frac{m^2}{2}\hat{\phi}^2\right)\right] \qquad \mathrm{Tr}\left(J^2\right) \equiv \sum_{i=1}^{3} \mathrm{Tr}\left(J_i^2\right)$$

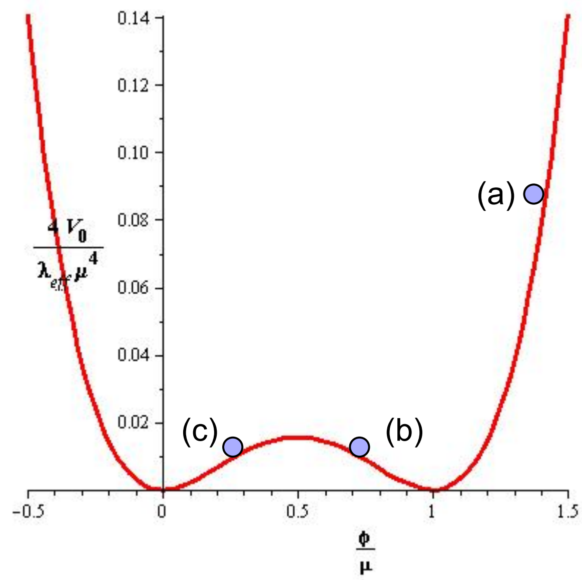
Defining $\phi \equiv \left(\mathrm{Tr}\, J^2\right)^{1/2} \hat{\phi}$ to make the kinetic term canonical, the potential takes the form

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2 \qquad \lambda_{eff} \equiv \frac{2\lambda}{\mathrm{Tr}\, \boldsymbol{J}^2} = \frac{8\lambda}{N(N^2-1)}, \qquad \kappa_{eff} \equiv \frac{\kappa}{\sqrt{\mathrm{Tr}\, \boldsymbol{J}^2}} = \frac{2\kappa}{\sqrt{N(N^2-1)}},$$

Analysis of the Gauged M-flation around the Single-Block Vacuum

$$V(\phi) = \frac{\lambda_{\rm eff}}{4}\phi^2(\phi-\mu)^2 \qquad \mu \equiv \frac{\sqrt{2}\boldsymbol{m}}{\sqrt{\lambda_{\rm eff}}}$$

Hill-top or Symmetry-Breaking inflation, Linde (1992)
Lyth & Boubekeur (2005)



(a) $\phi_i > \mu$

$\phi_i \approx 43.57\ M_P$ $\quad \phi_f \approx 27.07\ M_P$ $\quad \mu \approx 26\ M_P$

$\lambda_{eff} \approx 4.91\times 10^{-14}$ $\quad m \approx 4.07\times 10^{-6}\ M_P$

(b) $\mu/2 < \phi_i < \mu$

$\phi_i \approx 23.5\ M_P$ $\quad \phi_f \approx 35.03\ M_P$ $\quad \mu \approx 36\ M_P$

$\lambda_{eff} \approx 7.18\times 10^{-14}$ $\quad m \approx 6.82\times 10^{-6}\ M_P$

(c) $0 < \phi_i < \mu/2$

Due to symmetry $\phi \to -\phi + \mu$ this inflationary region has the same properties as $\mu/2 < \phi_i < \mu$

- $\lambda \approx 1 \Longrightarrow N \approx 5\times 10^4 \Longrightarrow \Delta\phi \leq 10^{-6}\boldsymbol{M_p}$

Mass Spectrum of Spectators

(a) $(N-1)^2 - 1$ α-modes $l \in Z \quad 0 \le l \le N-2$

Degeneracy of each l-mode is $2l+1$

$$M^2_{\alpha,l} = \frac{1}{2}\lambda_{\rm eff}(l+2)(l+3)\phi^2 - 2\kappa_{\rm eff}(l+2) + m^2$$

(b) $(N+1)^2 - 1$ β-modes $l \in Z \quad 1 \le l \le N$

Degeneracy of each l-mode is $2l+1$

$$M^2_{\beta,l} = \frac{1}{2}\lambda_{\rm eff}(l-2)(l-1)\phi^2 + 2\kappa_{\rm eff}(l-1) + m^2$$

(c) $N^2 - 1$ zero modes

$$M^2_0 = \lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2 = \frac{V'}{\phi}$$

These are infinitesimal gauge transformation $\Phi_i \to \Phi_i + ig[\Phi_i, \Lambda]$ where Λ is an arbitrary hermitian matrix. They are replaced by the following massive vector modes:

(d) $3N^2 - 1$ vector modes

$$M^2_{A,l} = \frac{\lambda_{eff}}{4}\phi^2 l(l+1)$$

$$\underbrace{\left[(N-1)^2 - 1\right]}_{\alpha-\text{modes}} + \underbrace{\left[(N+1)^2 - 1\right]}_{\beta-\text{modes}} + \underbrace{\left[3N^2 - 1\right]}_{\text{vector-field modes}} = 5N^2 - 1$$

- The ultra-light zero modes are replaced with massive vector modes that have a hierarchical mass structure.
- Number of contributing species to the cutoff varies between 3×10^5 and 10^6. Thus

$$10^{-3}M_{\rm pl} \lesssim \Lambda_{\rm dressed} \lesssim 5\times10^{-3}M_{\rm pl}$$

- Recall that field excursion, $\Delta\phi \le 10^{-6}M_{\rm pl}$ which is much smaller than $\Lambda_{\rm dressed}$
- Thus conformal breaking term is suppressed by a factor of $\sim 10^{-5}$ and could be safely ignored.

Cascade Inflation

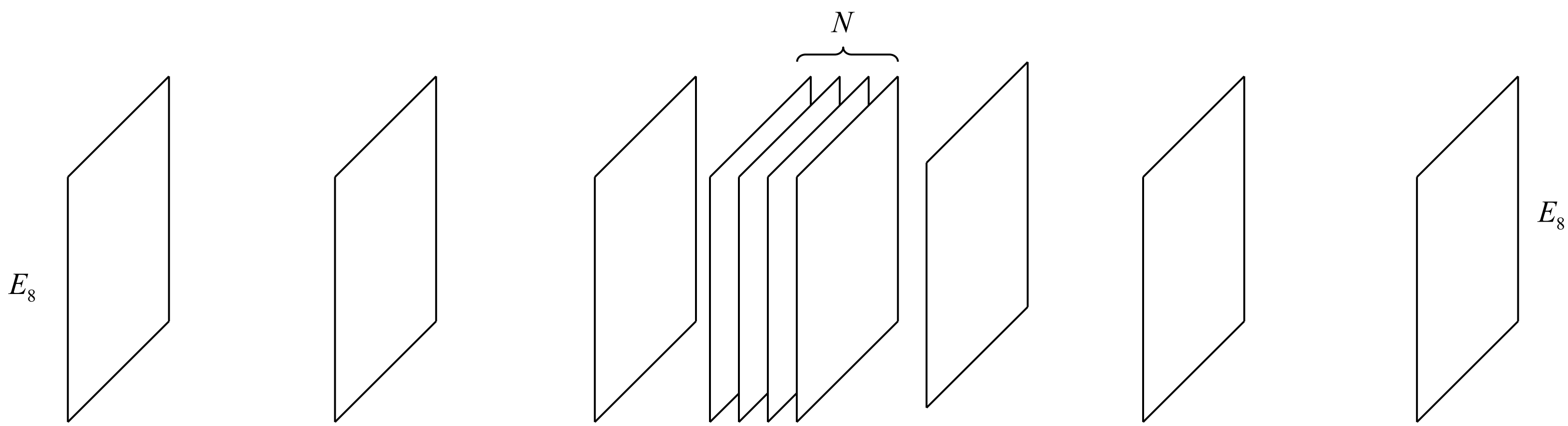
M. Becker, K. Becker, A. Krause (2005)
A.A., A. krause (2006)

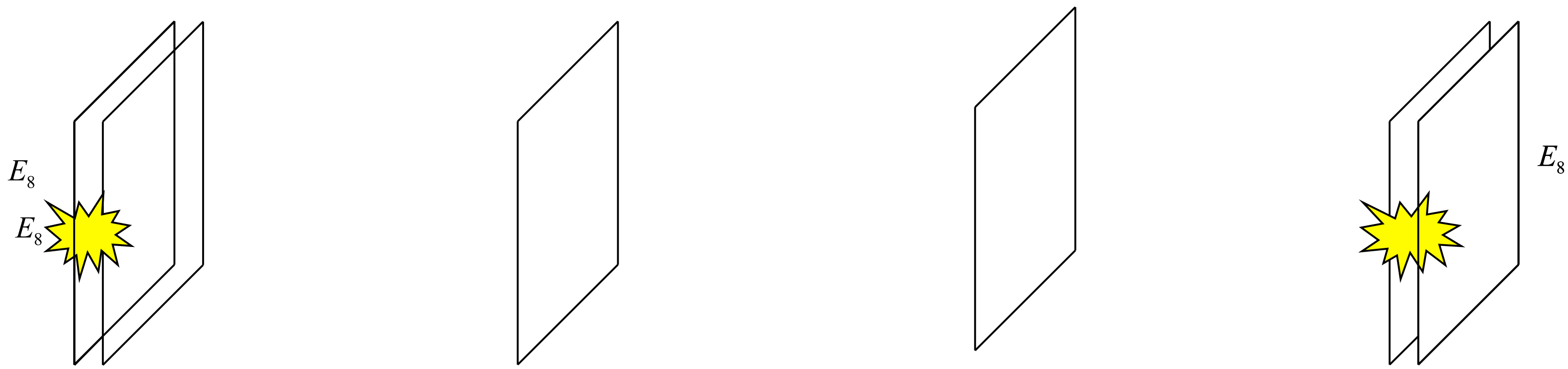
Starting point: M-theory in the presence of N parallel M5-branes distributed along the orbifold and compactified on a CY_3 preserving N=1 supersymmetry in 4D. Each M5-brane has wrapped the same 2-cycle Σ_2 on the CY_3 only once and fill the 4D space-time.

$$a \propto t^{p_N}$$

$$U_N = U_0 (N-1)^2 \exp\left(-\sqrt{\frac{2}{p_N}}\frac{\varphi}{M_{\rm Pl}}\right);$$

$$p_N \propto N^3$$
$$\Delta\varphi \propto N^{3/2}\Delta y$$





$$N \Rightarrow N-2$$

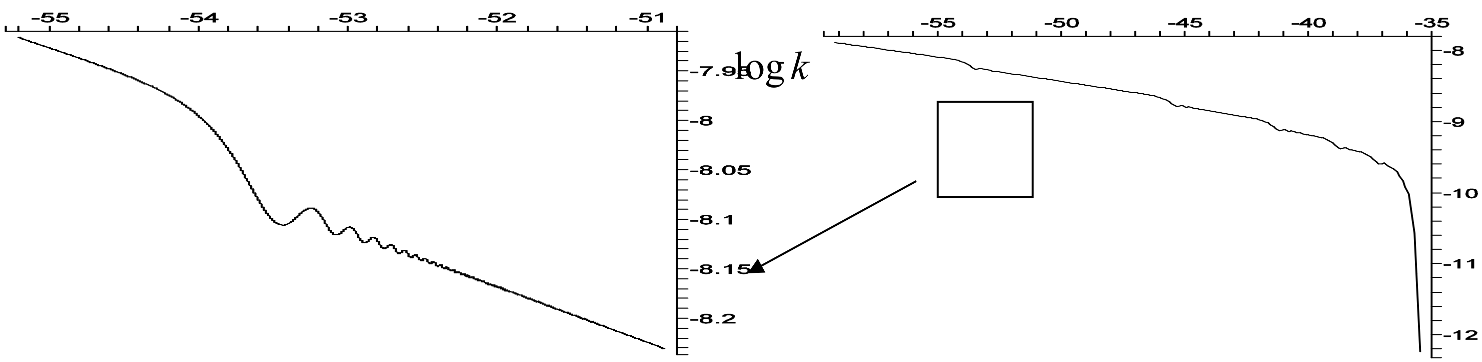
At the end, we have a cascade of power-law inflations during which the scale factor evolves as:

$$a_m(t) = a_m t^{p_m}, \qquad t_{m-1} \le t \le t_m, \qquad m = 1 \cdots K$$

Where the continuity of the scale factor at transition times determine a_m s.

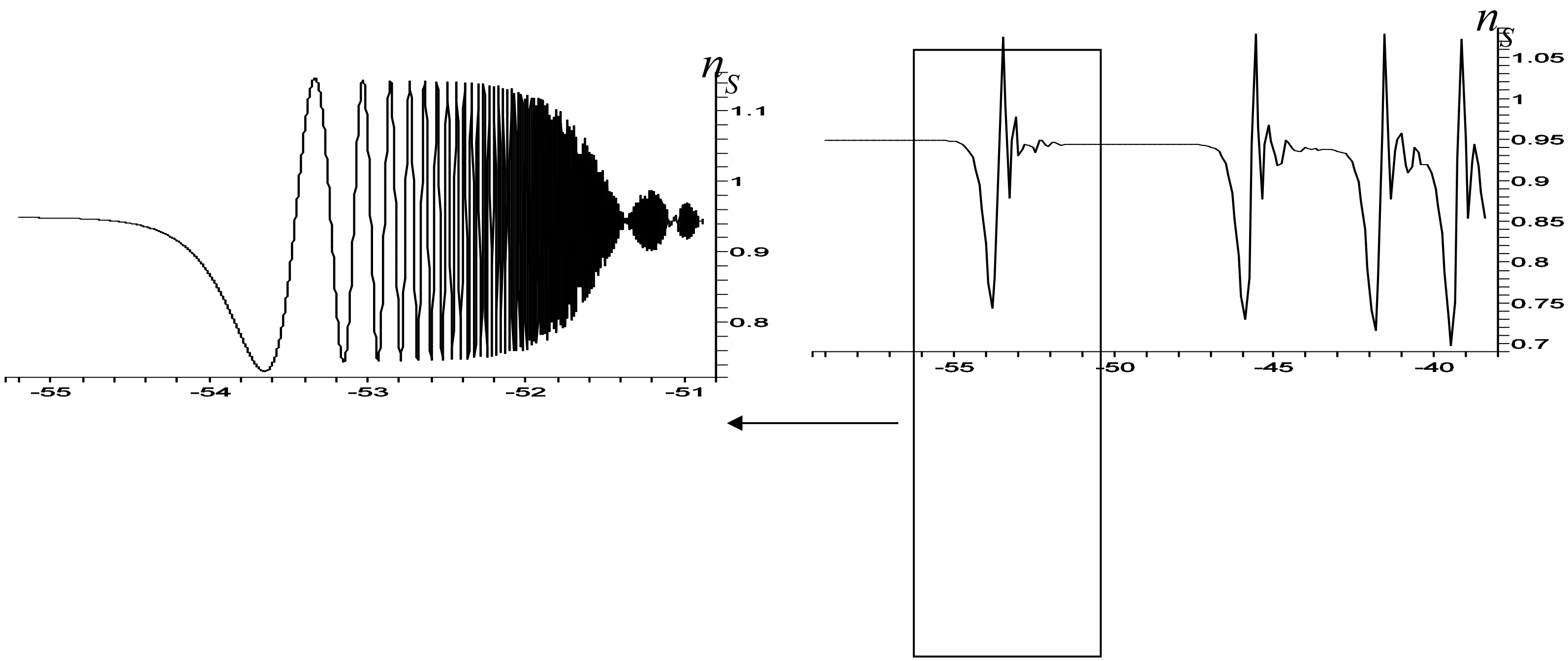
$$a_m = a_1 t_1^{p_{N_1}} \left(\frac{t_2}{t_1}\right)^{p_{N_2}} \left(\frac{t_3}{t_2}\right)^{p_{N3}} \cdots \left(\frac{t_{m-1}}{t_{m-1}}\right)^{p_{N_{m-1}}} \frac{1}{(t_{m-1})^{p_{N_m}}}$$

$N = 66$



Here for the 7% drop in the
potential the oscillations
last for three decades in *k.*

$\log P_S$ $\log P_S$



$\log k$ $\log k$

Summary and Conclusions

- The loop corrections that arise from interactions of the graviton with the scalar field create the quadratically divergent conformal mass type term which leads to the η-problem, if the UV cutoff of the theory is of order Planck mass.
- The problem seems to be commonplace in all inflationary models that use a scalar field to realize inflation.
- We suggest a resolution to this kind of η- problem in the context of many fields models.
- One example is N-flation which solves the problem at the price of super-species-cutoff excursion.
- Another example is M-flation which is qualitatively new third venue within string theory inflationary model-building, using the internal matrix degrees of freedom.
- Due to hierarchical mass spectrum of the isocurvature modes, one can avoid the "super-species-cutoff" problem.
- There are other bonuses like isocuravture perturbations, GWs, and embedded preheating.
- The other case is cascade inflation where again one has sub-species-cutoff excursion. The features of the model are power spectrum features and observable GWs.

Thank you