Three Generations of Higgses and the Cyclic Groups

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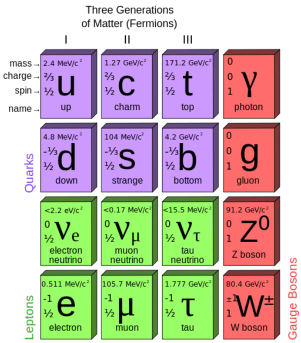
Matter content in the Standard Model (SM):



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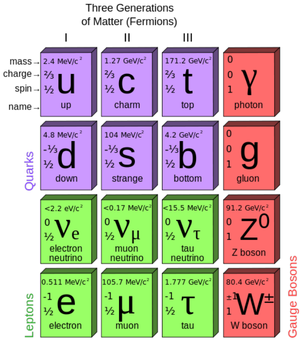

+

Scalar Field (H) responsible of mass generation.

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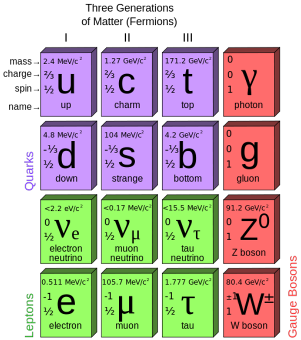

or +
several SU(2) scalar doublets (H_n).

Fermion Sector:

The info about the masses and mixing angles is encoded in the Yukawa terms,

$$\mathcal{L}_Y = Y^l \bar{L} e_R H + Y^d \bar{Q}_L d_R H + Y^u \bar{Q}_L u_R \tilde{H} + \text{h.c}, \quad (1)$$

which are allowed by the gauge symmetry and renormalizability.

- The Y^is are unspecified matrices.
- In the SM the **neutrinos do not have mass** ($m_\nu = 0$).

Warning signs

[1] Zhun, arXiv:1205.0761 [hep-ph]

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- $m_\nu \neq 0$, the neutrinos DO have mass and is pretty small,

$$\Delta m^2_{21} = 7.59^{+0.20}_{-0.18} \times 10^{-5}\mathrm{eV}^2 \text{ and } \Delta m^2_{31} = 2.5^{+0.09}_{-0.16} \times 10^{-3}\mathrm{eV}^2. \quad (2)$$

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2. no-renormalizable terms in the Lagrangian

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- the mixing angles are large for leptons[1], $U_{PMNS} = U_l^\dagger U_\nu$: and
- the mixing angles are small for quarks,

$$V_{CKM} = V_u^\dagger V_d \quad (3)$$

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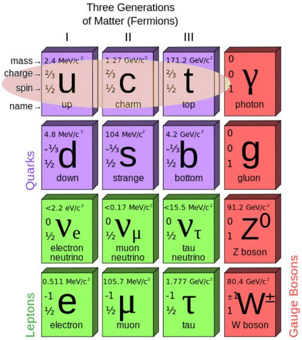
Then:

We need to go *beyond* the SM.
One possibility is $SM \times \mathcal{G}$, where $\mathcal{G}$ is an additional symmetry and it justifies:

- the presence of **other fields** (RH-neutrinos, H_n, flavons, etc.)
- the structure needed in mass matrices to explain **mixing angles** (especially in the quark sector).

A good way for explaining fermion masses and mixing angles is using **horizontal symmetries**, $\mathcal{G} = \mathcal{G}_{\mathcal{F}}$.



A good way for explaining fermion masses and mixing angles is using **horizontal symmetries**, $\mathcal{G} = \mathcal{G}_{\mathcal{F}}$.

1) $\Lambda_F >> \Lambda_{EW}$. Effective theories.
2) $\Lambda_F \sim \Lambda_{EW}$. Renormalizable models.

The additional symmetry group, $\mathcal{G}_F$ could be:

- *Continuous*: $U(1)$[2], $U(2)$;
- *Discrete*: Z_n, Q_4[3], D_6[4], $A_{4,5}$[5], $S_{3,4}$[6], T'[7]
    - which avoids undesirable Goldstone Bosons (Goldstone Theorem) and
    - is more restrictive,

and/or tensor products among them.

[2] Froggat and Nielsen...
[3] Frigerio, Hagedorn, Aranda, C.B., Rojas, Ramos...
[4] Babu...
[5] Altarelli, Feruglio, Merlo, Ma, Valle...
[6] Meloni, Mondragon, A. and M. , Morisi, Peinado...
[7] Aranda, Chen, Frampton, Merlo,...

U_{PMNS} with TBM-mixing assumption:

$$\tan^2\theta_{\rm atm} = 1$$

$$\tan^2\theta_{\rm sol} = 0.5$$

$$\sin^2\theta_{\rm reac} = 0$$

Favors discrete groups like:

- $\mathcal{G}_{TBM} = A_4,\, T',\, S_4,$ and/or
- $\mathcal{G}_F$ with three-dimensional irreps, **3**.

However,

$$\sin^2\theta_{\rm reac} \neq 0 \Longrightarrow \text{another } \mathcal{G}_F$$
$$\Longrightarrow \text{with } \mathcal{G}_{TBM} \text{ get TBMm as a limit.}$$

We want to explain the 6 quark masses, 3 mixing angles and a CP-violating phase. The CKM matrix is,

$$V_{CKM} = V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{4}$$

where $V_L^{u,d}$ come from,

$$M^{u,d} = V_L^{u,d} M_D^{u,d} (V_R^{u,d})^\dagger$$

Textures

- The fermion masses are determined by the flavor structure of the Yukawa couplings after EWSB, then if

$$Y_1^{u,d} \sim \begin{pmatrix} 0 & \star & 0 \\ \star & 0 & \star \\ 0 & \star & \star \end{pmatrix} \text{ or } Y_2^{u,d} \sim \begin{pmatrix} 0 & \star & 0 \\ \star & \star & \star \\ 0 & \star & \star \end{pmatrix},$$

one successfully reproduces data in the quark sector.

Textures

- Having

$$Y_1^{u,d} \sim \begin{pmatrix} 0 & \star & 0 \\ \star & 0 & \star \\ 0 & \star & \star \end{pmatrix} \text{ or } Y_2^{u,d} \sim \begin{pmatrix} 0 & \star & 0 \\ \star & \star & \star \\ 0 & \star & \star \end{pmatrix},$$

and $Y_1^{u,d} \neq (Y_1^{u,d})^{\dagger}$[8], one successfully reproduces data in the quark sector. $(V_{CKM})_{ij} = f(\frac{m_a}{m_b})$ con $a \neq b$.

The zeros (**textures**) are explained by:

- the group structure,
- alignment or
- both.

[8] Y_1 is known as NNI-texture.

Requirements:

- A *flavor model* (FM) with the minimal number of additions,
  - minimal number of $SU(2)$ scalar doublets (Higgs fields)
- A FM which explains **all** the fermion masses and mixing angles using **only one flavor symmetry** in both sectors, **i. e.** SM$\times\mathcal{G}_{\mathcal{F}}$.
- **Renormalizability**.

Quarks: $\mathcal{G}_F$ which explains $Y^{u,d}_{1,2}$.

Leptons: No special structure for (Y^l) and/or the mass matrices so far.[9]

[9] Attempts made by Fritzsch et al.,JHEP 1109:083,2011

C1) A FM that incorporates both sectors, quarks and leptons, with features mentioned above is hard to obtain in scenarios with only **2 or 3 Higgs** fields and **non-Abelian groups**.[10]

C2) *Therefore, we look at the* **Abelian Groups**.

[10] Aranda et. al, Phys.Rev.D84:016009,2011.

Focusing in the quarks:

- NNI texture in a 2HDM is generated with Z_4.[11]

Then,

(QI) What are the fermion fields charges under an Abelian group to create a 3HDM which provides a NNI-texture and what is the order of this Abelian group?

[11] This was shown by Branco et al, Phys.Lett.B690(2010)62-67.

To answer (QI)

Let H_1, H_2 and H_3 be the Higgs fields in the model and have taking into account the following considerations:

i) Each non-zero entry in the mass matrices contains contributions from a single vev.

ii) One of the Higgses will be assigned neutral charge while the other two will be related by conjugation.

iii) One of the Higgses contributes exclusively to the $3-3$ entry of the mass matrix for the up-type quark.

then with i)- iii) the possible textures are,

$$M^u_{A1} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_1 & 0 & v_2 \\ 0 & v_2 & v_3 \end{pmatrix}, \quad M^u_{A2} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_2 & 0 & v_1 \\ 0 & v_2 & v_3 \end{pmatrix},$$

$$M^u_{A3} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_2 & 0 & v_2 \\ 0 & v_1 & v_3 \end{pmatrix}, \quad M^u_{B1} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_1 & 0 & v_1 \\ 0 & v_2 & v_3 \end{pmatrix}, \quad (5)$$

$$M^u_{B2} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_2 & 0 & v_2 \\ 0 & v_2 & v_3 \end{pmatrix},$$

where v_i, $i = 1, 2, 3$ denote the Higgs vevs.

Denoting the charge assignments as

$$\overline{Q} \simeq (q_1, q_2, q_3),\ U_R \simeq (u_1, u_2, u_3),\ D_R \simeq (d_1, d_2, d_3), \quad (6)$$

$$\tilde{\mathcal{H}} \equiv (\tilde{H}_1, \tilde{H}_2, \tilde{H}_3) \simeq (h_1, h_2, h_3)\ , \quad (7)$$

where $q_i, u_i, d_i, h_i \in Z_N$.
The Z_N charges of the bilinears in the Yukawa terms of the lagrangian can be represented by

$$\mathcal{Y}^u_{ij} = (q_i + u_j)\ mod(N). \quad (8)$$

Case A1:

$$M^u_{A1} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_1 & 0 & v_2 \\ 0 & v_2 & v_3 \end{pmatrix}$$

From the point ii) above we have the following three possibilities for the Higgses charges:

a) $(h_1 = 0, h_2 = a, h_3 = -a)$,

b) $(h_1 = a, h_2 = 0, h_3 = -a)$ and

c) $(h_1 = -a, h_2 = a, h_3 = 0)$.

with $a \in Z_N$ and the following constraints

$$\mathcal{Y}^u_{11},\ \mathcal{Y}^u_{13},\ \mathcal{Y}^u_{22},\ \mathcal{Y}^u_{31} \neq (0, -a, a). \tag{9}$$

By the other hand, a) $(h_1 = 0, h_2 = a, h_3 = -a)$ leads to the following constraints:

$$\mathcal{Y}^u_{12} = 0,\ \mathcal{Y}^u_{21} = 0,\ \mathcal{Y}^u_{23} = -a,\ \mathcal{Y}^u_{32} = -a,\ \mathcal{Y}^u_{33} = a, \tag{10}$$

Therefore using Eq. (10) we find that the fermion assignments become

$$q_1 = -c,\quad q_2 = -c - 3a,\quad q_3 = -c - a, \tag{11}$$
$$u_1 = c + 3a,\quad u_2 = c,\quad u_3 = c + 2a, \tag{12}$$
$$d_1 = c + 3a,\quad d_2 = c,\quad d_3 = c + 2a, \tag{13}$$

where $c, a \in Z_N$.

Now we have to satisfy,

$$\mathcal{Y}^u_{11},\ \mathcal{Y}^u_{13},\ \mathcal{Y}^u_{22},\ \mathcal{Y}^u_{31} \neq (0, -a, a). \tag{14}$$

or given Eq.(11),

$$(3a, 2a, -3a) \neq \begin{cases} 0 \\ -a \quad mod(N) \\ a \end{cases}. \tag{15}$$

- $(2a = 0) \in Z_2$ and $(3a = 0) \in Z_3 \Longrightarrow Z_2, Z_3$ discarded.

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- Using similar arguments Z_4 is discarded.

Now, it is possible to satisfy,

$$(3a, 2a, -3a) \neq \begin{cases} 0 \\ -a \\ a \end{cases} mod(N) \ . \tag{16}$$

with Z_5.

Proof:

- If $a \in Z_5$ then $a \in (0, 1, 2, 3, 4)$,
  - since $a \neq 0 \Rightarrow a \in (1, 2, 3, 4)$.
  - If $a = 1$ then $-a = 4$, $2a = 2$, $3a = 3$, $-3a = 2$,

  and thus all conditions in Eq. (16) are satisfied.

Answer to (Q1)

- Z_5 is the smallest group that can be used in this setting:

$$M^u_{A1} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_1 & 0 & v_2 \\ 0 & v_2 & v_3 \end{pmatrix}$$

with a)$(h_1 = 0, h_2 = a, h_3 = -a)$.
Larger groups can also be used.

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- Z_6 is the smallest possibility for case:

$$M^u_{A1} \sim \begin{pmatrix} 0 & v_1 & 0 \\ v_1 & 0 & v_2 \\ 0 & v_2 & v_3 \end{pmatrix}$$

  with c)$(h_1 = -a, h_2 = a, h_3 = 0)$.
  Larger groups can also be used.

- All other cases do not satisfy the necessary conditions for any Z_N.

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- Thank you.

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- Taking $a \in Z_4$ then $a \in (0, 1, 2, 3)$:
  - Since $a \neq 0 \Rightarrow a \in (1, 2, 3)$
  - Since $2a \neq 0 \Rightarrow a \in (1, 3)$
  - If $a = 1$ one gets, $3a = 3 = -1 = -a \Rightarrow a \neq 1$

  then Z_4 is discarded.