Automated Systematic Generation of Flat Directions in Free Fermionic Heterotic Strings

Jared Greenwald
Baylor University

EUCOS
Early Universe Cosmology and Strings

CASPER
www.baylor.edu/CASPER

Outline

- (Very) brief look at WCFFH, NAHE
- EFT's
  - Scalar Potential, D-terms and F-terms
- Flatness
  - D- and F-flatness
- Computational Difficulties & Solutions
  - D- & F-flat
  - SVD, Linear Programming
- Conclusion

WCFFH (Weakly-coupled Free Fermionic Heterotic string theory)

- $E8_{obs}$ x $E8_{hidden}$
  - EFT:
    - Rich Phenomenology
    - NAHE (Nanopoulos, Antoniadis, Hagelin and Ellis)
      - $E8_{obs}$ -> SO(10) *(in D=4)*
        - SO(6) x SO(4)
        - SU(5) x U(1)
        - SM
        - Remain unbroken
      - SUSY: N=1
      - 3 generation
      - Tachyon-free

EFT: string model viability

- Given a string model, how do we test for phenomenological viability?
  - Construct EFT: Gauge group and matter analysis
  - The Lagrangian scalar potential for a supersymmetric model:

$$V(\varphi) = \frac{1}{2}\sum_{\alpha} g_{\alpha}^{2}\Big(\sum_{a=1}^{\dim(\mathcal{G}_{\alpha})} D_{a}^{\alpha} D_{a}^{\alpha}\Big) + \sum_{i} |F_{\varphi_i}|^{2}$$

φ = (scalar) bosonic fields

α = {groups}

a = {generators (*mediators*)}

EFT: Scalar Potential

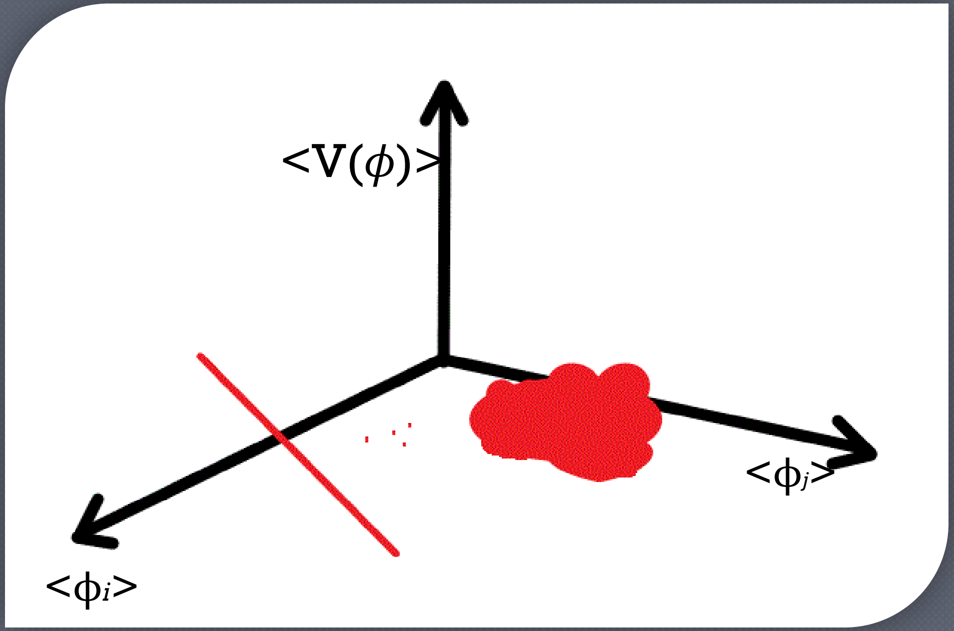
- *Supersymmetry exists if: $\langle V(\phi) \rangle = 0$*
  - Want SUSY above TeV scale
    - VEV's of D- & F-terms must be zero

$$\text{field VEV's} \in \{\text{moduli}\}$$

    - D- & F-flatness
      - *Essential part of moduli stabilization*

EFT: Scalar Potential

- Flat direction:
  - Locus of VEV'ed fields
    - Points, curves, regions in the $\langle \phi_i \rangle$ planes
    - $\langle V(\phi) \rangle = 0$



$$V(\varphi) = \frac{1}{2}\sum_{\alpha} g_{\alpha}^2 \left(\sum_{a=1}^{\dim(\mathcal{G}_\alpha)} D_a^\alpha D_a^\alpha\right) + \sum_i |F_{\varphi_i}|^2$$

EFT: D-terms

- Two main types:
  - Charged under $U(1)_i$ (type I)

$$D^i \equiv \sum_m Q_m^{(i)} |\varphi_m|^2$$

  - NA fields (type II)

$$D_a^\alpha \equiv \sum_m \varphi_m^\dagger T_a^\alpha \varphi_m$$

    - T: matrix generators

$$V(\varphi) = \frac{1}{2}\sum_{\alpha} g_{\alpha}^{2}\left(\sum_{a=1}^{\dim(\mathcal{G}_{\alpha})} D_{a}^{\alpha} D_{a}^{\alpha}\right) + \sum_{i} |F_{\varphi_i}|^2 \,,$$

EFT: F-terms

- F-terms:

$$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m}$$

  - W = superpotential = $\sum \ldots \Phi_i^{r_i} \Phi_j^{r_j} \Phi_k^{r_k} \Phi_l^{r_l} \ldots$

$$\text{order} \equiv \sum r_i$$

F-Flat fail	Order(W)	$3^{rd} – 5^{th}$	17^{th}
	SUSY broken	string scale *(1 to 1/10) [10^16 TeV]	1 TeV

- F-flatness usually tested: 3^{rd} - 5^{th} order
  - Less expensive computationally
  - More models discounted in lower orders than higher
    - more constraining

Systematic Flatness Overview

1. Find D-flat solution
   a) Must cancel FI-term
2. Check for F-flatness
   a) All-order flat: SUSY is not broken
   b) Fails at given order: SUSY broken at some scale
3. Rinse, wash and repeat

D-flatness: FI-Term and the U(1)$_A$

- **Build a D-flat direction to cancel FI-term**
  - Need anomalous U(1) VEV with opposite sign than ξ
    - FI-term breaks U(1)$_A$ which arises:
      - Compactifications - up to 6 (one from each)
      - 'freed' from breaking a gauge symmetry

$$\xi = \frac{g^2(\mathrm{Tr}Q_A)}{192\pi^2} M_{\mathrm{Pl}}^2$$

- **Find {VEV's} that satisfy D-flatness**
  - Restores SUSY

$$\langle D_A \rangle = \langle D_\alpha \rangle = 0$$

$$D_a^\alpha \equiv \sum_m \varphi_m^\dagger T_a^\alpha \varphi_m$$

$$D^{(A)} \equiv \sum_{m'} Q_{m'}^{(A)} |\varphi_m|^2 + \xi$$

F-flatness

- (Generally) Construct W to given order
  - Calculate F-terms

$$\begin{aligned} W_3 &= \ldots \\ &+ \Phi_{45}(\bar{\Phi}_{46}\bar{\Phi}'_{56} + \bar{\Phi}'_{46}\bar{\Phi}_{56}) + \bar{\Phi}_{45}(\Phi_{46}\Phi'_{56} + \Phi'_{46}\Phi_{56}) \\ &+ \ldots \end{aligned}$$

- F-flatness
  - Calculate F-terms
  - Multiple terms
    - Cancelling between
      - Complicated
      - Might lose flatness at next order

$$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m}$$

$$F_{\Phi_{45}} = \bar{\Phi}_{46}\bar{\Phi}'_{56} + \bar{\Phi}'_{46}\bar{\Phi}_{56}$$

$$< F_{\Phi_{45}} >= 0$$

$$\langle F_{\Phi_m} \rangle = \langle W \rangle = 0$$

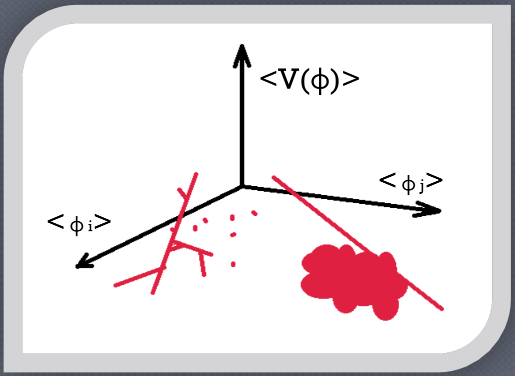
Stringent F-Flatness

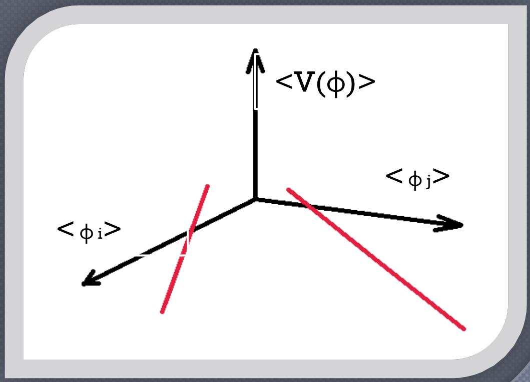
- Stringent F-flatness
  - Each term individually zero
  - All-order analysis
    - Look at potential 'dangerous' terms:
      - At most 1 unVEV'ed field in a term
      - When appearing, an unVEV'ed field is only to 1st power
  - Too constraining?
    - Stringent flat directions appear to be 'roots' of other flat directions
      - Not rigorously proved
      - General pattern

$$F_{\Phi_{45}} = \bar{\Phi}_{46}\bar{\Phi}'_{56} + \bar{\Phi}'_{46}\bar{\Phi}_{56}$$

$$< F_{\Phi_{45}} >= 0$$

$$< \bar{\Phi}_{46}\bar{\Phi}'_{56} >= 0, \; < \bar{\Phi}'_{46}\bar{\Phi}_{56} >= 0$$





Computational Problems

- WCFFHS
  - Previously: F77
    - Input: prebuilt models
      - Utilizes group theory program/results
      - Calculates superpotential (to any finite order)
      - Looks for flat directions (not without some "help")
  - Goal: C++
    - Completely automated
    - Expandable
      - Allows various types of searches
    - Systematic, large-scale searches
    - Beyond strings
      - Any EFT
- D-flatness
  - Matrix Solver
    - Infinite Solutions??
- F-flatness
  - All-order flatness
    - Creating superpotential
    - Check for potentially bad terms

Computational: D-flat

- D-flat
  - Matrix equation
    - Singular matrix?
      - SVD Solver
    - Infinite Solutions?
      - Constraints
        - Rescaling

$$\begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{pmatrix} \begin{pmatrix} \left|\langle \Phi_1 \rangle\right|^2 \\ \vdots \\ \left|\langle \Phi_n \rangle\right|^2 \end{pmatrix} = \begin{pmatrix} -\xi \\ \vec{0} \end{pmatrix}$$

$$Q\vec{x} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix}$$

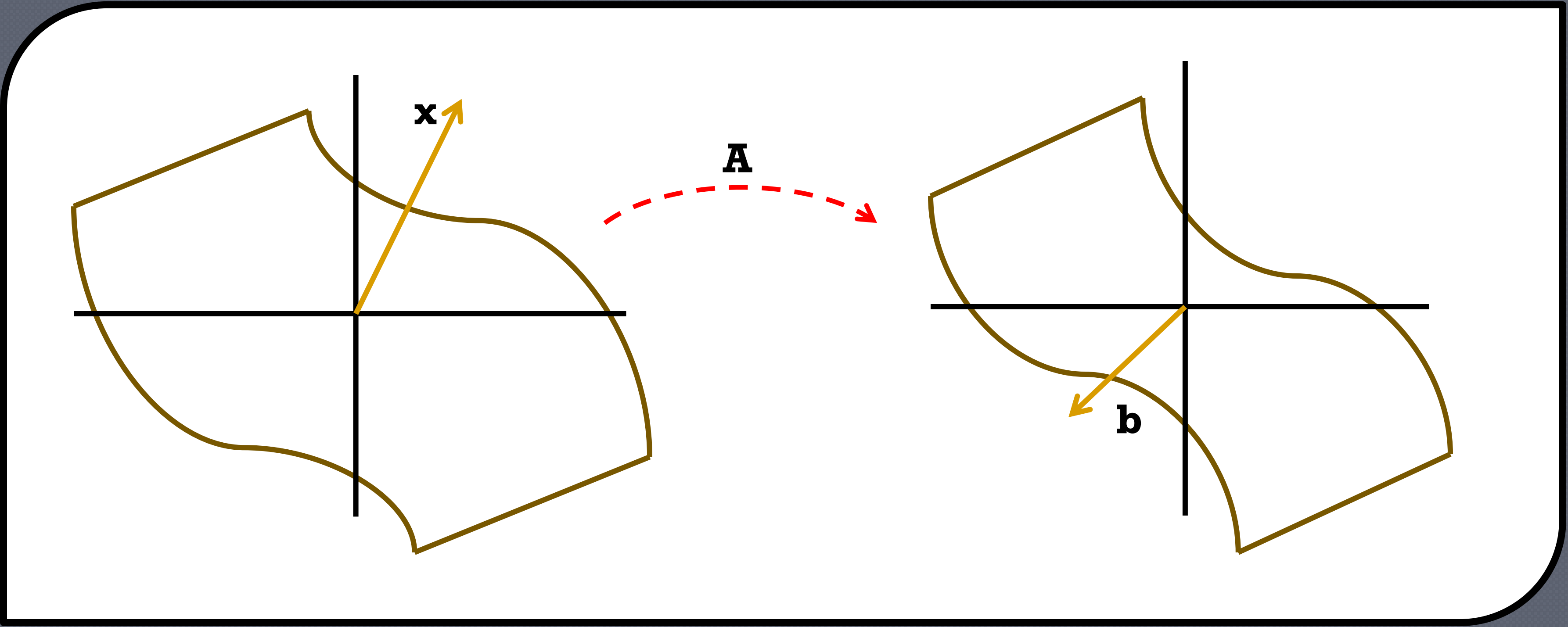
⇒ $\{x_i \text{ relatively prime } \forall i\}$

Computational: D-flat [SVD]

- Singular Value Decomposition
  - Decomposes matrix into
    - U, V unitary
    - Diagonal with singular values
  - A basis for the range of Q: $\{u_1, \cdots, u_l\}$
  - A basis for the nullspace of Q: $\{v_{l+1}, \cdots, v_n\}$
  - Decomposition
    - Easily invertible
      - Pseudoinverse
      - Qx=b

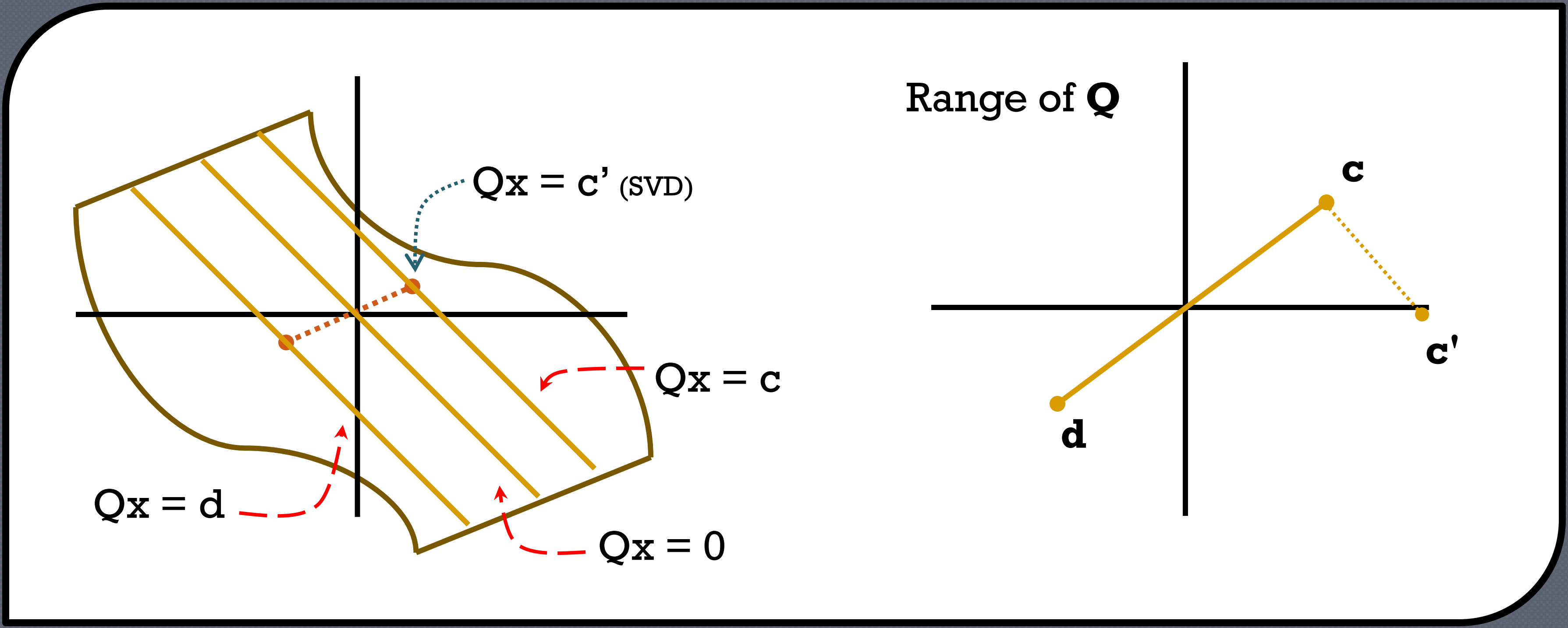
$$\begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{pmatrix} = \begin{pmatrix} u_1, & \cdots & u_m \end{pmatrix} \begin{pmatrix} \sigma_1 & & & & & 0 \\ & \ddots & & & & \\ & & \sigma_l & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{pmatrix} \begin{pmatrix} v_1^T \\ \vdots \\ v_n^T \end{pmatrix}$$

SVD Solver



- A nonsingular
  - Image, Domain: same dimension
- Q will be singular
  - Q maps to lower dimension

SVD Solver



- SVD solutions: Least square if c' is not in Range(Q)
  - All solutions: Qx=c,d
    - Accessible: x_{svd} + Nullspace Basis Vectors

$$\vec{x} = a_0 \vec{x}_{svd} + \sum a_i NS(Q)_i$$

D-flat Constraints

$$x = X\vec{a} \equiv \left(\vec{x}_{svd}, \vec{x}^{1}_{null}, \ldots, \vec{x}^{p}_{null}\right)\vec{a}$$

$$x^{i}_{null} \in NS(Q)$$

- VEV Ratio Rescaling: $\left\{X_{ij}a_j, \text{ relatively prime } \forall i\right\}$
- *Other possible constraints:

$$l.b. \leq \left|X_{ij}a_j\right| \leq u.b. \quad \forall i \in [1,n] \text{ (sumimplied)}$$

$$\left\|X_{ij}a_j\right| - \left|X_{lj}a_j\right\| \leq \alpha \quad \forall i, l \in [1,n]$$

$$f(\vec{a})$$

*applied if needed

D-flat Conjecture

- Conjecture:
  - The rescaling constraint can yield a minimal bounding surface in coefficient space
    - If not, then apply another
    - How to test?
- Linear Programming
  - Used in Operations research, Microeconomics, Cost/profit
    - Optimization over objectives and constraints
  - Uses Matrix (tableau) language
  - Many numerical solvers exist
  - Integer programming
  - Goal Programming
    - Multi-objective

Why Linear Programming?

- Systematic searches - We want to find every (reasonable) solution
  - Determines whether coefficient space is:
    - Bounded
      - Bounding surface yields full coefficient set
    - Unbounded
      - 'rough-tuning' to calculate and specify bounding surface
- LP can help determine interesting regions
  - Varying objective functions
    - D-flat constraints
  - Gather statistics for unbounded spaces

F-flatness

- LP + Recursive Loop
  - D-flat solution

$$\vec{x} = a_0 \vec{x}_{svd} + \sum a_i NS(Q)_i$$

- Given a D-flat solution
  - Stringent F-flatness
    - 'Dangerous' superpotential terms
      - Investigate gauge-invariant monomials

$$W_\alpha = \ldots \Phi_i^{r_i} \Phi_j^{r_j} \Phi_k^{r_k} \Phi_l^{r_l} \ldots \quad \text{is gauge invariant iff} \quad Q \begin{pmatrix} \vdots \\ r_i \\ r_j \\ \vdots \end{pmatrix} \equiv Q\,\vec{r} = \vec{0}$$

F-flatness: Dangerous terms

- Dangerous: $$\sum_{\left\{i \,\middle|\, \left|\langle \Phi_i \rangle\right|^2 = 0\right\}} r_i \leq 1$$
  - Why: $$W_\alpha = \ldots \Phi_j^{r_j} \Phi_k^{r_k} \Phi_l^{r_l} \Phi_m^{r_m} \ldots$$
    $$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m}$$
    $$\langle F_{\Phi_m} \rangle = \langle W \rangle = 0$$
  - Construct:
    $$NS(Q)_{red} \equiv \begin{pmatrix} \vdots \\ row(NS(Q))_i \\ \vdots \end{pmatrix}, \quad \left\{i \,\middle|\, \left|\langle \Phi_i \rangle\right|^2 = 0\right\}$$
    $$Q\,\vec{r} = \vec{0}$$

F-flatness: Dangerous Case I

- Case I: $\left\langle F_{\Phi_m} \right\rangle \neq 0$
  - Taking a derivative requires $r_i > 1$
  - Investigate coefficient space for matrix *B*:

$$NS(Q)_{red} B = I$$

$$B \equiv \left(\ldots, \vec{\beta}_i, \ldots \right), \left\{ i \middle| \left| \left\langle \Phi_i \right\rangle \right|^2 = 0 \right\}$$

    - Requires matrix solver
      - SVD readily provides a useful solution

F-flatness: Dangerous Case II

- Case II: $\langle W \rangle \neq 0$ $\quad NS(Q)_{red}\vec{b} = \vec{0},\ \vec{b} \neq \vec{0}$
  - Investigate Nullspace coefficient space for vector *b*
    - Since $NS[NS(Q)_{red}] \neq NS(Q)$
  - SVD approach
    - Generate allowed coefficients: $3 \leq \sum_i NS(Q)_{ij} b_j \leq 17$
      - Stringy models won't allow less than 3
      - Anything above 17 breaks SUSY lower than 1 TeV
        - Future searches can soften this
          - Kinetic Mixing etc…

F-flatness Result

- If no solutions can be found for either case:
  - All-order flat
- Otherwise
  - Find the order at which SUSY is broken
    - Sum the elements of the shortest surviving ‘power’ vector, r,

F-Flat fail	Order(W)	3rd – 5th	17th
	SUSY broken	string scale *(1 to 1/10) [10^16 TeV]	1 TeV

$$Min\left[\sum_i NS(Q)_{ij}\,\chi_j\right],\ \vec{\chi}\in\left\{\vec{b},\vec{\beta}_i\right\},\ \left\{i\,\middle|\,\left|\langle\Phi_i\rangle\right|^2=0\right\}$$

Conclusion

- Challenges for WCFFHS systematic D- & F-flat direction searches:
  - Bad
    - Complicated LP techniques are required
      - Difficulties with D-flat objective function determination
    - Maybe be computationally intensive to find coefficient bounds
  - Good
    - SVD is fast, reliable and more extensible than previously thought
    - Fully automated searches seems to be achievable
      - Dynamically using LP and/or 'rough-tuning' (forcing certain VEV ranges onto the system)
    - Some LP systems are parallelized already
    - Unbounded coefficient space for D-flat models can still be investigated
      - Population uncertainties can be estimated

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