The Systematic Construction of Free Fermionic Heterotic String Gauge Models

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Douglas Moore

Baylor University
Waco, TX 76798

Contents

FFHS Model Building

FFHS Models

- Heterotic Strings
  - Left- and Right- moving modes of a closed string are independent.
  - Left-movers are supersymmetric.
  - Right-movers are bosonic.
- Free-Fermionic Models
  - Free fermions "live" on the world sheet.
  - How do the fermions transform under parallel transport?
  - The boundary conditions (eventually) give us the particle states.
- What is needed to build a model?
  - A set of L basis vectors.
  - An $L \times L$ GSO coefficient matrix.

Basis Vectors - Boundary Conditions

The first input is a set of boundary conditions:

$$\mathbf{A} = \left\{ \vec{\alpha}_i \mid \vec{\alpha}_i \in \mathbb{Q}^{32} \cap (-1, 1]^{32},\ N_i \alpha_i^j = 0 \pmod 2 \right\}.$$

A valid set of basis vectors satisfies the modular invariance constraints:

$$N_i\ \vec{\alpha}_i^2 = \begin{cases} 0 \pmod 8), & \text{if } N_i \text{ even} \\ 0 \pmod 4), & \text{if } N_i \text{ odd} \end{cases}$$

$$N_{ij}\ \vec{\alpha}_i \cdot \vec{\alpha}_j = \quad 0 \pmod 4)$$

where $N_{ij} \equiv LCM(N_i, N_j)$.

$$\left\{ \begin{array}{l} (1\ (1\ 1\ 1)^3 \parallel 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) \\ (1\ (1\ 0\ 0)^3 \parallel 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \\ (0\ (0\ 0\ 0)^3 \parallel 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \\ (0\ (0\ 0\ 0)^3 \parallel 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \end{array} \right\}$$

Sectors and Charges - Phases and "States"

Within the Free-Fermionic formalism, we consider transport of fermions around non-contractable loops on the world-sheet.
Consistency requires:

$$\psi_j \rightarrow -e^{i\pi V^i_j}\psi_j$$

with $V^i_j \in (-1, 1] \cap \mathbb{Q}$. With 6 compactified dimensions, this is a 32 dimensional vector in the complex basis. We can write these sectors as linear combinations of basis vectors,

$$\vec{V}^i = \sum_{i=1}^{L} m^i_j \vec{\alpha}^j$$

with $m^i_j \in [0, N_j)$.
We can then find the charges to be

$$\vec{Q}^i = \frac{1}{2}\vec{V}^i + \vec{F}^i$$

with $F^i_j \in \{-1, 0, 1\}$, the fermion number operator.

Masslessness and GSO Projection

First, we need the states to be massless at the string scale:

$$\left(\vec{Q}^i_{left}\right)^2 = 1 \quad \left(\vec{Q}^i_{right}\right)^2 = 2$$

Second, we need the states to fit into representations. Additionally, there is a problem with the bosonic ground state: it is tachyonic. Hence, the GSO projection:

$$\vec{\alpha}_i \cdot \vec{Q}^j = \sum_{n=1}^{L} m^j_n k_{in} + s_i \pmod{2}$$

The k_{ij} matrix must also satisfy modular invariance constraints.

$$k_{ij} + k_{ji} = \frac{1}{2}\vec{\alpha}_i \cdot \vec{\alpha}_j \pmod{2}$$

$$k_{ii} + k_{i1} = \frac{1}{4}\vec{\alpha}^2_i + s_i \pmod{2}$$

$$N_j k_{ij} = 0 \pmod{2}$$

Supersymmetry

We assume an $SU(2)^6$ worldsheet SUSY:

$$(1\ \ (1\ 0\ 0)^3\ \|\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$$

- $\mathcal{N} = \mathcal{N}_{max}$
  - Results when $k_{2i} = 0$ for all $i > 2$.
  - The matter is completely determined by the gauge group.
  - The model is non-tachyonic.
- $\mathcal{N} = 0$
  - Results when $k_{2i} = 1$ for any $i > 2$.
  - Not all of the matter falls into adjoint representations of the gauge group.
  - The model MAY contain tachyons.

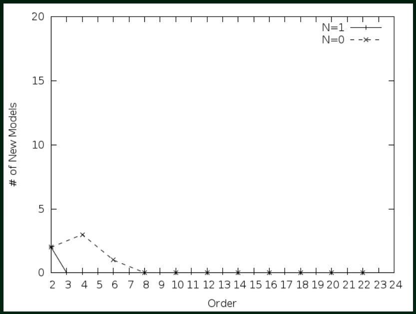
Layer 1 Survey

Systematic Layer 1 - Orders 2 to 22

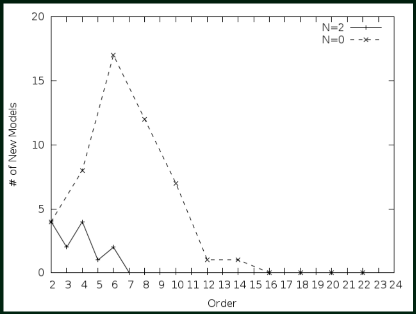
- $L = 1 \Rightarrow L = 3$.
  - All periodic basis vector.
  - SUSY generator.
- N ranged from 2 through 22, sequentially.
- Both $\mathcal{N} = \mathcal{N}_{max}$ and $\mathcal{N} = 0$ models were generated.
- "Unique" means unique gauge group and unique SUSY.

	$D = 10$	$D = 8$	$D = 6$	$D = 4$
$\mathcal{N} = \mathcal{N}_{max}$	2	13	18	68
$\mathcal{N} = 0$	6	50	73	502
Total Number of Models	$4,953,930$	$12,493,632$	$29,079,534$	$31,863,121$

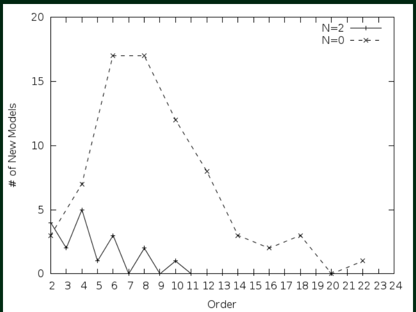
New Models Per Order



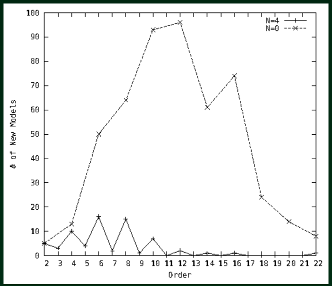
D = 10



D = 8



D = 6



D = 4

Figure 1

GUT Groups

	$\mathcal{N} = \mathcal{N}_{max}$					$\mathcal{N} = 0$			
	$D = 10$	$D = 8$	$D = 6$	$D = 4$		$D = 10$	$D = 8$	$D = 6$	$D = 4$
$\mathcal{F} - SU_5$	0%	0%	0%	5.9%		0%	0%	5.5%	20.9%
E_6	0%	7.6%	11.1%	8.8%		0%	8%	11.0%	9.6%
SO_{10}	0%	0%	11.1%	13.2%		0%	8%	13.7%	13.9%
PS	0%	0%	0%	5.9%		0%	2%	8.2%	16.3%
LRS	0%	0%	0%	0%		0%	0%	0%	8.7%
MSSM	0%	0%	0%	0%		0%	0%	0%	14.7%

Layer 2 Survey

Systematic, $D = 4$, Layer 2

- $L = 2 \Rightarrow L = 4$.
  - All periodic basis vector.
  - SUSY generator.
- Orders $2, 2 \rightarrow 2, 6$
- Orders $3, 3 \rightarrow 3, 6$
- Orders $4, 4$
- Both $\mathcal{N} = \mathcal{N}_{max}$ and $\mathcal{N} = 0$ models were generated.

No new models have been found.

Conclusions

What Comes Next?

- High order ($N > 22$) surveys
- Higher layer surveys
- Redundancy reductions
- Left-Movers
- "Reverse" surveys
- ...
- Whatever else we can think of.

Questions?

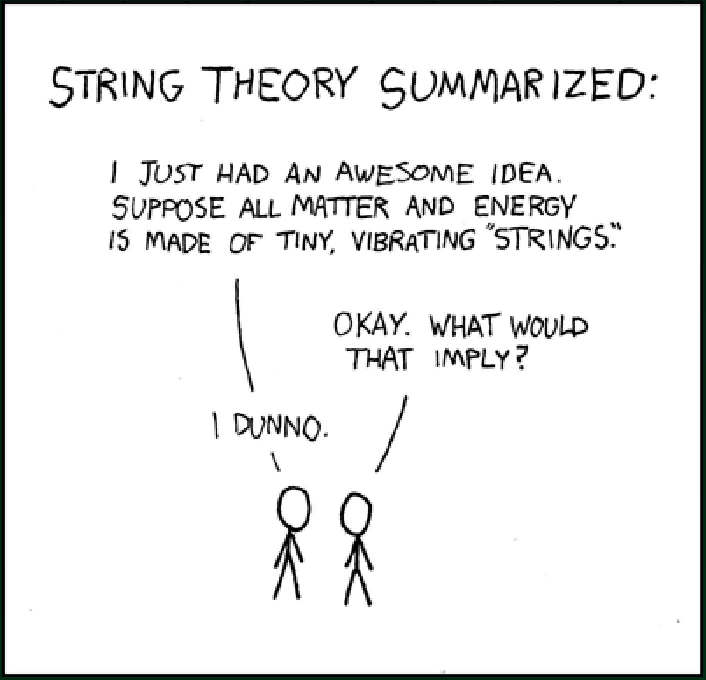


Figure 2 http://xkcd.com/171