Higgs mass finite GUT predictions

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Introduction

We consider N=1 supersymmetric Grand Unified Theories, which can be made all-loop finite:

- Dimensionless (gauge and Yukawa couplings) Sector
- dimensionful (soft supersymmetry breaking terms) Sector
- Reduction in the number of free parameters.
- Accurate prediction of the top quark mass in the dimensionless sector, and predictions for the Higgs boson mass and the supersymmetric spectrum in the dimensionful sector.

Finite Unified Theories

Finite Unified Theories are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite even to all-loop orders, including the soft supersymmetry breaking sector.

Building

- Renormalization Group Invariant (RGI)

Kubo, J., Mondragón, M. and Zoupanos, G. (1994) Nucl. Phys. **B424** *291*

Ma,E. (1978) Phys.Rev **D 17** *623;ibid (1985)* **D31** *1143.*

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- find RGI relations among couplings that guarantee finitenes to all-orders in perturbation theory.

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Building

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- In order to achieve the latter it is enough to study the uniqueness of the solutions to the one-loop finiteness conditions.

Ermushev, A.Z., Kazakov, D.I. and Tarasov, O.V. (1987) Nucl. Phys. 281 72; Kazakov, D.I. (1987) Mod. Phys. Lett. A9 663.

Building

- The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories which involves parameters of dimension one and two.
  Jack, I. and Jones, D.R.T. (1995) Phys. Lett. **B349** 294

  Kubo, J., Mondragón, M. and Zoupanos, G. (1996) Phys. Lett. **B389** 523.
- the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the supersymmetric spectrum (s-spectrum).

Reduction of Couplings and Finiteness

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu\, d\Phi/d\mu \;=\; \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i \;=\; 0.$$

g_i = *coupling*, β_i *its* β *function* Finding the $(N-1)$ independent Φ's is equivalent to solve the

reduction equations (RE)

$$\beta_g\,(dg_i/dg) = \beta_i \;,\;\; i = 1, \cdots, N$$

completely reduced theory contains only one independent coupling and its β function complete reduction: power series solution of RE uniqueness of the solution can be investigated at one-loop

Zimmermann, Oehme, Sibold

Reduction of Couplings and Finiteness

- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints

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$$\Rightarrow \quad \beta^N = 0$$

- In SUSY no-renormalization theorems

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- In SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - guarantee that is gauge and reparameterization invariant at all loops

Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2} C_{ipq}\, C^{jpq} = 2\delta_i^j g^2\, C_2(R_i)\,.$$

$C_2(G)$ = quadratic Casimir invariant, C_{ijk} = Yukawa coup., $T(R_i)$ Dynkin index of R_i.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

Finiteness

One-loop finiteness $\Rightarrow$ two-loop finiteness *Jack, Jones, Mezincescu and Yao*
One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$ The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

Theorem

Consider an $N = 1$ supersymmetric Yang-Mills theory, with simple gauge group. If the following conditions are satisfied

1. *There is no gauge anomaly.*
2. *The gauge β-function vanishes at one-loop*

$$\beta_g^{(1)} = 0 = \sum_i l(R_i) - 3\, C_2(G).$$

3. *There exist solutions of the form*

$$C_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in$$

to the conditions of vanishing one-loop matter fields anomalous dimensions

$$\gamma_j^{i\,(1)} \;=\; 0 = \frac{1}{32\pi^2}\,[\; C^{ikl}\; C_{jkl} - 2\; g^2\;\; C_2(R_i)\delta_{ij}].$$

4. *these solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β-functions:*

$$\beta_{ijk} = 0.$$

Then, each of the solutions can be uniquely extended to a formal power series in g, and the associated super Yang-Mills models depend on the single coupling constant g with a β function which vanishes at all-orders.

RGI in the Soft Supersymmetry Breaking Sector

Supersymmetry is essential. It has to be broken, though. . .

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

scalar h^{ijk} trilinear and b^{ij} bilinear couplings, M unified gaugino mass

The RGI method has been extended to the SSB of these theories.

One- and two-loop finiteness conditions for SSB have been known for some time *Jack, Jones, et al.* It is also possible to have all-loop RGI relations in the finite and non-finite cases *Kazakov; Jack, Jones, Pickering*

RGI in the Soft Supersymmetry Breaking Sector

SSB terms depend only on g and the unified gaugino mass M
universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

Very appealing! But too restrictive;
it leads to phenomenological problems:

- The lightest susy particle (LSP) is charged. *Yoshioka; Kobayashi et al*
- It is incompatible with radiative electroweak breaking.

Brignole, Ibáñez, Muñoz

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$\Rightarrow$ better phenomenology.

Kobayashi, Kubo, Mondragón, Zoupanos

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \ ,$$

The one- and two-loop finiteness for h gives

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5) \ .$$

Assume that lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \ , \qquad (m^2)^i_j = m^2_j \delta^i_j \qquad \text{for all } p \text{ and } q.$$

Soft scalar sum-rule for the finite case

We find the the following soft scalar-mass sum rule

$$(m_i^2 + m_j^2 + m_k^2)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(1)} + O(g^4)$$

for $i,\ j,\ k$ *with* $\rho_{(0)}^{ijk} \neq 0$, *where* $\Delta^{(1)}$ *is the two-loop correction,*

$$\Delta^{(1)} = -2\sum_l [(m_l^2/MM^\dagger) - (1/3)]\ T(R_l)\ ,$$

which vanishes for the universal choice.

All-loop sum rule

One can generalize the sum rule for finite and non-finite cases to all-loops!!

Possible thanks to renormalization properties of $N = 1$ susy gauge theories. *Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman*

The sum-rule in the NSVZ scheme is *Kobayashi, Kubo, Zoupanos*

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d (\ln g)^2} \} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln C^{ijk}}{d \ln g} .$$

$SU(5)$ Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3\ \overline{\mathbf{5}} + 3\ \mathbf{10} + 4\ \{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector.

$SU(5)$ Finite Models

In addition:

- *The soft scalar masses obey a sum rule*
- *At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM*
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the **24**

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \right] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4$$

$$+ g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries

For Model A the superpotetial has the discrete symmetry

$$Z_7 \times Z_3 \times Z_2$$

	$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_2$	$\overline{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	**24**
Z_7	4	1	2	1	2	4	5	3	6	-5	-3	-6	0	0	0
Z_3	0	0	0	1	2	0	1	2	0	-1	-2	0	0	0	0
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0

Whereas for Model B

$$Z_4 \times Z_4 \times Z_4$$

	$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_2$	$\overline{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	**24**
Z_4	1	0	0	1	0	0	2	0	0	0	-2	0	0	0	0
Z_4	0	1	0	0	1	0	0	2	0	3	0	-2	0	-3	0
Z_4	0	0	1	0	0	1	0	0	2	3	0	0	-2	-3	0

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\ g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\ g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{5}}^2 + m_{\mathbf{10}}^2 = M^2$

- **3 free parameters:**
  M, $m_{\mathbf{5}}^2$ and $m_{\mathbf{10}}^2$

The finiteness relations give at the M_{GUT} scale

Model B

- $g_t^2 = \frac{4}{5}\ g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\ g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$

- **2 free parameters:** $M,\ m_{\mathbf{5}}^2$

Phenomenology

The gauge symmetry is broken below M_{GUT}, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}, below that is the MSSM.

- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections Include radiative corrections to bottom and tau, plus resummation (very important!) Estimate theoretical uncertainties

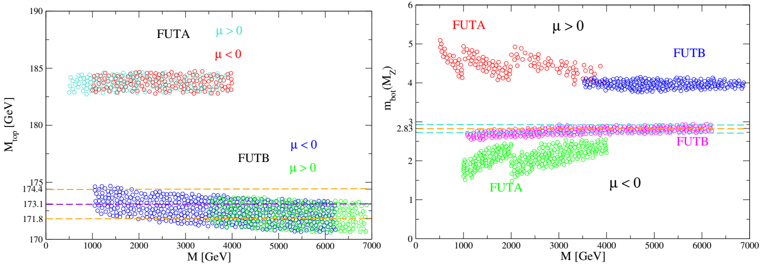
We look for the solutions that satisfy the following constraints:

Right masses for top and bottomfact of life *FeynHiggs* The decay $b \to s\gamma$ *MicroOmegas*fact of life The branching ratio $B_s \to \mu^+\mu^-$ *MicroOmegas*fact of life Cold dark matter density $\Omega_{CDM}h^2$ *MicroOmegas*loose constraint The anomalous magnetic moment of the muon $g-2$see what we get

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, Suspect, FUT

TOP AND BOTTOM MASS



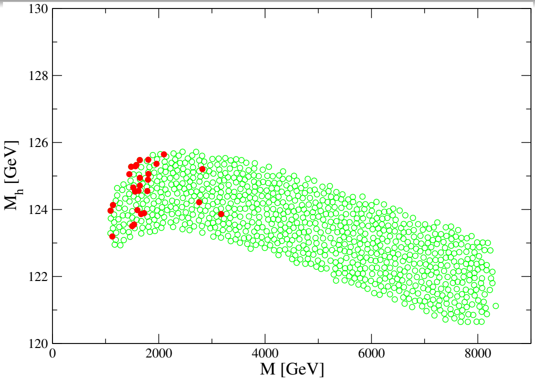
FUTA: $M_{top} \sim 183\ GeV$, FUTB: $M_{top} \sim 172\ GeV$

Theoretical uncertainties $\sim 4\%$

in M_{bot} Δb and Δtau included, resummation done

FUTB $\mu < 0$ favoured

uncertainties $\sim 8\%$



FUTB: $M_{Higgs} = 122 \sim 126\ GeV$

Uncertainties $\pm 3\ GeV$ (FeynHiggs)

$$\Omega_{\rm CDM} h^2 < 0.3$$

Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

- $M_{top} \sim 172\ GeV$ *4%*
- $m_{bot}(M_Z) \sim 2.8 GeV$ *8 %*
- $M_{Higgs} \sim 122 - 126\ GeV$ *3 GeV*
- $\tan\beta \sim 44 - 46$
- s-spectrum $> 500\ GeV$

Extension to 3 fams on its way with discrete flavour symmetry; with $\not{R} \Rightarrow$ neutrino masses

in this case dark matter candidate is not LSP, but probably gravitino

Conclusions

- Finiteness: powerful, interesting and intriguing principle $\Rightarrow$ **reduces greatly the number of free parameters**
- completely finite theories
  i.e. including the SSB terms, that satisfy the sum rule.
- Confronting the $SU(5)$ models with low-energy precision data does distinguish among models:
  - FUTB $\mu < 0$ survives (remarkably)
  - large $\tan\beta$
  - s-spectrum starts above ~ 400 GeV
  - a prediction for the Higgs $M_h \sim 122 - 126$ GeV

Thanks!